SET THEORY

A set is a collection of well-defined objects. The objects comprising the set are called elements. If x is an element of a set A, then we write $x \in A$. If x is not an element, then we write $x \notin A$.

Subset: Let A & B be two sets. A is said to be a subset of B, if $x \in A \Rightarrow x \in B$ & is denoted by $A \subset B$.

Equality of sets: Two sets A and B are said to be equal if $A \subseteq B$ & $B \subseteq A$ and we write $A = B$.

Universal set: All sets under consideration are taken to be subsets of a fixed set. This set is called universal set & is denoted by U.

Null set: A set containing no elements is called a null set and is denoted by ϕ.

Singleton set: A set containing a single element is called a singleton set.

SET OPERATIONS:

Let A and B be two sets.

Union: Union of A and B is denoted by $A \cup B$ and is defined as $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.

Intersection: Intersection of A and B is defined as $A \cap B = \{x \mid x \in A \ \& \ x \in B\}$. If $A \cap B = \phi$, then A & B are disjoint sets.

Difference: The difference $A - B$ is defined as $A - B = \{x \mid x \in A \ \& \ x \notin B\} = A \cap B'$

Complement: Complement of the set A is denoted by A' and is defined as $A' = \{x \mid x \in U \text{ but } x \notin A\}$.

Laws of Set Algebra:

1. $A \cap B = B \cap A$, $A \cup B = B \cup A$ **(Commutative laws)**
2. $A \cup (B \cup C) = (A \cup B) \cup C$, $A \cap (B \cap C) = (A \cap B) \cap C$ **(Associative laws)**
3. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ **(Distributive laws)**
4. $A \cup A = A$, $A \cap A = A$ **(Idempotent laws)**
5. $A \cup U = U$, $A \cap U = A$, $A \cup \phi = A$, $A \cap \phi = \phi$
6. $A \cap A' = \phi$, $A \cup A' = U$, $U' = \phi$, $\phi' = U$
7. $(A \cup B)' = A' \cap B'$, $(A \cap B)' = A' \cup B'$ **(De Morgan's laws)**

Cardinality: The number of elements in a set A is called cardinality of A and is denoted by n (A).

1. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
2. $n(A - B) = n(A) - n(A \cap B)$
3. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Methods of Enumeration:

Multiplication Principle: Suppose that a procedure, say procedure A can be done in n different ways and another procedure say B can be done in m different ways. Also suppose that any way of doing A can be followed by any way of doing B. Then, the procedure consisting of 'A followed by B' can be performed in mn ways.

Addition Principle: The number ways in which either A or B, but not both, can be performed is m + n.

Permutation: (Arrangement of given objects; Order is important)

The number of permutations of n distinct objects taken r at a time is

- $^nP_r = \frac{n!}{(n-r)!}, \ r \leq n$, if repetition is not allowed.
- n^r, if repetition is allowed.
- $\frac{n!}{k_1!k_2!\dots k_m!}$ where, of the n objects, k_1 are of one kind, k_2 are of a second kind, …,k_m are of m^{th} kind,

  $k_1 + k_2 + \dots + k_m = n$. (all objects are taken).
- $(n-1)!$, when arranged along a circle.
- $(n-1)!/2$, when clockwise and anticlockwise arrangements are indistinguishable

Combination: (Selection of objects; Order is not important)

- $^nC_r = \frac{^nP_r}{r!} = \frac{n!}{r!\,(n-r)!}$, if repetition is not allowed.
- $^{n+r-1}C_r$, if repetition is allowed.

Probability Theory

Random experiment: If the repetition of an experiment under identical conditions results in different possible outcomes, that experiment is called a random experiment.
Examples: Tossing of a coin, rolling of a die.

Sample Space: The set of all possible outcomes of a random experiment.
Examples:

- In tossing of a coin: S = {H, T}
- In tossing of two coins: S= {HH, HT, TH, TT}
- In tossing of two identical coins: S = {HH, HT, TT}
- In rolling of a die: S = {1,2,3,4,5,6}

If a sample space has finite number of elements, then it is called a finite sample space. Otherwise, the sample space is said to be an infinite sample space.
Example:

- Consider rolling of a die till a 5 appears: S = {5, 15,25,..65, 115, 125,…,215,225,….}

Event: An event is a subset of the sample space.
Null Event: An event, which does not contain any element, is called a null event or an impossible event, denoted by ϕ.
Certain or sure Event: If the event contains all the elements of the sample space, then it is called a certain event.

Definition: We say that an event has occurred if the outcome of the experiment belongs to the event. An event has not occurred if the outcome of the experiment is not in the event.

Mutually Exclusive Events: Two events A and B are said to be mutually exclusive if both of them cannot occur simultaneously. i.e., if occurrence of one event prevents the occurrence of the other, then the events are said to be mutually exclusive. A and B are mutually exclusive if $A \cap B = \phi$.

- ✓ In tossing of a coin head and tail are mutually exclusive
- ✓ In rolling of a die all six faces are mutually exclusive

Independent events:
Are **those events whose occurrence is not dependent on any other event**. For example, if we flip a coin in the air and get the outcome as Head, then again if we flip the coin but this time, we get the outcome as Tail. In both cases, the occurrence of both events is independent of each other.
If A and B are independent events, then $P(A \cap B) = P(A). P(B)$

Equally likely outcomes: If all outcomes of a random experiment have equal chances of occurrence, then the outcomes are said to be equally likely.

- ✓ In tossing of an unbiased coin, head and tail are equally likely.
- ✓ In rolling of an honest die, all six faces are equally likely.

Exhaustive cases: The total number of possible outcomes of a random experiment is called exhaustive cases for that experiment.

- In tossing of a coin, exhaustive cases = 2
- In tossing of 2 coins, exhaustive cases = 4
- In tossing n coins, exhaustive cases = 2^n
- In rolling of two dice, exhaustive cases = 6

Favourable cases: An outcome x is said to be favourable to an event A, if x belongs to A. The total number of outcomes favourable to A is called favourable cases to A.

- In tossing of two coins, favourable cases for getting 2 heads is 1, for getting exactly one head is 2 and for getting at least 2 heads is 3.
- In drawing a card from a pack, there are 4 cases favouring a king, 2 cases favouring a red queen and 26 cases favouring a black card.

Probability is a quantitative measure of chances of occurrence. There are 3 approaches to the study of probability.

1. Classical approach
2. Statistical or empirical approach
3. Axiomatic approach

Classical Definition of Probability: If an event A can occur in m different ways out of a total of n ways all of which are **equally likely** and mutually exclusive, then the probability of the event A is given by

$$P(A) = \frac{m}{n} = \frac{\text{favourable cases}}{\text{Exhaustive cases}}.$$

- For a null set, m = 0. Hence $P(\phi) = 0$
- For the sample space m = n. Hence P(S) = 1
- $0 \le m \le n$. Hence $0 \le m/n \le 1$ i.e. $0 \le P(A) \le 1$
- m outcomes are favourable to A $\Rightarrow$ remaining n – m are favourable to A′.
  Hence $P(A') = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$, i.e. $P(A) + P(A') = 1$

Statistical Definition of Probability:

If an experiment is repeated several times under essentially homogeneous and identical conditions, then the limiting value of the ratio of the number of times the event occurs to the number of trials, as the number of trials become indefinitely large, is called the probability of that event.

i.e. if an event A occurs m times in n trials then $P(A) = \lim_{n\to\infty} \frac{m}{n}$

Axiomatic approach:

1. For any event E, Probability P (E) ≥ 0.
2. When S is the sample space of an experiment; that is the set of all possible outcomes, P(S) = 1.
3. If A and B are mutually exclusive outcomes, then P (A ∪ B) = P (A) + P (B).

THOREMS:

Theorem 1.1:
If ϕ is the empty set, then P (ϕ) = 0.
Theorem 1.2:
If $\bar{A}$ is the complementary even of A, then P(A) = 1- P($\bar{A}$).
Theorem 1.3:
If A and B are any two events, then P($A \cup B$) = P(A) + P(B) – P ($A \cap B$)
Theorem 1.4: SAME as above for three events.
Theorem 1.5:
If $A \subset B$ then P(A) ≤ P(B).
Theorem 1.6:
Probability that exactly one of the event A or B occur i.e., P ($\{A \cap \bar{B}\} \cup \{\bar{A} \cap B\}$) = P(A)+P(B)-2 P ($A \cap B$)

Problems:

1. A pair of dice is rolled. What is the probability of getting a sum greater than 6? A pair of dice is rolled. What is the probability of rolling a sum neither 5 nor 10?

2. There are 8 positive numbers and 6 negative numbers. 4 numbers are chosen at random and multiplied. What is the probability that the product is a positive number?

3. A number is chosen between 1 and 50. What is the probability that it is divisible by 8?

4. An urn contains 5 red and 10 black balls. 8 of them are placed in another urn. What is the chance that the later then contains 2 red and 6 black balls?

5. A bag contains 8 white and 6 red balls. What is the probability of drawing two balls of the same color?

6. The coefficient A, b, c of the quadratic equation $ax^2 + \mathrm{bx} + \mathrm{c} = 0$ are determined by throwing a die 3 times find the probability that 1. Roots are real 2. Roots are complex.

7. Three group of children contain respectively 3 girls 1 boy, 2 girls 2 boys, 1 girl 3 boys. One child is selected at random from each group. Show that the chance that the 3 selected consist of 1 girl and 2 boys is 13/32.

8. A committee of 4 person is to be appointed from 3 offices of production department, 4 officers from purchase department, 2 officer from the sales department and 1 charted accountant. Find the probability of

1. There must be one from each category.
2. It should have at least one from the purchase department.
3. The CA must be in the committee.

9. What is the probability that a randomly selected year contains 53 Sundays?

10. Each of 2 person A and B tosses 3 fair coins. Find the probability that they get the same number of heads.

11. A and B throw a die alternatively till one of them gets a '6' and wins the game. Find their respective probabilities of winning if A starts first.

    Solution: Let S denote the success (getting a '6') and F denote the failure (not getting a '6').
    Thus, P(S) =1/6, P(F) =5/6
    P(A wins in the first throw) = P(S) = 1/6
    A gets the third throw, when the first throw by A and second throw by B result into failures.
    Therefore, P(A wins in the 3rd throw) = P(FFS) = P(F)P(F)P(S)= (5/6)(5/6)(1/6)
    P(A wins in the 5th throw) = P (FFFFS)= (5/6)(5/6) (5/6)(5/6)(1/6)
    Hence, P(A wins) =1/6 + (5/6)(5/6)(1/6) + (5/6)(5/6) (5/6)(5/6)(1/6) + ... = 6/11 (G.P infinite sum)
    P(B wins) = 1 – P (A wins) = 1- (6/11)= 5/11.

12. A and B throw alternatively a pair of die. A wins if he throws sum 6 before B throws sum 7 and B wins the other way. If A begins, find his chances of winning the game.

13. Six people toss a fair coin one by one. The game is win by the player who throws head. Find the probability of success of the 4^{th} player.

Answers:

1. 7/12, 29/36
2. 505/1001
3. 6/25
4. 140/429
5. 43/91
6. 43/216, 173/216
7. 13/32
8. 4/35, 195/210, 0.4
9. 1/7 and 2/7(leap year)
10. 5/16
11. Solution
12. For A: 30/61 For B:31/61
13. 4/63

Conditional Probability

Until now in probability, we have discussed the methods of finding the probability of events. If we have two events from the same sample space, does the information about the occurrence of one of the events affect the probability of the other event? Let us try to answer this question by taking up a random experiment in which the outcomes are equally likely to occur. Consider the experiment of tossing three fair coins. The sample space of the experiment is

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

Since the coins are fair, we can assign the probability 1/8 to each sample point. Let E be the event 'at least two heads appear' and F be the event 'first coin shows tail'.

Then, $E = \{HHH, HHT, HTH, THH\}$ *and* $F = \{THH, THT, TTH, TTT\}$

Therefore $P(E)=P(\{HHH\})+P(\{HHT\}+P(\{HTH\})+P(\{THH\})$

$=1/8+1/8+1/8+1/8=1/2$

and

$P(F) = P(\{THH\})+P(\{THT\}) + P(\{TTH\}) + P(\{TTT\}) =1/8+1/8+1/8+1/8=1/2$

Also $E \cap F = \{THH\}$ *with* $P(E \cap F) = P(\{THH\}) =1/8$

Now, suppose we are given that the first coin shows tail, i.e. F occurs, then what is the probability of occurrence of E? With the information of occurrence of F, we are sure that the cases in which first coin does not result into a tail should not be considered while finding the probability of E. This information reduces our sample space from the set S to its subset F for the event E. In other words, the additional information really amounts to telling us that the situation may be considered as being that of a new random experiment for which the sample space consists of all those outcomes only which are favourable to the occurrence of the event F.

Now, the sample point of F which is favourable to event E is THH.

Thus, Probability of E considering F as the sample space =1/4

or Probability of E given that the event F has occurred =1/4

This probability of the event E is called the conditional probability of E given that F has already occurred and is denoted by $P(E|F)$.

Thus $P(E|F) =1/4$

Note that the elements of F which favour the event E are the common elements of E and F, i.e., the sample points of $E \cap F$.

Thus, we can also write the conditional probability of E given that F has occurred as

$$P(E|F) = \frac{\text{Number of elementaryevents favourable to } E\cap F}{\text{Number of elementaryevents which are favourable to } F} = \frac{n(E\cap F)}{n(F)}$$

Dividing the numerator and the denominator by total number of elementary events of the sample space, we see that P(E|F) can also be written as

$$P(E|F) = \frac{n(E\cap F)/n(s)}{n(F)/n(s)} = \frac{P(E\cap F)}{P(F)}$$

If E and F are two events associated with the same sample space of a random experiment, the conditional probability of the event E given that F has occurred, i.e. P (E|F) is given by

$$P(E|F) = \frac{P(E\cap F)}{P(F)}, \text{ provided } P(F) \neq 0.$$

Example: A die is tossed if the number is odd on the face, what is the probability that it is a prime?
S = {1,2,3,4,5,6}
A= {1,3,5} reduction in the sample space because of the additional information that it is odd.
B= {3,5}

$$P(B|A) = \frac{P(B\cap A)}{P(A)} = \frac{2}{3}$$

NOTE:

If A and B are independent events, then $P(B|A) = \frac{P(B\cap A)}{P(A)} = \frac{P(B).P(A)}{P(A)} = P(B)$.

Theorem:

If A and B are 2 independent events of S then prove that A& $\bar{B}$, *B &* $\bar{A}$ *and* $\bar{A}$ *&* $\bar{B}$ *are also independent.*

Problems:

1. *If A and B are 2 independent events of S such that* $P(\bar{A} \cap B)=2/15$, $P(A \cap \bar{B})=1/6$ *then find P(B). Answer= 1\6 or 4\5.*
2. *If A and B are 2 independent events of S such that P(A)= 1/3, P(B)=1/4,* $P(A\cup B) =1/2$ *then find i) P(A|B) ii) P(B|A) iii)* $P(A\cap \bar{B})$ *iv)* $P(A|\bar{B})$. *Answer: i) 1/3 ii) 1/4 iii)1/4 iv) 1/3*
3. *In a certain town 40% have brown hair, 25% have brown eyes, 15% have both brown hair and brown eyes. A person is selected at random.*
   i. *If he has brown hair, then what is the probability that he has brown eyes also.*
   ii. *If he has brown eyes, then what is the probability that he has not have brown hair.*
   iii. *Determine the probability that he neither have brown hair nor brown eyes.*
   Answer: 3/8, 0.4 and 0.5

4. *A bag contains 10 gold coins and 8 silver coins. Two successive drawings of 4 coins are made such that.*
    i. *The coins are replaced before the second trial.*
    ii. *The coins are not replaced before the second trial.*

Find the probability that the first drawing will give 4 gold coins and second drawing will give 4 silver coins.

i) coins are replaced before the second trial

Gold = 10

Silver = 8

first drawing will give 4 gold = $^{10}C_4/^{18}C_4$

second drawing will give 4 silver = $^{8}C_4/^{18}C_4$

probability that the first drawing will give 4 gold and the second 4 silver coins. = $(^{10}C_4/^{18}C_4) * (^{8}C_4/^{18}C_4)$

$\mathbf{= {}^{10}C_4 * {}^{8}C_4 /({}^{18}C_4)^2}$

ii) the coins are not replaced before the second trial.

first drawing will give 4 gold = $^{10}C_4/^{18}C_4$

second drawing will give = $^{8}C_4/^{14}C_4$

probability that the first drawing will give 4 gold and the second 4 silver coins. = $(^{10}C_4/^{18}C_4) * (^{8}C_4/^{14}C_4)$

$\mathbf{=({}^{10}C_4 * {}^{8}C_4) /({}^{18}C_4 . {}^{14}C_4)}$

5. *Two defective tubes get mixed up with 4 good ones. The tubes are tested one by one, until both defective are found. What is the probability that the last defective tube is obtained on a) 2^{nd} test, b)3^{rd} test, c)6^{th} test.*

Total probability theorem, Bayes' theorem

Partition of a set:

The family of sets $C_1, C_2, \ldots, C_n$ is said to be a partition of S, if

i. $\bigcup_{i=1}^{n} C_i = S$ Collectively exhaustive

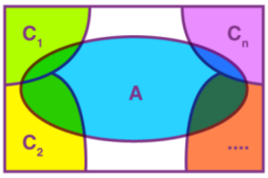
ii. $C_i \cap C_j = \phi, i \neq j, \forall\, i, j$ Mutually Exclusive

Law of Total Probability Statement

Let events $C_1, C_2 \ldots C_n$ form partitions of the sample space S, where all the events have a non-zero probability of occurrence. For any event, A associated with S, according to the total probability theorem,

$$P(A) = \sum_{k=0}^{n} P(C_k)P(A|C_k)$$

Proof:



$S = C_1 \cup C_2 \cup \ldots \cup C_n$

For any event A,

$A = A \cap S$

$= A \cap (C_1 \cup C_2 \cup \ldots \cup C_n)$

$= (A \cap C_1) \cup (A \cap C_2) \cup \ldots \cup (A \cap C_n) \ldots\ldots (1)$

We know that $A \cap C_i$ and $A \cap C_k$ are the subsets of C_i and C_k. Here, C_i and C_k are disjoint for $i \neq k$. since they are mutually events which implies that $A \cap C_i$ and $A \cap C_k$ are also disjoint for all $i \neq k$. Thus,

$P(A) = P[(A \cap C_1) \cup (A \cap C_2) \cup \ldots\ldots \cup (A \cap C_n)]$

$= P(A \cap C_1) + P(A \cap C_2) + \ldots + P(A \cap C_n) \ldots\ldots\ldots (2)$

We know that,

$P(A \cap C_i) = P(C_i)\, P(A|C_i)$(By multiplication rule of probability) . . . . (3)

Using (2) and (3), (1) can be rewritten as,

$P(A) = P(C_1)P(A|\, C_1) + P(C_2)P(A|C_2) + P(C_3)P(A|\, C_3) + \ldots\ldots + P(C_n)P(A|\, C_n)$

Hence, the theorem can be stated in form of equation as,

$$P(A) = \sum_{k=0}^{n} P(C_k)P(A|C_k)$$

Bayes Theorem Statement

Let $E_1, E_2, \ldots, E_n$ be a set of events associated with a sample space S, where all the events $E_1, E_2, \ldots, E_n$ have nonzero probability of occurrence and they form a partition of S. Let A be any event associated with S, then according to Bayes theorem,

$$P(E_i \mid A) = \frac{P(E_i)P(A \mid E_i)}{\sum_{k=1}^{n} P(E_k)P(A|E_k)}$$

for any k = 1, 2, 3, ..., n

Bayes Theorem Proof

According to the conditional probability formula,

$$P(E_i \mid A) = \frac{P(E_i \cap A)}{P(A)} \ldots (1)$$

Using the multiplication rule of probability,

$$P(E_i \cap A) = P(E_i)P(A \mid E_i) \ldots (2)$$

Using total probability theorem,

$$P(A) = \sum_{k=1}^{n} P(E_k)P(A|E_k) \ldots (3)$$

Putting the values from equations (2) and (3) in equation 1, we get

$$P(E_i \mid A) = \frac{P(E_i)P(A \mid E_i)}{\sum_{k=1}^{n} P(E_k)P(A|E_k)}$$

Problems:

1. A person has undertaken a construction job. The probabilities are 0.65 that there will be strike, 0.80 that the construction job will be completed on time if there is no strike, and 0.32 that the construction job will be completed on time if there is a strike. Determine the probability that the construction job will be completed on time.

   Solution: Let A be the event that the construction job will be completed on time, and B be the event that there will be a strike.
   We must find P(A).
   We have P(B) = 0.65, P(no strike) = P(B′) =1 − 0.65 = 0.35
   P(A|B) = 0.32, P(A|B′) = 0.80
   Since events B and B′ form a partition of the sample space S, therefore, by theorem on total probability, we have.
   P(A) = P(B) P(A|B) + P(B′) P(A|B′)
   = 0.65 × 0.32 + 0.35 × 0.8 = 0.208 + 0.28 = 0.488

2. suppose 3 companies x y z produce TVs x produces twice as many as y while y and z produce same number. It is known that 2% of x, 2% of y, 4% of z are defected. All the TVs are produced are put into 1 shop and then 1 tv is chosen at random what is the probability that the tv is defecated. Suppose a tv chosen is defective what is the probability that this tv is produced by company x.

Solution:

Let X denote the event "tv is produced by company x."

Let Y denote the event "tv is produced by company y."

Let Z denote the event "tv is produced by company z."

Let D denote the event "tv is defected".

Given

$$P(X)=0.5, P(Y)=P(Z)=0.25$$
$$P(D|X)=0.02, P(D|Y)=0.02, P(D|Z)=0.04$$

a) Theorem of total probability

$$P(D)=P(X)P(D|X)+P(Y)P(D|Y)$$
$$+P(Z)P(Z|D)$$
$$=0.5(0.02)+0.25(0.02)+0.25(0.04)=0.025$$

The probability that the tv is defected is 0.025.

ii) By the Bayes' Rule

$$P(X|D)$$
$$=\frac{P(X)P(D|X)}{P(X)P(D|X)+P(Y)P(D|Y)+P(Z)P(Z|D)}$$
$$=\frac{0.5(0.02)}{0.5(0.02)+0.25(0.02)+0.25(0.04)}$$
$$=0.4$$

The probability that this tv is produced by company x is 0.4.

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3. There are 3 boxes, the first one containing 1 white, 2 red and 3 black balls: the second one containing 2 white, 3 red and 1 black ball and the third one containing 3 white, 1 red and 2 black balls. A box is chosen at random and from it two balls are drawn at random. One ball is red and the other, white. What is the probability that they come from the second box? Ans: 6/11

4. A randomly selected year has 53 Sundays. Find the probability that it is a leap year. Ans: 0.4

5. Two factories produce identical clocks. The production of the first factory consists of 10,000 clocks of which 100 are defective. The second factory produces 20,000 clocks of which 300 are defective. What is the probability that a particular defective clock was produced in the first factory? Ans:0.25

6. One percent of the population suffers from a certain disease. There is blood test for this disease, and it is **99%** accurate, in other words, the probability that is gives the correct answer is **0.99**, regardless of whether the person is sick or healthy. A person takes the blood test, and the result says that he has the disease. The probability that he actually has the disease, is?

Solution:

A be the event of having the disease.

B be the event of testing positive.

$P(A) = 0.01$

$P(B) = P(B/A)P(A) + P(B/notA)P(notA)$

$P(B) = 0.01 * 0.99 + 0.01 * 0.99 = 0.0198$

$P(A \cap B) = 0.99 * 0.01$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow \frac{0.99 * 0.01}{0.0198}$$

=0.5 or 50%

7. If a machine is correctly set up, it produces 90% acceptable items. If it is incorrectly set up, it produces only 40% acceptable items. Past experience shows that 80% of the set ups are correctly done. If after a certain set up, the machine produces 2 acceptable items, find the probability that the machine is correctly setup.

Solution:
Let A be the event that the machine produces 2 acceptable items.
Also let B_1 represent the event of correct set up and B_2 represent the event of incorrect setup.
Now $P(B_1) = 0.8$, $P(B_2) = 0.2$
$P(A|B_1) = 0.9 \times 0.9$ and $P(A|B_2) = 0.4 \times 0.4$
Therefore $P(B_1|A) = 0.95$

8. It is suspected that a patient has one of the diseases A_1, A_2, A_3. Suppose that the population suffering from this illness are in the ratio 2:1:1. The patient is given a test which turns out to be positive in 25%of the cases of A_1, 50% of the cases of A_2 and 90% of the cases of A_3. Given that out of 3 tests taken by the patient two are positive, then find the probability for each of the diseases. Ans: 0.3128, 0.4170, 0.2703
Solution:
$A_i \rightarrow$ the patient has the illness A_i
$B \rightarrow$ two test results are positive.
$P(A_1) = 2/4$ $\quad$ $P(A_2) = 1/4$ $\quad$ $P(A_3) = 1/4$

$P(B|A_1) = [PPN+PNP+NPP]$
$= {}^3C_2(1/4)^2(3/4)$

$P(B|A_2) = {}^3C_2(1/2)^2(1/2)$

$P(B|A_3) = {}^3C_2(9/10)^2(1/10)$

$P(A_1|B) = ?$ $\quad$ $P(A_2|B) = ?$, $\quad$ $P(A_3|B) = ?$

9. An anchor with an accuracy of 75% fires 3 arrows at one target. The probability of the target falling is 0.6 if he hit once, 0.7 if he his twice. 0.8 if he hits thrice. Given that, the target has fallen find the probability that it was hit twice. Ans:0.411
Solution:
$B_i \rightarrow$ target is hit the i^{th} time.
$B \rightarrow$ target falls
P (anchor hits) =0.75
$P(B|B_1) = 0.6$, $P(B|B_2) = 0.7$, $P(B|B_3) = 0.8$

$P(B_1) = {}^3C_1(1/4)^2(3/4)$ $\quad$ $P(B_2) = {}^3C_2(1/4)(3/4)^2$ $\quad$ $P(B_3) = {}^3C_3(1/4)^0(3/4)^3$

$P(B_2|B) = ?$