3.2.2 The contrapositive

Consider the following two statements.

1. If this rectangle is a square, then its sides are all equal.
2. If the sides are not all equal, then this rectangle is not a square.

The second statement is called the **contrapositive** of the first statement. We see that the first statement has the form

$$p \rightarrow q$$

and its contrapositive has the form

$$\neg q \rightarrow \neg p.$$

Note that since $\neg(\neg p) = p$ and $\neg(\neg q) = q$, the contrapositive of the statement $\neg q \rightarrow \neg p$, is the statement $p \rightarrow q$.

We can show that every conditional statement is logically equivalent to its contrapositive. This means that we can make either a statement or its contrapositive, with the same meaning.

Result 3.7 $\neg q \rightarrow \neg p = p \rightarrow q$.

Proof. We can prove this result by constructing a truth table for each side of the equation.

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

p	q	$\neg q$	$\neg p$	$\neg q \rightarrow \neg p$
0	0	1	1	1
0	1	0	1	1
1	0	1	0	0
1	1	0	0	1

Figure 3.7.

Since the columns for $p \rightarrow q$ and $\neg q \rightarrow \neg p$ in Figure 3.7 contain the same entries, these two statements are logically equivalent. □.

Example 3.10

(a) The contrapositive of the statement "If your ticket has been drawn, then you win a prize" is the statement: "If you don't win a prize, then your ticket has not been drawn".

(b) The contrapositive of the statement "If $n = 17$, then $n > 10$" is "If $n \leq 10$, then $n \neq 17$ ", where we have expressed "not greater than" as "less than or equal to".

3.3 Laws of logic

Learning Objectives

After studying this section, you should be able to:

- use the laws of logic to simplify a given expression.

Introduction

By applying the laws of set algebra to truth sets, we can deduce equivalent laws for manipulating compound statements; these are known as *the laws of logic.* These laws can be used to prove the logical equivalence of two statements as an alternative to constructing truth tables.