Question 6

(a) (i) A simple, connected graph has 7 vertices, all having the same degree d. State the possible values of d and for each value also give the number of edges in the corresponding graph.

(ii) Another simple, connected graph has 6 vertices, all having the same degree, n. Draw such a graph when $n = 3$ and state the other possible values of n.

[4]

(b) The following adjacency matrix shows several European countries and an entry of 1 indicates the countries concerned share a common border, whereas a zero entry indicates they do not.

	Austria	Belgium	France	Germany	Italy
Austria	0	1	0	0	1
Belgium	1	0	1	1	1
France	0	1	0	1	1
Germany	0	1	1	0	1
Italy	1	1	1	1	0

(i) Write down the countries which share a border with Germany.

(ii) Is this matrix symmetric or not? Give an example to show what this means.

(iii) Draw the graph, G, associated with this matrix.

(iv) Explain how the number of edges of the graph can be calculated from the entries in the matrix and find this number.

(v) Draw another graph, H, which has 5 vertices and the same degree sequence as G but is not isomorphic to it. Give a reason why G and H are not isomorphic.

[6]