2004

Question 4 (a) List the following sets:

$$\{2^r \ : \ r \in \mathbb{Z} \text{ and } 0 \leq r \leq 5\}$$
$$\{r^2 \ : \ r \in \mathbb{Z} \text{and} 1 \leq r \leq \ 6\}.$$

[2]

(b) Let A, B and C be subsets of a universal set $\mathcal{U}$.

(i) Draw a labelled Venn diagram to illustrate the relationship between A, B and C such that they divide $\mathcal{U}$ into 8 separate regions. [1]

(ii) The subset $X \subseteq \mathcal{U}$ is defined by the following membership table.

A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Shade the area X on your Venn diagram. [2]

(iii) Use set operations to express the set X as a combination of the subsets A, B and C. [1]

(iv) The subset $Y \subseteq \mathcal{U}$ is defined as $Y = A \cap (B \cup C')$. Construct a membership table for Y. [2]

(v) For each of the following statements state whether it is true or false.
$X \subset Y$; $Y \subset X$, $Y = (A \cap B) \cap C'$. [2]

2005

Question 2 Let A and B and C be subsets of a universal set $\mathcal{U}$.

(a) Draw a labelled Venn diagram depicting A, B, C in such a way that they divide $\mathcal{U}$ into 8 disjoint regions. [1]

(b) The subset $X \subseteq \mathcal{U}$ is defined by the following membership table:

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

Shade the region X on your diagram. Describe the region you have shaded in set notation as simply as you can. [3]

(c) The subset $Y \subseteq \mathcal{U}$ is defined as $Y = A \cup (C - B)$. Construct a membership table for Y. [3]

(d) For each of the following statements say whether it is true or false, **justifying your answer**, using the Venn diagram you drew earlier.

(i) $Y \subseteq X$

(ii) $Y' \subseteq X'$

(iii) $Y - X = A \cap B \cap C$. [3]

2006 Set Theory

Question 2

(a) Describe the following sets using the listing method:

(i) $\{10^m : -2 \leq m \leq 3,\ m \in \mathbb{Z}\}$ (ii) $\{\frac{1}{n} : 1 < n < 6,\ n \in \mathbb{Z}\}$. [2]

(b) (i) Given 3 sets, A, B and C, subsets of a universal set $\mathcal{U}$, draw a labelled Venn diagram and shade the region corresponding to $A' \cap (B \cup C)$.

(ii) Show, using membership tables or Venn diagrams, that this region is equivalent to $(A' \cap B) \cup (A' \cap C)$.

(iii) What law does this illustrate? [5]

(c) Given the sets

$$\begin{aligned} \mathcal{U} &= sl\{1, 2, 3, ...9\} \\ A &= \{1, 2, 5, 6, 8\} \\ B &= \{3, 5, 7, 8\} \\ C &= \{5, 6, 7, 8, 9\}. \end{aligned}$$

(i) List separately the elements of $A' \cap B$ and $A' \cap C$.

(ii) Describe, as simply as you can in terms of set operations on A, B and C, the sets $\{5, 8\}$ and $\{1, 2, 3, 5, 6, 7, 8, 9\}$. [3]

2007

Question 2

(a) Let $A = \{2n : n \in \mathbb{Z}^+\}$ and $B = \{3, 6, 9, 12, ...\}$ be two sets of numbers.

(i) Describe the set A by the listing method.

(ii) Describe the set B by the rules of inclusion method.

(iii) Find the two sets $A \cap B$ and $A - B$, by the listing method. [5]

(b) Let P, Q and R be subsets of a universal set $\mathcal{U}$.

(i) Construct a membership table for the set $X = P' \cup (Q \cap R)$.

(ii) Draw a labelled Venn diagram showing P, Q, and R intersecting in the most general way.

(iii) Shade the region X on your diagram.

(iv) Is the set $Q \cap R \subseteq X$? Justify your answer. [5]