impossible to find a value of n that makes p true and q false. Thus this is the ONLY combination of truth values of p and q that would make the implications *false*.

Although we have determined the truth value of $p \to q$ only for a particular pair of propositions p and q, our conclusions hold in general: for any propositions p and q, $p \to q$ is false *only when p is true and q is false.* Thus we have the following truth table for $p \to q$.

p	q	$p \to q$
0	0	1
0	1	1
1	0	0
1	1	1

Figure 3.5.

To most people, this table does not seem to follow our intuition in the same way as the tables for $\neg$, $\wedge$ and $\vee$. You are therefore strongly advised to learn it carefully.

An alternative expression for $p \to q$

The truth table for $p \to q$ enables us to express this conditional statement by means of negations and the connective "or".

Result 3.5 $p \to q = \neg p \vee q$.

Proof. We use truth tables to show that $p \to q = \neg p \vee q$. The first table is for the left side of this equation and the second table is for the right side.

p	q	$p \to q$
0	0	1
0	1	1
1	0	0
1	1	1

p	q	$\neg p$	$\neg p \vee q$
0	0	1	1
0	1	1	1
1	0	0	0
1	1	0	1

Comparing the columns corresponding to the statements $p \to q$ and $\neg p \vee q$, we see they are identical. Hence these two expressions are logically equivalent. □

Truth tables for $q \to p$ and $p \leftrightarrow q$

From the table in Figure 3.5, we can deduce the truth table for $q \to p$, by interchanging the roles of p and q. Thus $q \to p$ is false only when q is true and p is false.

From the truth tables for $p \to q$ and $q \to p$, we can deduce the truth table for $p \leftrightarrow q$, because we have defined $p \leftrightarrow q$ as $(p \to q) \wedge (q \to p)$. Thus $p \leftrightarrow q$ is true only when either *both* p and q are true or *both* p and q are false.

p	q	$p \to q$	$q \to p$	$p \leftrightarrow q$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	1	1	1

Figure 3.6.

Result 3.5 gives us a way of expressing the truth set for the conditional statement $p \to q$ and hence for $q \to p$ and $p \leftrightarrow q$.

Result 3.6 *Let P, Q be the truth sets for propositions p and q respectively. Then the truth set for $p \to q$ is $P' \cup Q$ and the truth set for $p \leftrightarrow q$ is $(P' \cup Q) \cap (P \cup Q')$.* □