But notice that in either case, the "if" introduces the statement "$n = 17$".

A connective with a different meaning is *only if*. In order to link the two propositions "$n = 17$" and "$n$ is greater than 10" using *only if* with the same meaning as the statements (a) and (b), the "only if" must introduce the proposition "$n$ is greater than 10". Thus we would say:

(c) $n = 17$ *only if* $n$ is greater than 10.

Let $p$ denote the proposition "$n = 17$" and $q$ denote the proposition "$n$ is greater than 10". Then the statements above can be written as

(a) *If $p$, then $q$*; (b) $q$ *if* $p$ ; (c) $p$ *only if* $q$.

Without altering the meaning, we can rewrite each of these statements using the word *implies* as:

$n = 17$ *implies* $n$ is greater than 10.

This statement is written symbolically as

$$p \to q.$$

In general, the statements $p \to q$ and $q \to p$ have different meanings. When we want to make both of these statements, we often use the connectives *if* and *only if* together. For example, the following statement concerns an integer $n$ expressed in base 10:

(d) $n$ is divisible by 10 *if and only if* the last digit of $n$ is 0.

Let $r$ denote the proposition "the last digit of $n$ is 0" and $s$ denote the proposition "$n$ is divisible by 10". Then (d) can be written as

$r$ *if and only if* $s$.

In this case, each of the propositions $r$ and $s$ imply the other, so (d) has the same meaning as

$$r \to s \text{ and } s \to r.$$

This compound statement is written as

$$r \leftrightarrow s.$$

### 3.2.1 Truth tables for $p \to q$ and $p \leftrightarrow q$

We consider the truth value of $p \to q$ first in the special case of the propositions concerning a positive integer $n$ discussed in the previous section. Thus we let $p$ denote the proposition "$n = 17$", and $q$ denote the proposition "$n > 10$".

For this pair of propositions, the statement $p \to q$ is always true; that is, the statement "$n = 17$ implies $n > 10$" is true *whatever value of $n$ we choose*. For example, if $n$ happens to have the value 1, this does not make the statement "$n = 17$ implies $n > 10$" false.

To construct the truth table for $p \to q$, we need to find the truth value of $p \to q$ for each combination of $p$ false or true with $q$ false or true. Can we find an example of a value of $n$ to illustrate each of these four possibilities?

Now $p$ is true when $n = 17$ and false when $n \neq 17$; while $q$ is true when $n > 10$ and false when $n \leq 10$. Thus we can construct the following table.

| $p$ | $q$ | *example of $n$* |
|---|---|---|
| 0 | 0 | 2 |
| 0 | 1 | 14 |
| 1 | 0 | *none* |
| 1 | 1 | 17 |

As we have observed, the statement $p \to q$ is true for all integers $n$. Thus $p \to q$ is true when both $p$ and $q$ are false, when $p$ is false and $q$ is true and when both $p$ and $q$ are true. However, it is