3.1.2 The negation of a proposition

The **negation** of a proposition p, is the proposition that is true when p is false and false when p is true. We denote the negation of p by $\neg p$, read "not-p". The truth set for $\neg p$ is the complement of the truth set for p.

Example 3.4

(a) Let p denote the proposition "The integer n is prime"; then $\neg p$ is the proposition "The integer n is not prime".

(b) Let p denote the proposition "$x = x + 1$". Then $\neg p$ is the proposition "$x \neq x + 1$".

Notice that in Example 3.4(b), p is a contradiction and its negation is a tautology. This is a particular example of a general rule. Similarly, the negation of a tautology will always be a contradiction.

3.1.3 Compound statements

We can join two or more propositions together by such words as "and", "or", "if ... then", to form **compound statements**.

Example 3.5 We combine the propositions p and q of Example 3.2 to give the following compound statements.

(a) "The first toss is a head and the third toss is a tail" is denoted symbolically by $p \wedge q$.

(b) "The first toss is a head or the third toss is a tail" is denoted symbolically by $p \vee q$.

(c) "The first toss is a head or the third toss is a tail, **but not both**" is denoted symbolically by $p \oplus q$. □

Notice that or in mathematics is used in its inclusive sense, unless we specify otherwise, so that (b) includes the possibility that the first toss is a head *and* the third toss is a tail, whereas (c) does not. The symbol $\oplus$ is known as **exclusive or.**

Example 3.6 We have already found above the truth sets for the propositions p and q defined in Example 3.2. We now find the truth set for each of the compound statements (a), (b) and (c).

(a) The truth set for $p \wedge q$ is $\{HHT, HTT\} = P \cap Q$.

(b) The truth set for $p \vee q$ is $\{HHH, HHT, HTH, HTT, THT, TTT\} = P \cup Q$.

(c) The truth set for $p \oplus q$ is $\{HHH, HTH, THT, TTT\} = P \oplus Q$. □

3.1.4 Truth tables

We give each proposition a **truth value**, either 1 for true, or 0 for false. We may then determine the truth value of each of the compound statements $p \vee q$, $p \wedge q$, $p \oplus q$, from the truth values of their constituent propositions, p and q, by considering each combination of p true or false with q true or false. This is most easily done in the form of a **truth table**, as shown below.

p	q	$p \wedge q$	$p \vee q$	$p \oplus q$
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

Figure 3.1.

We see that each of the compound statements is logically distinct, because it has its own distinct pattern of 0's and 1's. Further, this pattern is identical to the pattern of 0's and 1's in the