神經網路與損失函數

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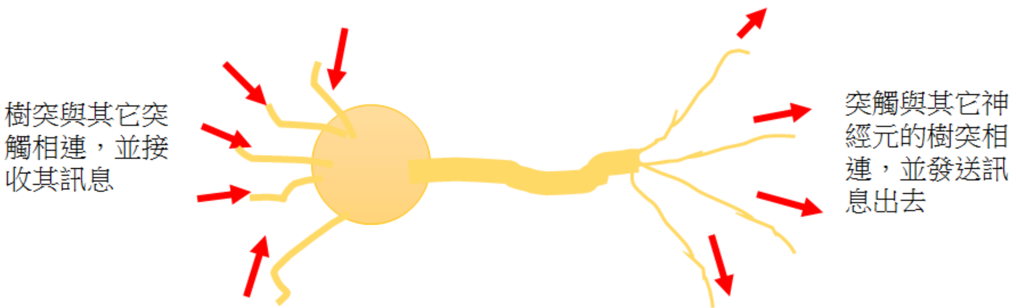
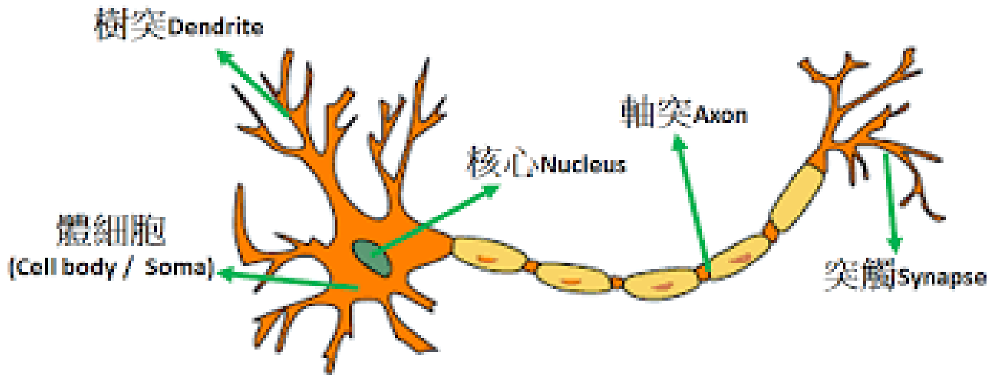
Outline

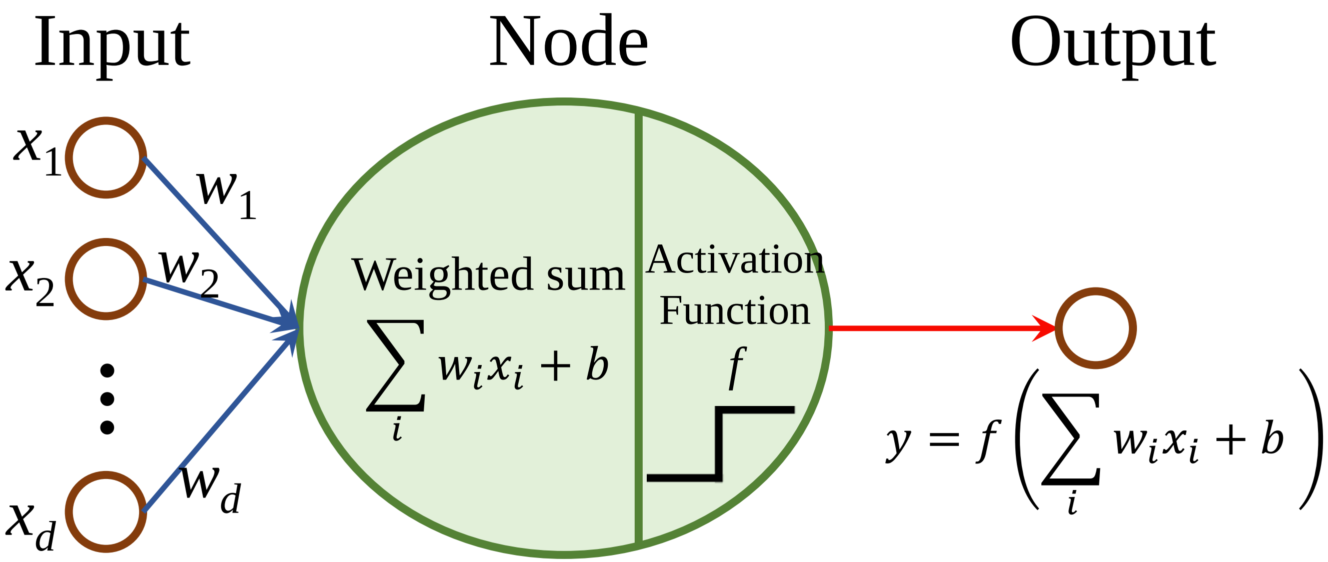
- 1.類神經網路(Neural Network, NN)
- 2.感知機(Perception)
- 3. Multi-layer perception (MLP)
- 4. How NN work?
- 5. Loss function

類神經網路

基本上神經網路是基於感知機(Perceptron)神經網路開始，主要是希望用數學模型去模擬神經細胞的運作模式。







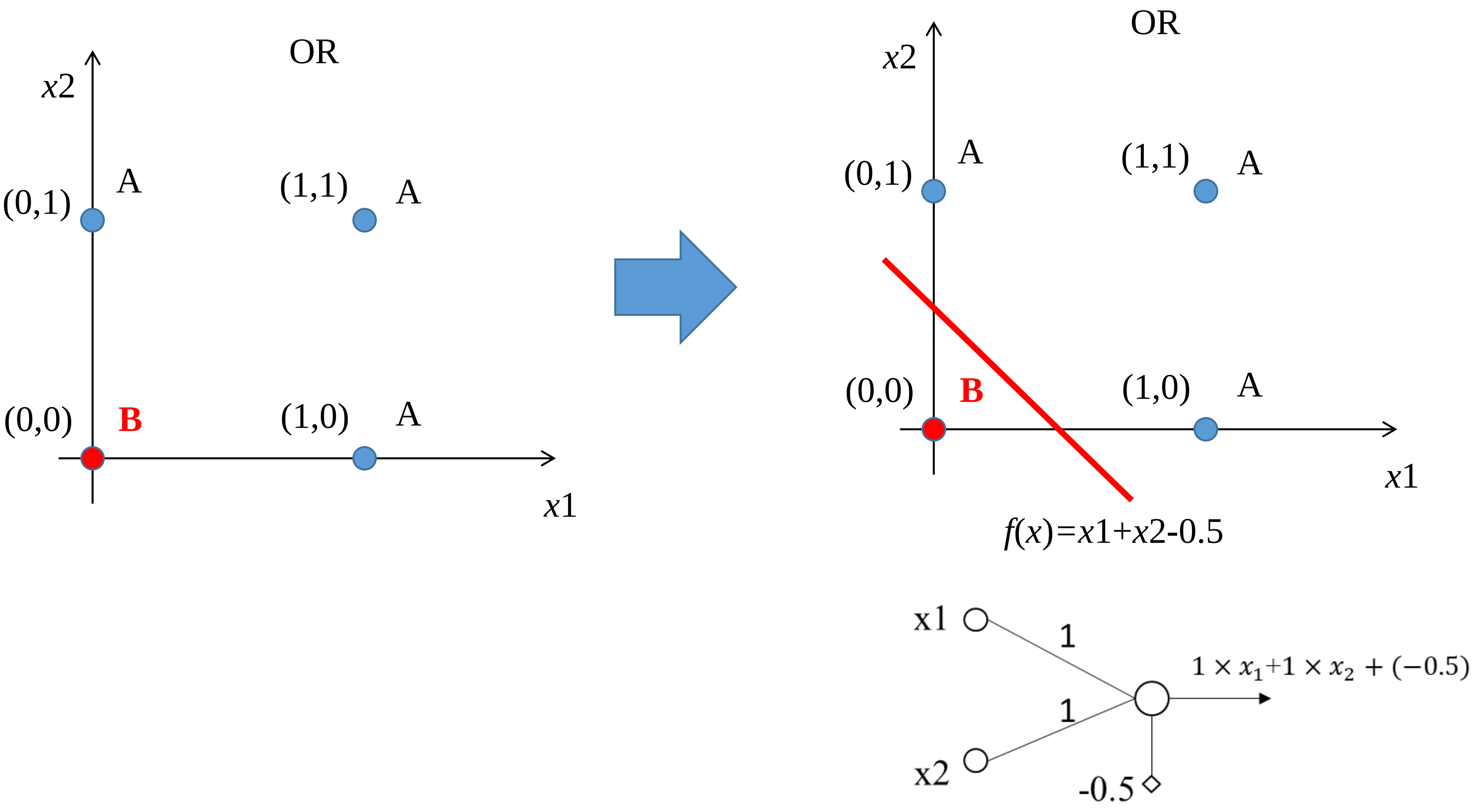
權重(w_i): Dendrite

Input(x_i) and output (y) node: Synapse

Node: Cell body

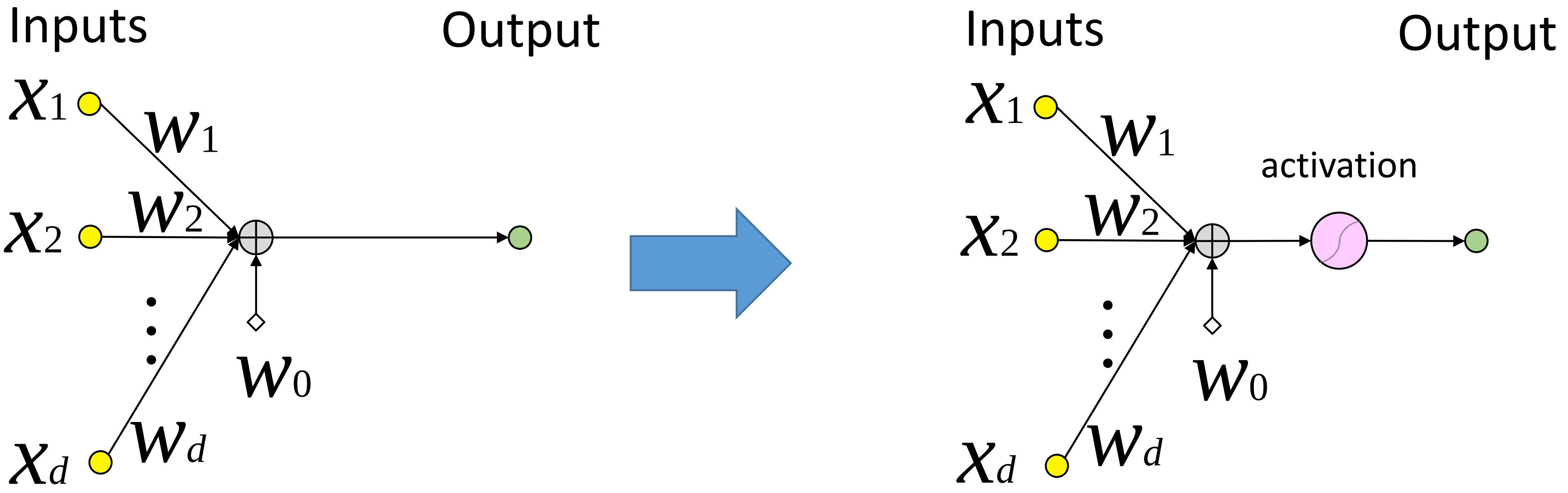
Output: Axon

NN for classification problem

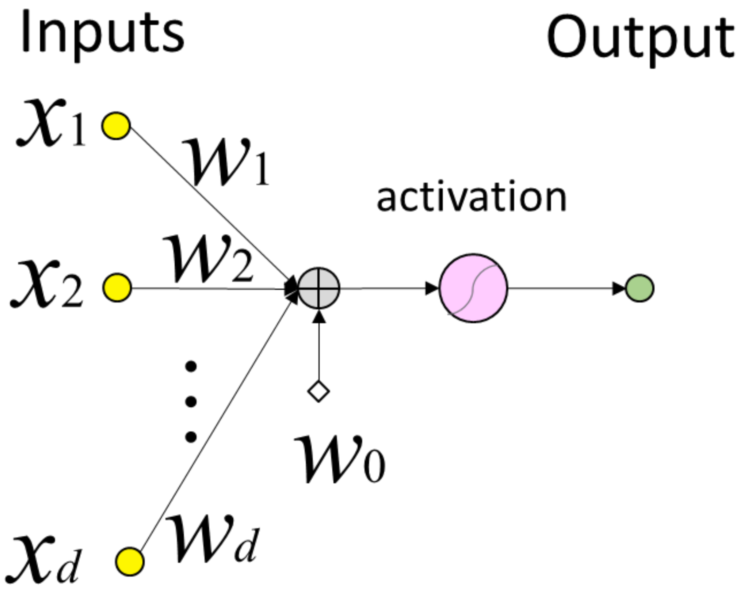


Perception

Perception can learn the nonlinear representation by activation function.



Perception



$$y = f(w_{10} + w_{11}x_{1+}w_{12}x_2 + \cdots + w_{1d}x_d) = f(\boldsymbol{W}^T\boldsymbol{x})$$

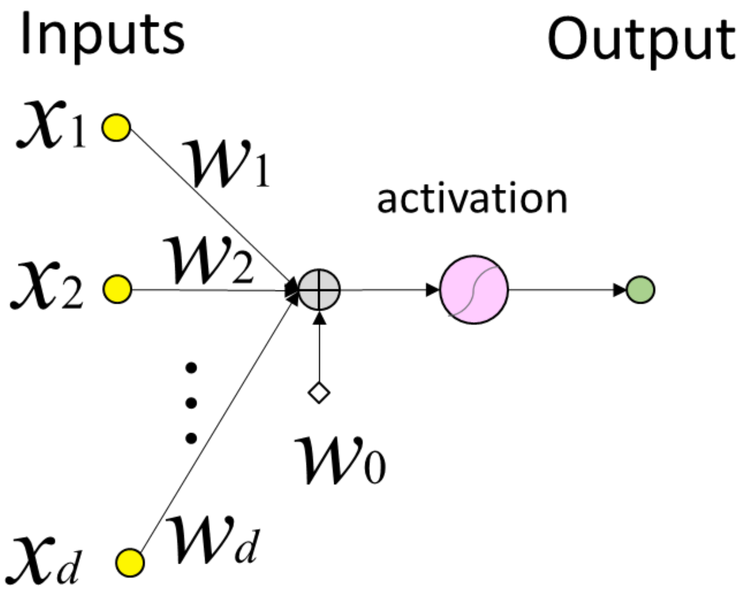
Classification: $f = \begin{cases} 1 & \boldsymbol{W}^T\boldsymbol{x} \geq 0 \\ 0 & O.W. \end{cases}$

Regression: $f(\boldsymbol{W}^T\boldsymbol{x})$

$$\boldsymbol{W} = \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_d \end{bmatrix}, \boldsymbol{x} = \begin{bmatrix} 1 \\ x_1 \\ \vdots \\ x_d \end{bmatrix}$$

神經元與回歸

神經元



回歸

$$f(x) = \beta_0 + \beta_1 x_1 + \cdots + \beta_d x_d$$

$$y = \sigma\left(f(\boldsymbol{x})\right) = \frac{1}{1 + e^{-f(\boldsymbol{x})}} = \frac{1}{1 + e^{-\boldsymbol{\beta}^T \boldsymbol{x}}}$$

Perception with linear output is the linear regression.

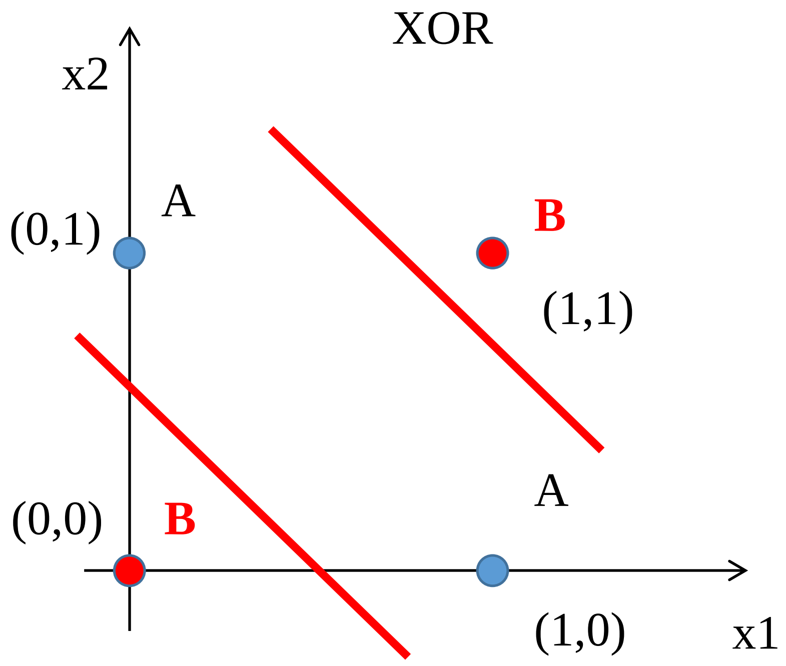
NN: backpropagation

Regression: OLSE (ordinary least squares estimator).

NN for XOR problem

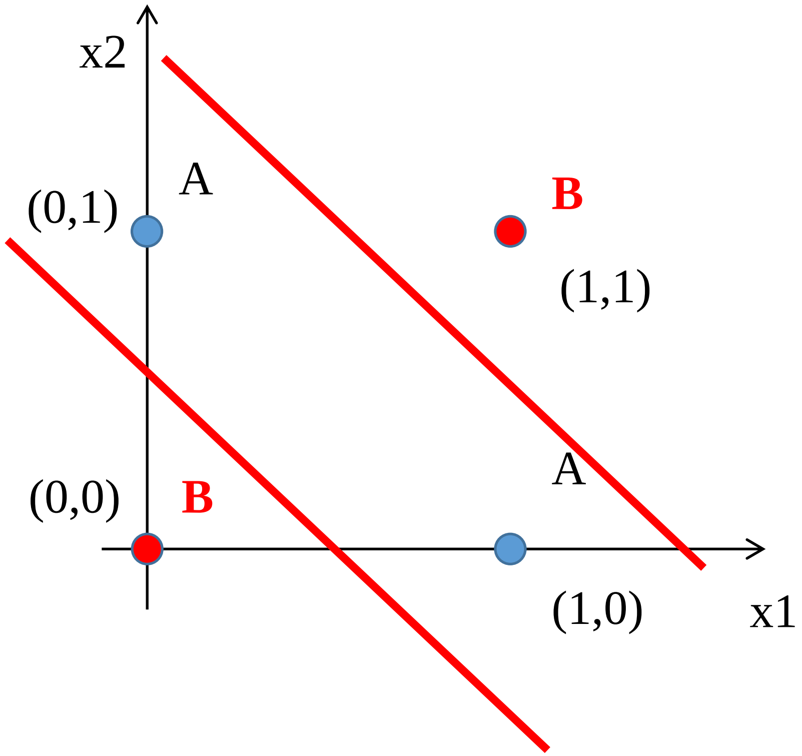
- Exclusive OR (XOR Boolean function)
- It's impossible to find a single straight line to separate two classes.

Truth Table for the XOR problem			
x1	x2	XOR	Class
0	0	0	B
0	1	1	A
1	0	1	A
1	1	0	B



NN for XOR problem

XOR



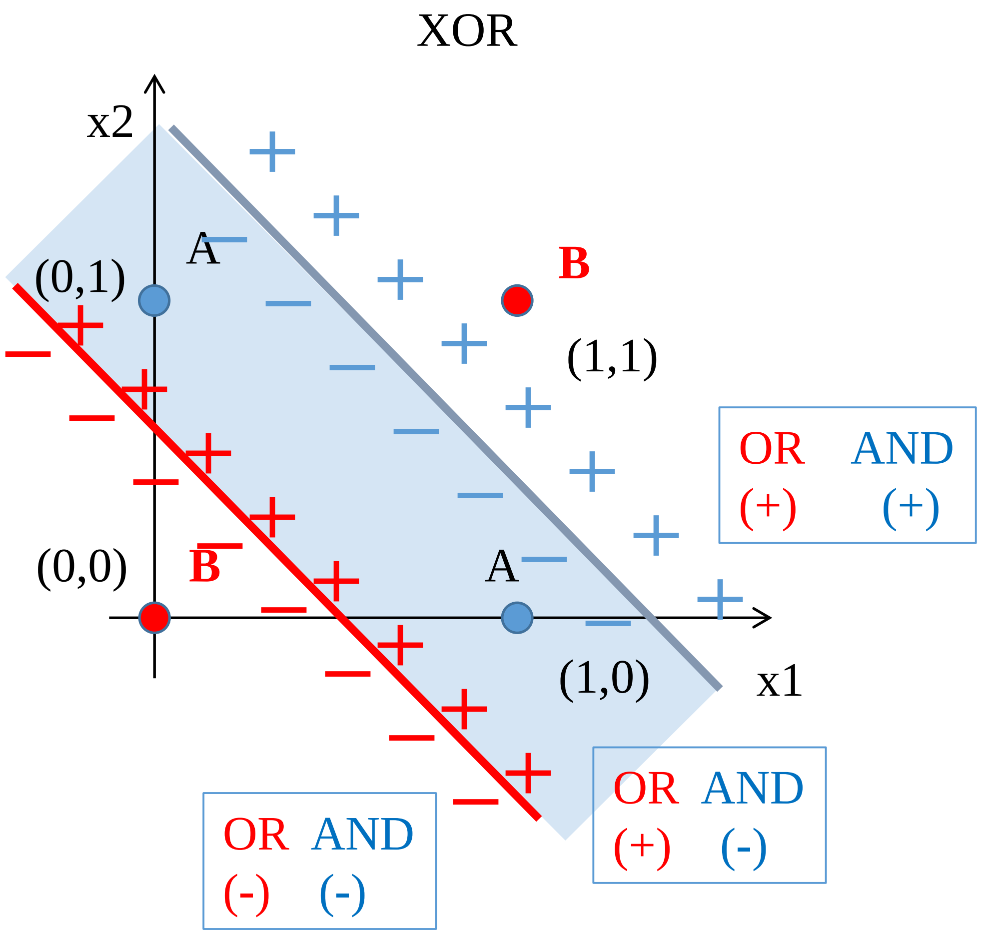
OR

$$h_1(x) = x_1 + x_2 - 0.5 = 0$$

AND

$$h_2(x) = x_1 + x_2 - 1.5 = 0$$

NN for XOR problem



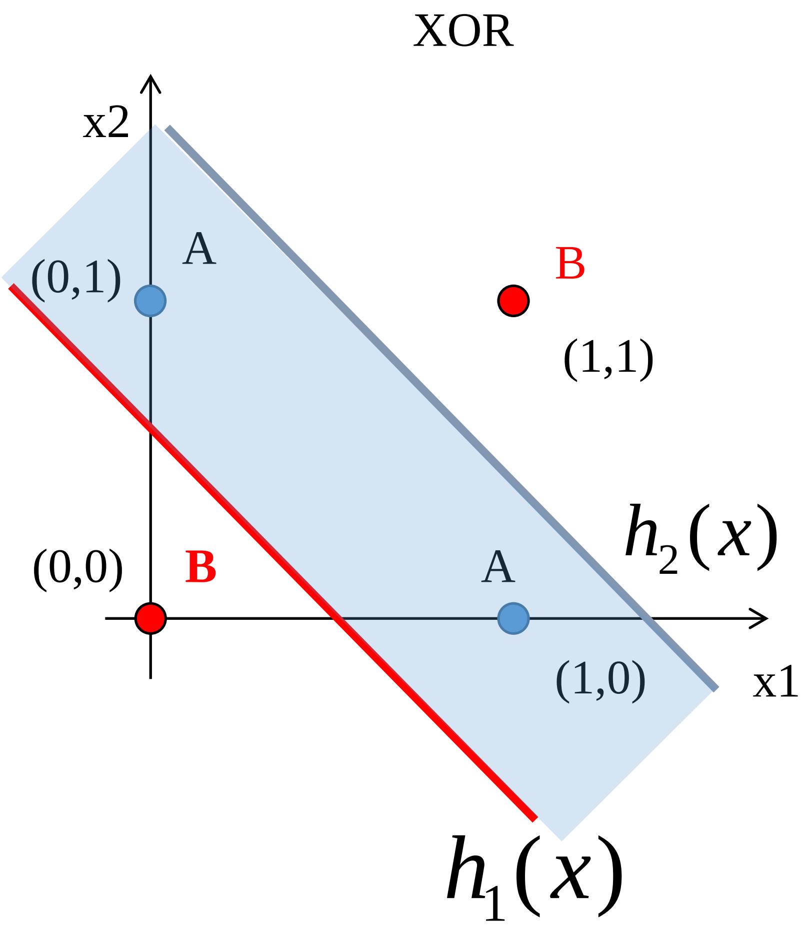
OR

$$h_1(x) = x_1 + x_2 - 0.5 = 0$$

AND

$$h_2(x) = x_1 + x_2 - 1.5 = 0$$

NN for XOR problem



OR

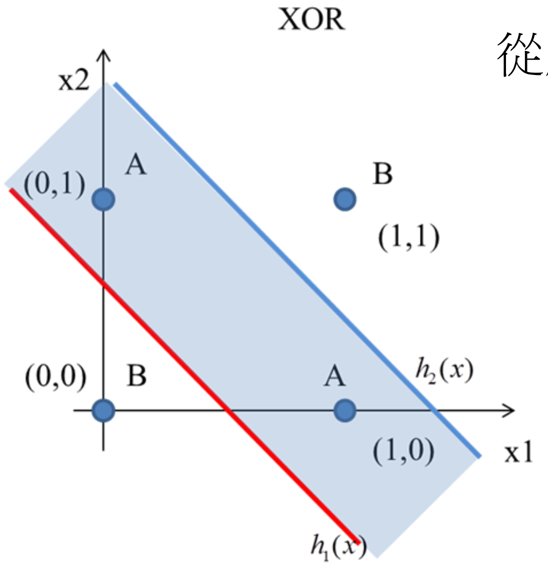
$$h_1(x) = x_1 + x_2 - 0.5 = 0$$

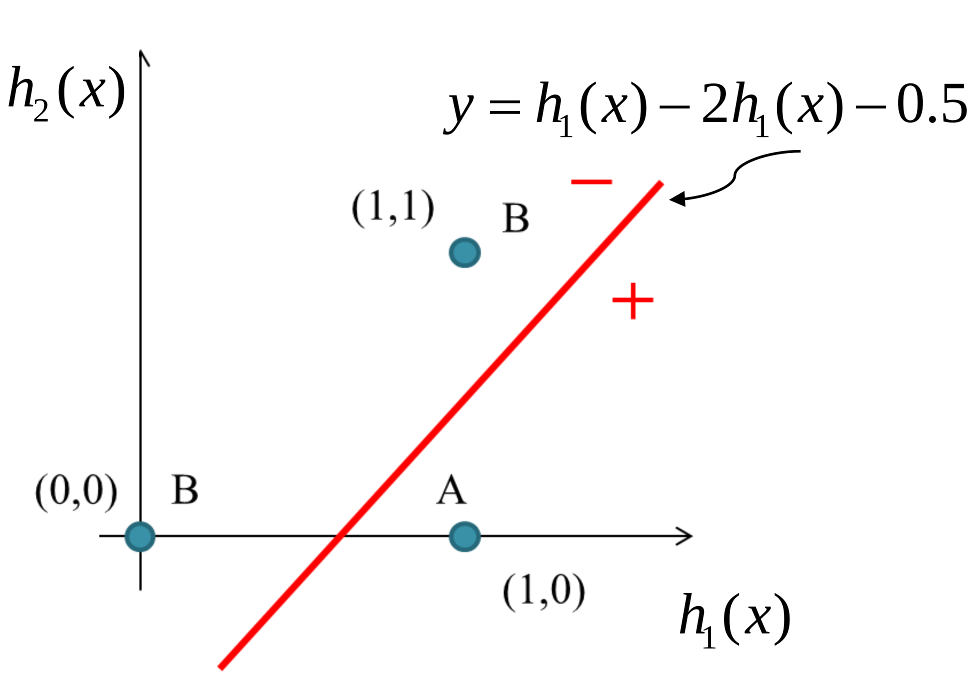
AND

$$h_2(x) = x_1 + x_2 - 1.5 = 0$$

NN for XOR problem

從原始特徵空間做特徵轉換(特徵萃取)

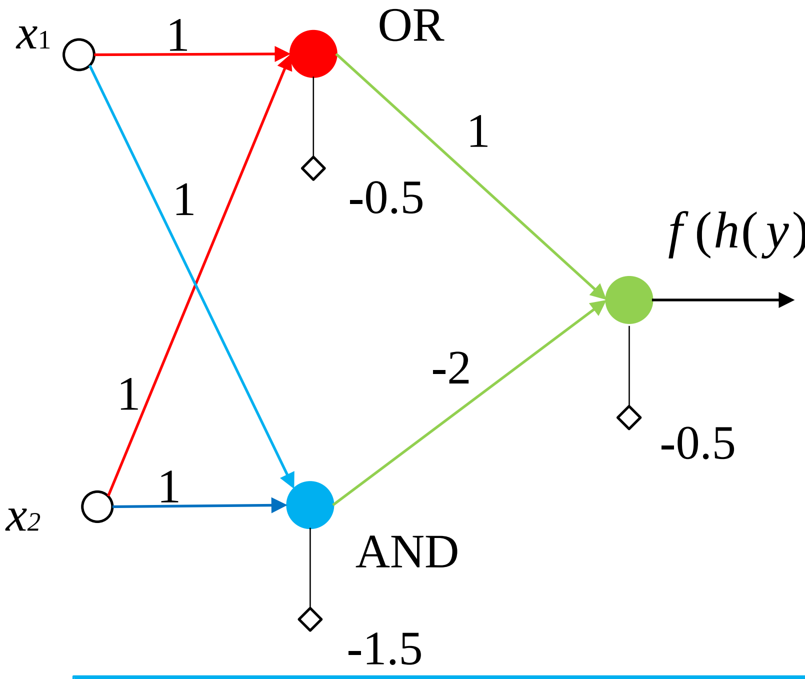




Truth Table for XOR problem				
x1	x2	$h_1(x)$	$h_2(x)$	Class
0	0	0(-)	0(-)	B
0	1	1(+)	0(-)	A
1	0	1(+)	0(-)	A
1	1	1(+)	1(+)	B

Two Layer Perception

$$y_1 = f(h_1(x)) = f(x_1 + x_2 - 0.5)$$



$$f(h(y)) = f(y_1 - 2y_2 - 0.5)$$

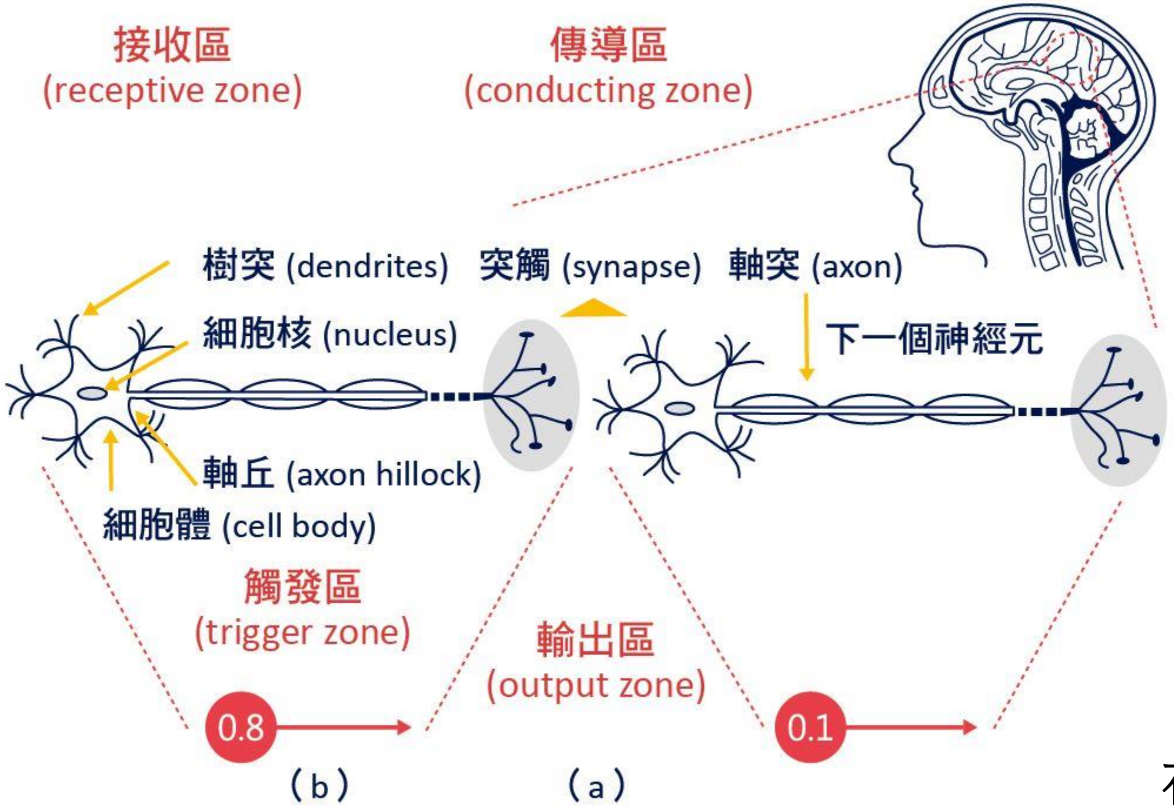
$$y_2 = f(h_2(x)) = f(x_1 + x_2 - 1.5)$$

$$f(v) = \begin{cases} 1 & if \;\; v \geq 0 \\ o & if \;\; v < 0 \end{cases}$$

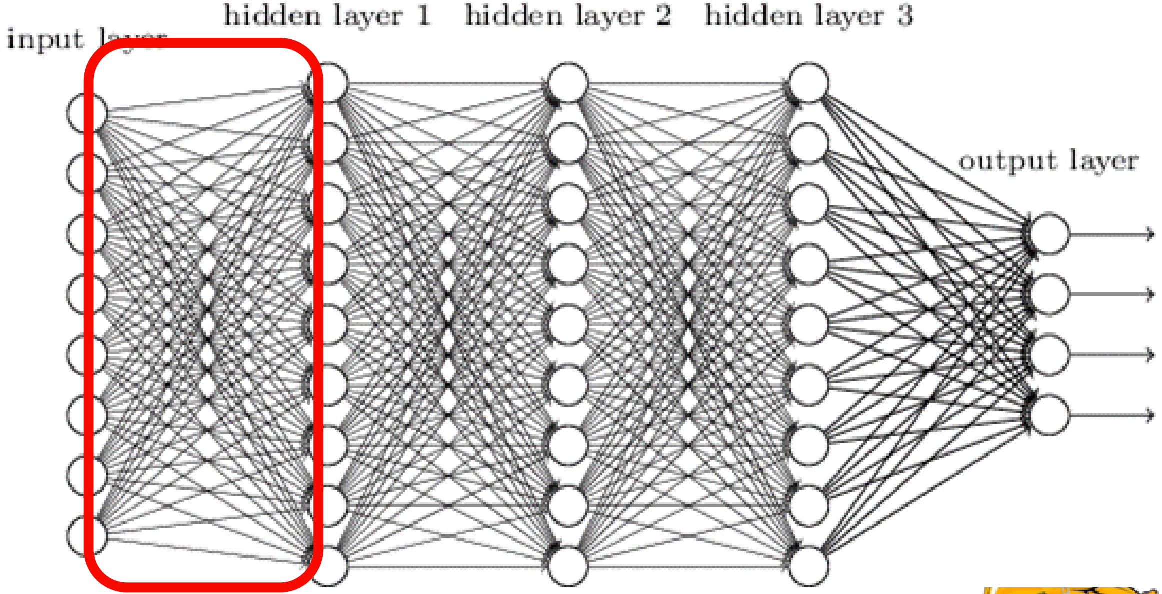
Step activation function

類神經網路

但神經訊息傳遞不會像只有一層Single layer perceptron運作，神經網路應該是多個細胞部段將訊息傳遞下去運作的模式，這就是Multilayer perception (MLP)，也就是一般認知的類神經網路。



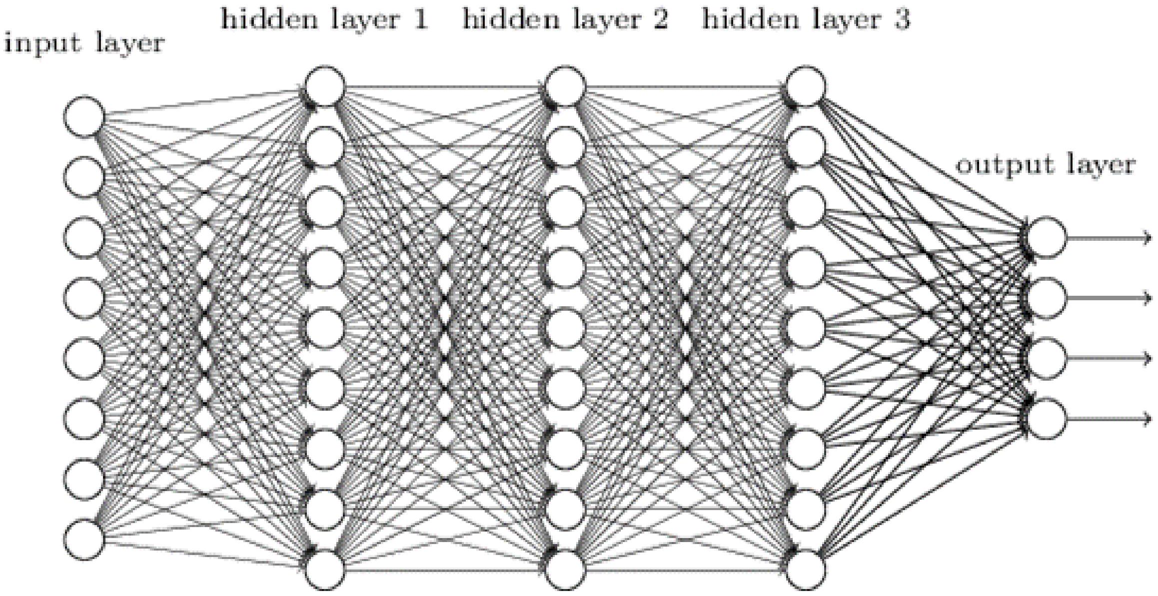
MLP



在深度學習這樣的層跟層叫做**fully connection**

Activation

- 如果沒有activation function會有什麼影響



$X_1 \quad W_1 \quad X_2 \quad W_2 \quad X_3 \quad W_3 \quad X_4 \quad W_4 \quad Y$

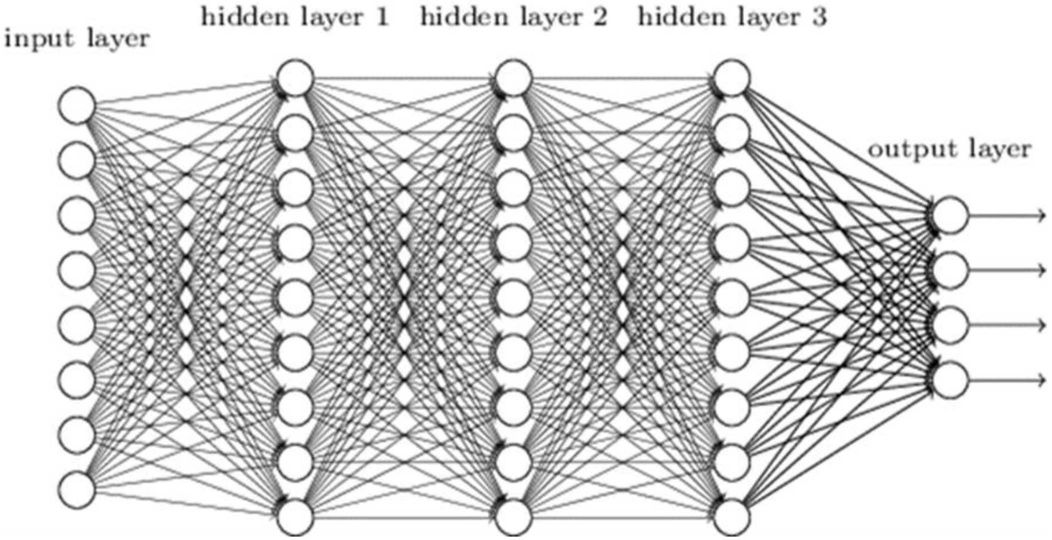
$$X_2 = W_1X_1$$
$$X_3 = W_2X_2$$
$$X_4 = W_3X_3$$
$$Y = W_4X_4$$

$$Y = W_4W_3W_2W_1X_1$$
$$\underline{Y = WX_1}$$
$$W = W_4W_3W_2W_1X_1$$

結果就只是一層的神經網路而已。

Activation

- 如果沒有activation function會有什麼影響



$X_1 \quad W_1 \quad X_2 \quad W_2 \quad X_3 \quad W_3 \quad X_4 \quad W_4 \quad Y$

$$X_2 = W_1 X_1$$
$$X_3 = W_2 X_2$$
$$X_4 = W_3 X_3$$
$$Y = W_4 X_4$$

$$X_2 = f(W_1 X_1)$$
$$X_3 = f(W_2 X_2)$$
$$X_4 = f(W_3 X_3)$$
$$Y = f(W_4 X_4)$$

$$Y = W_4 W_3 W_2 W_1 X_1$$
$$Y = W X_1$$
$$W = W_4 W_3 W_2 W_1 X_1$$

$$Y = f\left(W_4 f\left(W_3 f\left(W_2 f(W_1 X_1)\right)\right)\right)$$

Activation Function

- Sigmoid、Tanh。

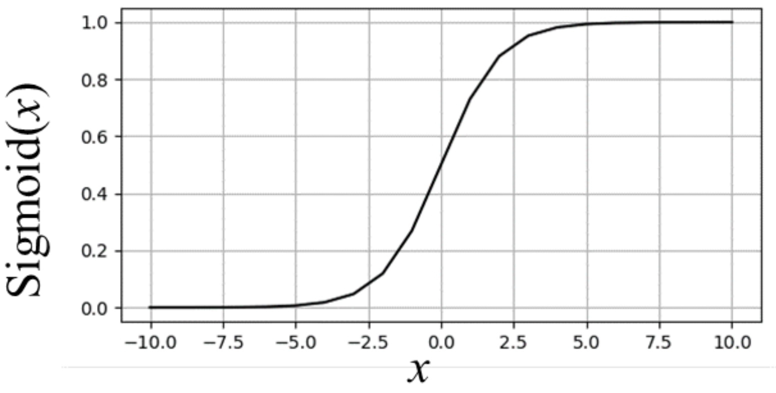
ReLU(Rectified Linear Unit)系列：

- ReLU
- Leaky ReLU
- ReLU6
- PReLU
- RReLU
- ELU (Exponential Linear Unit)
- SELU (Scaled Exponential Linear Units)

Activation function

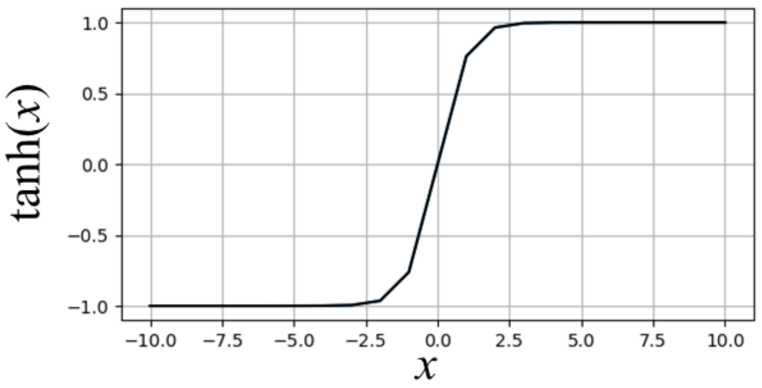
- $\mathrm{Sigmoid}(x) = \frac{1}{1+e^{-x}}$

將值壓到0~1之間



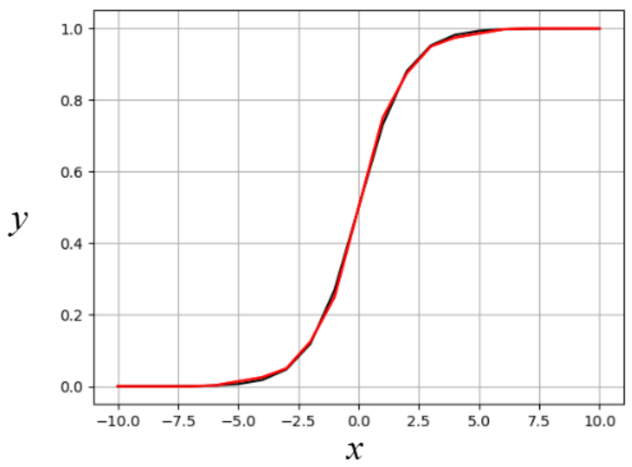
- $\tanh(x) = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$

將值壓到-1~1之間



Sigmoid近似法

$$
y = approxSigmoid(x) = \begin{cases} 0.5 * \left(1.5 * \left(\dfrac{\frac{x}{2}}{1+\frac{x}{2}}\right) + 1\right) & if\ \ 0 \le x < 3.4 \\ 0.5 * (0.9444 + 0.0459 * (\frac{x}{2} - 1.7) + 1) & if\ \ 3.4 \le x \le 5.8 \\ 0.9997 & x \ge 5.8 \end{cases}
$$



Black: sigmoid
Red: approximate sigmoid

```python
def fast_sigmoid2(x):
    x = 0.5 * x
    flag_neg=0
    if x<0:
        x=-x
        flag_neg=1

    if x < 1.7:
        z = (1.5 * x / (1 + x))
    elif x < 2.9:
        z = (0.9444 + 0.0459 * (x - 1.7))
    else:
        z = 0.9997
    if flag_neg==1:
        z=-z
    return 0.5 * (z + 1.)
```

Activation function: ReLU系列

- $\mathrm{ReLU}(x)=\max(0,x)=\begin{cases}x & if\ x>0\\ 0 & O.W.\end{cases}$
- $\mathrm{ReLU6}(x)=\min(\max(0,x),6)=\begin{cases}6 & if\ x\geq 6\\ x & if\ 0<x<6\\ 0 & O.W.\end{cases}$
- $\mathrm{LeakyReLU}(x)=\max(0,x)+a*\min(0,x)=\begin{cases}x & if\ x>0\\ ax & O.W.\end{cases}\ (default\ a=0.1)$
- $\mathrm{PReLU}(x)=\max(0,x)+a*\min(0,x)=\begin{cases}x & if\ x\geq 0\\ ax & O.W.\end{cases}$ (a是訓練得到，$init$設定: 0.25)
- $\mathrm{RReLU}(x)=\max(0,x)+a*\min(0,x)=\begin{cases}x & if\ x>0\\ ax & O.W.\end{cases}$ (a是隨機$U(lower=\frac{1}{8},upper=\frac{1}{3})$選取)
- $\mathrm{ELU}(x)=\max(0,x)+\min(0,\alpha*(e^x-1))=\begin{cases}x & if\ x>0\\ \alpha*(e^x-1) & O.W.\end{cases}$
- $\mathrm{SELU}(x)=\mathrm{scale}\big(\max(0,x)+\min\big(0,\alpha*(e^x-1)\big)\big)=\mathrm{scale}\begin{cases}x & if\ x>0\\ \alpha*(e^x-1) & O.W.\end{cases}$

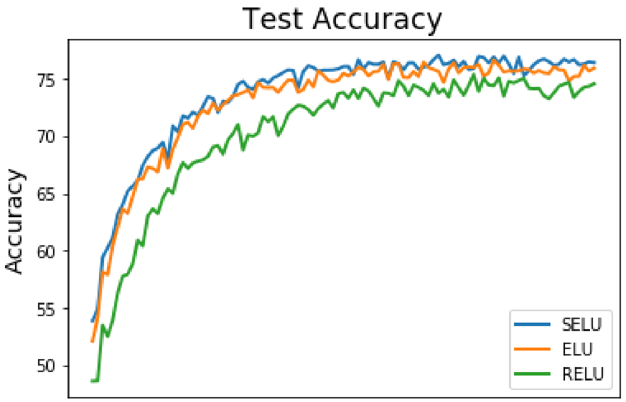
  α=1.6732632423543772848170429916717
  scale=1.0507009873554804934193349852946

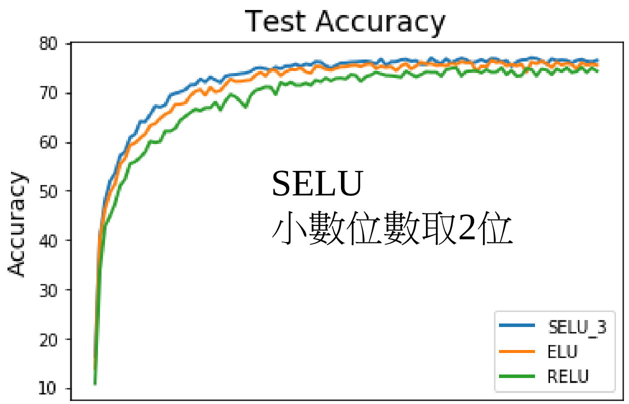
  實驗驗證，小數取兩位數即可，α=1.67, scale=1.05

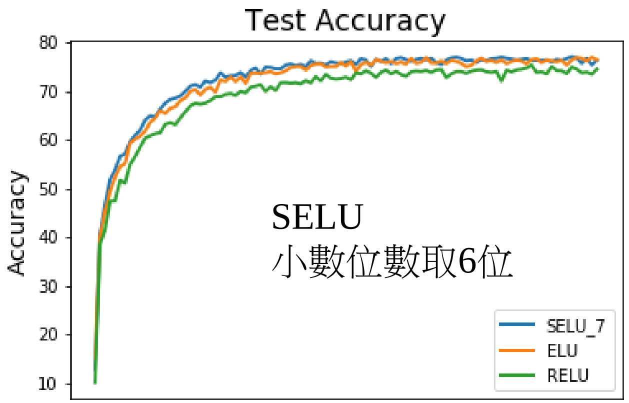
SELU係數精度實驗

- Model: LeNet 加強版
- Database: Cifar-10
- 看分類正確率隨著learning epoch變化的影響

SELU係數小數位數全取





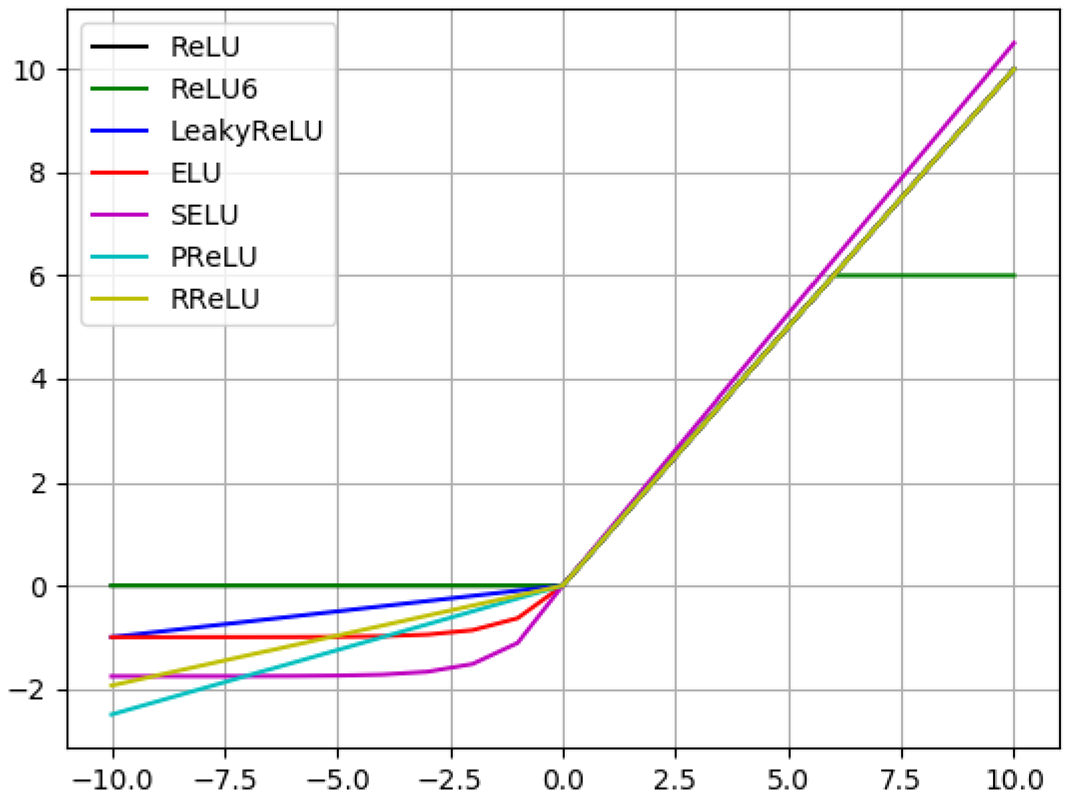


$$\mathrm{SELU}(x) = \mathrm{scale}\big(\max(0,x) + \min(0, \alpha * (e^{x} - 1))\big)$$

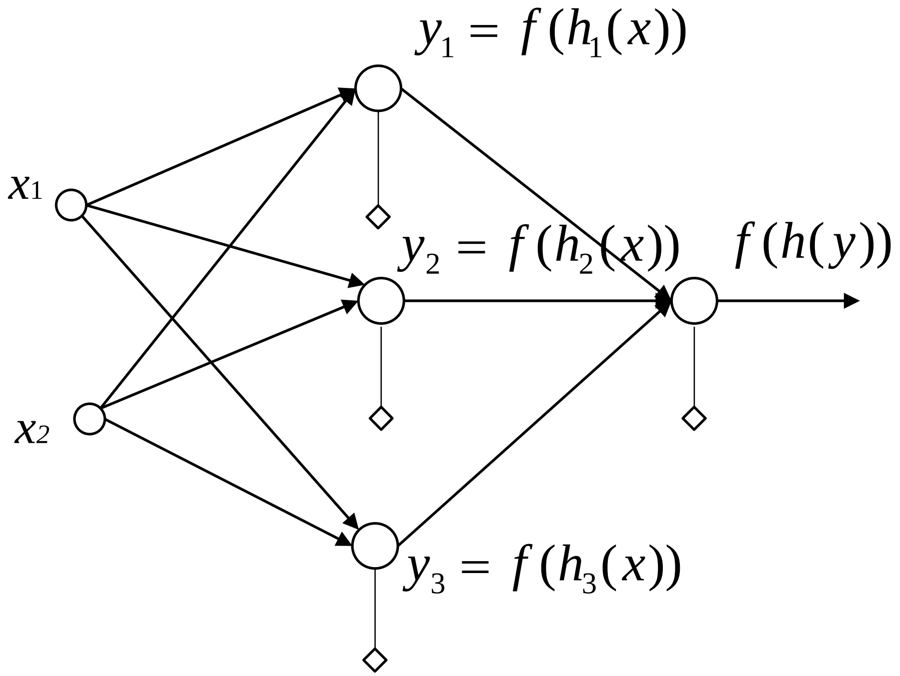
$$= \mathrm{scale}\begin{cases} x & if\ x > 0 \\ \alpha * (e^{x} - 1) & O.W. \end{cases}$$

α=1.6732632423543772848170429916717
scale=1.0507009873554804934193349852946

Activation function: ReLU系列

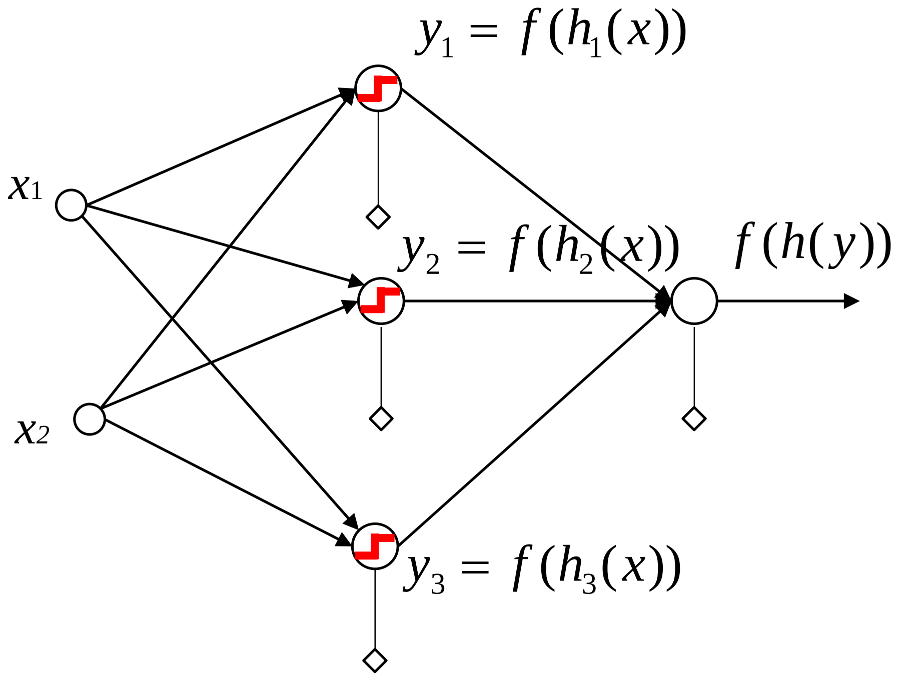


Polyhedral Regions

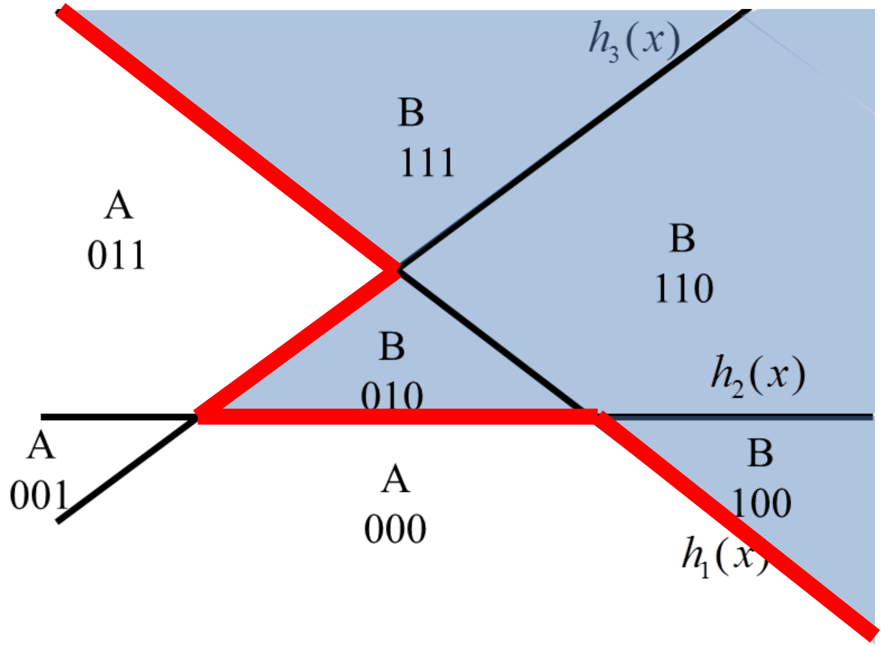


Polyhedral Regions

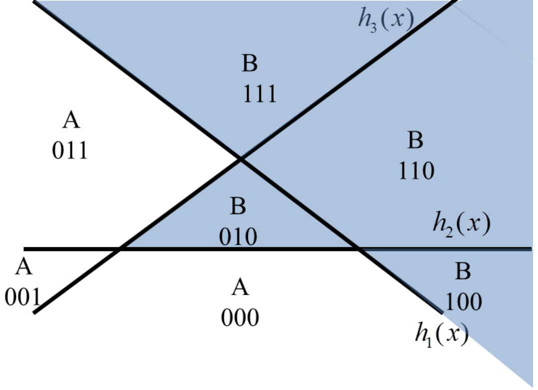
activation function = step function



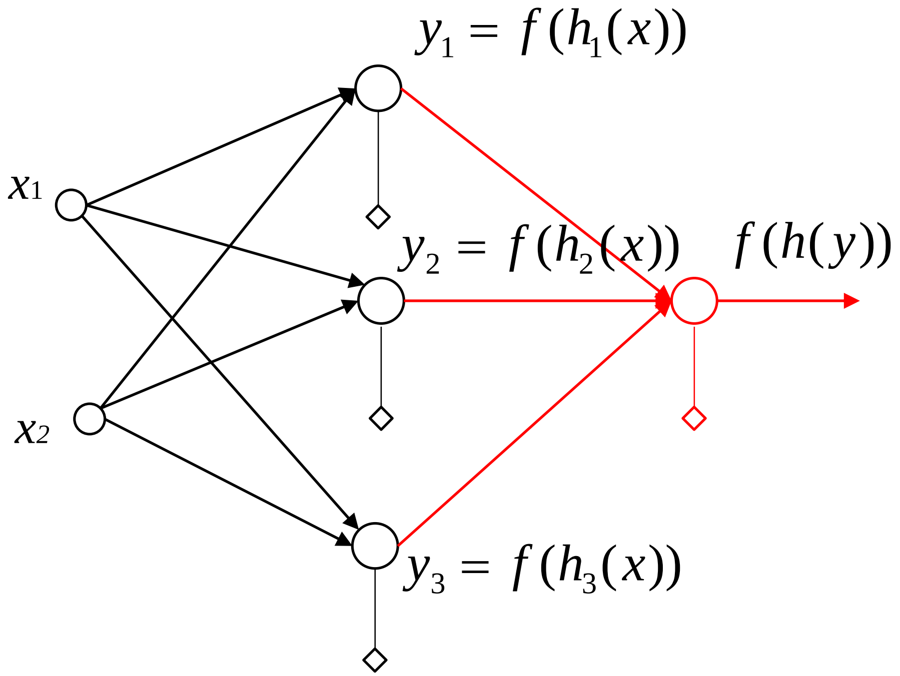
The first layer of neurons divides the input d-dimensional space into polyhedral, which are formed by hyperplane intersections.

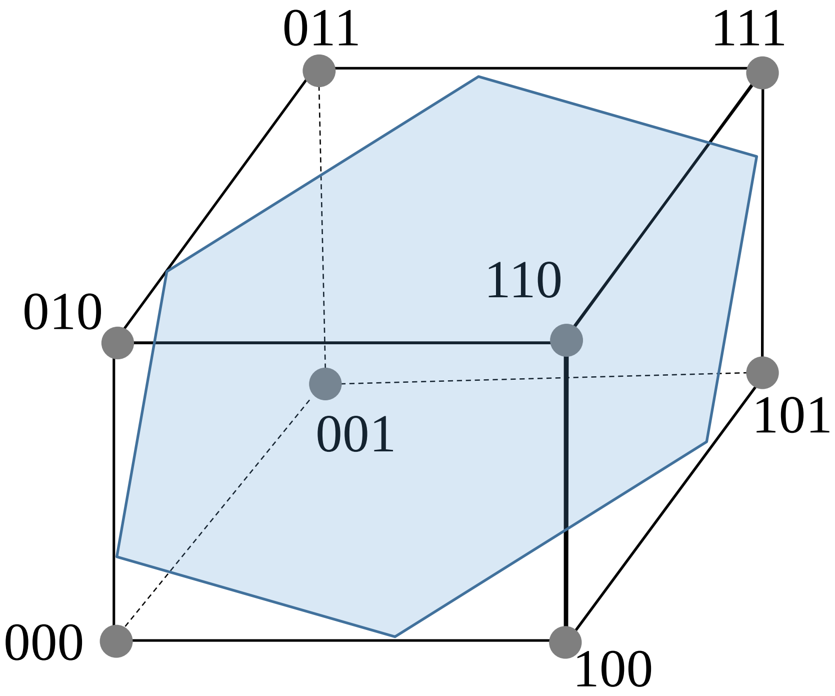


Polyhedral Regions

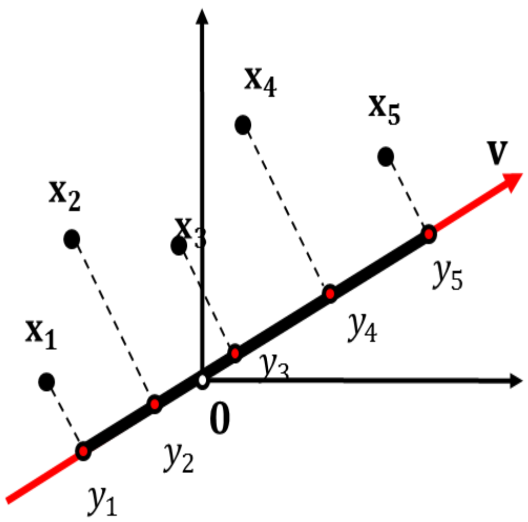


All vectors located within one of these polyhedral regions are mapped onto a specific vertex of the unit hypercube.



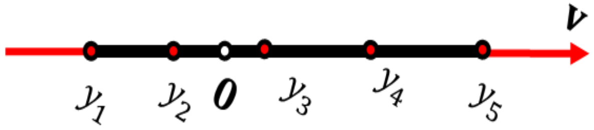


Dimension Reduction(Feature Projection)



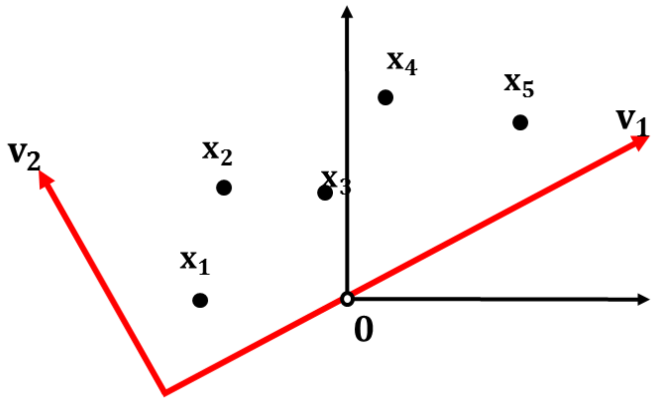
X空間轉換到特徵空間

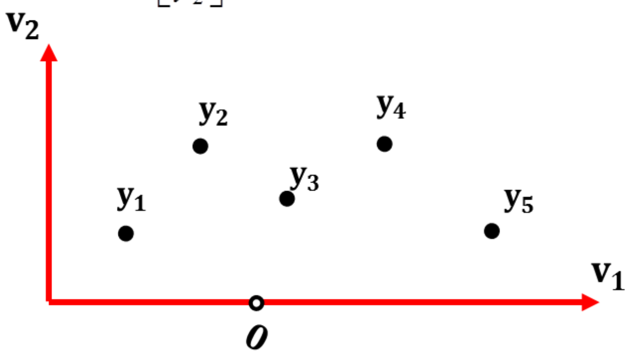
$y = \mathbf{v}^T \mathbf{x}$



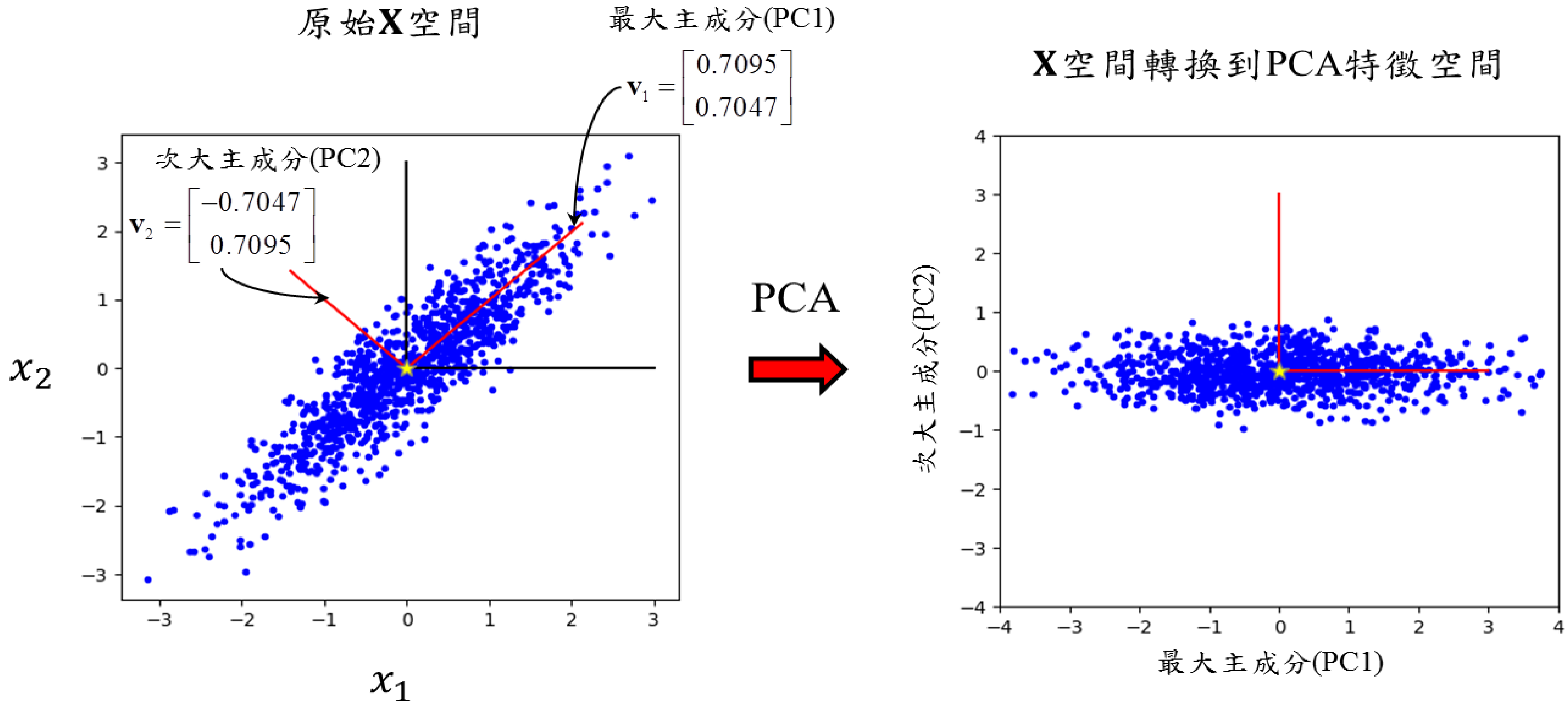
X空間轉換到**Y**空間

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathbf{V}^T \mathbf{x} = [\mathbf{v}_1 \quad \mathbf{v}_2]\mathbf{x}$$

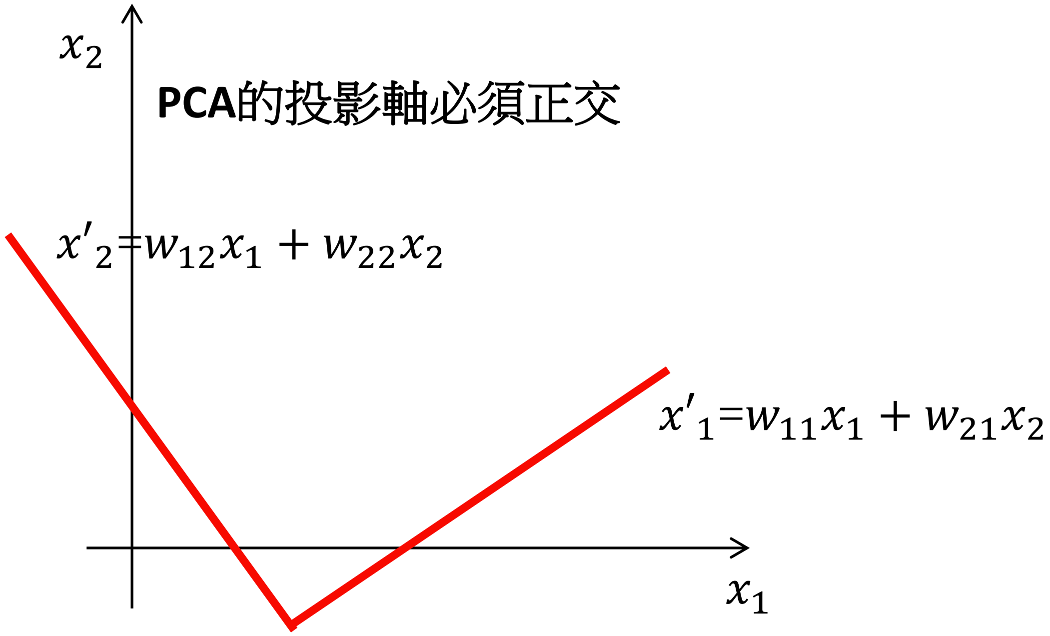




Principle Component Analysis



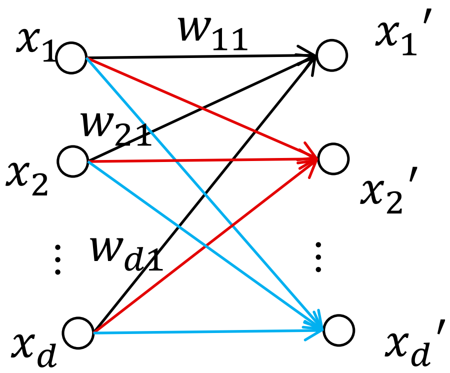
NN and Dimension Reduction



$$\boldsymbol{x}' = \begin{bmatrix} x'_1 \\ x'_2 \end{bmatrix} = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}^{\boldsymbol{T}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \boldsymbol{W}^T \boldsymbol{x}$$

拓展
為d維

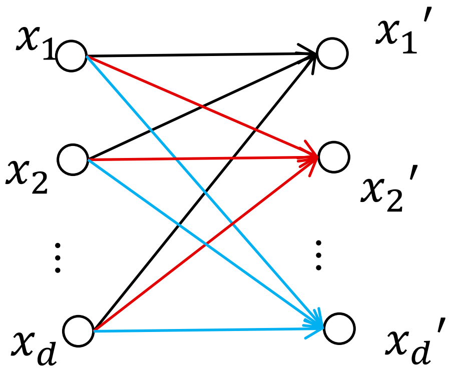
$$\boldsymbol{x}' = \begin{bmatrix} x_1' \\ \vdots \\ x_d' \end{bmatrix} = W^T \boldsymbol{x} = \begin{bmatrix} w_{11} & \cdots & w_{1d} \\ \vdots & \ddots & \vdots \\ w_{d1} & \cdots & w_{dd} \end{bmatrix}^{\boldsymbol{T}} \begin{bmatrix} x_1 \\ \vdots \\ x_d \end{bmatrix}$$

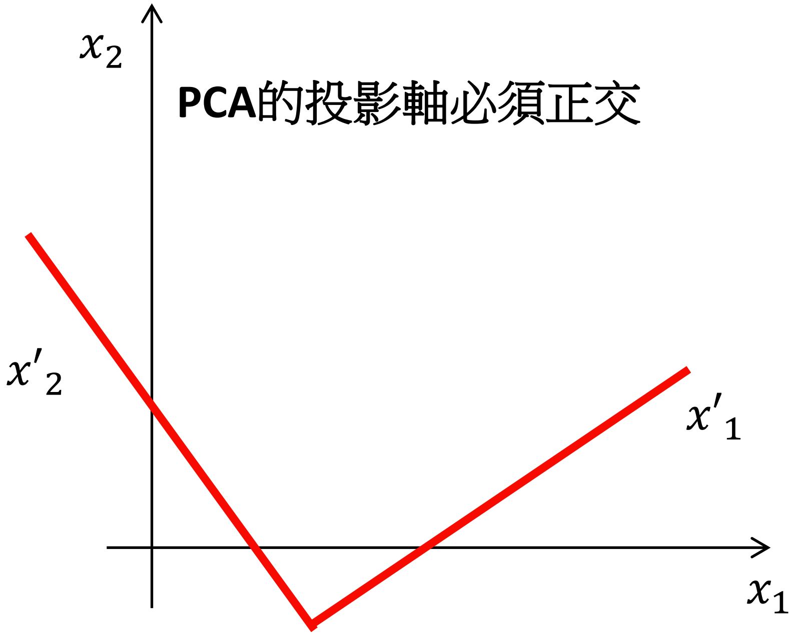


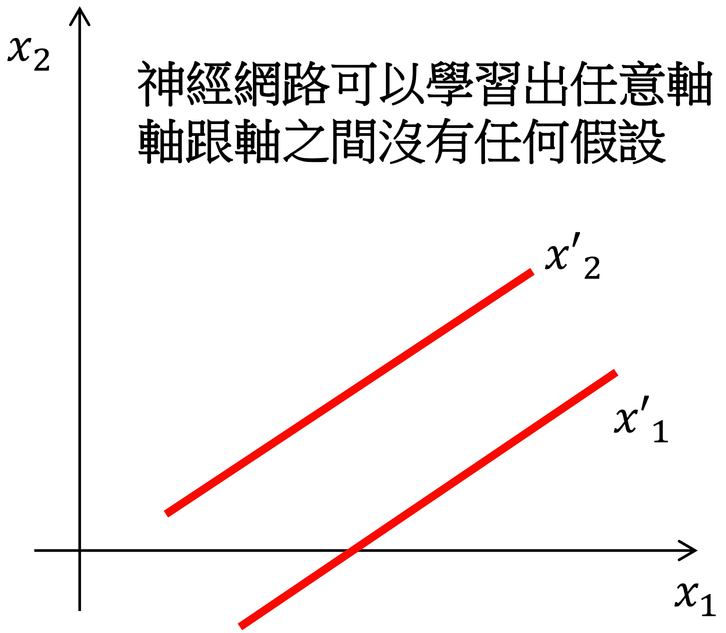
NN and Dimension Reduction

- **Principle Component Analysis**

$$\boldsymbol{x}' = \begin{bmatrix} x_1' \\ \vdots \\ x_d' \end{bmatrix} = W^T\boldsymbol{x} = \begin{bmatrix} w_{11} & \cdots & w_{1d} \\ \vdots & \ddots & \vdots \\ w_{d1} & \cdots & w_{dd} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_d \end{bmatrix}$$



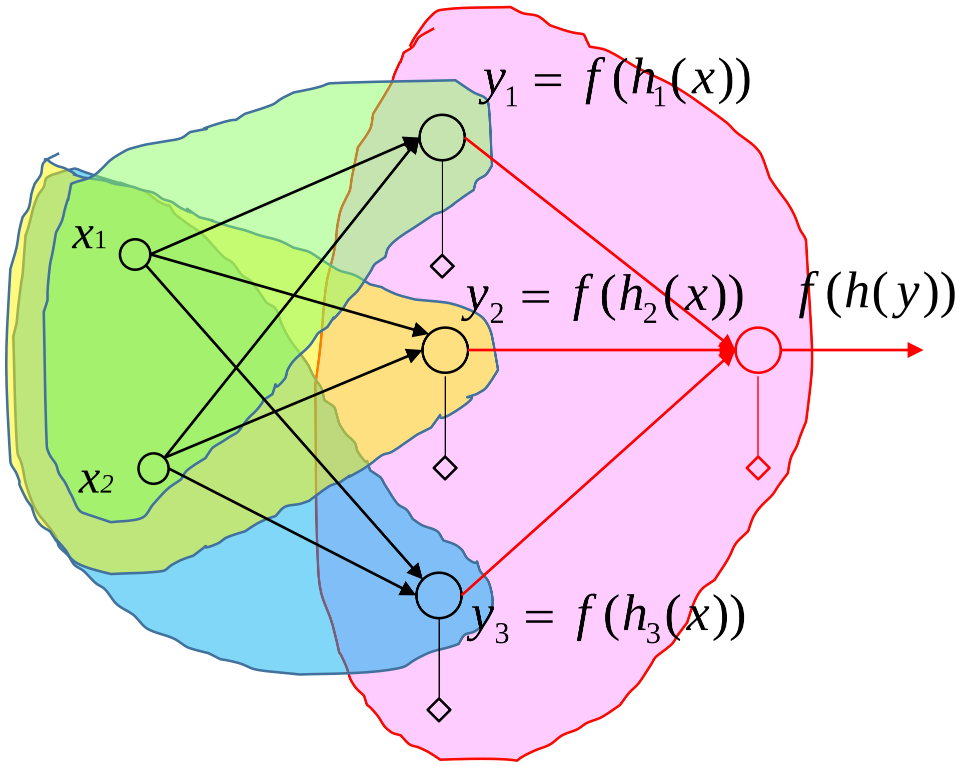




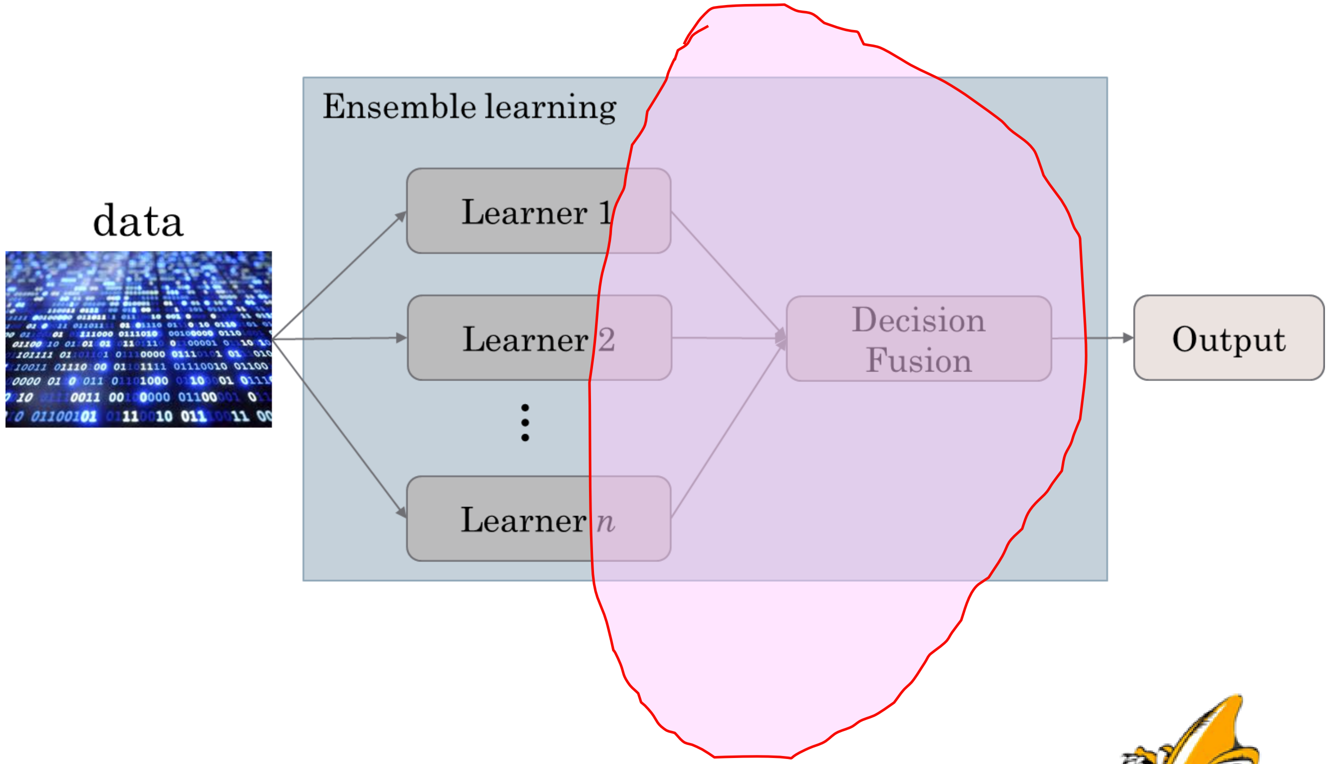
PCA是NN的一個special case

NN and Ensemble Learning

NN



Ensemble Learning



Introduction (loss function)

In learning algorithm, there is an assumption, which may accompany with an <u>object function</u>.

- **Regression**: minimizing the mean square errors (MSE)
- **K-mean**: minimizing the mean of square error between data and centers.
- **PCA**: maximizing the variance of project data.
- **SVM**: maximizing the margin.

Sometimes the same model but different object function can lead different results. (Linear regression and ridge regression)

Regression

$$\min_{\beta}\{\mathrm{MSE}(\hat{y}, y)$$

Ridge regression

$$\min_{\beta}\{\mathrm{MSE}(\hat{y}, y) + \lambda L_2\mathrm{norm}(\beta)\}$$

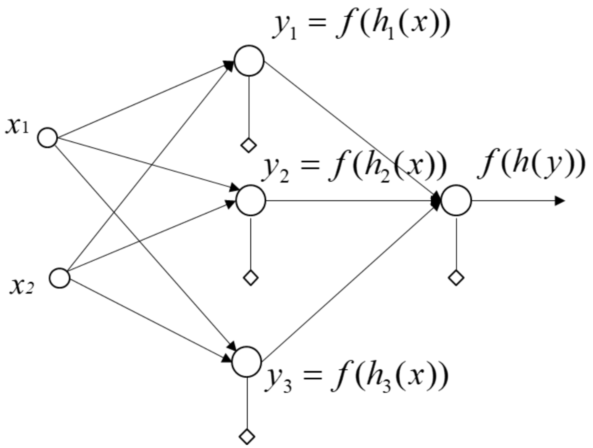
$$L_2\mathrm{norm}(\beta) = \sum_i \beta_i^2$$

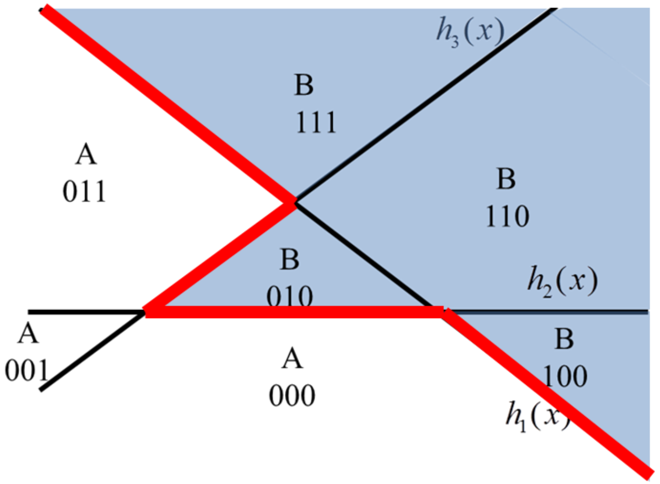
Neural Network
(Number of output node)

<u>**Regression case**</u>: Number of output node of NN architecture is 1.

<u>**Classification case**</u>: Number of output node of NN architecture is *L* (Number of class in define problem). ->**memberships (call logits in DL) for each class (such as posterior probability in ML).**

Q: Why number of output node in this toy case is 1?



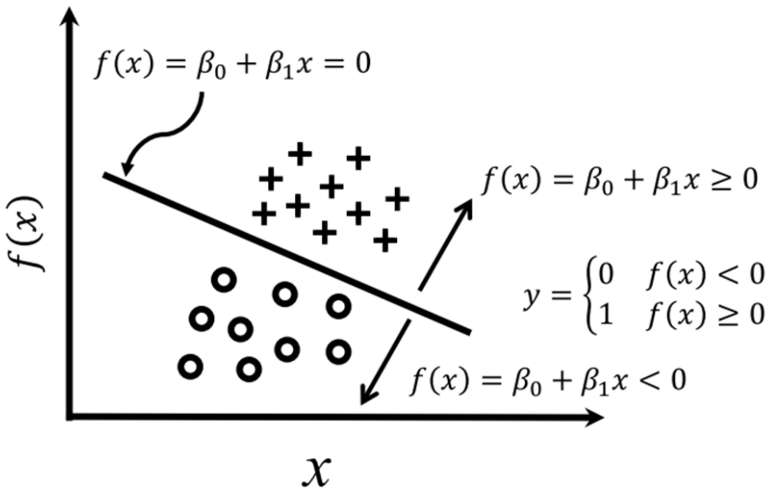


ANS: It's a binary classification problem.

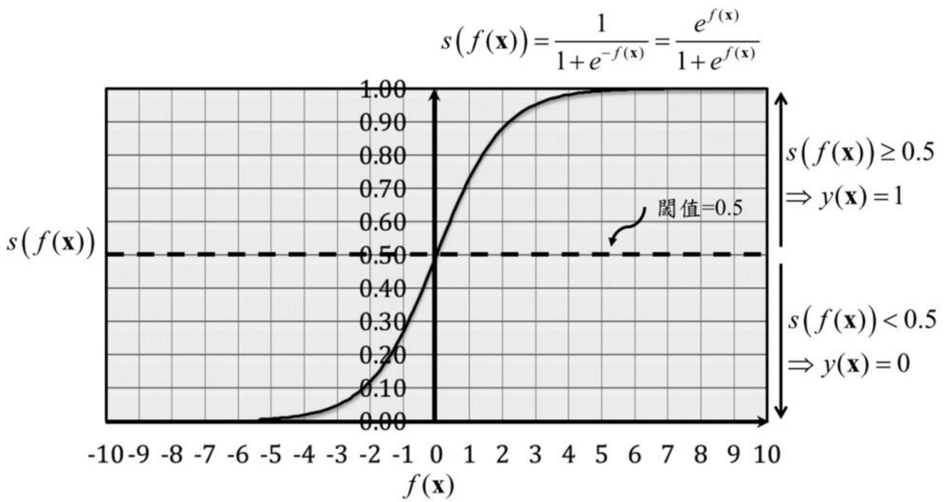
A simple threshold can be defined as decision boundary, see the Logistic regression.

Neural Network (Number of output node)

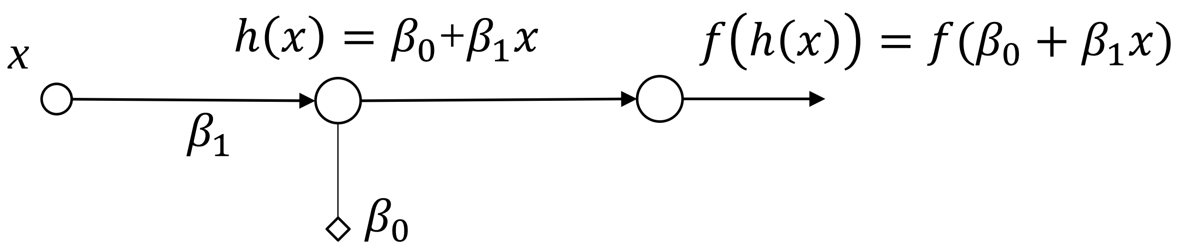
羅吉斯迴歸



$$\sigma\left(f(x)\right) = \frac{1}{1 + e^{-f(\boldsymbol{x})}} = \frac{e^{f(\boldsymbol{x})}}{1 + e^{f(\boldsymbol{x})}}$$



Neural Network



Logistic Regression	NN	Activation
$f(x)$	$h(x)$	None
$\sigma(f(x))$	$f(h(x))$	Sigmoid

Introduction

1. Regression

MSE, MAE, Huber Loss

2. Classification

Cross entropy, Focal loss

3. Triple loss

Residual

- Residual: predicted value v.s. target value.

Regression:

$$y - \hat{y}$$

Classification (error):

$$sign(\hat{y}, y) = \begin{cases} 1 & \hat{y} = \hat{y} \\ 0 & \hat{y} \neq \hat{y} \end{cases}$$

$$error\ rate = \frac{1}{n}\sum_{i=1}^{n} sign(\widehat{y_i}, y_i)$$

MSE & MAE

Mean Square Error (MSE)

Mean Absolute Error (MAE)

<u>Why square or absolute?</u>

Target value: $y_1 = 0, y_2 = 1$

Predicted value: $\hat{y}_1 = 100$，$\hat{y}_2 = 99$

Residual 1=$y_1 - \hat{y}_1 = 0 - 100 = -100$

Residual 2=$y_2 - \hat{y}_2 = 1 - (-99) = 100$

Residual 1+ Residual 2 $= -100 + 100 = 0$

MSE & MAE

Mean Square Error (MSE)

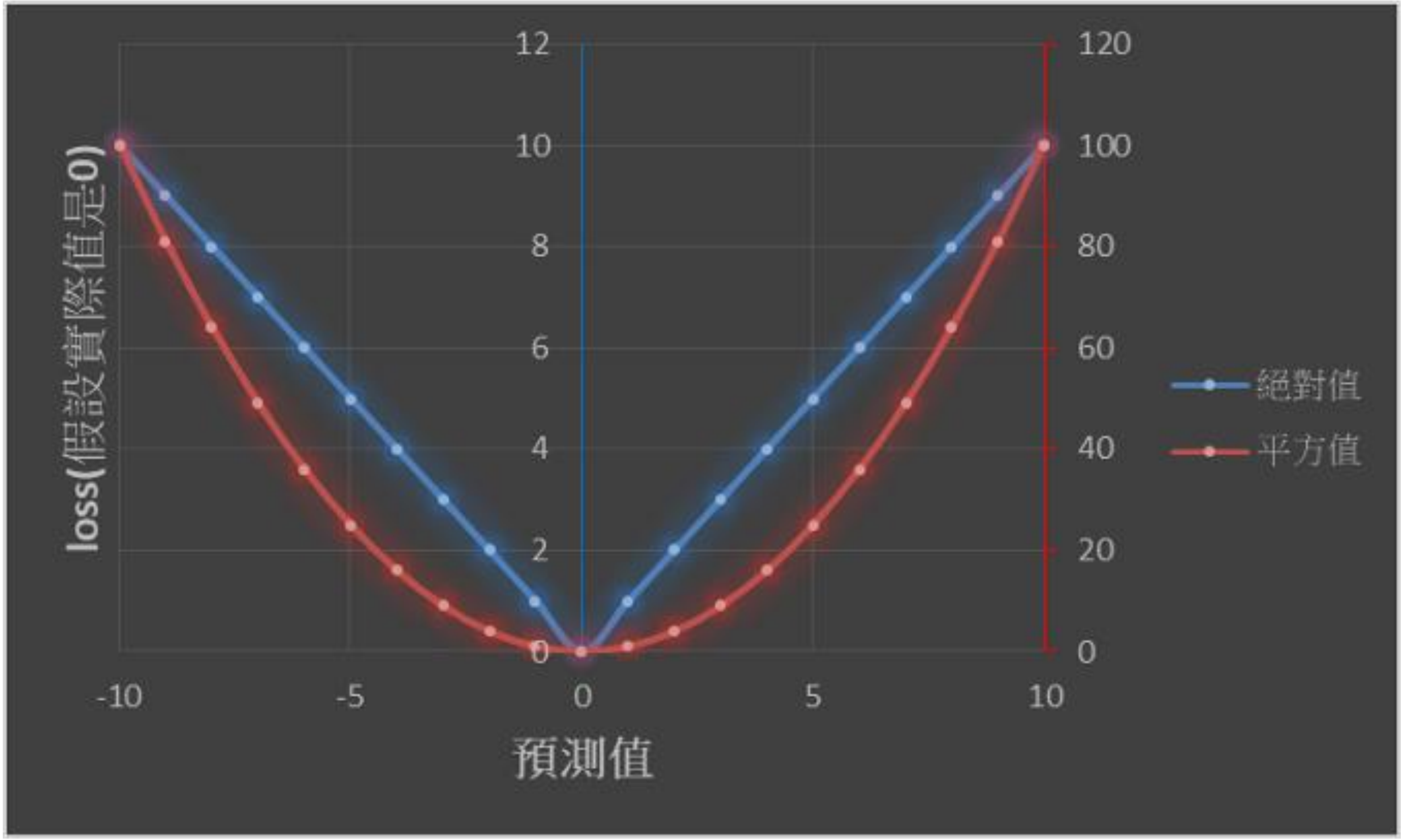
$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

Mean Absolute Error (MAE)

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|$$

MSE & MAE

Trend of residual (target value =0)



Comparison (MSE&MAE)

With a fair baseline, RMSE (root MSE)

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}$$

Comparison (MSE&MAE)

ID	residual	| residual |	$residual^2$
1	-10	10	100
2	-5	5	25
3	0	0	0
4	5	5	25
5	10	10	100
MAE=6, RMSE=7.07			

Change ID5 with a outliner

ID	residual	| residual |	$residual^2$
1	-10	10	100
2	-5	5	25
3	0	0	0
4	5	5	25
5	100	100	10000
MAE=24, RMSE=45.06			

Problem of MSE: more outlier sensitivity.

Comparison (MSE&MAE)

- Problem of MAE: same gradient value.
- When loss is small, it's difficult to reach the optimal target.

$$f_1(x) = x^2, f_1'(x) = 2x$$

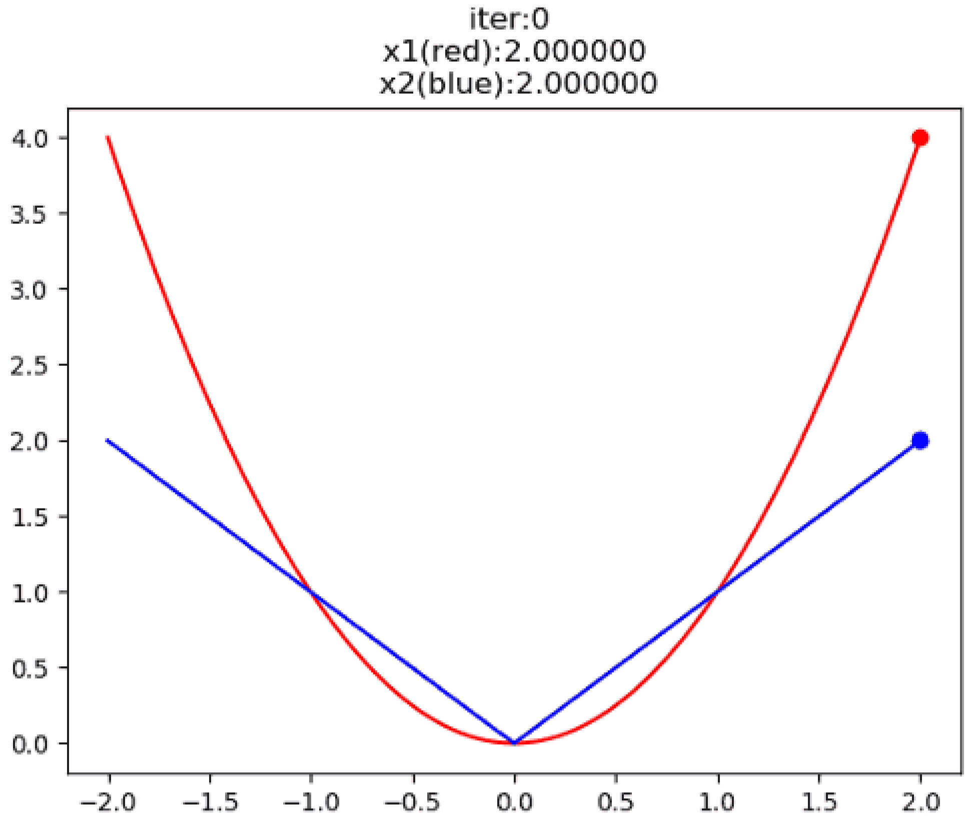
$$f_2(x) = |x|, f_2'(x) = \frac{x}{|x|}$$

Gradient update:

$$x^{t+1} \rightarrow x^t - rf'(x)$$

Suppose: $x^0 = 2, \ \ r = 0.3$

Comparison (MSE&MAE)



$f(x)=x^2$			
t	x^t	$f'(x)$	x^{t+1}
1	2	4	0.8
2	0.8	1.6	0.32
3	0.32	0.64	0.128
4	0.128	0.256	0.0512
5	0.0512	0.1024	0.02048
6	0.02048	0.04096	0.008192
7	0.008192	0.016384	0.003277
8	0.003277	0.006554	0.001311
9	0.001311	0.002621	0.000524
10	0.000524	0.001049	0.00021
11	0.000210	0.000419	0.000084
12	0.000084	0.000168	0.000034
13	0.000034	0.000067	0.000013
14	0.000013	0.000027	0.000005
15	0.000005	0.000011	0.000002
16	0.000002	0.000004	0.000001
17	0.000001	0.000002	0.000000
18	0.000000	0.000001	0.000000
19	0.000000	0.000000	0.000000
20	0.000000	0.000000	0.000000

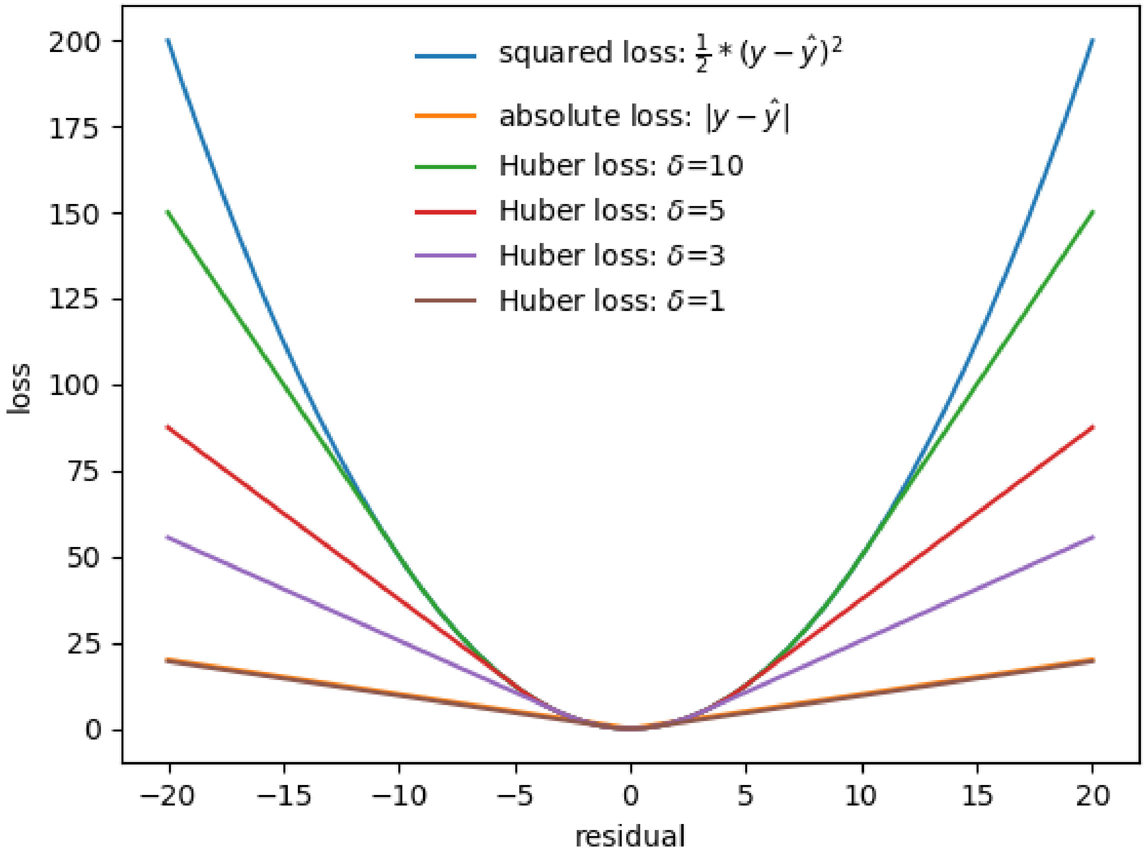
$f(x)=\|x\|$			
t	x^t	$f'(x)$	x^{t+1}
1	2	1	1.7
2	1.7	1	1.4
3	1.4	1	1.1
4	1.1	1	0.8
5	0.8	1	0.5
6	0.5	1	0.2
7	0.2	1	-0.1
8	-0.1	-1	0.2
9	0.2	1	-0.1
10	-0.1	-1	0.20
11	0.20	1	-0.1
12	-0.10	-1	0.20
13	0.20	1	-0.10
14	-0.10	-1	0.20
15	0.20	1	-0.10
16	-0.10	-1	0.20
17	0.20	1	-0.10
18	-0.10	-1	0.20
19	0.20	1	-0.10
20	-0.10	-1	0.20

Huber Loss

Huber loss:

$$Loss(y, \hat{y}) = \begin{cases} \frac{1}{2}(y - \hat{y})^2, & |y - \hat{y}| \le \delta \\ \delta(|y - \hat{y}| - \frac{1}{2}\delta), & O.W. \end{cases}$$

δ: parameter of Huber loss.



MAE, MSE & Huber Loss

ID	residual	| residual |	$residual^2$	Huber (δ=1)	Huber (δ=10)
1	-10	10	100	9.5	50
2	-5	5	25	4.5	12.5
3	0	0	0	0	0
4	5	5	25	4.5	12.5
5	10	10	100	9.5	50
MAE=6, RMSE=7.07 **MeanHuber(δ=1)=5.6, MeanHuber(δ=10)=25**					

ID	residual	| residual |	$residual^2$	Huber (δ=1)	Huber (δ=10)
1	-10	10	100	9.5	50
2	-5	5	25	4.5	12.5
3	0	0	0	0	0
4	5	5	25	4.5	12.5
5	100	100	10000	99.5	950
MAE=24, RMSE=45.06 **MeanHuber(δ=1)=23.6, MeanHuber(δ=10)=205**					

Classification

Classification:

$$sign(\hat{y}, y) = \begin{cases} 1 & y = \hat{y} \\ 0 & y \neq \hat{y} \end{cases}$$

$$error\ rate = 1 - \frac{1}{n}\sum_{i=1}^{n} sign(\hat{y}_i, y_i)$$

We hope less error rate more better in classification.

Can we use the classification error rate/accuracy as loss function?

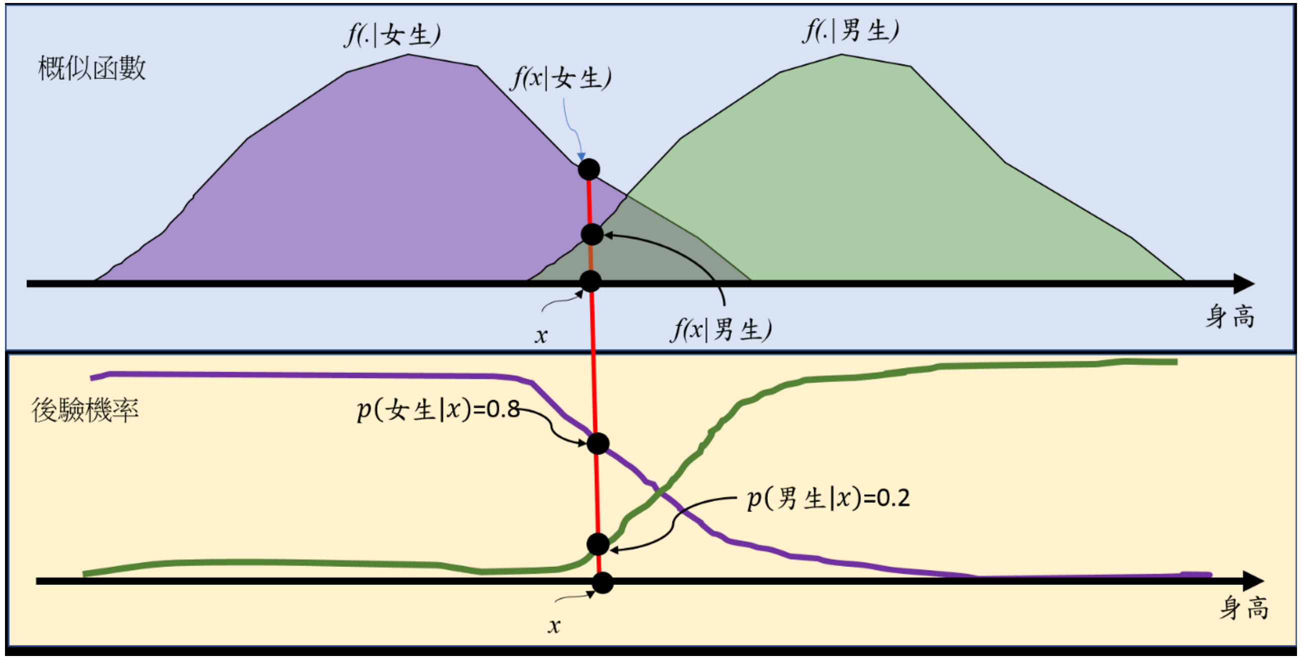
Classification

		Model 1 (輸出)				Model 2 (輸出)			
		機率輸出			判斷	機率輸出			判斷
	Target (Label)	男生	女生	其他		男生	女生	其他	
data 1	男生	0.4	0.3	0.3	男生 (正確)	0.7	0.1	0.2	男生 (正確)
data 2	女生	0.3	0.4	0.3	女生 (正確)	0.1	0.8	0.1	女生 (正確)
data 3	男生	0.5	0.2	0.3	男生 (正確)	0.9	0.1	0	男生 (正確)
data 4	其他	0.8	0.1	0.1	男生 (錯誤)	0.4	0.3	0.3	男生 (錯誤)
		模型1錯誤率: 1/4=0.25				模型2錯誤率: 1/4=0.25			

Can we observe any difference between model 1 & 2 from **error rate**?
NO…
BUT we can observe that model 2 has better probability outputs than model 1.
Error rate cannot as learning object for learning updating, it's just a metric for evaluating model performance.

Classification

- How do we make decision for a new sample in classification model?
- **ANS: posterior probability.**



Cross-entropy

- Cross-entropy is usually used in classification loss.
- **Entropy**: the average of information which is produced by a stochastic source of data.
- **Information gain**: (suppose X is a random variable)

$$I(x) = -log_2(p(x))$$

Information gain

A is stupid, and his grades usually are around 50 marks.

B is smart, and his grades usually are almost 100 marks.

Probability to pass the exam for A: $p(x_A) = 0.4$

$$I(x_A) = -log_2\big(p(x_A)\big) = 1.322$$

Probability to pass the exam for B: $p(x_B) = 0.99$

$$I(x_B) = -log_2\big(p(x_B)\big) = 0.014$$

Entropy

Entropy: the average of information which is produced by a stochastic source of data.

In information theory,

Entropy = Shannon entropy

$$H(X) = \sum_{i} -p_i log_2(p_i)$$

Generally, entropy refers to uncertainty for the random variable *X*.

Entropy

$$p(x_A = pass) = 0.4, \qquad p(x_A = fail) = 0.6$$

$$p(x_B = pass) = 0.99, \qquad p(x_B = fail) = 0.01$$

$$H(X) = \sum_i -p_i log_2(p_i)$$

$$H(X_A) = -0.4\log(0.4) - 0.6\log(0.6) = 0.971$$

$$H(X_B) = -0.99\log(0.99) - 0.01\log(0.01) = 0.081$$

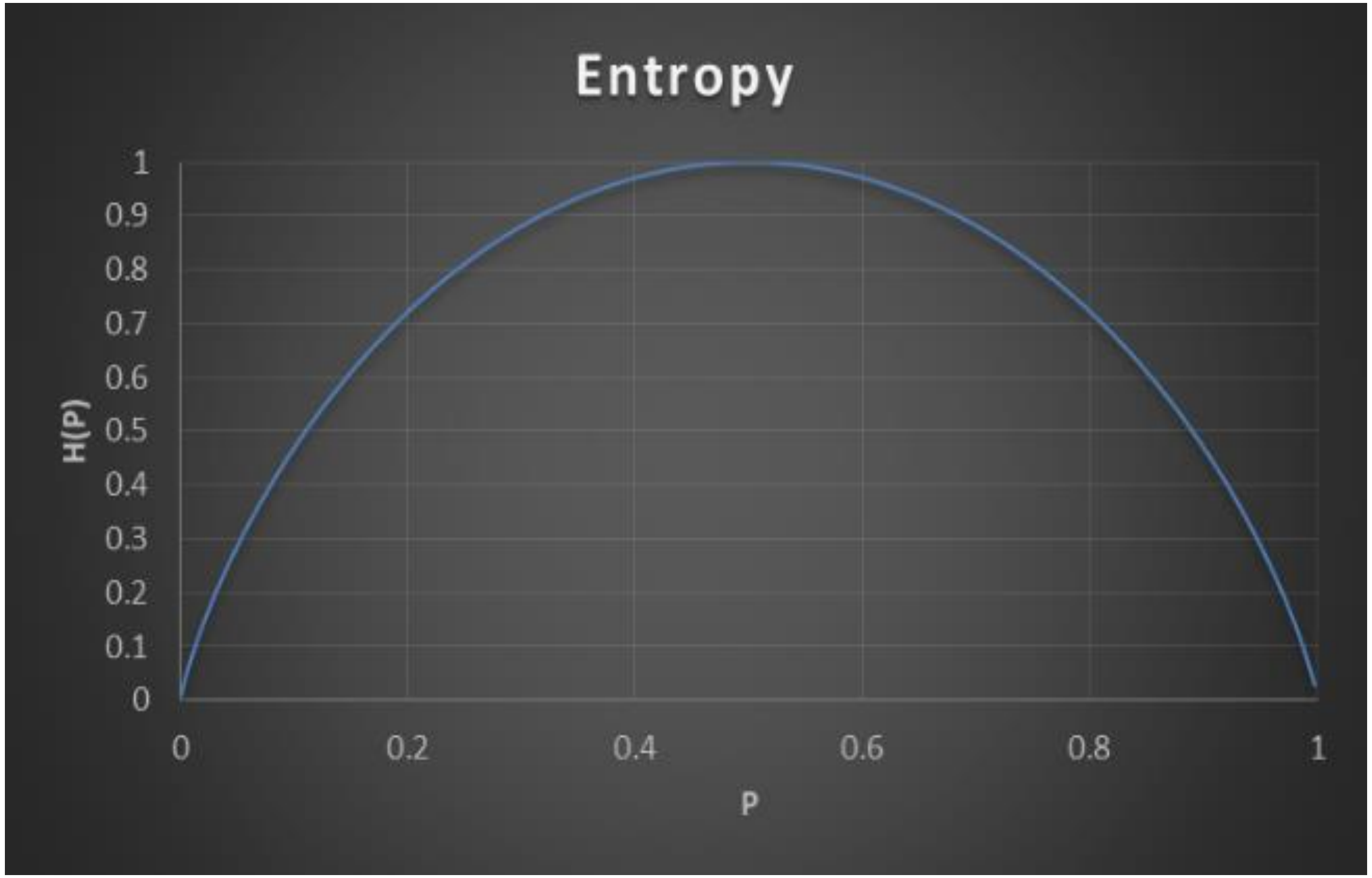
Same conclusion for information gain.

$$I(x_A) = -log_2\big(p(x_A)\big) = 1.322$$

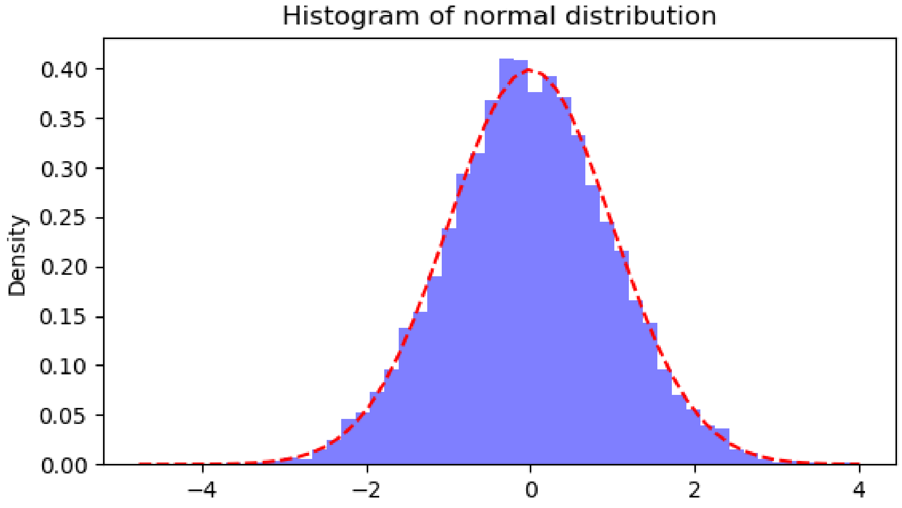
$$I(x_B) = -log_2\big(p(x_B)\big) = 0.014$$

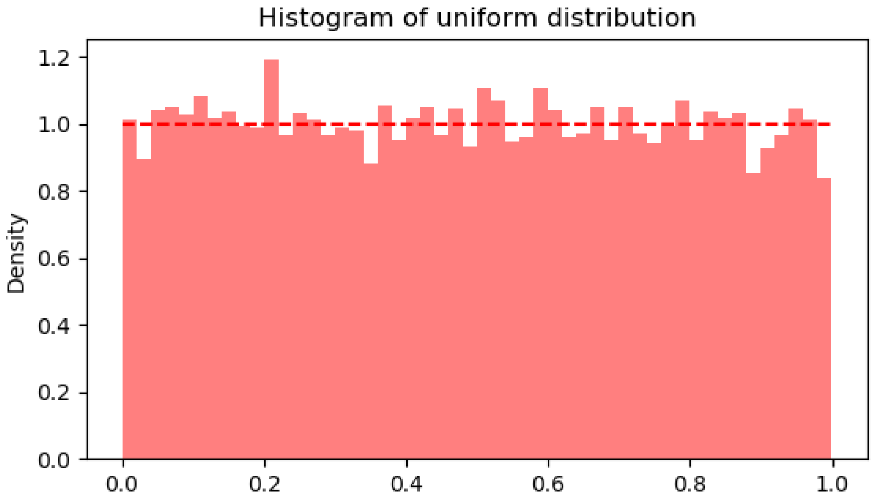
Entropy

When p=0.5 has the largest entropy.



Entropy





$p(X_A) = \begin{cases} 0.1 & x = 1 \\ 0.15 & x = 2 \\ 0.5 & x = 3 \\ 0.15 & x = 4 \\ 0.1 & x = 5 \end{cases}$	$p(X_A) = \begin{cases} 0.01 & x = 1 \\ 0.09 & x = 2 \\ 0.8 & x = 3 \\ 0.09 & x = 4 \\ 0.01 & x = 5 \end{cases}$	$p(X_B) = \begin{cases} 0.2 & x = 1 \\ 0.2 & x = 2 \\ 0.2 & x = 3 \\ 0.2 & x = 4 \\ 0.2 & x = 5 \end{cases}$
$H(X_A) = 1.985$	$H(X_A) = 1.016$	$H(X_B) = 2.322$

Cross-entropy

Formula of cross- entropy:

$$CE = \sum_{i=1}^{n} \sum_{c=1}^{C} -y_{c,i} log_2(p_{c,i})$$

C: number of class (male, female, other)

n: number of data

$y_{c,i}$: binary indicator (0 or 1) from one hot encode (i-th data assigns to c-class)

$p_{c,i}$: probability of i-th data assigns to c-class

One hot encode
(Dummy variable)

Data 1	Male
Data 2	Female
Data 3	Male
Data 4	Other

Male
Female
Other

	Male	Female	Other
Data 1	1	0	0
Data 2	0	1	0
Data 3	1	0	0
Data 4	0	0	1

Cross-entropy

$$CE = \sum_{i=1}^{n}\sum_{c=1}^{C} -y_{c,i} log_2(p_{c,i})$$

		Model 1 (輸出)					
		機率輸出			實際One-hot encode		
	Target (Label)	男生	女生	其他	男生	女生	其他
data 1	男生	0.4	0.3	0.3	1	0	0
data 2	女生	0.3	0.4	0.3	0	1	0
data 3	男生	0.5	0.2	0.3	1	0	0
data 4	其他	0.8	0.1	0.1	0	0	1
		模型1錯誤率: 1/4=0.25 Cross-entropy=6.966					

Data 1:

$$\sum_{c=1}^{C} -y_{c,1} log_2(p_{c,1})$$

$$= -1 * \log(0.4) - 0 * \log(0.3) - 0 * \log(0.3) = 1.322$$

Data 4:

$$\sum_{c=1}^{C} -y_{c,4} log_2(p_{c,4})$$

$$= -0 * \log(0.8) - 0 * \log(0.1) - 1 * \log(0.1) = 3.3219$$

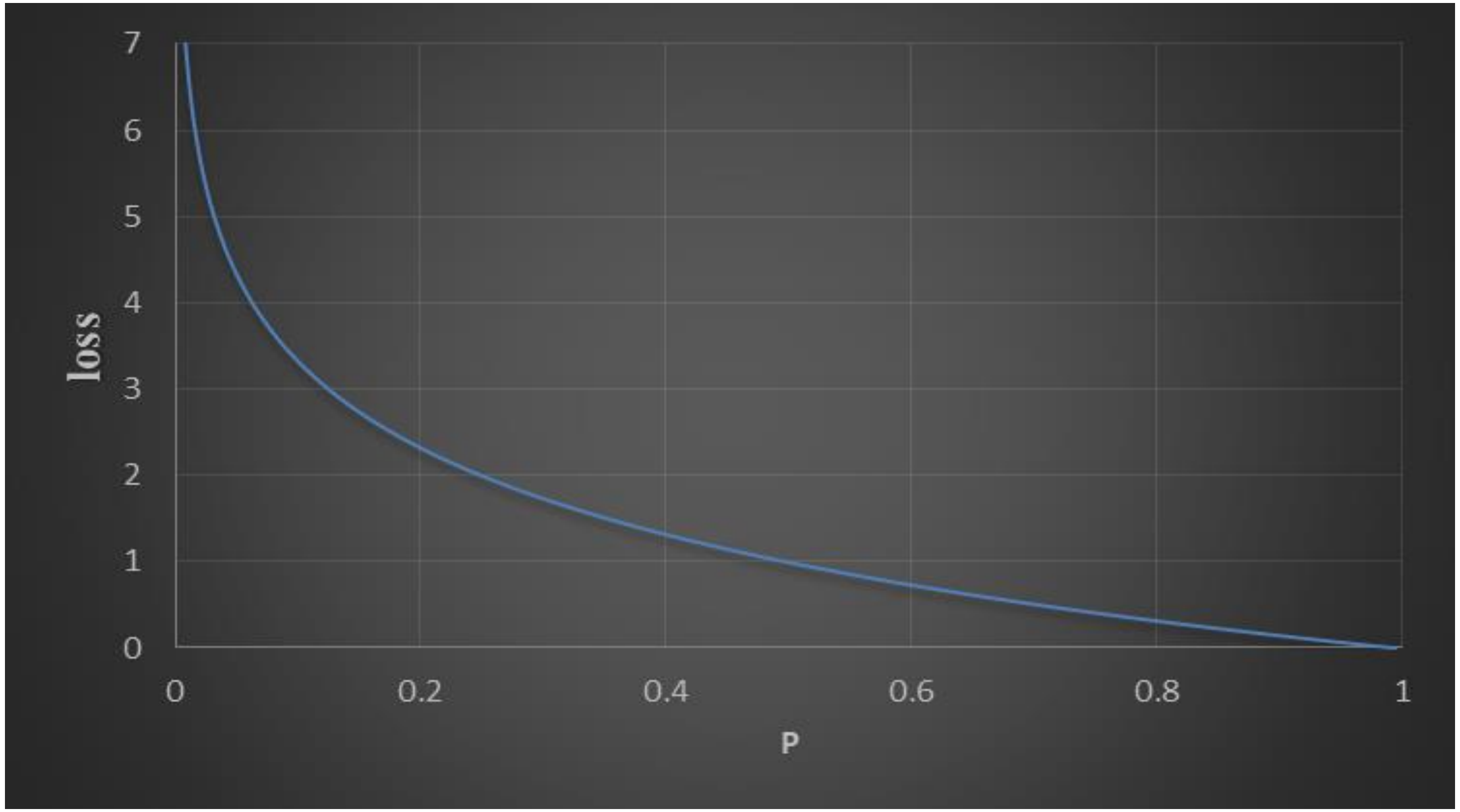
SO less probability data has larger loss function (entropy value)→learning target.

Cross-entropy
for evaluating the model performance

		Model 1 (輸出)					
		機率輸出			實際One-hot encode		
	Target (Label)	男生	女生	其他	男生	女生	其他
data 1	男生	0.4	0.3	0.3	1	0	0
data 2	女生	0.3	0.4	0.3	0	1	0
data 3	男生	0.5	0.2	0.3	1	0	0
data 4	其他	0.8	0.1	0.1	0	0	1
		模型1錯誤率: 1/4=0.25 **Cross-entropy=6.966**					

		Model 2 (輸出)					
		機率輸出			實際One-hot encode		
	Target (Label)	男生	女生	其他	男生	女生	其他
data 1	男生	0.7	0.1	0.2	1	0	0
data 2	女生	0.1	0.8	0.1	0	1	0
data 3	男生	0.9	0.1	0	1	0	0
data 4	其他	0.4	0.3	0.3	0	0	1
		模型1錯誤率: 1/4=0.25 **Cross-entropy= 2.310**					

Cross-entropy for loss function



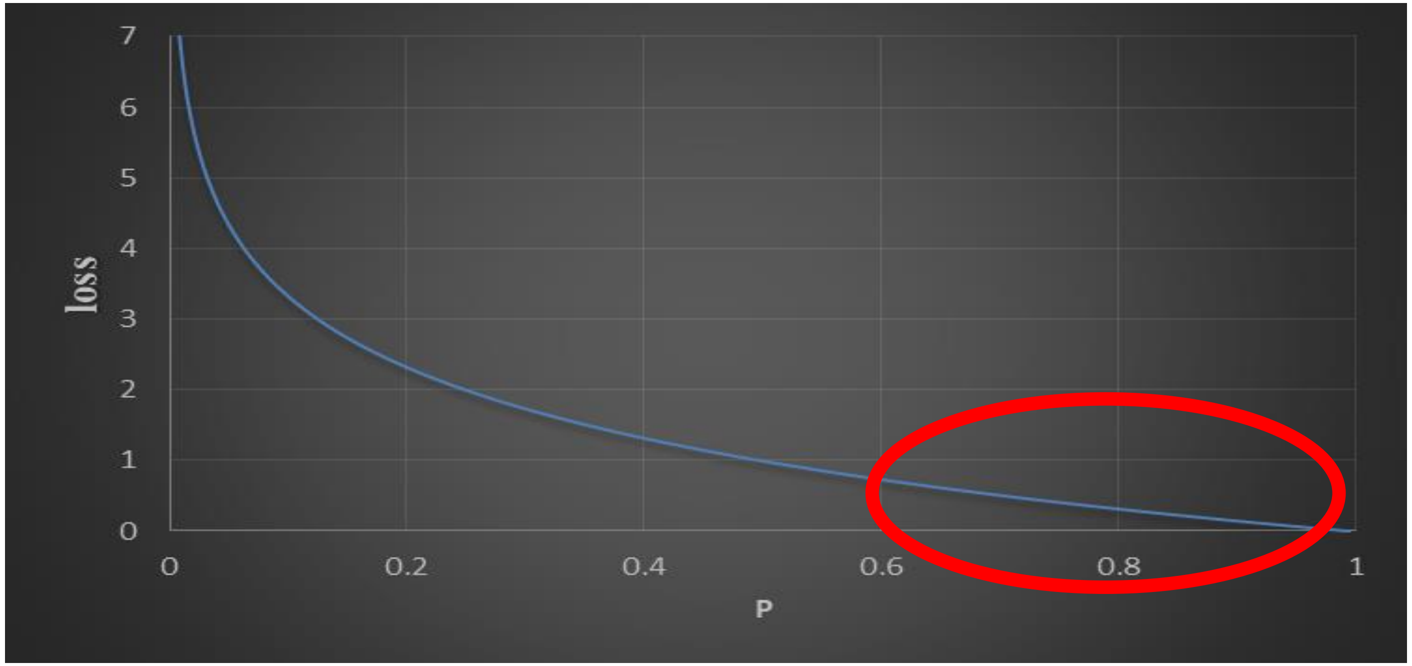
$$CE\ loss = \sum_{i=1}^{n} \sum_{c=1}^{C} -y_{c,i} log_2(p_{c,i})$$

Focal loss (1/3)

Cross-entropy (CE) for $y \in \{\pm 1\}$

$$CE(p,y) = \begin{cases} -\log(p), & if\ y = 1 \\ -\log(1-p), & if\ y = -1 \end{cases}$$

$$CE(p,y) = CE(p_t) = -\log(p_t), \qquad p_t = \begin{cases} p, & if\ y = 1 \\ 1-p, & if\ y = -1 \end{cases}$$



Focal loss (2/3)

α-balanced cross-entropy:

$$CE(p_t) = -\alpha \log(p_t)$$

BUT it’s not effect for larger class unbalance problem.

Modulating factor:

$$(1 - p_t)^r$$

r: focusing parameter, $r \geqq 0$.

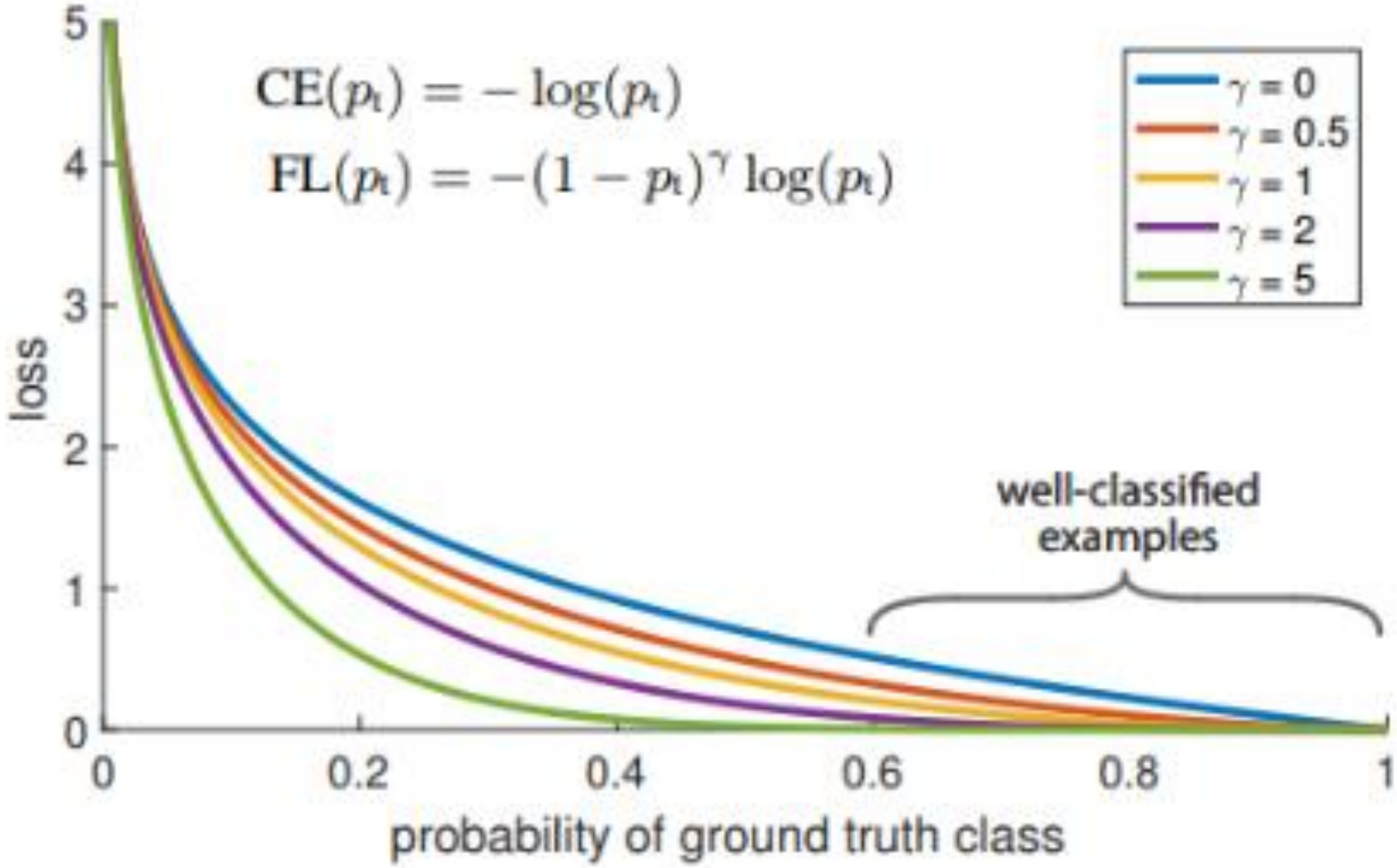
<u>**Focal loss:**</u>

$$FL(p_t) = -(1 - p_t)^r \log(p_t)$$

<u>**α-balanced focal loss:**</u>

$$FL(p_t) = -\alpha(1 - p_t)^r \log(p_t)$$

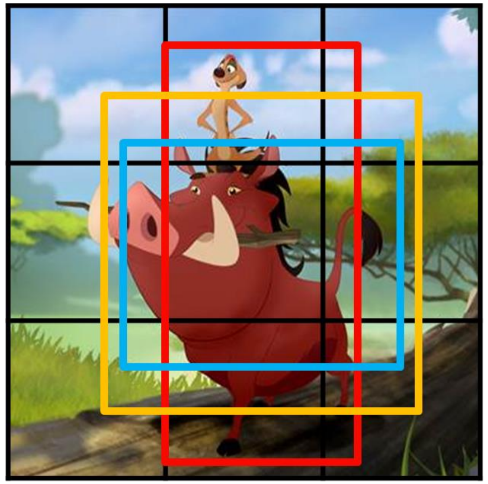
Focal loss (3/3)



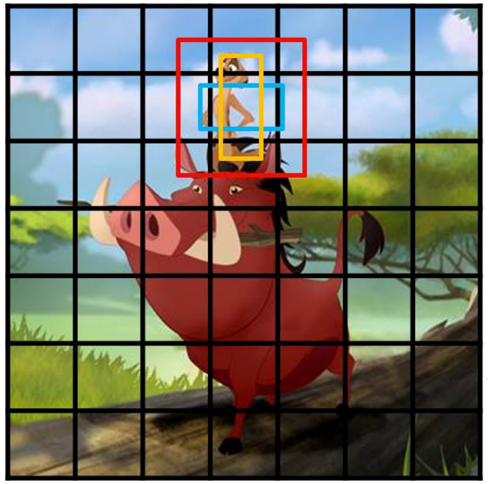
Why Focal loss? (Proposed in RetainNet)

在訓練階段的特徵圖對應的Anchor
有90%都是沒用的背景anchor ，所以要怎麼抑制背景的anchor。
怎麼強化判錯的Anchor的loss?
Focal loss。

3*3的特徵圖



7*7的特徵圖



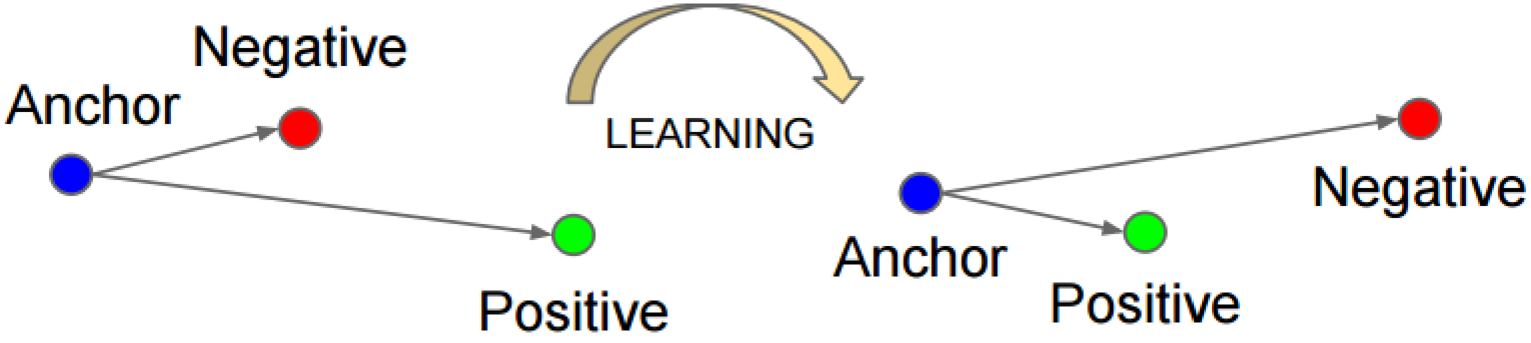
Triple Loss

Triple

Anchor: a randomly training data with label c

Positive: training data in label c

Negative: training data in other labels



Triple Loss

$$Anchor: x_i^a \qquad Positive: x_i^p \qquad Negative: x_i^n$$

$$\text{Encoder network}: f(x)$$

$$\text{Anchor}: f(x_i^a)$$

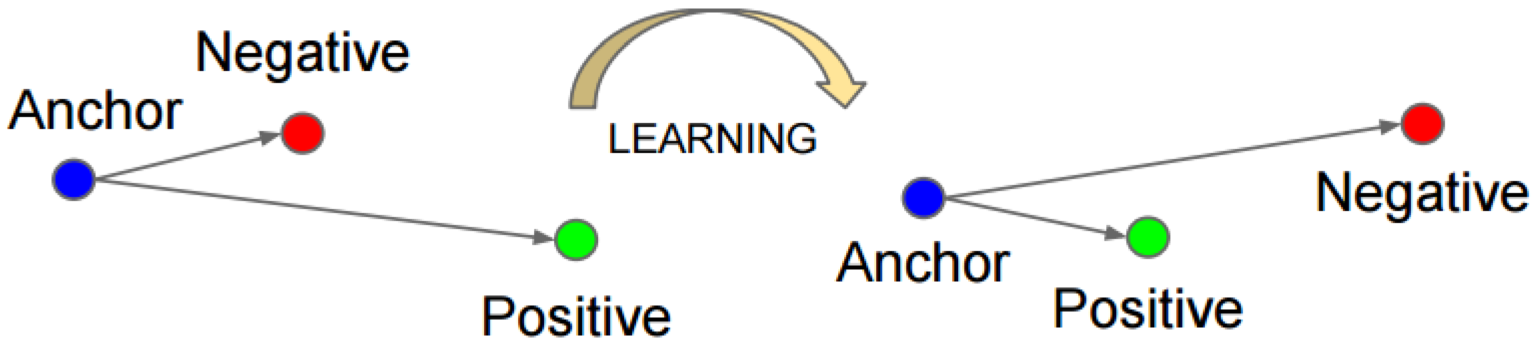
$$Positive: f(x_i^p)$$

$$Negative: f(x_i^n)$$

triple loss aims to

$$dist(f(x_i^a),(x_i^p))\downarrow$$

$$dist(f(x_i^a),(x_i^n))\uparrow$$



Triple Loss

$$dist(f(x_i^a), f(x_i^p)) + \alpha < dist(f(x_i^a), f(x_i^n))$$

$$\Rightarrow \left\| f(x_i^a) - f(x_i^p) \right\|_2^2 + \alpha < \left\| f(x_i^a) - f(x_i^n) \right\|_2^2$$

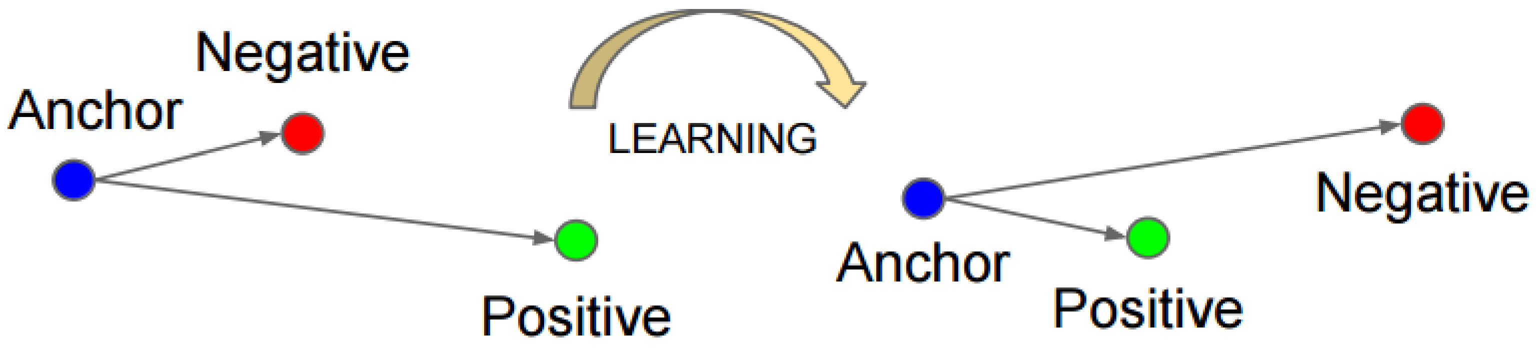
$$\arg\min\{\sum_i \left(\left\| f(x_i^a) - f(x_i^p) \right\|_2^2 + \alpha - \left\| f(x_i^a) - f(x_i^n) \right\|_2^2 \right)\}$$

$$\Downarrow$$

$$\arg\min \sum_i \left[\left\| f(x_i^a) - f(x_i^p) \right\|_2^2 + \alpha - \left\| f(x_i^a) - f(x_i^n) \right\|_2^2 \right]_+$$

Triple Loss

$$\left[d_p - d_n + \alpha\right]_+ = \begin{cases} d_p - d_n + \alpha & d_p - d_n + \alpha > 0 \\ 0 & d_p - d_n + \alpha < 0 \end{cases}$$

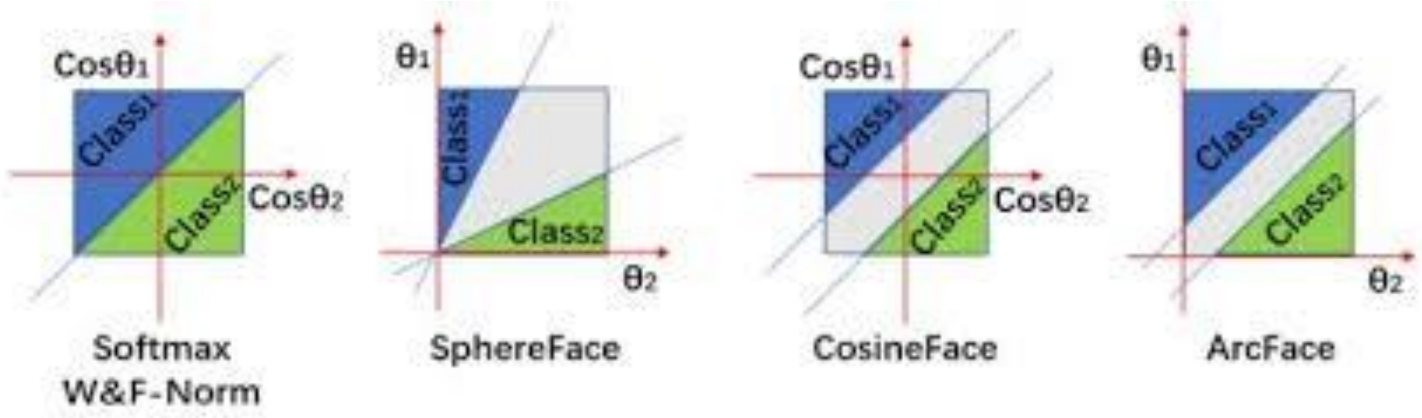


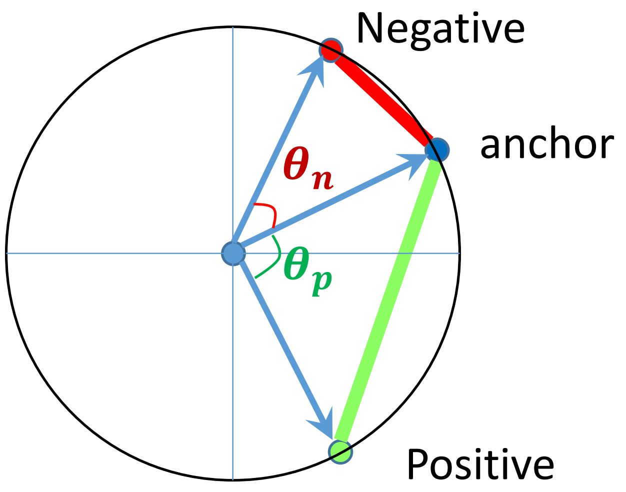
Conclusion

MSE, MAE, Huber loss, triple loss do the same thing.

Similarity measurement.

Cosine loss





Can MSE be a loss for classification?