Inverse of Functions

In the last example from the previous section we
looked at the two functions and and saw that
and as noted in that section this means that there is
a nice relationship between these two functions.
Lets see just what that relationship is. Consider the
following evaluations.

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In the first case we plugged into and got a value of -5. We then turned around and plugged into and got a value of -1, the number that we started off with.

In the second case we did something similar. Here we plugged into and got a value of , we turned around and plugged this into and got a value of 2, which is again the number that we started with. Note that we really are doing some function composition here. The first case is really, and the second case is really,

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Note as well that these both agree with the formula for the compositions that we found in the previous section. We get back out of the function evaluation the number that we originally plugged into the composition.

So, just what is going on here? In some way we can think of these two functions as undoing what the other did to a number. In the first case we plugged into and then plugged the result from this function evaluation back into and in some way undid what had done to and gave us back the original x that we started with.

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- Function pairs that exhibit this behavior are called inverse functions.
- Before formally defining inverse functions and the notation that were going to use for them we need to get a definition out of the way.
- A function is called **one-to-one** if no two values of x produce the same y.
- Mathematically this is the same as saying,

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So, a function is one-to-one if whenever we plug different values into the function we get different function values.

Sometimes it is easier to understand this definition if we see a function that isnt one-to-one. Lets take a look at a function that isnt one-to-one. The function is not one-to-one because both and . In other words there are two different values of x that produce the same value of y. Note that we can turn into a one-to-one function if we restrict ourselves to . This can sometimes be done with functions.

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Showing that a function is one-to-one is often tedious and/or difficult. For the most part we are going to assume that the functions that were going to be dealing with in this course are either one-to-one or we have restricted the domain of the function to get it to be a one-to-one function.

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Now, lets formally define just what inverse functions are. Given two one-to-one functions and if then we say that and are inverses of each other. More specifically we will say that is the inverse of and denote it by

Likewise we could also say that is the inverse of and denote it by
The notation that we use really depends upon the problem. In most cases either is acceptable.

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For the two functions that we started off this section with we could write either of the following two sets of notation.

Now, be careful with the notation for inverses. The -1 is NOT an exponent despite the fact that is sure does look like one! When dealing with inverse functions weve got to remember that

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This is one of the more common mistakes that students make when first studying inverse functions. The process for finding the inverse of a function is a fairly simple one although there are a couple of steps that can on occasion be somewhat messy. Here is the process

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Finding the Inverse of a Function Given the function we want to find the inverse function, . First, replace with y. This is done to make the rest of the process easier. Replace every x with a y and replace every y with an x. Solve the equation from Step 2 for y. This is the step where mistakes are most often made so be careful with this step. Replace y with . In other words, weve managed to find the inverse at this point!

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Verify your work by checking that and are both
true. This work can sometimes be messy making it
easy to make mistakes so again be careful.
Thats the process. Most of the steps are not all
that bad but as mentioned in the process there are
a couple of steps that we really need to be careful
with since it is easy to make mistakes in those steps.

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In the verification step we technically really do need to check that both and are true. For all the functions that we are going to be looking at in this course if one is true then the other will also be true. However, there are functions (they are beyond the scope of this course however) for which it is possible for only one of these to be true. This is brought up because in all the problems here we will be just checking one of them. We just need to always remember that technically we should check both.

Lets work some examples.

Example 1 Given find .

Solution Now, we already know what the inverse to this function is as weve already done some work with it. However, it would be nice to actually start with this since we know what we should get. This will work as a nice verification of the process.

So, lets get started. Well first replace with y.

Next, replace all xs with y and all ys with x.
Now, solve for y.

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Finally replace y with .

Now, we need to verify the results. We already took care of this in the previous section, however, we really should follow the process so well do that here. It doesnt matter which of the two that we check we just need to check one of them. This time well check that is true.

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Example 2 Given find .

Solution The fact that were using instead of doesnt change how the process works. Here are the first few steps.

Now, to solve for y we will need to first square both sides and then proceed as normal.

This inverse is then,

Finally lets verify and this time well use the other one just so we can say that weve gotten both down somewhere in an example.

So, we did the work correctly and we do indeed have the inverse.

The next example can be a little messy so be careful with the work here.

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Example 3 Given find . Solution The first couple of
steps are pretty much the same as the previous
examples so here they are,

Now, be careful with the solution step. With this
kind of problem it is very easy to make a mistake
here.

So, if weve done all of our work correctly the inverse
should be,

Finally well need to do the verification. This is also a fairly messy process and it doesnt really matter which one we work with.

Okay, this is a mess. Lets simplify things up a little bit by multiplying the numerator and denominator by .

Wow. That was a lot of work, but it all worked out in the end. We did all of our work correctly and we do in fact have the inverse.

There is one final topic that we need to address quickly before we leave this section. There is an interesting relationship between the graph of a function and the graph of its inverse.

Here is the graph of the function and inverse from the first two examples.

In both cases we can see that the graph of the inverse is a reflection of the actual function about the line . This will always be the case with the graphs of a function and its inverse.