Assignment-3 : III CS A(MAT 2155)

1. Show that a lattice (A, $\leq$) is distributive if and only if for any elements a, b, c in the lattice , $(a \vee b) \wedge c \leq a \vee (b \wedge c)$.
2. Show that a lattice (A, $\leq$) is distributive if and only if for any elements a, b, c in the lattice , $(a \vee b) \wedge (b \vee c) \wedge (c \vee a) = (a \wedge b) \vee (b \wedge c) \vee (c \wedge a)$.
3. A lattice (A, $\leq$) is called a modular lattice if for any a, b, c $\in A$ where a $\leq c$, $a \vee (b \wedge c) = (a \vee b) \wedge c$. Show that a lattice is modular if and only if the following condition holds:
$$a \vee (b \wedge (a \vee c)) = (a \vee b) \wedge (a \vee c)$$
4. Draw the Hasse diagram representing the positive divisors of 24.
5. How many of the first thousand positive integers have distinct digits?
6. How many ways are there to distribute eight balls into six boxes with the first two boxes collectively having at most 4 balls if
    a. The balls are identical?
    b. The balls are distinct?
7. How many ways are there to collect \$24 from 4 children and 6 adults if each person gives at least \$1,but children can give at most \$4 and each adult at most \$7?
8. Prove that number of partitions of n is equal to the number of partitions of 2n which have exactly n parts.
9. Show that the proportion of permutations of {1,2,…n} which contains no consecutive pair (i, i+1) for any i,approximately $\frac{n+1}{ne}$.
10. Given n=5 and the five marks 0,1,2,3,4 what are the 50^{th} and 100^{th} permutations in the following orders: 1. Lexicographical 2. Reverse lexicographical 3. Fike's