Graph Theory

- Graphs
- Edges
- Vertex (Vertices)
- Degree
- Degree Sequence
- Sum of Degree Sequence
- Isomorphism

Graph Theory (2002)

2002 Question 6

- A company operates an express coach service between seven cities; **c1,c2,c3,…,c7**
- The number of other cities to which each city is directly linked by a coach is given in the following table.

CITY	C1	C2	C3	C4	C5	C6	C7
No. of Connections	3	2	3	5	4	4	1

Graph Theory (2002)

Describe how such as communications network can be modelled by a graph, saying what the vertices represent and a rule for determining when two vertices are adjacent.

Calculate how many pairs of cities have a direct coach link between them. giving at brief explanation of your method.

Graph Theory (2002)

- What is meant by saying that a graph is simple?
- Say why a graph model of this communications network would be simple.

- Is it possible to construct a graph with degree sequence (4,4, 4, 3, 3, 2, l)?
- Either construct an example of such a graph or say why it is not possible to do so

Graph Theory (2007)

Question 6 Given the following definitions for simple, connected graphs:

- K_n is a graph on n vertices where each pair of vertices is connected by an edge;
- C_n is the graph with vertices $v_1, v_2, v_3, ..., v_n$ and edges $\{v_1, v_2\}$, $\{v_2, v_3\}, ...\{v_n, v_1\}$;
- W_n is the graph obtained from C_n by adding an extra vertex, v_{n+1}, and edges from this to each of the original vertices in C_n.

Graph Theory (2007)

(a) Draw K_4, C_4, and W_4. $[2\frac{1}{2}]$

(b) Giving your answer in terms of n, write down an expression for the number of edges in K_n, C_n, and W_n. $[2\frac{1}{2}]$

Graph Theory (2007- Part C not part of course)

(c) (i) Find the number of different paths of length two in each of the graphs in part (a), where a path does not contain the same edge more than once, and a path from v_x to v_y is different from a path from v_y to v_x.

(ii) Giving your answer in terms of n, write down an expression for the number of different paths of length two there are in K_n. [5]

Graph Theory (2006)

Question 5 (a) Let G be a simple graph with vertex set $V(G) = \{v_1,\ v_2,\ v_3,\ v_4,\ v_5\}$ and adjacency lists as follows:

$$\begin{array}{lcl} v_1 & : & v_2\ v_3\ v_4 \\ v_2 & : & v_1\ v_3\ v_4\ v_5 \\ v_3 & : & v_1\ v_2\ v_4 \\ v_4 & : & v_1\ v_2\ v_3. \\ v_5 & : & v_2 \end{array}$$

Graph Theory (2006)

$$\begin{array}{lcl} v_1 & : & v_2\ v_3\ v_4 \\ v_2 & : & v_1\ v_3\ v_4\ v_5 \\ v_3 & : & v_1\ v_2\ v_4 \\ v_4 & : & v_1\ v_2\ v_3. \\ v_5 & : & v_2 \end{array}$$

(i) List the degree sequence of G.

(ii) Draw the graph of G.

(iii) Find two distinct paths of length 3, starting at v_3 and ending at v_4.

(iv) Find a 4 cycle in G.

Graph Theory (2006)

(b) In the following cases either construct a graph with the specified properties or say why it is not possible to do so.

(i) A graph with degree sequence 3,2,2,1.

(ii) A simple graph with degree sequence 4,3,2,2. [4]

Graph Theory 2010

Question 6

Let G be a graph and let u and v be vertices of G.

(a) i. Say what is meant by the degree of v.

ii. Say what is meant by saying u and v are adjacent. [2]

Graph Theory 2010

(b) State, without proof, a result connecting the number of edges of G with the degrees of its vertices. [1]

Graph Theory 2010

(c) A graph is called k-regular if each of its vertices has degree k.

i. Use the result from (b) to find the number of edges in a 7-regular graph with 8 vertices.

ii. Explain why it is not possible to construct a 7-regular graph on 9 vertices.

iii. Construct an example of a 2-regular graph on 5 vertices.

Graph Theory 2010

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Graph Theory 2010

iv. Construct an example of a 3-regular graph on 6 vertices.

v. Given a 7-regular graph on $2n$ vertices where $n \geq 4$ find how many edges there are in this graph.

[7]

Graph Theory 2010

iv. Construct an example of a 3-regular graph on 6 vertices.

v. Given a 7-regular graph on $2n$ vertices where $n \geq 4$ find how many edges there are in this graph.

[7]

Graph Theory 2009

Question 4 (a) What properties should a graph possess if it is

(i) simple (ii) connected? [2]

Graph Theory 2009

(b) Given a graph G with degree sequence

$$4, 3, 3, 2, 1, 1.$$

(i) How many vertices are there in G?

(ii) Find the number of edges in G, explaining how you obtain your answer.

Graph Theory 2009

(iii) Draw an example of a simple graph G with the degree sequence

$$4, 3, 3, 2, 1, 1.$$

[6]

Graph Theory 2009

(c) (i) Say why it is not possible to construct a simple graph with the degree sequence

$$4, 2, 2, 2.$$

(ii) Show it is possible to construct a graph with this degree sequence if we do not require it to be simple. [2]

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