Question 5 (a) Let G be a simple graph with vertex set $V(G) = \{v_1,\ v_2,\ v_3,\ v_4,\ v_5\}$ and adjacency lists as follows:

$$\begin{array}{lcl} v_1 & : & v_2\ v_3\ v_4 \\ v_2 & : & v_1\ v_3\ v_4\ v_5 \\ v_3 & : & v_1\ v_2\ v_4 \\ v_4 & : & v_1\ v_2\ v_3. \\ v_5 & : & v_2 \end{array}$$

(i) List the degree sequence of G.

(ii) Draw the graph of G.

(iii) Find two distinct paths of length 3, starting at v_3 and ending at v_4.

(iv) Find a 4 cycle in G. [6]

(b) Let K_n be the simple graph with vertices $v_1, v_2, v_3, ..., v_n$ in which each vertex is joined to every other vertex by an edge.

(i) Draw K_6.

(ii) Determine the number of edges of K_6.

(iii) Determine the number of paths from v_1 to v_2 of length two.

(iv) Find an expression in terms of n for the number of paths from v_1 to v_2 of length two in k_n. [5]

(c) Draw two different (that is non-isomorphic) connected graphs each having the degree sequence $3, 3, 2, 1, 1, 1, 1$. Give one reason why the graphs you have drawn are not isomorphic. [3]

(a) Let G be a simple graph. Explain why the sum of the degrees of the vertices of G is twice the number of its edges. [2]

(b) Justifying your answer, say why it is not possible to construct a simple graph G with degree sequence

$$6, 5, 3, 2, 2, 1, 1, 1.$$

[2]

(c) Justifying your answer, say whether it is possible to construct a simple graph with degree sequence $3, 3, 3, 3, 3, 3$. [2]