Solutions to test 2. Sets. October 2007

1. (a) {-2,-1,0,1,2} (b) {4,7,10,13}

2. (a) $\{2^n : n \in \mathbb{Z}^+\}$ (b) $\{5n : n \in \mathbb{Z}, -\nVdash \leq \ltimes \leq \not\triangleleft\}$.

3. (a) $|A| = 6$ (b)$2^6 = 64$ subsets of A.
   (c) $3 \in B$, $B \subseteq A$, $\emptyset \in A$, $B \in P(A)$.

4. $Q' = \{6, 7, 8, 9, 10\}$, $P \cup Q = \{1, 2, 3, 4, 5, 6, 7\}$
   $P \cap Q = \{3, 5\}$, $P \oplus Q = \{1, 2, 4, 7\}$.

5.

A	B	A'	B'	$A' \cup B'$	$A \cap B$	$(A \cap B)'$
0	0	1	1	1	0	1
0	1	1	0	1	0	1
1	0	0	1	1	0	1
1	1	0	0	0	1	0

   Since the coonlumns of the membership table are equal we see that $(A \cap B)' = A' \cup B'$.

$$(A \cup B)' = A' \cap B'.$$

6. (a) If we code the regions of the Venn diagram as follows then the shading is in the ticked regions.

A	B	C	*Shaded Area*
0	0	0	0
0	0	1	0
0	1	0	$\surd$
0	1	1	0
1	0	0	$\surd$
1	0	1	0
1	1	0	$\surd$
1	1	1	0

   (b) See shaded area. (c) Shaded area $(A \cup B) - C$ or equivalent.

7. Shading as ticked area

(a)

A	B	C	$Shaded\,Area$	A	B	C	$Shaded\,Area$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	0	0	1	1	0
1	0	0	$\surd$	1	0	0	$\surd$
1	0	1	0	1	0	1	0
1	1	0	0	1	1	0	$\surd$
1	1	1	0	1	1	1	0

(b) (i) $B \cup C = \{s, p, a, i, n, w, l, e, s\}$ $A - (B \cup C) = \{f, r, c\}$

(ii) $A - B = \{f, r, c, e\}$ $A - C = \{f, r, n\}$

$(A - B) \cup (A - C) = \{f, r, n, c, e\}$.

(c) So $A-(B\cup C) \neq (A-B)\cup(A-C)$ and set difference is not distributive over set union **in this case.**