5. Construct a truth table for each of the following compound statements and hence find simpler propositions to which each is equivalent.

$$\text{(a) } p \vee F; \qquad \text{(b) } p \wedge T.$$

6. Let p, q, r be the following propositions concerning an integer n.

$$p\text{: } n = 20; \qquad q\text{: } n \text{ is even}; \qquad r\text{: } n \text{ is positive.}$$

Express each of the following conditional statements symbolically, using the symbol $\rightarrow$.

(a) If $n = 20$, then n is positive.

(b) n is even if $n = 20$.

(c) $n = 20$ only if n is even.

7. Let q and r be the propositions defined in the previous exercise. Complete the following table by giving the truth value of each of the statements $q, r, q \rightarrow r, r \rightarrow q$ and $q \leftrightarrow r$ corresponding to each value of n.

n	q	r	$q \rightarrow r$	$r \rightarrow q$	$q \leftrightarrow r$
-8					
-3					
10					
17					

8. Use truth tables to prove that $\neg p \leftrightarrow \neg q$ is logically equivalent to $p \leftrightarrow q$.

9. Write the contrapositive of each of the following statements.

(a) If $n = 12$, then n is divisible by 3.

(b) If $n = 5$, then n is positive.

(c) If the quadrilateral is a square, then its four sides are equal.

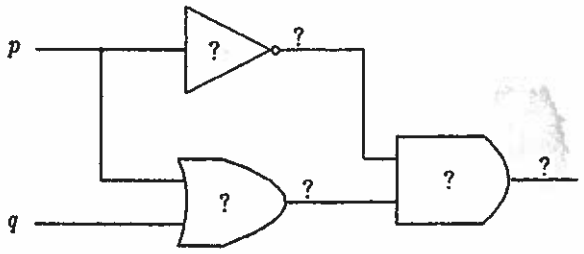
10. The basis for logical argument is that given propositions p, q, r such that p imples q and q implies r, then we can deduce that p implies r. The validity of this argument depends upon the fact that the statement $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is always true, that is, it is a tautology.

Construct a truth table with columns for p, q, r and $p \rightarrow q, q \rightarrow r, (p \rightarrow q) \wedge (q \rightarrow r), p \rightarrow r, [(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$. Hence prove that

$$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$$

is indeed a tautology.

11. The following logic network accepts inputs p and q, which may each independently have the value 0 or 1.



(a) Copy the network and label each of the gates appropriately with one of the words "NOT", "AND" or "OR". Label the diagram also with a symbolic expression for the output from *each* gate.

(b) Construct a logic table to show the value of the output corresponding to each combination of values (0 or 1) for the inputs p and q.

(c) Find a simpler expression that is logically equivalent to the final output.