Name: Disha Andre

PRN: 123B1B078

Assignment No. 1

Title:

You are working as a data analyst for a financial institution that handles high precision transactions involving large, multi-digit monetary values. Recently, the institution needed to calculate the product of two exceptionally large numbers (each over 15 digits) representing aggregated transaction amounts for quarterly reports. Due to the size of these values, traditional multiplication methods are computationally inefficient. Implement the algorithm to perform high-precision multiplication of these large integers accurately and efficiently. Test your implementation with multiple large-number pairs to ensure reliability and precision in financial reporting.

Objectives:

1. To study the problem of multiplying very large numbers used in financial systems.
2. To understand and implement the Karatsuba multiplication algorithm.
3. To improve efficiency compared to traditional multiplication methods.
4. To ensure accurate results for large monetary values.

Theory:

In financial institutions, calculations often involve very large numbers, such as total transaction amounts or quarterly financial data. Using the traditional multiplication method for such large values is inefficient because it takes more time as the number of digits increases.

To solve this problem, the Karatsuba multiplication algorithm is used. It is a divide and conquer algorithm that multiplies large numbers faster by reducing the number of multiplication operations.

Karatsuba Algorithm:

Karatsuba's algorithm splits each large number into two smaller parts.

Let:

$x = a \times 10^m + b$

$y = c \times 10^m + d$

Instead of four multiplications, the algorithm performs only three:

- $z_0 = b \times d$
- $z_2 = a \times c$
- $z_1 = (a + b)(c + d) - z_2 - z_0$

The final product is calculated as: $x \times y = z_2 \times 10^{2m} + z_1 \times 10^m + z_0$

This reduces the time complexity and makes the algorithm efficient for large numbers.

Example (Karatsuba Method):

Multiply: 1234×5678

Step 1: Split the numbers

Number of digits = 4, so $m = 2$

$a = 12,\ b = 34$
$c = 56,\ d = 78$

Step 2: Perform three multiplications

$z_0 = 34 \times 78 = 2652$
$z_2 = 12 \times 56 = 672$
$z_1 = (12 + 34)(56 + 78) = 46 \times 134 = 6164$
$z_1 = 6164 - 672 - 2652 = 2840$

Step 3: Combine results

$= (672 \times 10^4) + (2840 \times 10^2) + 2652$
$= 7006652$

Final Answer: $1234 \times 5678 = 7006652$

Time Complexity:

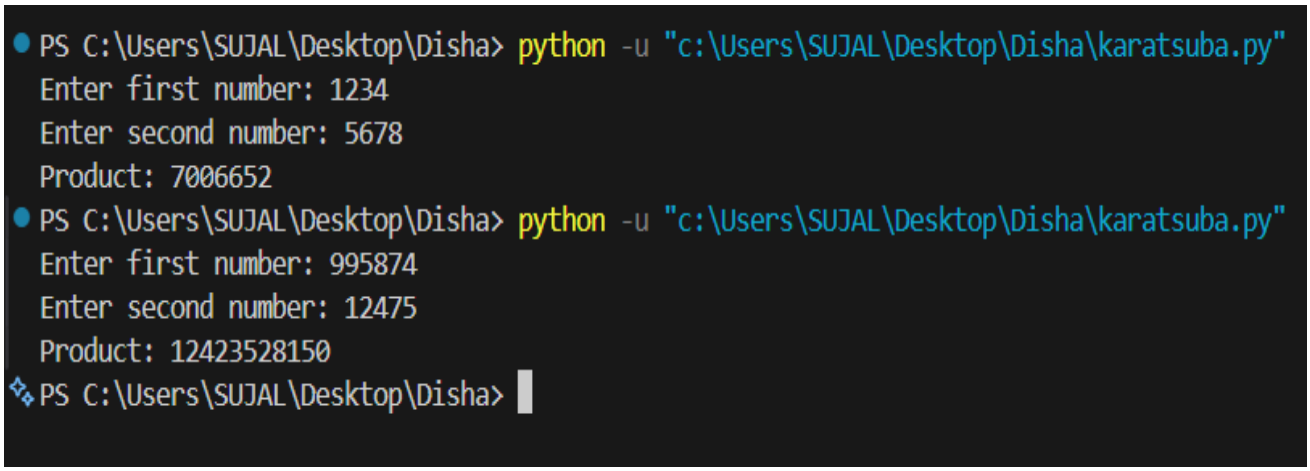
- Traditional multiplication: $O(n^2)$
- Karatsuba multiplication: $O(n^{1.5})$

Karatsuba is faster for large numbers.

Code:

```
def karatsuba(x, y):
    if x < 10 or y < 10:
        return x * y

    # Calculate the number of digits
    n = max(len(str(x)), len(str(y)))
    m = n // 2

    # Split the numbers
    a, b = divmod(x, 10**m)
    c, d = divmod(y, 10**m)

    # Recursive calls
    z0 = karatsuba(b, d)
    z1 = karatsuba(b + a, d + c)
    z2 = karatsuba(a, c)

    # Combine the results
    return (z2 * 10**(2*m)) + ((z1 - z2 - z0) * 10**m) + z0

# Take input from user
num1 = int(input("Enter first number: "))
num2 = int(input("Enter second number: "))

result = karatsuba(num1, num2)
print("Product:", result)
```

Output:



Conclusion:

Karatsuba multiplication is an efficient algorithm for multiplying large numbers accurately. It reduces computation time compared to traditional methods and is suitable for financial applications involving high-precision values. The implementation provides reliable and correct results for large inputs.