and

"not (p or q)" is the same as saying "not p and not q."

Example 3.8 Let p be the proposition: "the last digit of n is 5" and q be the proposition: "the last digit of n is 0". Then $\neg(p \vee q)$ is the statement

"the last digit of n is not 0 or 5."

This has the same meaning the as:

"the last digit of n is not 0 and the last digit of n is not 5"

which is the statement $\neg p \wedge \neg q$. □

Tautologies and contradictions

All tautologies are logically equivalent because every tautology has only the truth value 1. Similarly, all contradictions are logically equivalent, since each takes only the truth value 0. We can therefore use the one symbol T to denote any tautology and the symbol F to denote any contradiction.

Example 3.9 The truth table for $p \vee T$ is shown in Figure 3.4. Note that we need only two rows for this table, because T has only one truth value.

p	T	$p \vee T$
0	1	1
1	1	1

Figure 3.4.

The table shows that $p \vee T = T$. This means that the compound statement

(proposition p) or (tautology)

is always true, whatever the truth value of p. □

3.2 The Conditional Connectives

Learning Objectives

After studying this section, you should be able to:

- construct the truth table for the conditional connectives *if*, *only if* and *if and only if*;
- interpret alternative ways of wording conditional statements;
- state the contrapositive of a given statement.

Introduction

In mathematics and when writing computer programs, as well as in everyday language, we often use the word "if" to connect two statements. However, "if" can be used in more than one way. Consider, for example, the following statements concerning an integer n.

(a) *If* $n = 17$, *then* n is greater than 10.

We sometimes say this with the "if" clause second, as:

(b) n is greater than 10 *if* $n = 17$.