

## Session 06: Digraphs and Relations

6A.1 In-degree and out-degree

6A.2

6A.3

### Relations

6B.1 Equivalence Relations (6.2.2)

6B.2

6B.3 Relations and Cartesian Products (6.3)

Reflexive :

Symmetric :

Transitive :

Anti-symmetric :

Equivalence Relation :

Partial Order :

Order :

## Directed Graphs

A directed graph or digraph is a graph, or set of nodes connected by edges, where the edges have a direction associated with them. In formal terms a digraph is a pair  $G = (V, A)$  (sometimes  $G = (V, E)$ ) of a set  $V$ , whose elements are called vertices or nodes, and a set  $A$  of ordered pairs of vertices, called arcs, directed edges, or arrows (and sometimes simply 'edges' with the corresponding set named  $E$  instead of  $A$ ).

## Recurrence Relation

In mathematics, a recurrence relation is an equation that recursively defines a sequence, once one or more initial terms are given: each further term of the sequence is defined as a function of the preceding terms. The term difference equation sometimes (and for the purposes of this article) refers to a specific type of recurrence relation. However, "difference equation" is frequently used to refer to any recurrence relation.

### 2008 Zone B question 7

Given the set  $S = \{g, e, r, b, i, l\}$ .

- Describe how each subset of  $S$  can be represented by using a 6 digit binary string
- Write down the string corresponding to the subset  $\{g, r, l\}$  and the subset corresponding to the string 010101.
- What is the total number of subsets of  $S$ ?
- A relation  $R$  from a set  $A$  to a set  $B$  is a subset of the **cartesian product**  $A \times B$ .
- Thus  $R$  is a set of **ordered pairs** where the first element comes from  $A$  and the second element comes from  $B$  i.e.  $(a, b)$
- If  $(a, b) \in R$  we say that  $a$  is related to  $b$  and write  $aRb$ .
- If  $(a, b) \notin R$ , we say that  $a$  is not related to  $b$  and write  $a \not R b$ .  
CHECK
- If  $R$  is a relation from a set  $A$  to itself then we say that " $R$  is a relation on  $A$ ".
- Let  $A = \{2, 3, 4, 6\}$  and  $B = \{4, 6, 9\}$
- Let  $R$  be the relation from  $A$  to  $B$  defined by  $xRy$  if  $x$  divides  $y$  exactly.

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- Let  $R$  be the relation from  $A$  to  $B$  defined by  $\mathbf{xRy}$  if  $x$  divides  $y$  exactly.
- Then

$$R = (2, 4), (2, 6), (3, 6), (3, 9), (4, 4), (6, 6)$$