$V(c(x)) = c^2 V(x)$

Page :
Date :

K is uniformly distributed

$$f(x) = \begin{cases} 1/5 & 0 \le x \le 5 \\ 0 & \text{else} \end{cases}$$

$$P(\text{real}) = \int_2^5 \frac{1}{5}\, dx = \frac{3}{5}$$

Method 2 $\quad P = P[K \geq 2] + P[K \leq -1]$

$$= \int_2^5 \frac{1}{5}\, dx + \int_{-1}^{\infty} 0\, dx = \frac{3}{5}$$

Ex. $\quad V(2+3x) = E[(2+3x)^2] + [E(2+3x)]^2$

$$= E(9x^2 + 12x + 4) + [E(2) + 3E(x)]^2$$

$$= 9E(x^2) + 12E(x) + 4\cancel{E(x)}\ 4 + (2 + 3E)^2$$

$$= 9E(x^2) + 12E + 4 + 4 + 9E^2 + 12E$$

$$= 9E(x^2) + 24E(x) + 8 + 9(E(x))^2$$

Chebyshev's Inequality (only MCQ or verification)

Let x be a random variable with $E(X) = \mu$, $V(x) = \sigma^2$ then for any +ve real no. K

i) $P\{|X - \mu| \geq K\} \leq \frac{\sigma^2}{K^2}$

and

ii) $P\{|X - \mu| < K\} \geq 1 - \frac{\sigma^2}{K^2}$

$$\Rightarrow P\{\mu - K < X < \mu + K\} \geq 1 - \frac{\sigma^2}{K^2}$$

variation

$K = K\sigma$

i) $P\{|X - \mu| \geq K\sigma\} \leq \frac{1}{K^2}$

ii) $P\{|X - \mu| < K\sigma\} \geq 1 - \frac{1}{K^2}$

Camlin

$V(c(x)) = c^2 V(x)$

X is uniformly distributed

$$f(x) = \begin{cases} 1/5 & 0 \le x \le 5 \\ 0 & \text{else} \end{cases}$$

$$P(\text{real}) = \int_2^5 \frac{1}{5}\, dx = \frac{3}{5}$$

Method 2 $P = P[X \ge 2] + P[X \le -1]$

$$= \int_2^5 \frac{1}{5}\, dx + \int_{-1}^{\infty} 0\, dx = \frac{3}{5}$$

Ex. $V(2+3x) = E[(2+3x)^2] + [E(2+3x)]^2$

$= E(9x^2 + 12x + 4) + [E(2) + 3E(x)]^2$

$= 9E(x^2) + 12E(x) + 4\cancel{E(x)}\ 4 + (2 + 3E)^2$

$= 9E(x^2) + 12E + 4 + 4 + 9E^2 + 12E$

$= 9E(x^2) + 24E(x) + 8 + 9(E(x))^2$

Chebyshev's Inequality (only MCQ or verification)

Let x be a random variable with $E(X) = \mu$, $V(x) = \sigma^2$ then for any +ve real no. k

i) $P\{|x-\mu| \ge k\} \le \frac{\sigma^2}{k^2}$

and

ii) $P\{|x-\mu| < k\} \ge 1 - \frac{\sigma^2}{k^2}$

$\Rightarrow P\{\mu - k < X < \mu + k\} \ge 1 - \frac{\sigma^2}{k^2}$

variation

$k = k\sigma$

i) $P\{|X-\mu| \ge k\sigma\} \le \frac{1}{k^2}$

ii) $P\{|X-\mu| < k\sigma\} \ge 1 - \frac{1}{k^2}$

~~med~~ $m = \frac{1}{2}, 1.36, -0.36$

$\boxed{m = 1/2}$ as $1/2$ is in domain

v) Mode : highest no. of occurrencies.
that value for which probability is highest
The value of x for which the probability is highest

max $6x(1-x)$ → need to find $f' = 6x - 6x^2$

$$\frac{d(6x(1-x))}{dx} = 0 = \frac{d\,6(x-x^2)}{dx} = 6(1-2x)$$

$$\boxed{x = \frac{1}{2}}$$

$$\frac{d^2 f}{dx^2} = -2 < 0 \rightarrow \text{maximum}$$

if > 0 → min.

$= 0$ → further investigation

Q. A random variable k is uniformly distributed over the interval $(0,5)$. What is the probability that the roots of the equation

$$4x^2 + 4xk + k + 2$$ are real?

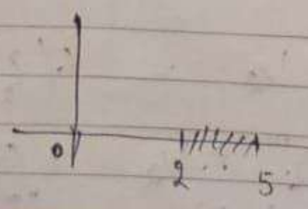
$b^2 - 4ac = (4k)^2 - 4(4)(k+2) = 16(k^2 - k - 2)$

If real $k^2 - k - 2 \geq 0$.

$(k - 2k + k - 2) \geq 0$

$(k-2)(k+1) \geq 0$ →

$k \in (-\infty, -1] \cup [2, \infty)$.



$\frac{3}{5}$

$3-2=1$

$6x^2-6x^3 \to 2x^2-2x^3 \Big|_0^1$

$2m^2-2m-1=0$

$\cancel{(i)\ \int_0^1 6x^2(1-x)\,dx = 6\left(\frac{x^2}{2}-\frac{x^3}{3}\right) = 6\left(\frac{1}{2}-\frac{1}{3}\right) = 1}$

$\cancel{(ii)}$ X

(i) $\int_0^1 6x^2(1-x)\,dx = 6\left(\frac{x^3}{3}-\frac{x^4}{4}\right)_0^1 = \frac{6}{12} = \frac{1}{2}$

$V = E(x^2) - (E(x))^2 = \frac{3}{10} - \frac{1}{4} = \frac{12-10}{40} = \boxed{\frac{1}{20}}$ Ans

(ii) $\int_0^1 6x^3(1-x)\,dx = 6\left(\frac{x^4}{4}-\frac{x^5}{5}\right) = \frac{6}{20} = \frac{3}{10}$

~~(iii) $E(2+3x) = \int_2^5 (2+3x)^2(1-3x)\,d(2+3x) = 3\int_2^5 (9x^2+12x+4)(1-3x)\,dx$~~

~~$= 3\int_2^5 9x^2+12x+4-27x^3-36x^2-12x\,dx.$~~

~~$= 3\int_2^5 -27x^3-27x^2+4\,dx.$~~

~~$= 3\left(-27\frac{x^4}{4}-27\frac{x^3}{3}+4x\right)_2^5$~~

~~$= 3\left(-\frac{(625-16)}{4} - 9(125-8) + 12\right)$~~

~~$= -609 - 9\times 117\times 4 + 12\times 4.$~~

$E(2+3x) = E(2) + E(3x) = 2 + 3E(x) = 2+\frac{3}{2}$

$= \frac{7}{2} = 3.5$

(iv) + Median : value M such that $f(x \geq m) \geq 1/2$

$f(x \leq m) \geq 1/2$

$f(x \leq m) = 1/2 \to$ to get value of m.

$m = 1/2$

$\int_0^m 6x(1-x)\,dx = 1/2 = 6\left(\frac{x^2}{2}-\frac{x^3}{3}\right)\Big|_0^m$

$= (3x^2-2x^3)_0^m = 1/2 \quad 2(3m^2-2m^3) = 1$

$= 6m^2 - 4m^3 = 1 \Rightarrow \quad 4m^3-6m^2+1=0$

$= 4m^3-2m^2-4m^2+1 = 2m^2(2m-1) - (2m-1)(2m+1)$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 \dots$$
$$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 \dots$$

iii) $P(|x| < 1) = 1 - P(|x| < 1)$

$\hookrightarrow 2P(|x|<1) = 1 \rightarrow P(|x|<1) = 1/2$

$\int_{-1}^{1} \frac{1}{2\alpha} dx = \frac{1}{2} = \boxed{\alpha = 2}$

ii) $P(x < \frac{1}{2}) = \frac{1}{2\alpha}(\alpha - 1) = \frac{1}{2} \hookrightarrow$ no α exists,

Q. A coin is tossed till the first head appears. let x denotes the no. of tosses find expectation of x and variance of x. Take probability of head = P. ($P(\text{head}) = P$)

$R_x = \{1, 2, 3, \dots\}$

x	1	2	3	$\dots$	k
$P(x)$	P	$(1-P)P$	$(1-P)^2P$		$(1-P)^{k-1}P$

$E(x) = \sum x_i P(x_i) = P + 2P(1-P) + 3(1-P)^2P \dots + k(1-P)^{k-1}P$
$= P(1 + 2(1-P) + 3(1-P)^2 \dots)$

Taking $(1-P) = P'$

$= P(1 + 2P' + 3(P')^2 \dots)$

$= \frac{P}{(1-P')^2} = \frac{P}{P^2} = \frac{1}{P}$ Answer.

Note $\frac{1}{1-x} = 1 + x + x^2 \dots$

$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 \dots$

$E(x^2) = \sum x_i^2 P(x_i)$
$= P + 4P(1-P) + 9(1-P)^2P \dots$
$= P(1 + 2^2(1-P) + 3^2(1-P)^2 \dots)$
$=$

Given that x has the pdf

$$f(x) = \begin{cases} 6x(1-x) & 0 < x < 1 \\ 0 & \text{else} \end{cases}$$

i) $E(x)$ (iii) $E(2+3x)$
(ii) $V(x)$ (iv) Median (v) Mode.

$E(P) = \left(-e^{-x}\right)_{0.9}^{\infty} \times 3 + 2\left(+e^{-x}\right)_{0}^{0.9} = 3e^{-0.9} + 2\left(e^{-0.9} - 1\right)$

$E(P) = 5e^{-0.9} - 2$

P	3	-2
P(P)	0.4065	0.5934

$\hookrightarrow \quad 3(0.4065) - 2(0.5934) = 0.032$

Uniformly Distributed Random Variable : a random variable X is said to be uniformly distributed in the interval (a, b) if its pdf is given by

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{if } a \le x \le b \\ 0 & \text{else} \end{cases}$$

$$E(x) = \int_a^b x f(x)\, dx = \int_a^b \frac{x}{b-a}\, dx = \frac{1}{2}\frac{b^2-a^2}{(b-a)} = \frac{b+a}{2}$$

$$= \frac{a+b}{2}$$

$$V(x) = \frac{(b-a)^2}{12}$$

Q. Suppose x is uniformly distributed in the interval $(-\alpha, \alpha)$ when $\alpha > 0$ whenever possible find value of α for which
i) $P(x > 1) = \frac{1}{3}$ ii) $P(x < \frac{1}{2})$ iii) $P(|x| < 1) = P(|x| > 1)$

Solution $f(x) = \begin{cases} \frac{1}{2\alpha} & -\alpha < x < \alpha \\ 0 & \text{else} \end{cases}$

$|x| > 1$
$\hookrightarrow$
$-\infty, -1 \cup 1, \infty$

i) $P(x > 1) = \int_1^{\alpha} \frac{1}{2\alpha}\, dx = \frac{1}{2\alpha}(\alpha - 1) = \frac{1}{3}$

$3\alpha - 3 = 2\alpha \rightarrow \boxed{\alpha = 3}$

ii) $\cancel{\int_0^{1/2} \frac{1}{2\alpha}\, dx = \frac{1}{2\alpha}\left(\frac{1}{2}\right) = \frac{1}{2}}$ $\quad \boxed{\alpha = 2}$

iii) $\cancel{\int_1^{\infty} \frac{1}{2\alpha}\, dx + \int_{-\infty}^{-1} \frac{1}{2\alpha}\, dx = \int_{-1}^{1} \frac{1}{2\alpha}\, dx}$

$\hookrightarrow \quad \cancel{\frac{1 - \infty - 1 + \infty}{2\alpha} = \frac{2}{2\alpha} = \left(\frac{1}{2}\right)}$ satisfies for all α

$E(x) = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3)$
$= 0 \cdot \frac{8}{15} + 1 \cdot \frac{6}{15} + 2 \cdot \frac{1}{15}$

$\boxed{E(x) = \frac{8}{15}}$

$V(x) = E(x^2) - [E(x)]^2 = \frac{10}{15} - \frac{64}{15^2} = \frac{150 - 64}{15^2} = \frac{86}{15^2}$
$E(x^2) = \sum x_i^2 p(x_i) = 0^2 \times \frac{8}{15} + 1^2 \times \frac{6}{15} + 2^2 \times \frac{1}{15} = \frac{10}{15}$

$\boxed{V(x) = \frac{86}{225}}$

Q. $Q = \{1, 2, 3 \cdots\}$. Let $f(x) = \frac{1}{2^x}$ be the pdf.
↳ X is a random variable taking values from Q set

Let $A = \{1, 3, 5 \cdots\}$ find $P(A)$.

$2n-1$ $P(A) = \frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^5} \cdots \frac{1}{2^{2n-1}} = \frac{1/2}{1 - (1/2)^2} = \frac{2}{3}$

Q. Suppose that an electronic device has life length X (in 1000 hrs) which is considered as a crv with pdf $f(x) = e^{-x}$, $x > 0$. Suppose that the cost of manufacturing such an item is \$2 and the manufacturer sells it for \$5 but guarantees the total refund if $X \leq 0.9$. What is the expected profit of the manufacturers.

E (Profit) $P \leftarrow$ profit
$R_P = \{3, -2\} \rightarrow$ range space of profit
$E(P) = 3 \cdot P(x) - 2P(x)$

pdf of profit

P	3	-2
P(Profit)	$\int_{0.9}^{\infty} e^{-x} dx$	$\int_{0}^{0.9} e^{-x} dx$

3W	3	$R = \{-3, -2, -1, 0, 1, 2, 3\}$
3B	0	
3R	-3	3W 3R 5B
WBB	1	$^3C_1 \times {}^5C_2$
RBB	-1	
WWR	1	
WWB	2	
RRW	-1	
RRB	-2	
WBR	0	

X	-3	-2	-1	0	1	2	3
P(X)	$\frac{^3C_3}{^{11}C_3}$	$\frac{^7C_2\, ^5C_1}{^{11}C_3}$	$\frac{^3C_1 \times {}^5C_2}{^{11}C_3}$ +	$\frac{^5C_3 + {}^5C_1 {}^3C_1 {}^3C_1}{^{11}C_3}$	$\frac{^3C_1 \times {}^5C_2}{^{11}C_3}$	$\frac{^3C_2 \times {}^5C_1}{^{11}C_3}$	$\frac{^3C_3}{^{11}C_3}$

* $E(X) = \sum x_i P(x_i) = \int x f(x)\, dx$

  $E(c) = c$

  $E(cx) = c\,E(x)$

* $E(x^3) = \int x^3 f(x)\, dx$
* $V(x) = E(x^2) - (E(x))^2$
* $V(c) = 0$
* $V(cX) = c^2 V(x)$
* $F(x) = \int_{-\infty}^{x} f(x)\, dx = \sum_{-\infty}^{x} P(x_i)$

Q. A student takes multiple choice test containing 2 problems. The first one has 3 possible answers and second one has 5 possible answers. The student selects 1 answer at random as the right answer. Let X denote the no. of right answers. Find E(x) and V(x).

I → 3 II → 5 $X \leftarrow$ no. of right ans

$R_X = \{0, 1, 2\}$

pdf:-

X	0	1	2
P(X)	$\frac{2}{3} \times \frac{4}{5}$	$\frac{2}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{4}{5}$	$\frac{1}{5} \times \frac{1}{3}$

X	0	1	2
P(X)	$\frac{8}{15}$	$\frac{6}{15}$	$\frac{1}{15}$

Expectation (mean) (μ)

$\mu = E(x) = \sum_{-\infty}^{\infty} x_i P(x_i) = \int_{-\infty}^{\infty} x f(x)\, dx$

$E(c) = c$

Variance (σ^2)

$\sigma^2 = E(X-\mu)^2 = E(X^2 + \mu^2 - 2X\mu)$

$= E(X^2) + E(\mu^2) - E(2X\mu)$

$= E(X^2) + \mu^2 - 2\mu E(X)$

$= E(X^2) + \mu^2 - 2\mu^2$

$V(X) = E(X-\mu)^2 = E(X^2) - [E(X)]^2$

$\sigma \rightarrow$ standard deviation

$\sigma^2 \rightarrow$ variance

Q. 3 balls are randomly selected from a box containing 8 white, 3 red and 5 black balls. The person who selects the ball brings 1 rupee for each white ball selected and loses 1 rupee for each red ball selected. 3 balls are selected. Let X denote the total winnings from the experience. Find the pdf and expectation of X.

-2 rup	-1 rupee	0 rupee	1 rupee	2 rupee	3 rupee.
2R 1B	1R 2B	1B+1R+1W			
	2R 1W	3B			

$\Big\} x$

ii) $x < 0 \rightarrow F(x) = \int_{-\infty}^{x} 0 = 0$

$0 \le x < 1 \rightarrow F(x) = \int_{-\infty}^{0} 0 + \int_{0}^{x} 6x(1-x)\,dx$

$x \ge 1 \rightarrow 1$

$$F(x) = \begin{cases} 0 & x < 0 \\ 3x^2 - 2x^3 & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

(iii) $P(x<b) = 2P(x>b) = 2[1 - P(x<b)]$

$3P(x<b) = 2$

$P(x<b) = 2/3$

$\int_{0}^{b} 6x(1-x)\,dx = \frac{2}{3}$ $\quad \frac{b^2}{2} - \frac{b^3}{3} = \frac{\cancel{2}}{\cancel{18}}\frac{1}{9}$

$3b^2 - 2b^3 = \frac{2}{3} = 9b^2 - 6b^3 = 2 \Rightarrow 6b^3 - 9b^2 + 2 = 0$

$b = 0.61$

Q. A coin is known to come up 3 times head as often as tail. This is tossed thrice. Let x be the no. of heads. This coin is tossed 3 times. Let x denote the no. of heads find the pdf.

$X \leftarrow$ no. of H

$R_x = \{0, 1, 2, 3\}$

$P(x=0) \Rightarrow TTT \rightarrow (1/4)^3$

X	0	1	2	3
$P(x)$	$\frac{1}{64}$	$\frac{9}{64}$	$\frac{27}{64}$	$\frac{27}{64}$

Expectation

$6 \int x - x^2 \, dx$

$\frac{x^2}{2} - \frac{x^3}{3}$

8. Given that x is a crv with pdf $f(x) = kx^3$ $0 < x < 1$.
find (i) $P(x > 0.8)$ (ii) $P(x < 1/2)$ $P(1/4 < x < 1/2)$

(i) $\int_{0.8}^{1} kx^3 \, dx = \frac{kx^4}{4} = \frac{k}{4}(1 - 0.8^4) = \frac{4}{4}(1 - 0.8^4)$ .5904

To find k: $\int_0^1 kx^3 dx = 1 \rightarrow kx^4 \Big|_0^1 = 4 \rightarrow k = 4$

(ii) $\int_0^{1/2} 4x^3 dx = x^4 \Big|_0^{1/2} = \left(\frac{1}{2}\right)^4 = 0.0625 = \frac{1}{16}$

(iii) $\int_{1/4}^{1/2} 4x^3 \, dx = x^4 \Big|_{1/4}^{1/2} = \left(\frac{1}{2}\right)^4 - \left(\frac{1}{4}\right)^4 = \frac{3}{64}$

(iv) $P(1/4 < x < 2) = \int_{1/4}^{1} 4x^3 \, dx = 1 - \left(\frac{1}{4}\right)^4 = \frac{63}{64}$

Q. The diameter of an electric cable say x is ~~is said to be~~ assumed to be a continuous random variable with

$$f(x) = \begin{cases} 6x(1-x) & 0 \le x \le 1 \\ 0 & \text{else} \end{cases}$$

i) $P(x < 1/2 \,(1/3 < x < 2/3))$
ii) cdf
iii) find 'b' st $P(x < b) = 2P(x > b)$

Solution

i) $\frac{P(x < 1/2 \cap 1/3 < x < 2/3)}{P(1/3 < x < 2/3)} = \frac{P(1/3 < x < \frac{1}{2})}{P(1/3 < x < 2/3)}$

$= \frac{\int_{1/3}^{1/2} 6x(1-x)\,dx}{\int_{1/3}^{2/3} 6x(1-x)\,dx} = \frac{\left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_{1/3}^{1/2}}{\left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_{1/3}^{2/3}}$

$x < 0 \quad : \quad F(x) = 0$

$0 \leq x < 1 \quad : \quad F(x) = 1/3$

$1 \leq x < 2 \quad : \quad F(x) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

$\cancel{2 \leq x \leq 2}\ x \geq 2 \quad : \quad F(x) = 1$

$$F(x) = \begin{cases} 0 & x < 0 \\ 1/3 & 0 \leq x < 1 \\ 1/2 & 1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$

$F(0.5^-) = 1/3$

Q If x is a crv with a pdf $f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & \text{else} \end{cases}$

find the cdf

$0 \quad x = 0$

$2x \quad 0 < x < 1$

$0 \quad x = 1$

$$F(x) = \begin{cases} 0 & x = 0 \\ \int_0^1 f(x)\,dx & 0, 1 \\ \int_0^1 f(x)\,dx & x = 1 \end{cases}$$

$\hookrightarrow \int_0^1 2x\,dx = x^2 \Big|_0^1 = 1 \qquad \hookrightarrow F(x) = \begin{cases} 0 & x = 0 \\ 1 & 0 < x < 1 \\ 0 & x = 1 \end{cases}$ Sol

$$f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & \text{else} \end{cases}$$

$x < 0 \quad : \quad F(x) = \int_{-\infty}^{x} 0\,dx = 0$

$0 \leq x < 1 \quad : \quad F(x) = \int_{-\infty}^{0} 0 + \int_0^x 2x\,dx = x^2$

$x \geq 1 \quad : \quad F(x) = \int_{-\infty}^{0} 0\,dx + \int_0^1 2x\,dx + \int_1^x 0 = 1$

$$F(x) = \begin{cases} 0 & x < 0 \\ x^2 & 0 \leq x < 1 \\ 1 & x > 1 \end{cases}$$

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Discrete random function or probability mass function

Probability distributn fn $P(x)$

(i) $P(x_i) \geq 0$

(ii) $\sum_{-\infty}^{\infty} P(x_i) = 1$

(table)

Continuous random function

Probability distributn fn $f(x)$

(i) $f(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

* prob at a pt is zero
* $P(a < x < b) = \int_a^b f(x)dx$

* $P(a < x < b) = P(a \leq x \leq b) = P(a < x \leq b) = P(a \leq x < b)$

(Prob density fn)

Cumulative density function $F(x)$

$$F(x) = P(x \leq x) = \sum_{i=-\infty}^{x} P(x_i) \quad \text{drv}$$

$$= \int_{-\infty}^{x} f(x)dx \quad \text{crv}$$

i) $F(-\infty) = 0$ ii) $F(\infty) = 1$

iii) drv :- $P(x_j) = F(x_j) - F(x_{j-1})$

(iv) crv :- $f(x) = \dfrac{d[F(x)]}{dx}$

(v) $P(a < x < b) = F(b) - F(a)$

Q. Suppose the random variable x takes the value 0, 1, 2 with probabilities $1/3$, $1/6$, $1/2$ respectively. find the cumulative density function

pdf:

$X = x_i$	0	1	2
$P(x_i)$	$1/3$	$1/6$	$1/2$

Page :
Date :

Q. 2 1×1 squares are selected from 8×8 chessboard. What are the probability that they're adjacent?

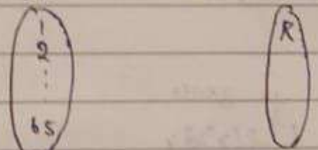
Random variables – 1 Dimensional.

A random variable x is a function that assigns some real nos to all the elements in 's'.

Ex. $S = \{1, 2 \cdots 65\}$. E: pick a student.

X :- square of his roll no.

R_x : $\{1, 4, 9, 16 \cdots 65^2\}$



Range of X set of all possible values X takes.

Types of random variables :

1. Discrete random variable (drv) : R_x is either finite or countably infinite.
2. Continuous random variable (crv) : R_x is uncountably infinite

* Prob mass fn / prob dist fn) (pmf/pdf).

* Let X be a drv and R_x be its range space then for each x_i belongs to R_x,

$$P(X = x_i) = P(x_i)$$

is said to be probability function of drv if it satisfies the following conditions

~~the pair x_i~~ $P(x_i) \geq 0$ $\Sigma P(x_i) = 1$

The $(x_i, P(x_i))$ is called probability distribution function of X.

$f(x)$
i) $f(x) \geq 0$.
ii) $\int f(x) dx = 1$

$\rightarrow$ when fn is continuous.

Camlin

Q. Consider a family of n children. Let e be an event that the family has both boy and girl child. Let b be the event that there is atmost 1 girl in the family. Find the value of n for which e and b are independent assuming $P(B) = 1/2$ (probability of getting boy child).

E: Both B & G

B: Atmost 1 G

$P(E) = 1 -$ only girl $-$ only boys

$$1 - \left(\frac{1}{2}\right)^n \times 2 = 1 - \frac{1}{2^{n-1}}$$

$$P(B) = 0G + 1G = \frac{1}{2^n} + \frac{{}^nC_1}{2^n} = \frac{n+1}{2^n}$$

$$\cancel{\text{Ans)} \quad \frac{2^{n-1}-1}{2^{n-1}} = \frac{n+1}{2 \times 2}}$$

$$\cancel{2^n - 2 = n+1.}$$

$$\cancel{2^n = n+3} \qquad n = 1, 3, 5, 13 \, \alpha$$

$$P(E \cap B) = 1G + 1B = n\left(\frac{1}{2}\right)^n$$

$$n\left(\frac{1}{2}\right)^n = \left(\frac{n+1}{2^n}\right)\left(\frac{2^{n-1}-1}{2^{n-1}}\right)$$

$$n \cdot \cancel{2^{n-1}} = n\cancel{2^{n-1}} - n + 2^{n-1} - 1$$

$$2^{n-1} = n+1 \qquad n = 1, 3$$

$$\boxed{n = 3}$$

Q. $P(A \cup B) = 0.7$ $\quad P(\bar{B}/\bar{A}) = 0.5$ $\quad$ find $P(A)$

$$\frac{P(\bar{B} \cap \bar{A})}{P(\bar{A})} = 0.5$$

$$\frac{1 - 0.7}{1 - P(A)} = 0.5$$

$$\frac{0.3}{0.5} = 1 - P(A) \quad \rightarrow \quad P(A) = 0.4$$

P(A wins) = $\frac{}{\cancel{36}}$

$1+5,\ 2+4,\ 3+3,\ 4+2,\ 5+1 \quad P = 5/36 = 5/36 = A_6$

$1+6,\ 2+5,\ 3+4,\ 4+3,\ 5+2,\ 6+1 \quad P = 6/36 = 1/6 = B_7.$

$P(A\ wins) = A_6 + \overline{A_6}\,\overline{B_7}\,A_6 + \overline{A_6}\,\overline{B_7}\,\overline{A_6}\,\overline{B_7}\,A_6 \ldots$

$= A_6 (1 + \overline{A_6}\,\overline{B_7} + (\overline{A_6}\,\overline{B_7})^2 \ldots)$

$= A_6 \frac{1}{1 - \overline{A_6}\,\overline{B_7}} = \frac{5}{36} \cdot \frac{1}{1 - \frac{31}{36} \times \frac{5}{6}} = \cancel{\frac{216}{216-5}} \times \cancel{\frac{5}{36}}$

$= \cancel{\frac{30}{21}} = \frac{5}{36 - \frac{155}{6}} = \cancel{\frac{36}{201}}\ \cancel{\frac{10}{67}}$

$= \boxed{\frac{30}{61}}$

Q. ~~Box 1 has~~ There are 2 boxes A and B. The Box A has 8 white and 7 black balls and B has 9 black and 7 white balls. 1 ball is randomly drawn from box A and placed in box B. Then a ball is transferred from B to A. finally 1 ball is selected from Box A. What is the probability that it is white?

$A \rightarrow 8W\ 7B \qquad B \rightarrow 7W\ 9B$ ($10B$)

$B_t B_t + B_t W_t + W_t B_t + W_t W_t$ then white drawn ($7W\ 6B$)

$\frac{7}{15} \times \frac{10}{17} \times \frac{8}{15} + \frac{7}{15} \times \frac{7}{17} \times \frac{9}{15} + \frac{8}{15} \times \frac{8}{17} \times \frac{7}{15} + \frac{8}{15} \times \frac{8}{17} \times \frac{8}{15}$

$= \frac{7\times10\times8 + 7\times7\times9 + 8\times8\times7 + 8\times8\times8}{15\times17\times15} = 0.51$

B1: selected 1st factory → 10,000/30,000

B2: selected 2nd factory → 20,000/30,000.

A: selected defective clock → $P(A|B1) = \frac{100}{10000}$ $P(A|B2) = \frac{300}{20000}$

$$P(B1|A) = \frac{P(A|B1)\,P(B1)}{P(A|B1)P(B1) + P(A|B2)P(B2)}$$

$$= \frac{\frac{1}{3} \times \frac{1}{100}}{\frac{2}{3} \times \frac{300}{20{,}000} + \frac{1}{3} \times \frac{1}{100}} = \frac{\frac{1}{3}}{1 + \frac{1}{3}} = \frac{1}{4}$$

Q. An officer is in hurry to reach the airport to catch the flight scheduled at 1 AM. Probability that he gets the taxi at such an early hour is 0.23. However, if he gets the taxi, he would catch the flight with probability 0.85. If he doesn't get a taxi he will catch the flight with a probability 0.43 by some other mode of transportation. Given that he got the flight, what is the probability that he came by taxi?

B1: got taxi $= 0.23$

B2: didn't get taxi $= 0.77$

A: caught flight. $P(B1/A) = 0.23 \times 0.85$

$P(B2/A) = 0.77 \times 0.43$.

Ans $\frac{0.23 \times 0.85}{0.23 \times 0.85 + 0.77 \times 0.43} = 0.37$

Q. 2 people A and B toss a pair of dice alternatively. The person A wins if he gets the sum 6 before B gets the sum 7. B wins if he gets the sum 7 before A gets the sum 6. (i) If A starts the game, what is the probability that A wins?

0.1518 $\frac{n!}{r!(n-r)!}$

Baye's Theorem: Let S be the sample space associated with experiment E, let A be element & $B_1 \cdots B_k$ form a partition of S if the conditional probabilities $P(A/B_i)$ and $P(B_i)$ are known for $i=1$ to k then $P(B_i/A) = \frac{P(A/B_i)\,P(B_i)}{\sum_{j=1}^{k} P(A/B_j)P(B_j)}$

Proof
LHS = $P(B_i/A) = \frac{P(A\cap B_i)}{P(A)} = \frac{P(A/B_i)\,P(B_i)}{\sum P(A/B_i)P(B_i)}$ ($\because$ x^n theorem) ($\because$ total prob)

Q. A bag has 3 coins, one of which is a coin with 2 heads and other 2 are normal coins. One coin is selected at random from the bag and tossed 4 times in succession. If head turns up each time, what is the probability that it is a 2 headed coin?

~~P = +~~ Bag: 2N + 1 Double Head
B1: selecting normal coin
B2: 2 headed coin } Step marks.
A: got Head 4 times, when tossed

$P(B_2/A) = \frac{\frac{1}{3}\times(1)^4}{\frac{2}{3}\times\left(\frac{1}{2}\right)^4 + \frac{1}{3}\times(1)^4} = \frac{1}{\frac{1}{8}+1} = \frac{8}{9}$

$= \frac{P(A/B_2)\,P(B_2)}{P(A/B_1)P(B_1) + P(A/B_2)P(B_2)}$

Q. 2 factories produce identical clocks. The products of the first factory consists of 10,000 clocks out of which 100 are defective. The second factory contains 20,000 clocks out of which 300 are defective. What is the probability that a particular selected defective clock is produced by 1st factory.

$P = \frac{\frac{1}{2}\times\frac{100}{10000}}{\frac{1}{2}\times\frac{100}{10000} + \frac{1}{2}\times\frac{300}{20000}} = \frac{1}{1+\frac{3}{2}} = \boxed{\frac{2}{5}}$

Camlin

Q. A student takes his examination for the subjects P, Q, R, S. He estimates his chances of passing in:

$P \to 4/5$ $\quad Q \to 3/4$ $\quad R \to 5/6$ $\quad S \to 2/3$. To qualify the exam he must pass in the subject P and atleast 2 other subjects. What is the probability that he qualifies

$$\frac{4}{5} \times \left(\frac{\cancel{3}}{4} \times \frac{5}{\cancel{6}_2} + \frac{5}{\cancel{6}_3} \times \frac{\cancel{2}}{3} + \frac{2}{\cancel{3}} \times \frac{\cancel{3}}{4} \right)$$

$$= \frac{4}{5}\left(\frac{5}{8} + \frac{5}{9} + \frac{1}{2} \right)$$

$$= \frac{1}{2} + \frac{4}{9} + \frac{2}{5} = \frac{45+40+36}{90} =$$

$$\frac{4}{5} \times \left(\frac{\cancel{3}}{4} \times \frac{5}{6} \times \frac{1}{\cancel{3}} + \frac{1}{\cancel{4}_2} \times \frac{5}{6} \times \frac{\cancel{2}}{3} + \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{_2\cancel{4}} \times \frac{1}{6} \right) + \frac{4}{5} \times \frac{3}{4} \times \frac{5}{6} \times \frac{2}{3}$$

$$= \frac{4}{5} \times \left(\frac{5}{24} + \frac{5}{36} + \frac{1}{12} \right) + \frac{4}{5} \times \frac{3}{4} \times \frac{5}{6} \times \frac{2}{3}$$

$$= \frac{\cancel{4}}{5}\left(\frac{15+10+6}{\cancel{72}\ 18} \right) = \frac{31}{90} + \frac{\cancel{4}}{\cancel{5}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{5}}{6} \times \frac{2}{\cancel{3}} \cdot \frac{1}{3}$$

$$= \boxed{\frac{61}{90}}$$

0.1518

Solution: A : till 4th test $(T_1 T_2 T_3 T_4) = 3B, 1G$

B : 4th bad tube in 6^{th} test ⇒ B

$P(A \cap B) = P(A) \cdot P(B|A)$

$P(A \cap B) = \frac{{}^6C_1 \times {}^4C_3}{{}^{10}C_4} \times \frac{1}{{}^6C_1}$

$P(A \cap B) = \frac{{}^4C_3}{{}^{10}C_4}$

(ii) A : till 9th test : 6G 3B

B : 4th bad tube in 10th test.

$P(A \cap B) = \frac{{}^6C_6 \times {}^4C_3}{{}^{10}C_9} \times \frac{{}^1C_1}{{}^1C_1} = \frac{{}^4C_3}{{}^{10}C_9} = \frac{4}{10} = \frac{2}{5}$

~~P(A∩B~~

Q. A problem in statistics is given to 3 students A, B, C whose chances of solving are 1/2, 3/4, 1/4 respectively. The problem is solved if atleast 1 student will solve the problem. What is the probability that the problem is solved?

$P = 1 - \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} = 1 - \frac{3}{32} = \frac{29}{32}$

Q. Each of 2 people toss 3 coins. What is the probability that they get same no. of heads?

~~No head, 3~~ 0, 1, 2, 3.

$\frac{1}{8} \times \frac{1}{8} + \frac{3}{8} \times \frac{3}{8} + \frac{3}{8} \times \frac{3}{8} + \frac{1}{8} \times \frac{1}{8}$

$\left(\frac{20}{8 \times 8}\right) = \cancel{\frac{5}{32}} = \cancel{\frac{27}{32}} \quad \cancel{1 - \frac{5}{16} = \frac{11}{16}}$

$\frac{5}{16}$

0.1518 $\quad \underline{n!}$

$b = \frac{4}{5}, \frac{1}{6}$

$a - ab = \frac{1}{6}$

$a\left(\frac{1}{5}\right) = \frac{1}{6}$ or $a\left(\frac{5}{6}\right) = \frac{1}{6}$

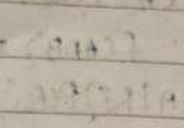
$a = \frac{5}{6}$ or $\frac{1}{5}$

$b, a = \frac{4}{5}, \frac{5}{6}$ or $b, a = \frac{1}{6}, \frac{1}{5}$

(2) Given that $P(A) = 1/3$, $P(B) = 1/4$, $P(A \cup B) = 1/2$ find
(i) $P(A \cap \bar{B})$ (ii) ~~$P(A \cap \bar{B})$~~ $P(A|B)$

(i) $P(A \cap \bar{B}) = P(A) - P(A \cap B) = 1/3 - \frac{1}{12} = 1/4$

$\cancel{P(A) P(B)} - P(A \cap B) = \frac{1}{2} - \frac{1}{3} - \frac{1}{4}$



$P(A \cap B) = -\frac{1}{12} \qquad \frac{6 - 4 - 3}{12} \Rightarrow$

(ii) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/12}{1/4} = \frac{1}{3}$

Q. A box has 4 bad and 6 good tubes. The tubes are tested
** 1 by 1 by drawing 1 tube at random. Testing and repeating the process is done until all the bad tubes are found. What is the probability that 4th bad tube is found in (i) 5th test (ii) 10th test

$\underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \overset{B}{\underline{\quad}}$

3B 1G

(i) $\frac{\frac{4!}{3!\,1!}}{{}^{10}C_5}$

(ii)

Multiplication Theorem - $P(A \cap B) = P(A)\,P(B/A)$

if A and B are independent : $P(A \cap B) = P(A)\,P(B)$

Total Probability Theorem :- $P(A) = \sum_{i=1}^{k} P(A \mid B_i)\,P(B_i)$, $\underbrace{B_1 \ldots B_k}_{\text{partition}}$

If A and B are indepent : A & $\bar{B}$ are independent

$P(A \cap \bar{B}) = P(A) - P(A \cap B)$

Let A, B are independent , $P(A \cap B) = P(A)\,P(B)$

consider , $P(A \cap \bar{B}) = P(A) - P(A \cap B)$:

$P(A \cap \bar{B}) = P(A) - P(A) \cdot P(B)$

$= P(A)\,(1 - P(B)) = P(A)\,P(\bar{B})$

* If A, B are independent , then $\bar{A}, B$, $\bar{B}, A$, $\bar{A}, \bar{B}$ are also independent (proof may come)

Q. What is the probability that atleast 1 head occurs in 4 tosses of coin.

$1 - \frac{1}{16} = \frac{15}{16}$

$1 - P(\text{no heads}) = 1 - \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{15}{16}$

Q. If A and B are 2 independent events such that $P(\bar{A} \cap B) = 2/15$, $P(A \cap \bar{B}) = 1/6$ find $P(B)$

$(1 - P(A))\,P(B) = \frac{2}{15}$ $\rightarrow$ $P(B) - P(A)P(B) = \frac{2}{15}$

$P(A)(1 - P(B))$ $\rightarrow$ $P(A) - P(A)P(B) = \frac{1}{6}$

$b - ab = \frac{2}{15}$ $\quad a - ab = \frac{1}{6}$ $\quad 30b^2 - 24b - 5b + 4 = 0$

$6b(5b - 4)$

$b - a = \frac{2}{15} - \frac{1}{6} = \frac{4 - 5}{30}$ $\quad a - b = \frac{1}{30}$

$\left(\frac{1}{30} + b\right)(1 - b) = \frac{1}{6}$ $\quad \frac{1}{30}$

$(30b + 1)(1 - b) = 5$ $\quad 29b - 30b^2 + 1 = 5$

$30b^2 - 29b - 4$

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~~Partitio.~~

Partition in the sample space

Let S be the sample space then we say that the events $B_1, B_2 \cdots B_k$ represent a partition of S if the following conditions are satisfied:

(i) $B_i \cap B_j = \phi \quad \forall\, i, j \,\&\, i \neq j$

(ii) $\bigcup_{i=1}^{n} B_i = S$

(iii) $P(B_i) > 0 \quad \forall\, i$

Total Probability Theorem / Addition Theorem of Probability

Let A be an event with respect to the sample space S. Let B_1 to B_k form a partition of S, then

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) \cdots + P(A|B_k)P(B_k)$$

Proof: Since A is an event

$$A = S \cap A$$
$$= (B_1 \cup B_2 \cup \cdots \cup B_k) \cap A$$

$$P(A) = P((B_1 \cup B_2 \cdots B_k) \cap A)$$

$$P(A) = \cancel{P(A \cap B)}\, P(B_1 \cap A) + P(B_2 \cap A) \cdots P(B_k \cap A)$$

$$P(A) = P(B_1|A) \cdot P(B_1) + P(B_2|A) \cdot P(B_2) \cdots P(B_k|A) \cdot P(B_k)$$

$\frac{64}{2} = 8$

~~Conditional probability~~

Conditional Probability

The conditional probability of B given A denoted by $P(B|A)$ is the probability of B when it is known that A has already occurred.

$$\boxed{P(B|A) = \frac{P(A \cap B)}{P(A)}}$$

(i) $0 \le P(B|A) \le 1$

(ii) $P(S|A) = 1$

(iii) $P(B_1 \cup B_2 | A) = P(B_1|A) + P(B_2|A)$

provided B_1 & B_2 are mutually exclusive.

Note:

(i) When $P(A) = 0$, $P(B|A)$ is not defined.

(ii) When A, B are indepent.

$P(B|A) = P(B)$

↳ $P(B|A) = \frac{P(A \cap B)}{P(A)}$

↳ $P(A \cap B) = P(A) \cdot P(B|A)$

Multiplication Theorem

Let A and B are 2 events with respective probabilities $P(A)$ and $P(B)$. Let $P(B|A)$ be the conditional probability then the probability of simultaneous occurrences of A and B

$P(A \cap B) = P(A) P(B|A)$

Page :
Date :

Q. A 2n digit no. is formed which starts with no. 2 and all other digits are prime no.s. find the probability the sum of consecutive integers are also prime.

2 , 3, 5, 7.   2, 3 ✓
                  2, 5 ✓

$\overset{1}{\uparrow} \overset{2}{\uparrow} \overset{1}{\uparrow} \uparrow$

$Ex = 4^{2n-1}$ $fav = 2^n$ ~~Ans: $\frac{2^{4n}}{2^n} = \frac{2^n}{2^{4n}} = \frac{1}{2^{3n}}$~~

Ans: $\frac{2^n}{4^{2n-1}}$

Q Coefficients of a quadratic equation $ax^2+bx+c=0$. coefficients are obtained by rolling a die thrice. what is the probability that roots of the equation are real.

$b^2 - 4ac > 0$ $9-4ac$ $16-4ac$. 25

b	1	2	3	4	5	6
(A, C)	x	(1,1)	(1,1)	(1,1)		
			(1,2)	(1,2)		
			(2,1)	(2,1)		
				(1,3)		
				(3,1)		
				(4,1)		
				(1,4)		

Ans: $\frac{43}{216}$

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$= \sum P(A_i) - \sum P(A_i \cap A_j) + \sum P(A_i \cap A_j \cap A_k)$
$- \sum P(A_1 \cap A_2 \cap A_3 \cap A_4)$

3 2 1

$^4C_1 \times$

$P = \frac{^4C_1}{4!}$

$P = \frac{3!}{4!} =$

$P(A_1) = \frac{3!}{4!} = P(A_2) = P(A_3) = P(A_4)$.

$P(A_1 \cap A_2) = \frac{2!}{4!}$

$P = 4\left[\frac{3!}{4!}\right] - {^4C_2}\left(\frac{2!}{4!}\right) + {^4C_3}\frac{1!}{4!} - 1$

$\frac{4}{4!} - \frac{6 \times 2!}{4!} = \frac{5}{8}$

Q) 2 squares of size 1×1 are selected from an 8×8 chessboard what is the probability that they're adjacent?

Ex : $^{64}C_2$

fav: ~~7×8×2~~ | corner : 4 × 2 → choices
7×8×2 | side : (8−2)×4 ×3 → choices
$P = \underline{224}$

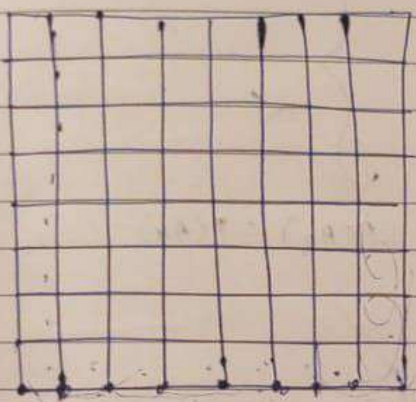
18×7.
2
×2

Ans: 224

Ans: $\frac{112}{^{64}C_2}$

→try with repetition

8. A rectangle is selected from 8×8 chessboard. What is the probability that it a square?



$^9C_2 \times {}^9C_2 \to$ exhaustive

$64+49 \cdots +1 \to$ fav

$$P = \frac{1+4+9+16+25+36+49+64}{^9C_2 \times {}^9C_2}$$

$$P = \frac{204}{^9C_2 \times {}^9C_2}$$

9. n people throw their n hats to the center of the hall and pick them later. Then show that the probability that nobody will get their own hat is $P=\sum_{k=2}^{n} \frac{(-1)^k}{k!}$ Also find the probability that exactly k people will get their own hat.

(i) ~~nobody gets their own hat~~

~~1 - 1 get~~

Sol

Q. Suppose that 4 digits 1,2,3,4 are written in random order, what is the probability that atleast one digit will occupy its proper position.

$$P[\text{none}] = \sum_{k=2}^{4} \frac{(-1)^k}{k!} = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} = \frac{3}{8}$$

$$\text{Atleast 1 dig} = 1 - \frac{3}{8} = \frac{5}{8}$$

$A_i \to$ ith digit occupies its proper position $i=1,2,3,4$

$P(A_1 \cup A_2 \cup A_3 \cup A_4) = P$(atleast 1 digit in its proper position)

$\frac{[illegible]}{3} + \frac{1}{24} \quad \frac{9}{24} = \frac{3}{8}$

Page:
Date:

Q. 10 chits are no.ed 1-10 mixed in a bowl. 2 chits x, y are drawn from the bowl successively without replacement what is the probability that $x+y=10$?

Exhaustive cases: ${}^{10}C_2 \times 2 = {}^{10}P_2$

$1+9,\ 2+8,\ 3+7,\ 4+6,\ 6+4,\ 7+3,\ 8+2,\ 9+1$

$P = \frac{8}{{}^{10}P_2} = \frac{8^4}{9\times 10_5} = \frac{4}{45} = \frac{8}{90}$

Q. What is the probability that in a group of 7 people

(i) No 2 were born on the same day of the week.

(ii) Atleast 2 were born on the same day.

(iii) 2 on Sunday and 2 on wednesday.

(i) Exhaustive cases: 7^7

$P = \frac{7!}{7^7}$

(ii) $1 -$ no two on same day $= 1 - \frac{7!}{7^7}$

(iii) $\frac{5^3 \times {}^7C_2 \times {}^5C_2}{7^7}$

$\times 5 \times 5 \times 5$

W	W	S	S			

Q. If there are n less than 365 people what is the probability that no 2 will have their bday on the same date

$365-n$

365

$\cancel{\frac{(365-n)}{(365)^{(365-n)}}}$ $\Big\}\ \alpha$

365, 364, 365-(n-1)

		...	

I - - - n^{th}

Ex :- 365^n

Fav :- $365\times 364 \cdots 365-(n-2)$

↳ Fav = $\frac{365\times 364 \cdots 365-(n-1)}{365^n}$

$n < 365$

Q. Out of the digits 0, 1, 2, 3, 4 a 5 digit no. is formed w/o repetition. What is the probability it is divisible by 4

_ _ _ _ _

12 24 20

32 04 40

$3\times 2\times 2\times 1 + 3\times 3\times 2\times 1$

$= 3\times 2(5) = 30$

Ex $\to 4\times 4\times 3\times 2$

$P = \frac{30}{4\times 4\times 3\times 2}$

Page :
Date :

Q. From 6 positive & 8 negative no.s, 4 no.s are selected at random and then multiplied. What is the probability that the product is positive?

$$\cancel{P = \frac{{}^6C_2 + {}^8C_2}{{}^{14}C_2}}$$

$$P = \frac{{}^6C_4 + {}^8C_4 + {}^6C_2 \times {}^8C_2}{{}^{14}C_4}$$

Q. A box has ~~stacks~~ tags which are marked 1 to n. 2 tags are selected at random, what is the probability that the no.s on the tags will be consecutive integers if the tags are
(i) Selected with replacement (ii) without replacement.

(i) $\{1,2\}$ $\{2,3\}$ .... $\{n-1, n\}$ → $n-1$ cases.
↳ $\{n, n-1\}$ .... $\{2,1\}$.

$$\frac{n-1}{n^2} \times 2 = \frac{2(n-1)}{n^2}$$

(ii) $$\frac{n-1}{{}^nC_2} = \frac{\cancel{(n-1)}\,2}{\cancel{(n-1)}\,(n)} = \frac{2}{n}$$

no. of favourable cases $= 2(n-1)$
i) no. of total cases $= n^2 \rightarrow 2(n-1)/n^2$
ii) no. of total cases $= {}^nC_2 \times 2 \rightarrow 2(n-1)/{}^nC_2 \times 2 = n-1/{}^nC_2$
$= 2/n$

Q. A bag has 40 tickets no.ed 1 to 40 out of which 4 are selected at random and then later arranged in ascending order what is the probability that t_3 is 25 ? $t_1 < t_2 < t_3 < t_4$

1, 2 ... 24   25   26 ... 40 $40 - 26 + 1 = 15$

$$\frac{{}^{24}C_2 \times 1 \times {}^{15}C_1}{{}^{40}C_4} = \frac{{}^{24}C_2 \times {}^{15}C_1}{{}^{40}C_4}$$

(i) $A \cup B \geq A$

$P(A \cup B) \geq P(A) \rightarrow P(A \cup B) \geq 3/4$

(ii) $(A \cap B) \leq B$

$P(A \cap B) \leq P(B) \rightarrow P(A \cap B) \leq 3/8$ — ①

$P(A \cup B) = P(A) + P(B) - P(A \cap B) \leq 1$

$= \frac{3}{4} + \frac{3}{8} - P(A \cap B) \leq 1$

$\frac{1}{8} \leq P(A \cap B)$ — ②

By ① & ②

$\frac{1}{8} \leq P(A \cap B) \leq \frac{3}{8}$

(iii) H.W.

Q. 10 people in a room are wearing tags which are marked 1-10. 3 people are selected at random and asked to leave the room simultaneously and their tag no. is noted

i) what is the probability that the smallest no. is 5

(ii) " " the largest no. is 5

(i) 10   6, 7, 8, 9, 10.

$\frac{{}^5C_2}{{}^{10}C_3}$

1, 2, 3, 4.

(ii) $\frac{{}^4C_2}{{}^{10}C_3}$

Solution: $Ex = {}^{10}C_3$.

(i) ${}^5C_2 / {}^{10}C_3 \times 1$ (ii) ${}^4C_2 / {}^{10}C_3 \times 1$

② $P(\bar{A}) = 1 - P(A)$

Proof: $S = A \cup \bar{A}$

$P(S) = P(A \cup \bar{A}) = P(A) + P(\bar{A})$

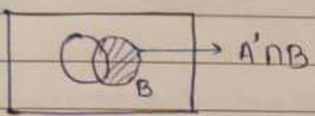
$\hookrightarrow P(\bar{A}) = 1 - P(A)$ Hence proved.

2/2/2023

③ for any two events A and B

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Proof:



$A \cup B = A \cup (A' \cap B)$

$P(A \cup B) = P(A \cup (A' \cap B))$

$P(A \cup B) = P(A) + P(A' \cap B)$ — (i)

$B = (A' \cap B) \cup (A \cap B)$

$P(B) = P(A' \cap B) + P(A \cap B)$

$\hookrightarrow P(A' \cap B) = P(B) - P(A \cap B)$ — (ii)

Sub (ii) in (i)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ Hence proved.

④ $P(A \cup B \cup C) = P(A) + P(B) + P(C) - \{P(A \cap B) + P(A \cap C) + P(B \cap C)\} + P(A \cap B \cap C)$

Proof: $P(A \cup B \cup C) = 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$

⑤ If $A \subseteq B$, $P(A) \leq P(B)$

⑥ $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

Q. Given if $P(A) = 3/4$, $P(B) = 3/8$, then prove that

i) $P(A \cup B) \geq 3/4$

ii) $1/8 \leq P(A \cap B) \leq 3/8$

iii) $3/8 \leq P(A \cap \bar{B}) \leq 5/8$

i) $\cancel{P(A \cup B) = P(A) + P(B) - P(A \cap B) = 3/4 + 3/8 - P(A \cap B)}$

$\cancel{P(A \cup B) =}$

Page :
Date :

30/1/23 write null set like ϕ or {}

PROBABILITY

Random expt : o/p cannot be predicted

Sample space : set of all possible outcomes (denoted by S)

Event : subset of S

Null event : don't contain any element

Elementary event : contains exactly 1 element

Exhaustive cases : all possible outcomes

favourable case to an event : An element is said to be favourable to event A if that element belongs to the set A.

Mutually exclusive events : two events A and B are said to be mutually exclusive if $A \cap B = \phi$ (intersection) (Null set)

Equally likely outcomes :

Indepent events : Occurrence of 1 event doesn't effect the occurrence of other.

classical approach

Probability : if A is an event

$P(A) = m/n$

$m \to$ no. of favourable cases to A, $n \to$ 1st total no. of possible outcomes $\hookrightarrow$ no. of exhaustive cases

Axiomatic approach – KALMOGOROV's

(i) $0 \le P(A) \le 1$ (probability of every event is b/w 0 and 1)

(ii) $P(S) = 1$ (probability of sample space is 1)

(iii) If A and B are mutually exclusive

$P(A \cup B) = P(A) + P(B)$

(iv) If $A_1 \dots A_n$ are pairwise mutually exclusive then

$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$

Theorems

1. $P(\phi) = 0$

Proof : $A = A \cup \phi$

$P(A) = P(A \cup \phi) = P(A) + P(\phi)$

$\hookrightarrow P(\phi) = 0$ Hence proved

Camlin