2002

Question 4 (a) The function $f : \mathbf{Z} \to \mathbf{Z}$ is defined by the rule

$$f(m) = |m + 2|,$$

where $|m|$ denotes the absolute value of m.

(i) Calculate $f(-3)$. [1]

(ii) Calculate the set of ancestors (pre-images) of 10. [2]

(iii) Decide whether the function f has the *one-to-one* property, justifying your answer. [1]

(iv) Decide whether the function f has the *onto* property, justifying your answer. [1]

(b) Give the conditions to be satisfied by a function $f : X \to Y$ for it to have an inverse function $f^{-1} : Y \to X$. [1]

(c) The following functions f are both invertible. In each case, give a formula for the inverse function f^{-1}.

(i) $f : \mathbf{R}^+ \cup \{0\} \to \mathbf{R}^- \cup \{0\}$ defined by $f(x) = x^3$. [2]

(ii) $f : \mathbf{R} \to \mathbf{R}$ defined by $f(x) = 4x + 3$. [2]

Question 4 (a) The function $f : \mathbf{Z} \to \mathbf{Z}$ is defined by the rule

$$f(m) = |m + 2|,$$

where $|m|$ denotes the absolute value of m.

(i) Calculate $f(-3)$. [1]

(ii) Calculate the set of ancestors (pre-images) of 10. [2]

(iii) Decide whether the function f has the *one-to-one* property, justifying your answer. [1]

(iv) Decide whether the function f has the *onto* property, justifying your answer. [1]

(b) Give the conditions to be satisfied by a function $f : X \to Y$ for it to have an inverse function $f^{-1} : Y \to X$. [1]

(c) The following functions f are both invertible. In each case, give a formula for the inverse function f^{-1}.

(i) $f : \mathbf{R}^+ \cup \{0\} \to \mathbf{R}^- \cup \{0\}$ defined by $f(x) = x^3$. [2]

(ii) $f : \mathbf{R} \to \mathbf{R}$ defined by $f(x) = 4x + 3$. [2]

2003

Question 4 (a) The function $f : \mathbb{R} \to \mathbb{Z}$ is defined by the rule $f(x) = \lfloor \frac{x}{3} \rfloor$.

(i) Find $f(2)$, $f(10)$.

(ii) Find the range of f.

(iii) Find the set of pre-images of 1.

(iv) Say, with reason, whether or not f is invertible. [6]

(b) Copy and complete the following table of values for the functions $g(x) = \log_3 x$ and $h(x) = \sqrt[3]{x}$.

x	1				81	
$g(x)$		1	2			5
$h(x)$ to 2 d.p.				3.00		

Is $\log_3 x = O(\sqrt[3]{x})$? Give a reason for your answer. [4]

2004

Question 3 Given any number $x \in \mathbb{R}$ the floor value is denoted by $\lfloor x \rfloor$ and the absolute value is denoted by $|x|$.

(a) Find $\lfloor \sqrt{2} \rfloor$ and $|-2|$. [2]

(b) Find the set of values of a such that $\lfloor a \rfloor = 1$ and the set of values of b such that $|b| = 1$. [2]

(c) Consider the functions $f : \mathbb{R} \to \mathbb{Z}$ and $g : \mathbb{R} \to \mathbb{R}$ given by:

$$f(x) = \lfloor x - 1 \rfloor\ and g(x) = |x - 1|.$$

(i) Write down the domain, co-domain and range of f and g. [2]

(ii) For each function, say whether or not it is one to one, justifying your answer. [2]

(iii) For each function, say whether or not it is onto, justifying your answer. [2]

2005

Question 4 (a) Let S be the set of all 4 bit binary strings. The function $f : S \to \mathbb{Z}$ is defined by the rule:

$$f(x) = thenumberofzerosinx,\ foreachbinarystringx \in S.$$

Find:

(i) the number of elements in the domain

(ii) $f(1000)$

(iii) the set of pre-images of 1

(iv) the range of f. [4]

(b) Decide whether the function f defined in part (a) has either the one to one or the onto property, justifying your answers. [2]

(c) State the condition to be satisfied by a function $f : X \to Y$ for it to have an inverse function $f^{-1} : Y \to X$. [1]

(d) Define the inverse functions for each of the following:

(i) $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = 3x + 5$;

(ii) $h : A \to B$ where $A = \{1, 2, 3, 4, 5\}$, $B = \{u, v, w, x, y, z\}$ and h is defined by the following table:

x	1	2	3	4	5
$h(x)$	w	v	y	x	u

[3]

2006

Question 4

(a) Given $A = \{a, b, c, d\}$ and $B = \{2, 4, 6, 8, 10\}$, a function f is defined as a subset of $A \times B$ where f consists of the ordered pairs:

$$(a, 6), (b, 8), (c, 6), (d, 10).$$

(i) Illustrate this function using an arrow diagram.

(ii) List the domain, co-domain and range of this function,

(iii) Say whether or not the function f has the onto property, justifying your answer.

(iv) Which pair or pairs should be altered in order to make the function have the one-to-one property?

[-]

(b) Another function g is given by

$$g(n) = nmod3whereg : \mathbb{Z}^+ \to \mathbb{Z}$$

i.e $g(n)$ is the remainder when n is divided by 3, so that $g(6) = 0$ and $g(7) = 1$.

(i) Find $g(5)$ and $g(10)$

(ii) List the ancestors of 0.

(iii) List the range of g, and say whether or not g is onto, justifying your answer.

(iv) Say whether or not g is a one-to-one function, giving a reason for your answer. [5]

2007

Question 5

(a) There are 16 different 2 by 2 matrices whose entries may consist only of zeroes and ones, for example

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} and \mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} aretwosuchmatrices.$$

Let S be the set all such matrices. We define a function f on S by the rule

$$f(\mathbf{X}) = thenumberofzeroesin\mathbf{X}wheref : S \to \mathbb{Z}and\mathbf{X} \in S.$$

(i) Find a numerical value for both $f(\mathbf{A})$ and $f(\mathbf{B})$.
(ii) Write down the set of pre-images or ancestors of 1.
(iii) Write down the range of f.
(iv) Say whether or not this function is one to one, justifying your answer.
(v) Say whether or not this function is onto, justifying your answer. [6]

(b) Say whether or not each of the following functions has an inverse, justifying your answer. In the cases where there is an inverse define it.

(i) $f : S \to \mathbb{Z}$ defined in part (a).
(ii) $g : \mathbb{R} \to \mathbb{Z}$ defined by $g(x) = \lfloor x \rfloor$.
(iii) $h : \mathbb{R} \to \mathbb{R}$ defined by $h(x) = 2x + 5$. [4]