1 SPSS Output - Block 0: Beginning Block.

Block 0 presents the results with only the constant included before any coefficients (i.e. those relating to family size and mortgage) are entered into the equation. Logistic regression compares this model with a model including all the predictors (family size and mortgage) to determine whether the latter model is more appropriate. The table suggests that if we knew nothing about our variables and guessed that a person would not take the offer we would be correct 53.3% of the time. The variables not in the equation table tells us whether each IV improves

Block 0: Beginning Block

Classification Table[a,b]

			Predicted		
			take solar panel offer		Percentage Correct
	Observed		decline offer	take offer	
Step 0	take solar panel offer	decline offer	0	14	.0
		take offer	0	16	100.0
	Overall Percentage				53.3

a. Constant is included in the model.
b. The cut value is .500

Figure 1: Classification table

the model. The answer is yes for both variables, with family size slightly better than mortgage size, as both are significant and if included would add to the predictive power of the model. If they had not been significant and able to contribute to the prediction, then termination of the analysis would obviously occur at this point

This presents the results when the predictors family size and mortgage are included. Later SPSS prints a classification table which shows how the classification error rate has changed from the original 53.3we can now predict with 90% accuracy (see Classification Table later). The model appears good, but we need to evaluate model fit and significance as well. SPSS

Variables in the Equation

		B	S.E.	Wald	df	Sig.	Exp(B)
Step 0	Constant	.134	.366	.133	1	.715	1.143

Variables not in the Equation

			Score	df	Sig.
Step 0	Variables	Mortgage	6.520	1	.011
		Famsize	14.632	1	.000
	Overall Statistics		15.085	2	.001

Figure 2: Variables in / not in the equation

Block 1: Method = Enter

Omnibus Tests of Model Coefficients

		Chi-square	df	Sig.
Step 1	Step	24.096	2	.000
	Block	24.096	2	.000
	Model	24.096	2	.000

Model Summary

Step	-2 Log likelihood	Cox & Snell R Square	Nagelkerke R Square
1	17.359[a]	.552	.737

a. Estimation terminated at iteration number 8 because parameter estimates changed by less than .001.

Hosmer and Lemeshow Test

Step	Chi-square	df	Sig.
1	6.378	8	.605

Figure 3: Test Outcomes

will offer you a variety of statistical tests for model fit and whether each of the independent variables included make a significant contribution to the model.

1.1 Omnibus Test for Model Coefficients

The overall significance is tested using what SPSS calls the ***Model Chi-square***, which is derived from the likelihood of observing the actual data under the assumption that the model that has been fitted is accurate. There are two hypotheses to test in relation to the overall fit of the model:

H_0 The model is a good fitting model.

H_1 The model is not a good fitting model (i.e. the predictors have a significant effect).

In our case model chi square has 2 degrees of freedom, a value of 24.096 and a probability of $p < 0.000$.

Thus, the indication is that the model has a poor fit, with the model containing only the constant indicating that the predictors do have a significant effect and create essentially a different model. So we need to look closely at the predictors and from later tables determine if one or both are significant predictors.

This table has 1 step. This is because we are entering both variables and at the same time providing only one model to compare with the constant model. In stepwise logistic regression there are a number of steps listed in the table as each variable is added or removed, creating different models. The step is a measure of the improvement in the predictive power of the model since the previous step. (I will revert to this next class).

1.2 Model Summary Table

The likelihood function can be thought of as a measure of how well a candidate model fits the data (although that is a very simplistic definition). The AIC criterion is based on the Likelihood function. The likelihood function of a fitted model is commonly re-expressed as -2LL (i.e. The log of the likelihood times minus 2). The 2LL value from the Model Summary table below is 17.359.

Although there is no close analogous statistic in logistic regression to the coefficient of determination R^2 the Model Summary Table provides some approximations. Cox and Snells R-Square attempts to imitate multiple R-Square based on likelihood, but its maximum can be (and usually is) less than 1.0, making it difficult to interpret. Here it is indicating that 55.2% of the variation in the DV is explained by the logistic model. The Nagelkerke modification that does range from 0 to 1 is a more reliable measure of the relationship. Nagelkerkes R^2 will normally be higher than the Cox and Snell measure. Nagelkerkes R^2 is part of SPSS output in the Model Summary table and is the most-reported of the R-squared estimates. In our case it is 0.737, indicating a moderately strong relationship of 73.7% between the predictors and the prediction.

1.3 Hosmer and Lemeshow Statistic

An alternative to model chi square is the Hosmer and Lemeshow test which divides subjects into 10 ordered groups of subjects and then compares the number actually in the each group (observed) to the number predicted by the logistic regression model (predicted). The 10 ordered groups are created based on their estimated probability; those with estimated probability below .1 form one group, and so on, up to those with probability .9 to 1.0.

Each of these categories is further divided into two groups based on the actual observed outcome variable (success, failure). The expected frequencies for each of the cells are obtained from the model. A probability (p) value is computed from the chi-square distribution with 8 degrees of freedom to test the fit of the logistic model.

If the H-L goodness-of-fit test statistic is greater than .05, as we want for well-fitting models, we fail to reject the null hypothesis that there is no difference between observed and model-predicted values, implying that the models estimates fit the data at an acceptable level. That is, well-fitting models show non-significance on the H-L goodness-of-fit test. This desirable outcome of non-significance indicates that the model prediction does not significantly differ from the observed.

The H-L statistic assumes sampling adequacy, with a rule of thumb being enough cases so that 95% of cells (typically, 10 decile groups times 2 outcome categories = 20 cells) have an expected frequency > 5. Our H-L statistic has a significance of .605 which means that it is not statistically significant and therefore our model is quite a good fit.

Hosmer and Lemeshow Test

Step	Chi-square	df	Sig.
1	6.378	8	.605

Figure 4: Hosmer and Lemeshow Statistic

Contingency Table for Hosmer and Lemeshow Test

		take solar panel offer = decline offer		take solar panel offer = take offer		
		Observed	Expected	Observed	Expected	Total
Step 1	1	3	2.996	0	.004	3
	2	3	2.957	0	.043	3
	3	3	2.629	0	.371	3
	4	2	2.136	1	.864	3
	5	2	1.881	1	1.119	3
	6	0	.833	3	2.167	3
	7	0	.376	3	2.624	3
	8	1	.171	2	2.829	3
	9	0	.019	3	2.981	3
	10	0	.000	3	3.000	3

Figure 5: Hosmer and Lemeshow Table

1.4 Classification Table

Rather than using a goodness-of-fit statistic, we often want to look at the proportion of cases we have managed to classify correctly. For this we need to look at the classification table printed out by SPSS, which tells us how many of the cases where the observed values of the dependent variable were 1 or 0 respectively have been correctly predicted.

In the Classification table, the columns are the two predicted values of the dependent, while the rows are the two observed (actual) values of the dependent. In a perfect model, all cases will be on the diagonal and the overall percent correct will be 100%. In this study, 87.5% were correctly classified for the take offer group and 92.9% for the decline offer group. Overall 90% were correctly classified. This is a considerable improvement on the 53.3% correct classification with the constant model so we know that the model with predictors is a significantly better mode.

Classification Table[a]

			Predicted		
			Take solar panel offer		
	Observed		Decline offer	Take offer	Percentage correct
Step 1	take solar panel offer	decline offer	13	1	92.9
		take offer	2	14	87.5
	Overall Percentage				90.0

[a] The cut value is .500.

Figure 6: Classification Table

1.5 Classification Table

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1.6 Variables in the Equation

The Variables in the Equation table has several important elements. The Wald statistic and associated probabilities provide an index of the significance of each predictor in the equation. The simplest way to assess Wald is to take the significance values and if less than 0.05 reject the null hypothesis as the variable does make a significant contribution. In this case, we note that family size contributed significantly to the prediction (p = .013) but mortgage did not (p

Classification Table[a]

Observed			Predicted: Take solar panel offer — Decline offer	Predicted: Take solar panel offer — Take offer	Predicted: Percentage correct
Step 1	take solar panel offer	decline offer	13	1	92.9
		take offer	2	14	87.5
	Overall Percentage				90.0

[a] The cut value is .500.

Figure 7: Classification Table

Variables in the Equation

		B	S.E.	Wald	df	Sig.	Exp(B)
Step 1[a]	Mortgage	.005	.003	3.176	1	.075	1.005
	Famsize	2.399	.962	6.215	1	.013	11.007
	Constant	-18.627	8.654	4.633	1	.031	.000

a. Variable(s) entered on step 1: Mortgage, Famsize.

Figure 8: Variables in the Equation

= .075). The researcher may well want to drop independents from the model when their effect is not significant by the Wald statistic (in this case mortgage).

The ***Exp(B)*** column in the table presents the extent to which raising the corresponding measure by one unit influences the odds ratio. We can interpret ***Exp(B)***) in terms of the change in odds. If the value exceeds 1 then the odds of an outcome occurring increase; if the figure is less than 1, any increase in the predictor leads to a drop in the odds of the outcome occurring. For example, the ***Exp(B)*** value associated with family size is 11.007. Hence when family size is raised by one unit (one person) the odds ratio is 11 times as large and therefore householders are 11 more times likely to belong to the take offer group.

The ***B*** values are the logistic coefficients that can be used to create a predictive equation (similar to the b values in linear regression) formula seen previously.

Here is an example of the use of the predictive equation for a new case. Imagine a householder whose household size including themselves was seven and paying a monthly mortgage of 2,500 euros. Would they take up the offer, i.e. belong to category 1? Substituting in we get:

Therefore, the probability that a householder with seven in the household and a mortgage of 2,500 p.m. will take up the offer is 99%, or 99% of such individuals will be expected to take up the offer. Note that, given the non-significance of the mortgage variable, you could be justified in leaving it out of the equation. As you can imagine, multiplying a mortgage value by B adds

$$\text{Probability of a case} = \frac{e\{(2.399 \times \text{family size}) + (.005 \times \text{mortgage}) - 18.627\}}{1 + e\{(2.399 \times \text{family size}) + (.005 \times \text{mortgage}) - 18.627\}}$$

Figure 9: images/Logistic Regression Equation

$$\begin{aligned}
\text{Probability of a case taking offer} &= \frac{e\{(2.399 \times 7) + (.005 \times 2500) - 18.627\}}{1 + e\{(2.399 \times 7) + (.005 \times 2500) - 18.627\}} \\
&= \frac{e^{10.66}}{1 + e^{10.66}} \\
&= 0.99
\end{aligned}$$

Figure 10: images/Logistic Regression Equation : Example

a negligible amount to the prediction as its B value is so small (.005).

Variables in the Equation

		B	S.E.	Wald	df	Sig.	Exp(B)
Step 1[a]	Mortgage	.005	.003	3.176	1	.075	1.005
	Famsize	2.399	.962	6.215	1	.013	11.007
	Constant	-18.627	8.654	4.633	1	.031	.000

a. Variable(s) entered on step 1: Mortgage, Famsize.

Figure 11: Variables in the Equation

$$\text{Probability of a case} = \frac{e\{(2.399 \times \text{family size}) + (.005 \times \text{mortgage}) - 18.627\}}{1 + e\{(2.399 \times \text{family size}) + (.005 \times \text{mortgage}) - 18.627\}}$$

Figure 12: Logistic Regression Equation

1.7 Variables in the Equation

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The ***Exp(B)*** column in the table presents the extent to which raising the corresponding measure by one unit influences the odds ratio. We can interpret ***Exp(B)***) in terms of the change in odds. If the value exceeds 1 then the odds of an outcome occurring increase; if the figure is less than 1, any increase in the predictor leads to a drop in the odds of the outcome occurring. For example, the ***Exp(B)*** value associated with family size is 11.007. Hence when family size is raised by one unit (one person) the odds ratio is 11 times as large and therefore householders are 11 more times likely to belong to the take offer group.

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Therefore, the probability that a householder with seven in the household and a mortgage of 2,500 p.m. will take up the offer is 99%, or 99% of such individuals will be expected to take up the offer. Note that, given the non-significance of the mortgage variable, you could be justified in leaving it out of the equation. As you can imagine, multiplying a mortgage value by B adds a negligible amount to the prediction as its B value is so small (.005).

$$\begin{aligned}
\text{Probability of a case taking offer} &= \frac{e\{(2.399\times 7)+(.005\times 2500)-18.627\}}{1+e\{(2.399\times 7)+(.005\times 2500)-18.627\}}\\
&= \frac{e^{10.66}}{1+e^{10.66}}\\
&= 0.99
\end{aligned}$$

Figure 13: Logistic Regression Equation : Example