Telescoping Series

- In mathematics, a telescoping series is a series whose partial sums eventually only have a fixed number of terms after cancellation.
- Such a technique is also known as the method of differences.

Telescoping Series

- A telescoping series does not have a set form, like the geometric and p-series do.
- A telescoping series is any series where nearly every term cancels with a preceeding or following term.
- For instance, the series

$$\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1}$$

is telescoping.

Telescoping Series

For example, the series

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

simplifies as

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right) \tag{1}$$

$$= \lim_{N\to\infty} \sum_{n=1}^{N} \left(\frac{1}{n} - \frac{1}{n+1}\right) \tag{2}$$

$$= \lim_{N\to\infty} \left[\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \cdots + \left(\frac{1}{N} - \frac{1}{N+1}\right)\right. \tag{3}$$

$$= \lim_{N\to\infty} \left[1 + \left(-\frac{1}{2} + \frac{1}{2}\right) + \left(-\frac{1}{3} + \frac{1}{3}\right) + \cdots + \left(-\frac{1}{N}\right.\right. \tag{4}$$

Telescoping Series

Look at the partial sums:

$$\sum_{i=1}^{n} \frac{1}{i} - 1/(i+1) = (1/1 - 1/2) + (1/2 - 1/3) + \ldots + (1/n - 1$$

$$= 1 - 1/(n+1)$$

because of cancellation of adjacent terms.
So, the sum of the series, which is the limit of the partial sums, is 1.

Telescoping Series

You do have to be careful; not every telescoping series converges. Look at the following series:

$$\sum_{i=1}^{\infty} n-(n+1)$$

You might at first think that all of the terms will cancel, and you will be left with just 1 as the sum.. But take a look at the partial sums:

$$\sum_{i=1}^{n} -(i+1) = (1-2)+(2-3)+\ldots+(n-(n+1)) =$$

$$1-(n+1) =$$

Telescoping Series

This sequence does not converge, so the sum does not converge. This can be more easily seen if you simplify the expression for the term. You find that

$$\sum_{i=1}^{\infty} n - (n+1) = \sum_{i=1}^{\infty} -1$$

and any infinite sum with a constant term diverges.