Contents

Arrow Diagrams for Functions

- Domain of a Function
- Co-Domain of a Function
- Range of a function
- one-one (surjective)
- onto (bijective)

Question Sheet

Questions are taken from past papers of the “Mathematics for Computing” syllabus. They are re-arranged to correspond to the syllabus of this section.

Question 1 : 2010 part A

(a) Consider the two sets $X = \{a, b, c, d\}$ and $Y = \{1, 2, 3, 4, 5\}$. A function $f : X \to Y$ is given by $\{(c, 3), (a, 2), (b, 5), (d, 3)\}$, a subset of $X \times Y$.

i. Show f as an arrow diagram.

ii. State the domain, co-domain and range of f.

iii. Say why f does not have the one-to-one property and why f does not have the onto property, giving a specific counter example in each case.

[5]

Question 2 : 2009 part A

(a) Given the ceiling function: $\lceil x \rceil = n$ where $n-1 < x \leq n$, $n \in \mathbb{Z}$.

Let $A = \{1, 2, 3, 4, 5\}$ and $f(x) = \left\lceil \frac{x^2-1}{4} \right\rceil$ where $f : A \to \mathbb{Z}$

(i) Find $f(2)$ and the ancestor of 0.

(ii) Find the range of f.

(iii) Is f invertible? Justify your answer. [3]

Question 3 : 2009 part B

(b) Let $g(n) = \lceil \frac{n-1}{4} \rceil$ where $g : \mathbb{Z} \to \mathbb{Z}$.

(i) Find $g(4)$ and the set of ancestors of 0.

(ii) Find the range of g.

(iii) Is g invertible? Justify your answer. [4

Question 4 : 2009 part B

(b) i. State the condition to be satisfied in order for a function to have an inverse.

ii. Given $f : \mathbb{R} \rightarrow \mathbb{R}$ where $f(x) = 2x - 1$, define fully the inverse function f^{-1} and state the value of $f^{-1}(1)$.

iii. Given $g : \mathbb{R} \rightarrow \mathbb{R}^{+}$ where $g(x) = 3^{x}$, define fully the inverse function g^{-1} and state the value of $g^{-1}(1)$.

[5]

Question 5 : 2009 part B

(a) Showing your working, use the rules of indices and logarithms to give the following two expressions in their simplest possible form.

i. $4 \cdot 2^x - 2^{x+1}$; ii. $\dfrac{\ln(2) + \ln(2^2) + \ln(2^3) + \ln(2^4) + \ln(2^5)}{\ln(4)}$.

[2]

Question 6 : 2009 part B

(b) i. Given a positive real number x, say how $\log_2(x)$ and $\log_4(x)$ are defined.

ii. Compute $\log_2(16)$ and $\log_2(\frac{1}{16})$.

iii. Explain what it means for a function to be $O(\log_2(x))$.

iv. Justifying your answer, say whether $\log_4(x)$ is $O(\log_2(x))$. [5]

Question 7 : 2009 part B

(c) Consider the function $f : \mathbb{A} \to \mathbb{A}$ where $\mathbb{A} = \{1, 2, 3, 4, 5, 6\}$ and f is defined by the table

x	1	2	3	4	5	6
$f(x)$	3	1	2	5	6	4

Let $g : \mathbb{A} \to \mathbb{A}$ be the function defined by $g(x) = f(f(x))$.

i. Complete the following table so it defines the function $g(x)$:

x	1	2	3	4	5	6
$g(x)$						

ii. Show that the function g is the inverse of f. [3]

Note: Invertible Functions

A function is invertible if it fulfils two criteria

- The function is ***onto***,
- The function is ***one-to-one***.

State the conditions to be satisfied by a function $f : X \leftarrow Y$ for it to have an inverse function $f^{-1} : Y \leftarrow X$.

1 Part 2

Encoding and Decoding Functions (4.2)

Onto Functions (4.2.2)

$\lceil \frac{x^2+1}{4} \rceil$ where $f : A \rightarrow \mathbf{Z}$

(i) Find $f(4)$ and the ancestors of 3.

(ii) Find the range of f.

(iii) Is f invertible? Justify your answer

Given $f : \mathbf{R} \rightarrow \mathbf{R}$ where f(x) =3x-1,define fully the inverse of the function f ,i.e.f^{-1}. State the value of $f^{-1}(2)$

2 Part 3

2.1 Exponentials Functions

$$e^a \times e^b = e^{a+b}$$

$$(e^a)^b = e^{ab}$$

2.2 Logarithmic Functions

2.2.1 Laws for Logarithms

The following laws are very useful for working with logarithms.

1. $\log_b(X) + \log_b(Y) = \log_b(X \times Y)$
2. $\log_b(X)$ - $\log_b(Y) = \log_b(X/Y)$
3. $\log_b(X^Y)$= $Y\log_b(X)$

Question1.3 Compute the Logarithm of the following

- $\log_2(8)$
- $\log_2(\sqrt{128})$
- $\log_2(64)$
- $\log_5(125) + \log_3(729)$
- $\log_2(64/4)$

Question 4

Part A : Functions

Given a real number x, say how the floor of x $\lfloor x \rfloor$ is defined.

(i) Find the values of $\lfloor 2.97 \rfloor$ and $\lfloor -2.97 \rfloor$.

(ii) Find an example of a real number x such that $\lfloor 2x \rfloor \neq 2\lfloor x \rfloor$, justifying your answer.

Part B : Logarithms

Evaluate the following expression.

$$\mathrm{Log}_4 64 + \mathrm{Log}_5 625 + \mathrm{Log}_9 3$$

2.3 Precision Functions

- Absolute Value Function $|x|$
- Ceiling Function $\lceil x \rceil$
- Floor Function $\lfloor x \rfloor$

Question1.2: State the range and domain of the following function

$$F(x) = \lfloor x - 1 \rfloor$$

One-to-One Functions (4.2.3)

One-to-One Functions (4.2.3)

$f(x)$, must be *One-to-One* and *Onto*

Exponential and Logarithmic Functions (4.3)

The Laws of Logarithms

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- $log_b(x^y) = y \times log_b(x)$
-
-

Comparing the size of Functions (4.4)

Using O-notations

Power Notation (4.4.2)

Section 8 Exercises

- $8^{\frac{1}{3}}$ Recall $a^{\frac{b}{c}} = a^{\frac{b}{c}}$
-
-

3 Mathematics for Computing: Onsite Tutorial two

Today's Class

- Chapter 3: Logic
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  -
- Chapter 4: Functions
  - Inverse of a Function
  - One-to-One and Onto
  - Special Functions

The Asbolute Value, Floor and Ceiling Functions

- The Absolute Value Function
- Floor
- Ceiling
-
-
-

Power functions and Polynomials

Consider the function $f : Z \to Z$ defined by f(n) = 3n-1. Does this function have the onto property?

$$ax^2 + bx + c$$

Summer 2003 Question 4

x	
f(x)	
g(x)	