Binomial coefficients.

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

1) Show that

$$r\binom{n}{r} = n\binom{n-1}{r-1}$$

Solution

$$r\binom{n}{r} = \frac{r \times n!}{r! \times (n-r)!}$$

$r! = r \times (r-1)!$

$$= \frac{\cancel{r} \times n!}{\cancel{r} \times (r-1)! \times (n-r)!}$$

$$= \frac{n \times (n-1)!}{(r-1)! \times (n-r)!}$$

$(r-1)+(n-r)$
$= n-1$

$$= n\binom{n-1}{r-1}$$

11) Show that

$$\binom{n+1}{n-r} = \frac{n+1}{r+1}\binom{n}{r}$$

Solution

$$\binom{n+1}{n-r} = \frac{(n+1)!}{(n-r)!(r+1)!}$$

$$\boxed{\text{useful} \quad (n+1)-(n-r) = r+1}$$

$$\frac{(n+1) \times n!}{(n-r)! \times (r+1) \times r!}$$

$$= \frac{n+1}{r+1} \times \frac{n!}{(n-r)!\,r!}$$

$$= \frac{n+1}{r+1}\binom{n}{r}$$