Permutations

- Given a set of distinct numbers, $\{1, 2, 3, 4, 5, 6\}$, find all permutations containing 3 numbers. All the permutations have to be in ascending order.
- For example, some correct permutations would be $\{1, 2, 3\}$, $\{2, 4, 6\}$, etc.
- $\{2, 3, 1\}$ would not be acceptable because it is not in ascending order.

Permutations

Given a set of distinct numbers, $\{1, 2, 3, 4\}$, find all permutations containing 2 numbers.

$$\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 1\}, \{2, 3\}, \{2, 4\},$$

$$\{3, 1\}, \{3, 2\}, \{3, 4\}, \{4, 1\}, \{4, 2\}, \{4, 3\},$$

When all the permutations have to be in ascending order.

$$\{1, 2\}, \{1, 3\}, \{1, 4\}, \qquad , \{2, 3\}, \{2, 4\},$$

$$, \{3, 4\},$$

Permutations

$$\{1,2\},\{1,3\},\{1,4\},\{2,1\},\{2,3\},\{2,4\},$$
$$\{3,1\},\{3,2\},\{3,4\},\{4,1\},\{4,2\},\{4,3\},$$
$$^4P_2 = \frac{4!}{2!} = 12$$

$$\{1,2\},\{1,3\},\{1,4\},\qquad\qquad,\{2,3\},\{2,4\},$$
$$,\{3,4\},$$

$$^4C_2 = \frac{4!}{2! \times 2!} = 6$$

Permutations

Given a set of distinct numbers, $\{1, 2, 3, 4\}$, find all permutations containing 3 numbers. (Strictly ascending permutations in red).

$$\mathbf{\{1, 2, 3\}}, \mathbf{\{1, 2, 4\}}, \{1, 3, 2\}, \mathbf{\{1, 3, 4\}}, \{1, 4, 2\}, \{1, 4, 3\},$$

$$\{2, 1, 3\}, \{2, 1, 4\}, \{2, 3, 1\}, \mathbf{\{2, 3, 4\}}, \{2, 4, 1\}, \{2, 4.3\},$$

$$\{3, 1, 2\}, \{3, 1, 4\}, \{3, 2, 1\}, \{3, 2, 4\}, \{3, 4, 1\}, \{3, 4, 2\},$$

$$\{4, 1, 2\}, \{4, 1, 3\}, \{4, 2, 1\}, \{4, 3, 1\}, \{4, 3, 1\}, \{4, 3, 2\}$$

$$^4P_3 = \frac{4!}{1!} = 24$$

$$^4C_3 = \frac{4!}{3! \times 1!} = 4$$

Permutations

- In general, when you must select k numbers from n, and those k numbers must be in ascending order, then there are ${}^{n}C_{k}$ ways permutations.

$$ {}^{C}_{k} = \binom{n}{k} = \frac{n!}{k!(n-k)!} $$