Question 3

- (a) (i) Draw a logic network that accepts independent inputs p and q and gives as output ¬p ∧ (p ∨ q). Label your diagram to show the symbolic output after each gate.
 - (ii) Make a table to show the truth value of the output from the network corresponding to each combination of truth values of p and q. [2]

[4]

- (b) (i) Construct the truth table for the proposition $p \rightarrow q$. [2]
 - (ii) Let n be an element of the set {1, 2, 3, 4, 5, 6, 7}. Let p, q be the propositions

p: n is even;
$$q: n > 4$$
.

Find the values of n for which $p \rightarrow q$ is true.

Logic (BLANK)

Question 3 (a) Let n be an element of the set $\{10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$, and p and q be the propositions:

p: niseven, q: n>15.

Draw up truth tables for the following statements and find the values of n for which they are true:

- (i) (i) p ∨ ¬q (ii) ¬p ∧ q
- (ii) Use truth tables to find a statement that is logically equivalent to ¬p → q.
 [6]

(b) Let p, q be the following propositions:

$$p: this apple is red, q: this apple is ripe.$$

Express the following statements in words as simply as you can:

(i)
$$p \rightarrow q$$
 (ii) $p \land \neg q$.

Express the following statements symbolically:

- (iii) This apple is neither red nor ripe.
- (iv) If this apple is not red it is not ripe.

[4]

Logic (BLANK)

Question 3 (a) Let n be a positive integer and p and q be the following propositions:

 $p: n \leq 12$

q: nisodd.

 Express each of the three following compound propositions concerning positive integers symbolically by using p, q and appropriate logical symbols.

 $n \leq 12$ andniseven.

 $ifn \leq 12 thenniseven$

n > 12 and n is odd.

Logic (BLANK)

- (ii) Construct the truth table for the statement q → p. Hence find a value of n that makes this statement false.
- (iii) Write in logical symbols the contrapositive of the statement:

 $ifnisoddthenn \leq 12.$

[6]

(b) Construct a logic network that accepts as inputs p and q, which may independently have the value 0 or 1, and gives as final output

$$\neg(\neg p \land q)$$
.

Show the truth table for this output and hence give a simple expression (without using negation) that is equivalent to $\neg(\neg p \land q)$. [4]

Question 3

(a) Let S = {10, 11, 12, 13, 14, 15, 16, 17, 18, 19} and let p, q be the following propositions concerning the integer n ∈ S.

p : n is a multiple of two

q: n is a multiple of three.

 For each of the following compound statements find the set of values n for which it is true.

$$p \land q$$
; $p \lor q$; $\neg p \oplus q$

ii. Express the following statement using logic symbols.

n is not a multiple of either two or three.

iii. List the elements of S which are in the truth set for the statement in (ii).

[6]

[0]

(b) (i) Let p and q be propositions. Use truth tables to prove that

$$p \to q \equiv \neg q \to \neg p.$$

(ii) Write the contrapositive of the following statement concerning an integer n.

If the last digit of n is 0, then n is divisible by 5.

[4]

Logic (20YY)

Question 3

Let p, q and r be the following propositions concerning integers n.

 $p \quad : \quad n \text{ is a multiple of two}$

q : n < 20

 $r : n \leq 20.$

(a) List the truth set of the compound proposition $\neg q \land p$.

[2]

Logic (20YY)

- (b) Express each of the following statements using the propositions p, q and r and logical symbols.
 - n is an integer less than 20 which is even;
 - ii. n is an integer larger than 20 which is odd;

iii.
$$n = 20$$
. [3]

Logic (20YY)

(c) i. Use truth tables to prove that

$$p \to (q \lor r) \equiv (\neg q \land \neg r) \to \neg p.$$

ii. Write in plain English the contrapositive of the statement

"If n is a positive integer and n < 2 then n = 1".

Logic (BLANK)