Mathematics for Business

Sequences and Series

Arithmetic Progressions

www.MathsResource.com

Arithmetic Progressions

Find the sum of the arithmetic progression

$$11 + 13 + 15 + \ldots + 49 + 51$$

Arithmetic Progressions

First recall two useful equations for working with arithmetic progressions.

For the arithmetic sequence $a, (a+d), (a+2d), \ldots$

(i) t_n is the $n-th$ term of series.

$$t_n = a + (n-1)d$$

(ii) S_n is the sum of the first n terms

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Arithmetic Progressions

Find the sum of the arithmetic progression

$$11 + 13 + 15 + \cdots + 49 + 51$$

- Note that $a = 11$ and $d = 2$.
- We need to find out what n (the number of terms) is.
- The last term is 51.

Arithmetic Progressions

- The last term is 51.
- $t_n = 51 = [\ 11 + (\text{n-1})2\ ]$
- $t_n = 51 = 2\text{n} + 9$
- $n = 21$

There are 21 terms in the series.

Arithmetic Progressions

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Recall $a = 11, d = 2$ and $n = 21$

Arithmetic Progressions

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Recall $a = 11, d = 2$ and $n = 21$

$$S_n = \frac{21}{2}\left[(2.11) + [(21-1)2]\right]$$

$$S_n = 10.5\left[22 + 40\right] = 10.5 \times 62$$

$$S_n = 651$$

Arithmetic Progressions

End Slide

Mathematics for Business

Sequences and Series

Geometric Series

www.Stats-Lab.com

Summations

Find S_n, the sum of n terms, of the geometric series

$$2 + \frac{2}{3} + \frac{2}{3^2} + \frac{2}{3^3} + \ldots + \frac{2}{3^{n-1}}$$

If $S_n = 242/81$, find the value of n.

Summations

$$2 + \frac{2}{3} + \frac{2}{3^2} + \frac{2}{3^3} + \ldots + \frac{2}{3^{n-1}}$$

Summation Theorem

$$\sum_{r=0}^{n} x^r = \frac{x^{n+1} - 1}{x - 1}$$

Summations

$$2 + \frac{2}{3} + \frac{2}{3^2} + \frac{2}{3^3} + \ldots + \frac{2}{3^{n-1}}$$

$$2 \times \left[1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots + \frac{1}{3^{n-1}} \right]$$

Summation Theorem

$$\sum_{r=0}^{n} x^r = \frac{x^{n+1} - 1}{x - 1}$$

$$k \sum_{r=0}^{n} x^r = k \left(\frac{x^{n+1} - 1}{x - 1} \right)$$

Here $k = 2$ and $x = 1/3$

Summations

$$k\sum_{r=0}^{n} x^r = k\left(\frac{x^{n+1}-1}{x-1}\right)$$

Here $k = 2$ and $x = 1/3$

Summations

$$k\sum_{r=0}^{n} x^r = k\left(\frac{x^{n+1}-1}{x-1}\right)$$

Here $k = 2$ and $x = 1/3$

$$2\sum_{r=0}^{n}(1/3)^r = 2\left(\frac{(1/3)^{n+1}-1}{(1/3)-1}\right) = \frac{242}{81}$$

Summations

$$2\left(\frac{(1/3)^{n+1}-1}{(1/3)-1}\right)=\frac{-3}{4}\left[(1/3)^{n+1}-1\right]=\frac{242}{81}$$

$$\frac{-3}{4}\left[(1/3)^{n+1}-1\right]=\frac{242}{81}$$

$$\left[(1/3)^{n+1}-1\right]=\frac{-4}{3}\times\frac{242}{81}$$