PROBABILITY

- **Random Experiment** (E) : An experiment which can give multiple outcomes
- **Sample Space** (S) : Set of all outcomes of experiment
- **Event** : subset of sample space which is outcome of an experiment

- **Mutually Exclusive Events** : let $A, B \subset S$, A and B are mutually exclusive if $A \cap B = \phi$

Axiomatic Definition (Kolomogorov's ~~Th~~ Axioms) :

- $0 \le P(A) \le 1$
- $P(S) = 1$
- if A & B are Mutually exclusive (M.E), $P(A \cup B) = P(A) + P(B)$
- if $A_1, A_2, \ldots A_n$ M.E events, $P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$

$\Rightarrow S = A \cup \bar{A}$

$P(S) = P(A \cup \bar{A})$

$= P(A) + P(\bar{A})$

$P(A) + P(\bar{A}) = 1$

$\Rightarrow A \cup \phi = A$

$P(A \cup \phi) = P(A)$

$P(A) + P(\phi) = P(A)$

$P(\phi) = 0$

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$\Rightarrow$ If $A \subset B$

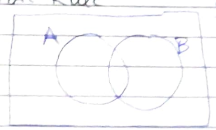
$P(A) \leq P(B)$

$B = A \cup (\bar{A} \cap B)$

$P(B) = P(A) + P(\bar{A} \cap B)$

$\therefore P(A) \leq P(B)$

$\rightarrow$ Addition Rule



$A = (A \cap \bar{B}) \cup (A \cap B)$ — ①

$B = (B \cap \bar{A}) \cup (A \cap B)$ — ②

$A \cup B = (A \cap \bar{B}) \cup (\bar{A} \cap B) \cup (A \cap B)$

$P(A \cup B) = P(A \cap \bar{B}) + P(\bar{A} \cap B) + P(A \cap B)$ — ③

$P(A) = P(A \cap \bar{B}) + P(A \cap B)$ — ④

$P(B) = P(\bar{A} \cap B) + P(A \cap B)$ — ⑤

from ③, ④, ⑤

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

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$P(A \cup (B \cup C))$ $\qquad D = B \cup C$

$= P(A) + P(D) - P(A \cap D)$

$= P(A) + P(B \cup C) - [P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)]$

$= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$

Q. Show that probability of ... A or B ...

= Probability ...

$P(A \cap \bar{B}) + P(\bar{A} \cap B)$ ...

$P(B) = \frac{3}{8}$

ii) $\frac{3}{4} \le P(A \cup B)$

$\frac{1}{8} \le P(A \cap B) \le \frac{3}{8}$

iii) $\frac{3}{8} \le P(A \cap \bar{B}) \le \frac{5}{8}$

$A \cup B \subseteq S$

$P(A \cup B) \le P(S)$

$P(A \cup B) \le 1$

$P(A) + P(B) - P(A \cap B) \le 1$

$P(A \cap B) \ge \frac{3}{4} + \frac{3}{8} - 1$

$\ge \frac{3}{4} - \frac{5}{8} \ge \frac{1}{8}$ — ①

$A \subseteq A \cup B$

$P(A) \le P(A \cup B)$

$\boxed{\frac{3}{4} \le P(A \cup B)}$

$A \cap B \subseteq B$

$P(A \cap B) \le P(B)$

$P(A \cap B) \le \frac{3}{8}$ — ②

from ① & ②.

$\boxed{\frac{1}{8} \le P(A \cap B) \le \frac{3}{8}}$

Q. A pair dice are rolled. What is probability that

i) sum > 10 and

ii) sum neither 8 nor 10

i) $P(A) = \frac{3}{36} = \frac{1}{12}$ $\{(5,6), (6,5), (6,6)\}$

ii) $P(\bar{A}) = 1 - P(\text{sum } 8 \text{ or } 10)$

$= 1 - \frac{8}{36} = \frac{28}{36} = \frac{7}{9}$

Q. From 6 +ve and 8 -ve integers, 4 numbers are chosen at random and are multiplied. F... probability that product is positive

$\Rightarrow$ Total ways $= {}^{14}C_4 = 1001$

Case 1 : all +ve $= {}^{6}C_4 = 15$

Case 2 : all -ve $= {}^{8}C_4 = 70$

Case 3 : 2 +ve, 2 -ve $= {}^{6}C_2 \times {}^{8}C_2 = 420$

Probability $= \frac{505}{1001}$

Q. 4-person committee is to be appointed from 4 officers of Purchase Dept, 3 officers of Production Dept, 2 officers of Sales Dept and 1 CA

i) 1 from each category
ii) atleast 1 from purchase dept.
iii) should have CA.

$\Rightarrow$ Total cases $= {}^{10}C_4 = 210$

i) No. of ways $= {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times {}^1C_1$
$= 24$

Probability $= \frac{24}{210} = \frac{4}{35}$

ii) Probability $= 1 -$ None from purchase
$= 1 - \frac{{}^6C_4}{{}^{10}C_4}$

$= 1 - \frac{15}{210}$
$= 1 - \frac{1}{14} = \frac{13}{14}$

iii) $\frac{{}^9C_3}{{}^{10}C_4} = \frac{84}{210} = 0.4$

Q. 4 people are chosen at random from a group consisting of 4 men, 3 women & 2 children. Find the chance that the selected group contains atleast 1 child.

$\Rightarrow$ Total ways $= {}^9C_4 = 126$

Probability $= 1 - P(\text{no child})$
$= 1 - \frac{{}^7C_4}{{}^9C_4}$
$= 1 - \frac{35}{126} = \frac{91}{126} = 0.72$

Q. Determine probability that atleast 1 head appears in 4 tosses of a fair coin

$\Rightarrow$ Probability $= 1 - P(\text{no head})$
$= 1 - \left(\frac{1}{2}\right)^4 = \frac{15}{16}$

Q. A pair of dice is tossed 6 times. What is the probability that all faces ~~with~~ will appear twice.

$\Rightarrow$ Total ways $= (36)^6$
Total faces after 6 rolls $= 12$
No of ways $= \frac{12!}{(2!)^6}$ because each face appears twice

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Q. The coefficients a, b, c of quadratic eqⁿ $ax^2 + bx + c = 0$ are determined by throwing a die 3 times. Find probability that:

i) roots real
ii) roots complex

$\Rightarrow$ i) $b^2 - 4ac \geq 0$

$b^2 \geq 4ac$

Total ways = 216

b	a	c	
1	x	x	— 0 cases
2	1	1	— 1 case
3	1 2	2 1	— 2 + 1 cases
4	1 1 4 1 2	3 4 1 4 2	5 + 3 cases
5	1 5 2 3 1 6	5 1 3 2 6 1	6 + 8 cases
6	3 4 2	3 2 4	3 + 14 cases

Probability of real roots $= \frac{1+3+8+14+17}{216} = \frac{43}{216} = 0.199$

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Probability of complex roots $= \frac{173}{216} = 0.801$

Q. If 4 squares are chosen at random on a chess board, find the chance that they should be in a diagonal line.

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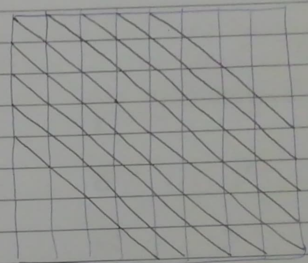


Q A bag contains 40 tickets numbered 1 to 40 of which 4 are drawn at random and arranged in ascending order. What is the probability that t_3 is 25.

$= \frac{{}^{24}C_2 \times {}^{15}C_1}{{}^{40}C_4}$

first 24 tickets

last 15 tickets

= 0.045

Q. 3 groups of children contains (3G, 1B), (2G, 2B) and (1G, 3B). One child is selected at random from each group. Show that the chance that the 3 children picked consist (1G, 2B) is $\frac{13}{32}$

⇒ Total $= {}^4C_1 \times {}^4C_1 \times {}^4C_1$
$= 64$

	G	B	B	
Group 1	${}^3C_1$	${}^2C_1$	${}^3C_1$	= 18
Group 2	${}^2C_1$	${}^1C_1$	${}^3C_1$	= 6
Group 3	${}^1C_1$	${}^1C_1$	${}^2C_1$	= 2

Probability $= \frac{26}{64} = \frac{13}{32}$

Q. The odds that a book be reviewed favourably by 3 independent critics are in the ratio 5 to 2, 4 to 3, 3 to 4. What is the probability that of the 3 reviews, a majority will be favorable

	A	B	C
Favour	5/7	4/7	3/7
No favour	2/7	3/7	4/7

$P(\text{Majority}) = P(ABC) + P(AB\bar{C}) + P(A\bar{B}C) + P(\bar{A}BC)$

$= \frac{209}{343}$

Q. Find the probability that among 7 person

i) no 2 are born are born on same day of week
ii) atleast 2 were born on same
iii) 2 are born on Sunday & 2 are born on tuesday

i) Prob. $\frac{{}^7C_1}{7} \times \frac{{}^6C_1}{7} \times \frac{{}^5C_1}{7} \times \frac{{}^4C_1}{7} \times \frac{{}^3C_1}{7} \times \frac{{}^2C_1}{7} \times \frac{{}^1C_1}{7}$

$= \frac{7!}{7^7}$

ii) P(atleast 2 on same day) = 1 − P(no 2 on same day)
$= 1 - \frac{7!}{7^7}$

iii) Probability $= \frac{{}^7C_2}{7 \times 7} \times \frac{{}^5C_2}{7 \times 7} \times \frac{5}{7} \times \frac{5}{7} \times \frac{5}{7}$

Q. A box containing 6 red, 4 white, 5 black balls. A person draws 4 balls. Find prob. that there is atleast 1 ball of each color.

$\Rightarrow$ Total ways $= {}^{15}C_4 =$

1R 1W 2B
${}^6C_1 \times {}^4C_1 \times {}^5C_2 = 240$

1R 2W 1B
${}^6C_1 \times {}^4C_2 \times {}^5C_1 = 180$

2R 1W 1B
${}^6C_2 \times {}^4C_1 \times {}^5C_1 = 300$

Prob $= \frac{720}{1365} = 0.527$

Q. Person A & B throw a pair of dice. A wins if he throws sum 6 before B throws sum 7 and vice versa. If A begins, find his chances of winning.

$\frac{30}{61}$

(without repitition)

Q. A five figure number is formed by 0, 1, 2, 3, 4. Find the prob. that no. is formed and divisible by 4.

$\Rightarrow$ numbers ending with : 04, 12, 20, 24, 32, 40

Total $= 4 \times 4! = 96$

no. of ways $\Rightarrow$ $3!$ $(2\times2\times1)$ $3!$ $(2\times2\times1)$ $(2\times2\times1)$ $3!$

Req. prob. $= \frac{(3\times3!) + (3\times4)}{96} = \frac{30}{96} = \frac{5}{16}$

Q. 3 winning tickets are drawn from an urn containing 100 tickets. What is prob of winning if a person buys:

i) 4 tickets
ii) 1 ticket

⇒ i) Total = $^{100}C_4$

$P(\text{loosing}) = \frac{^{97}C_4}{^{100}C_4}$

Req. prob. $= 1 - \frac{^{97}C_4}{^{100}C_4} = 0.1164$

ii) Req. prob. $= 1 - \frac{^{97}C_1}{^{100}C_1} = 0.03$

Q. By induction, show that for arbitrary events $A_1, A_2, \ldots A_n$.

$$P\{A_1 A_2 \ldots A_n\} \geq \sum_{i=1}^{n} P(A_i) - (n-1)$$

$= P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

$P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2)$

$= \sum_{i=1}^{2} (A_i) - 1$

$= \sum_{i=1}^{2} (A_i) - (2-1)$

$= P[(A_1 \cap A_2 \cap \ldots A_k) \cap A_{k+1}]$

$= P(A_1 \cap A_2 \ldots \cap A_k) + P(A_{k+1})$

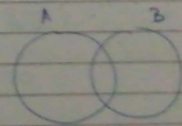
$= \sum_{i=1}^{k} P(A_i) - (k-1) - 1$

$= \sum_{i=1}^{k} P(A_i) - k$

Independent Events

2 sets A & B are independent iff $P(A \cap B) = P(A)P$

- A & $\bar{B}$ are independent
- $\bar{A}$ & B are independent
- $\bar{A}$ & $\bar{B}$ are independent



To prove: $P(A \cap \bar{B}) = P(A)\,P(\bar{B})$

$(A \cup B) = (A \cap \bar{B}) \cup B$

$P(A \cup B) = P[(A \cap \bar{B}) \cup B]$

$P(A) + \cancel{P(B)} - P(A \cap B) = P(A \cap \bar{B}) + \cancel{P(B)}$

$P(A) - P(A)P(B) = P(A \cap \bar{B})$

$P(A)\,P(\bar{B}) = P(A \cap \bar{B})$

To prove: $P(\bar{A} \cap B) = P(\bar{A})\,P(B)$

$(A \cup B) = (\bar{A} \cap B) \cup A$

$P(A \cup B) = P[(\bar{A} \cap B) \cup A]$

$\cancel{P(A)} + P(B) - P(A \cap B) = P(\bar{A} \cap B) + \cancel{P(A)}$

$P(B) - P(A)\,P(B) = P(\bar{A} \cap B)$

$P(\bar{A})\,P(B) = P(\bar{A} \cap B)$

To prove: $P(\bar{A} \cap \bar{B}) = P(\bar{A})\,P(\bar{B})$

$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - P(A) - P(B) + P(A)\,P(B)$

$= [1 - P(A)] - P(B)\,[1 - P(A)]$

$= [1 - P(A)]\,[1 - P(B)]$

$= P(\bar{A})\,P(\bar{B})$

Conditional Probability

$P(B/A) \Rightarrow$ read as "B given A"

means "Probability that B occurs given that A has already occurred"

$$\boxed{P(B/A) = \frac{P(B \cap A)}{P(A)}}$$

$P(A \cap B) = P(B/A)\,P(A)$ [Multiplication Rule]

Q. In a certain town, 40% have brown hair, 25% have brown eyes & 15% have both. A person is selected at random. What is the probability that

i) If he has brown hair, he has brown eyes.

ii) If he has brown eyes, he doesn't has brown hair

iii) He has neither brown hair nor brown eyes.

$\Rightarrow$ i) Let A = brown hair, B = brown eyes

$$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{15/100}{40/100} = \frac{15}{40} = 0.375$$

ii) $P(\bar{A}|B) = \dfrac{P(\bar{A}\cap B)}{P(B)}$

$= \dfrac{\frac{25}{100} - \frac{15}{100}}{\frac{25}{100}} = \dfrac{\frac{10}{100}}{\frac{25}{100}} = \underline{0.4}$

iii) $P(\bar{A}\cap\bar{B}) = 1 - P(A\cup B)$

$= 1 - [P(A) + P(B) - P(A\cap B)]$

$= 1 - (0.4 + 0.25 - 0.15)$

$= 1 - 0.5 = 0.5 \;\; \underline{(50\%)}$

Q. A bag contains 10 gold & 8 silver coins. 2 successive drawings of 4 coins are made such that :

i) coins are replaced before second trial
ii) coins aren't replaced before second trial

Find prob. that first drawing will give 4 gold and second drawing will give 4 silver coins

$\Rightarrow$ Let A = drawing 4 gold coins in first draw
B = drawing 4 silver coins in second draw

i) $P(A) = \dfrac{^{10}C_4}{^{18}C_4}$; $P(B) = \dfrac{^{8}C_4}{^{18}C_4}$

$P(B|A) = \dfrac{P(B\cap A)}{P(A)}$

$P(B\cap A) = P(B|A)\,P(A)$

$= \dfrac{^{8}C_4}{^{18}C_4} \times \dfrac{^{10}C_4}{^{18}C_4}$

ii) $P(A) = \dfrac{^{10}C_4}{^{18}C_4}$; $P(B|A) = \dfrac{^{8}C_4}{^{14}C_4}$

$P(A\cap B) = \dfrac{^{10}C_4}{^{18}C_4} \times \dfrac{^{8}C_4}{^{14}C_4}$

Q. A box contains 4 bad and 6 good tubes. Tubes are checked by drawing at random, testing it and repeati[ng] until 4 tubes are located. What is prob that 4th bad tube will be located on

i) 5th test
ii) 10th test

$\Rightarrow$ i) A: drawing 3 bad & 1 good tube
B: drawing 1 bad tube in 5th test.

$P(A\cap B) = P(B|A) \times P(A)$

$= \dfrac{^{1}C_1}{^{6}C_1} \times \dfrac{^{4}C_3 \times {}^{6}C_1}{^{10}C_4} = \dfrac{^{4}C_3}{^{10}C_4} = \dfrac{[illegible]}{^{10}C_4}$

4 tubes removed

ii) Req. prob. $= \frac{{}^4C_3}{{}^{10}C_9} = \frac{4}{10} = 0.4$

Partition

Q.

B_i is event of sample space S

- $\bigcup_{i=1}^{n} B_i$
- $B_i \cap B_j = \phi$

$A = A \cap S$
$= A \cap (B_1 \cup B_2 \cup \dots B_n)$
$A = (A \cap B_1) \cup \dots (A \cap B_n)$

- $P(A) = \sum_{i=1}^{n} (A \cap B_i)$. — ①
- $P(B_i / A) = \frac{P(A \cap B_i)}{P(A)}$
- $P(A / B_i) = \frac{P(A \cap B_i)}{P(B_i)}$
- $P(A \cap B_i) = P(A / B_i)\, P(B_i) = P(B_i / A)\, P(A)$ — ②

①, ② : Total probability theorem

→ Baye's Theorem

$$P(B_i / A) = \frac{P(A / B_i)\, P(B_i)}{\sum_{i=1}^{n} P(B_i)\, P(A / B_i)}$$

★ Prove Total Probability Theorem to prove Baye's Theorem.

★★★ Q. 3 machines A, B, C produce respectively 40%, 10%, 50% of items in a factory. The % of defective items produced at 2%, 3%, 4% respectively. An item is selected at random.

i) Find prob. that item is defective
ii) If item is defective, find prob. that item was produced by
a) A b) B c) C

⇒ D : defective item B : items from B
A : items from A C : items from C

$P(D) = P(A)\, P(D/A) + P(B)\, P(D/B) + P(C)\, P(D/C)$
$= (0.4 \times 0.02) + (0.1 \times 0.03) + (0.5 \times 0.04)$
$= 0.008 + 0.003 + 0.02 = 0.031$
$= 3.1\%$

ii) $P(A/D) = \frac{P(D/A)\,P(A)}{P(D)}$

$= \frac{0.02 \times 0.4}{0.031} = 0.26 = 26\%$

$P(B/D) = \frac{P(D/B)\,P(B)}{P(D)}$

$= \frac{0.03 \times 0.1}{0.031} = 0.097 = 9.7\%$

$P(C/D) = \frac{P(D/C)\,P(C)}{P(D)}$

$= \frac{0.5 \times 0.04}{0.031} = 0.64\cancel{5}^{3} = 64.\cancel{5}^{3}\%$

Q. 3 companies X, Y, Z produce TV. X produce twice as Y. Y and Z produced the same. 2% of X, 2% of Y and 4% of Z are defective. All TV produced are put into 1 shop and 1 TV selected at random.

i) What prob. that TV defective
ii) If TV is defective, what prob. that it is produced by X.

$P(X) + P(Y) + P(Z) = 1$
$2k + k + k = 1$
$k = \frac{1}{4}$

$\Rightarrow$ D: defective

$P(D) = P(X)\,P(D/X) + P(Y)\,P(D/Y) + P(Z)\,P(D/Z)$
$= 0.5 \times 0.02 + 0.25 \times 0.02 + 0.25 \times 0.04$
$= 0.01 + 0.005 + 0.01$
$= 0.025 = 2.5\%$

$P(X/D) = \frac{P(D/X)\,P(X)}{P(D)}$

$= \frac{0.02 \times 0.5}{0.025} = 0.4 = 40\%$

Q. What is the prob. that a randomly contain 53 sundays.

Normal - 1/7
Leap - 2/7

Q. Randomly selected year is observed to have 53 sundays. Find prob. that it's a leap year.

= S: 53 sundays
L: leap year
NL: not leap year

$P(L) = \frac{1}{4}$; $P(NL) = \frac{3}{4}$

$P(S/L) = \frac{2}{7}$; $P(S/NL) = \frac{1}{7}$

$\therefore P(L/S) = \frac{P(S/L)\,P(L)}{P(S/L)\,P(L) + P(S/NL)\,P(NL)}$

$$P(L/S) = \frac{\frac{2}{7}\times\frac{1}{4}}{\frac{2}{7}\times\frac{1}{4} + \frac{1}{7}\times\frac{3}{4}} = \frac{2}{5} = 0.4$$

Q. A box contains 10 coins where 5 coins are two-headed, 3 coins are 2-tailed & 2 fair coins. A coin is chosen at random and tossed. Find prob. that:

i) head appears
ii) if head appears, it is fair coin

⇒ H: head appears
i) TH: 2 headed
TT: 2 tailed
F: fair coin

$$P(H) = P(H/TH)\,P(TH) + P(H/TT)\,P(TT) + P(H/F)\,P(F)$$
$$= 1\times\frac{1}{2} + 0 + \frac{1}{2}\times\frac{2}{10}$$
$$= \frac{1}{2} + \frac{1}{10} = 0.5 + 0.1 = 0.6$$

ii) $P(F/H) = \frac{P(H/F)\,P(F)}{P(H)}$

$$= \frac{\frac{1}{2}\times\frac{2}{10}}{} = \frac{0.1}{0.6} = 0.167$$

Q. From a vessel containing 3 white & 5 black balls, 4 balls are transferred to empty vessel. A ball is drawn and found to be white. What is probability that out of 4 balls, 3 are white & 1 is black.

= E_1 : 4B
E_2 : 3B + 1W
E_3 : 2B + 2W
E_4 : 1B + 3W
E : drawing white ball from second vessel

$$P(E_2/E) = \frac{P(E/E_2)\,P(E_2)}{P(E)}$$

$$P(E) = P(E/E_1)P(E_1) + P(E/E_2)P(E_2) + P(E/E_3)P(E_3) + P(E/E_4)P(E_4)$$

$$= 0 + \frac{({}^1C_1)({}^3C_3)({}^5C_1)}{{}^4C_1 \times {}^8C_4} + \frac{({}^3C_2)({}^5C_2)({}^2C_1)}{{}^8C_4 \times {}^4C_1} + \frac{({}^5C_1)({}^3C_3)({}^3C_1)}{{}^8C_4 \times {}^4C_1}$$

$$= \frac{5 + 60 + 15}{{}^8C_4 \times {}^4C_1} = \frac{80}{280} = \frac{2}{7}$$

$$P(E_2/E) = \frac{P(E/E_2)\,P(E_2)}{P(E)}$$

$$= \frac{\frac{{}^1C_1}{{}^4C_1} \times \frac{{}^3C_3 \times {}^5C_1}{{}^8C_4}}{\frac{2}{7}} = \frac{\frac{5}{\cancel{280}\,40}}{\frac{2}{\cancel{7}}} = \frac{5}{80}$$

Q. Box 1 → 10 white + 3 black

Box 2 → 3 white + 5 black

2 balls are drawn at random from box 1 and placed in box 2. Then 1 ball is drawn from box 2, what is prob. that it is white

Q. It is suspected that a patient has one of the diseases A_1, A_2, A_3. Suppose that people suffering are in ratio 2:1:1. The patient is given a test which turns out to be +ve 25% of A_1, 50% of A_2 and 90% of A_3.
Given that out of 3 tests, 2 were +ve, then find prob. for each of these illness.

⇒ A_i = patient has i^{th} illness
$P(A_1) = \frac{1}{2}$; $P(A_2) = \frac{1}{4}$; $P(A_3) = \frac{1}{4}$

B : patient took test (2 +ve)

$$P(B) = P(B/A_1)P(A_1) + P(B/A_2)P(A_2) + P(B/A_3)P(A_3)$$
$$= {}^3C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)\left(\frac{1}{2}\right) + {}^3C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)\left(\frac{1}{4}\right) + {}^3C_2\left(\frac{9}{10}\right)^2\left(\frac{1}{10}\right)\left(\frac{1}{4}\right)$$

$$= 3\left(\frac{3}{128} + \frac{1}{32} + \frac{81}{4000}\right) = 0.2248$$

$$P(A_1/B) = \frac{P(B/A_1)\,P(A_1)}{0.2248}$$

$$= \frac{\frac{9}{128}}{0.2248} = 0.3128$$

$$P(A_2/B) = \frac{P(B/A_2)\,P(A_2)}{0.2248} = \frac{3/32}{0.2248} = 0.417$$

$$P(A_3/B) = \frac{P(B/A_3)\,P(A_3)}{0.2248} = \frac{243/4000}{0.2248} = 0.2702$$

Q. An urn A contains 5B + 6W balls, urn B contains 8B + 4W balls. 2 balls are transferred from B to A. Then a ball is drawn from urn A.

i) What is prob. that it is white ball.
ii) given that ball is drawn is white, what is prob. that atleast 1 white ball transferred.

⇒ E_1 : 2B   I : A → 7B + 6W
E_2 : 1B + 1W   II : A → 6B + 7W
E_3 : 2W   III : A → 5B + 8W

$P(E_1) = \frac{{}^8C_2}{{}^{12}C_2}$; $P(E_2) = \frac{8\times4}{{}^{12}C_2}$; $P(E_3) = \frac{{}^4C_2}{{}^{12}C_2}$

W : drawing white ball from A.

$$P(W) = P(E_1)P(W/E_1) + P(E_2)P(W/E_2) + P(E_3)P(W/E_3)$$
$$= \frac{{}^8C_2}{{}^{12}C_2}\times\frac{{}^6C_1}{{}^{13}C_1} + \frac{32}{{}^{12}C_2}\times\frac{{}^7C_1}{{}^{13}C_1} + \frac{{}^4C_2}{{}^{12}C_2}\times\frac{{}^8C_1}{{}^{13}C_1}$$

$$P(W) = \frac{168 + 224 + 48}{{}^{12}C_2 \times {}^{13}C_1}$$

$$= \frac{440}{858} = 0.5128$$

ii) $P(E_2/W) + P(E_3/W)$

$$= \frac{P(W/E_2)\,P(E_2)}{P(W)} + \frac{P(W/E_3)\,P(E_3)}{P(W)}$$

$$= \frac{\frac{32}{{}^{12}C_2} \times \frac{7}{13} + \frac{{}^{4}C_2}{{}^{12}C_2} \times \frac{8}{13}}{0.5128}$$

$$= 0.6182$$

Q. 2 absent minded roommates forget their umbrellas some way or another. 'A' always take umbrella when go out, 'B' forget to take with prob. $= \frac{1}{2}$.

Each of them forgets umbrella at a shop with prob. $= \frac{1}{4}$. After visiting 3 shops, they return home. Find prob. that :

a) They have both umbrella
b) They have only 1 umbrella
c) B lost umbrella given that there's only 1 umbrella after their return.

⇒ A_i : A forget umbrella at i^{th} shop
B_i : B forget umbrella at i^{th} shop
B_0 : B leave umbrella at home.

$P(A_i) = \frac{1}{4}$; $P(B_i) = \frac{1}{4}$; $P(B_0) = \frac{1}{2}$

i)
⇒ $P\{\text{Both umbrellas}\} = P(\bar{A}_1 \bar{A}_2 \bar{A}_3)P(B_0) + P(\bar{A}_1 \bar{A}_2 \bar{A}_3)\,P(\bar{B}_0 \bar{B}_1 \bar{B}_2 \bar{B}_3)$

$$= \left(\frac{3}{4}\right)^3\left(\frac{1}{2}\right) + \left(\frac{3}{4}\right)^3\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)^3$$

$$= \frac{27}{128} + \frac{\cancel{4}729}{8192}$$

$$= 0.2999$$

ii) $P\{\text{only 1 umbrella}\} =$

$P(\bar{A}_1 \bar{A}_2 \bar{A}_3)\left[P(\bar{B}_0 B_1) + P(\bar{B}_0 \bar{B}_1 B_2) + P(\bar{B}_0 \bar{B}_1 \bar{B}_2 B_3)\right]$

B loses umbrella

$$= \left(\frac{3}{4}\right)^3\left(\frac{1}{2}\right)\left[\frac{1}{4} + \left(\frac{3}{4}\right)\left(\frac{1}{4}\right) + \left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)\right]$$

$$= \frac{27}{128}\left(\frac{1}{4} + \frac{3}{16} + \frac{9}{64}\right) = 0.1219$$

$P(\overline{B_0}\,\overline{B_1}\,\overline{B_2}\,\overline{B_3})\left[P(A_1) + P(\overline{A_1}\,A_2) + P\,\overline{A_1}\,\overline{A_2}(A_3)\right] +$

$P(B_0)\;\; P(A_1) + P(\overline{A_1}\,A_2) + P(\overline{A_1}\,\overline{A_2}\,A_3)$

$= 0.1219 + 0.1219 + 0.289$

$= 0.5328$

Q. Consider families of n-children.
A: family has children of both sexes
B: atmost 1 girl

show that the only 1 value of n for which the event A & B are independent is $n=3$.
assuming each child has prob. $= \frac{1}{2}$

Q. An archer with accuracy 75% fires 3 arrows at a target. ~~...~~ prob. of target falling is 0.6 (hit once), 0.7 (hits twice), 0.8 (hit thrice)

given that target has fallen, find prob. it was hit twice.