membership tables of $P \cap Q$ in the case of $p \wedge q$, of $P \cup Q$ in the case of $p \vee q$ and of $P \oplus Q$ in the case of $p \oplus q$. Thus we have:

Result 3.1 *Let P, Q be the truth sets for the propositions p and q respectively. Then the truth set for $p \wedge q$ is $P \cap Q$, for $p \vee q$ is $P \cup Q$ and for $p \oplus q$ is $P \oplus Q$.*

Definition 3.2 *When two statements p and q have the same truth table, they are called logically equivalent and we write $p = q$.*

Statements involving negation

Result 3.3 (Double negative law) $\neg(\neg p) = p$.

Proof. To prove this result, we construct the truth table for $\neg(\neg p)$ and compare it with the truth values of p.

p	$\neg p$	$\neg(\neg p)$
0	1	0
1	0	1

Figure 3.2.

We see that whatever the truth value of p, the truth value of $\neg(\neg p)$ is the same. This proves that $\neg(\neg p) = p$. □

Example 3.7 Let p denote the proposition "This program is in Pascal". Then $\neg p$ denotes the proposition "This program is not in Pascal", and $\neg(\neg p)$ denotes the proposition "It is not true that this program is not in Pascal". This means the same as "This program is in Pascal". □

The double negative in Example 3.7 makes the sentence awkward and its meaning less obvious. Rule 3.3 says that we can always avoid double negatives, and in the interests of clarity we should do so.

We also have to take great care when negating expressions involving the connectives *and* or *or*, as ordinary English usage is often rather inexact. We have the following equivalents of De Morgan's Laws for $\vee$ and $\wedge$.

Result 3.4 (De Morgan's laws) *For all propositions p and q, we have*

(i) $\neg(p \wedge q) = \neg p \vee \neg q$;

(ii) $\neg(p \vee q) = \neg p \wedge \neg q$.

Proof. These laws can be proved using truth tables. The general method is similar to the way we proved set laws using membership tables. As an example, we prove law (i) by constructing a truth table for each side of the equation. For the left side, we first construct a column for $p \wedge q$ and from this deduce the column for $\neg(p \wedge q)$. Similarly, for the right side, we first construct columns for $\neg p$ and $\neg q$, and from these deduce the column for $\neg p \vee \neg q$.

p	q	$p \wedge q$	$\neg(p \wedge q)$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	0

Figure 3.3

Since the columns corresponding to the statements $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are identical, these two statements are logically equivalent. □

Result 3.4 states that

"not (p and q)" is the same as saying "not p or not q;"