DM07 Sequences and Series

Session 07: Sequences and Series

7A.1 Sequences

7A.2 Induction

7A.3 Series and the Sigma Notation

Recurrence Relations(7.1.1)

- $u_1 = 2$
- $u_2 = u_1 + 3 = 2 + 3 = 5$
- $u_3 = u_2 + 3 = 5 + 3 = 8$

Proof by Induction(7.2.2)

Step 1 Base case

Step 2 Induction hypothesis

Step 3 Induction step

Series and Sigma Notation(7.2.3)

Mathematical notation uses a symbol that compactly represents summation of many similar terms: the summation symbol, $\sum$, an enlarged form of the upright capital Greek letter *Sigma*. This is defined as:

$$\sum_{i=m}^{n} a_i = a_m + a_{m+1} + a_{m+2} + \cdots + a_{n-1} + a_n.$$

Where, i represents the index of summation; ai is an indexed variable representing each successive term in the series; m is the lower bound of summation, and n is the upper bound of summation. The "i = m" under the summation symbol means that the index i starts out equal to m. The index, i, is incremented by 1 for each successive term, stopping when i = n. Here is an example showing the summation of exponential terms (all terms to the power of 2):

$$\sum_{i=3}^{6} i^2 = 3^2 + 4^2 + 5^2 + 6^2 = 86.$$

Informal writing sometimes omits the definition of the index and bounds of summation when these are clear from context, as in:

$$\sum a_i^2 = \sum_{i=1}^{n} a_i^2.$$