2003

Question 3 (a) Let n be an element of the set $\{10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$, and p and q be the propositions:

$$p : niseven, \quad q : n{>}15.$$

Draw up truth tables for the following statements and find the values of n for which they are true:

(i) (i) $p \vee \neg q$ (ii) $\neg p \wedge q$

(ii) Use truth tables to find a statement that is logically equivalent to $\neg p \rightarrow q$. [6]

(b) Let p, q be the following propositions:

$$p : thisappleisred, \quad q : thisappleisripe.$$

Express the following statements in words as simply as you can:
(i) $p \rightarrow q$ (ii) $p \wedge \neg q$.
Express the following statements symbolically:
(iii) This apple is neither red nor ripe.
(iv) If this apple is not red it is not ripe. [4]

2004

Question 2 Let p and q be the following propositions about the positive integer n:

$$p : n < 20 \qquad q : nisprime.$$

(a) List the truth sets for: (i) p (ii) $p \wedge q$. [2]

(b) Express each of the following compound propositions symbollically using p, q :

(i) $n < 20$ and n is not prime
(ii) $n < 20$ if n is prime
(iii) $n < 20$ or n is prime.

[3]

(c) For each of the compound expressions in (b) give ONE example of n for which the proposition is FALSE. [3]

(d) Write the contrapositive of the folowing proposition:

$$"ifn = 14thennisdivisibleby7."$$

[2]

2005

Question 3 (a) Given propositions p and q, construct the truth tables for:

$$p \wedge q; \quad q \rightarrow p; \quad \neg p \wedge (p \vee q).$$

[3]

(b) Given n is a positive integer and p and q the following statements about n :

$$p : nisodd; \quad q : n < 12.$$

(i) List the elements of the truth set for each of the compound statements: $p \wedge q$; $\neg p \wedge (p \vee q)$.

(ii) Find one value of n which makes $q \rightarrow p$ **false**. [3]

(c) Draw a logic network that accepts independent inputs p and q and gives as output

$$\neg p \wedge (p \vee q)$$

Find a simpler expression that is logically equivalent to this final output. [4]

2006

Question 5

(a) A logic network accepts inputs p and q, which may each independently have the value 0 or 1, and gives as the final output

$$(p \vee q) \wedge \neg q.$$

(i) Draw this network. Label each of the gates appropriately and also label the diagram with a symbolic expression for the output after each gate.

(ii) Construct a truth table to show the value of the output corresponding to each combination of values (0 or 1) for the inputs p and q.

(iii) Hence, or otherwise, find a simpler expression that is logically equivalent to the final output. [5]

(b) Let p be the proposition "this animal is a cat" and q be the proposition "this animal has a tail".

(i) Expain in words the meaning of the logical statement $p \rightarrow q$.

(ii) Write the contrapositive of this statement in logical symbols and explain its meaning in an English sentence.

(iii) Write each of the following as a logical statement involving p and q :

"This animal is a cat and it does not have a tail";
"This animal neither is a cat nor has a tail". [5]

2007

Question 3 (a) Let n be a positive integer and p and q be the following propositions:

$$\begin{aligned} p &: n \leq 12 \\ q &: nisodd. \end{aligned}$$

(i) Express each of the three following compound propositions concerning positive integers symbolically by using p, q and appropriate logical symbols.

$$\begin{aligned} n &\leq 12andniseven. \\ ifn &\leq 12thenniseven \\ n &> 12andnisodd. \end{aligned}$$

(ii) Construct the truth table for the statement $q \rightarrow p$. Hence find a value of n that makes this statement false.

(iii) Write in logical symbols the contrapositive of the statement:

$$ifnisoddthenn \leq 12.$$

[6]

(b) Construct a logic network that accepts as inputs p and q, which may independently have the value 0 or 1, and gives as final output

$$\neg(\neg p \wedge q).$$

Show the truth table for this output and hence give a simple expression (without using negation) that is equivalent to $\neg(\neg p \wedge q)$. [4]