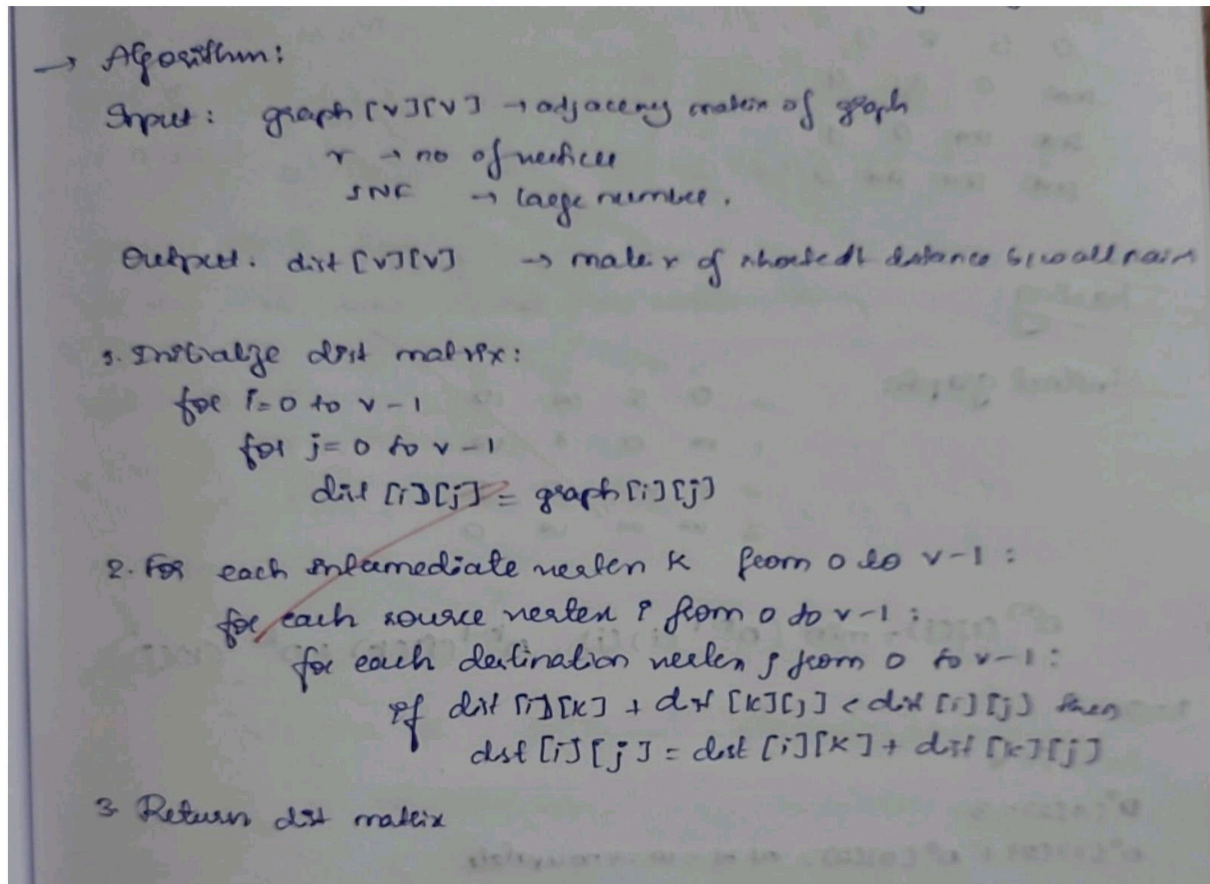


PROBLEM 6

Implement All Pair Shortest paths problem using Floyd's algorithm.

ALGORITHM



CODE

```
#include <stdio.h>
```

```
#define INF 99999
```

```
#define V 4 // Number of vertices
```

```
// Function to print the shortest distance matrix
```

```
void printSolution(int dist[V][V]) {
```

```
    printf("Shortest distances between every pair of vertices:\n");
```

```
    for (int i = 0; i < V; i++) {
```

```
        for (int j = 0; j < V; j++) {
```

```
            if (dist[i][j] == INF)
```

```

        printf("%7s", "INF");
    else
        printf("%7d", dist[i][j]);
    }
    printf("\n");
}
}

// Floyd-Warshall algorithm
void floydWarshall(int graph[V][V]) {
    int dist[V][V];

    // Initialize the solution matrix with input graph
    for (int i = 0; i < V; i++)
        for (int j = 0; j < V; j++)
            dist[i][j] = graph[i][j];

    // Update dist[i][j] to the shortest path
    for (int k = 0; k < V; k++) {
        for (int i = 0; i < V; i++) {
            for (int j = 0; j < V; j++) {
                if (dist[i][k] + dist[k][j] < dist[i][j])
                    dist[i][j] = dist[i][k] + dist[k][j];
            }
        }
    }

    printSolution(dist);
}

```

```
// Driver code

int main() {
    // Example graph with 4 vertices
    int graph[V][V] = {
        {0, 5, INF, 10},
        {INF, 0, 3, INF},
        {INF, INF, 0, 1},
        {INF, INF, INF, 0}
    };

    floydWarshall(graph);

    return 0;
}
```

OUTPUT:

Shortest distances between every pair of vertices:

0	5	8	9
INF	0	3	4
INF	INF	0	1
INF	INF	INF	0

TRACING

Tracing:

Initial graph:

	0	1	2	3
0	0	5	∞	10
1	∞	0	3	∞
2	∞	∞	0	1
3	∞	∞	∞	0

$$D^{(k)}(i)(j) = \min(D^{(k-1)}(i)(j), D^{(k-1)}(i)(k) + D^{(k-1)}(k)(j))$$

$k=0$

$i=1, j=2$

$$D^0(1)(2) = 3$$

$$D^0(1)(0) + D^0(0)(2) = \infty + \infty = \infty \rightarrow \text{no update}$$

$i=0, j=2$

$$D^0(0)(2) = \infty$$

$$D^0(0)(0) + D^0(0)(2) = 0 + \infty = \infty \rightarrow \text{no update}$$

no update since going through 0 doesn't improve any distance

S2: $k=1$

update matrix to $D^{(1)}$:

check if $D^{(1)}(i)(j) > D^{(1)}(i)(1) + D^{(1)}(1)(j)$ - yes

$$i=0, j=2 \quad D^{(1)}(0)(2) = \infty, \quad D^{(1)}(0)(1) + D^{(1)}(1)(2) = 5 + 3 = 8$$

$$8 < \infty, \text{ update } D^{(1)}(0)(2) = 8$$

for other pairs, similarly checked & no updates here.

S3: $k=2$

update to $D^{(2)}$:

$i=0, j=3$

$$D^{(2)}(0)(3) = 10, \quad D^{(2)}(0)(2) + D^{(2)}(2)(3) = 8 + 1 = 9$$

$$9 < 10 \Rightarrow D^{(2)}(0)(3) = 9$$

$i=1, j=3$:

$$D^{(2)}(1)(3) = \infty, \quad D^{(2)}(1)(2) + D^{(2)}(2)(3) = 3 + 1 = 4$$

$$D^{(2)}(1)(3) = 4$$

S4: $k=3$

update to $D^{(3)}$:

when checked $D^{(3)}(i)(j) > D^{(3)}(i)(3) + D^{(3)}(3)(j)$

$$\text{for } i=0, j=0 \quad D^{(3)}(0)(0) = 0 \quad D^{(3)}(0)(3) + D^{(3)}(3)(0) = 9 + \infty = \infty \rightarrow \text{no update}$$

$$i=0, j=3 \quad D^{(3)}(0)(3) = 9 \quad D^{(3)}(0)(3) + D^{(3)}(3)(3) = 9 + 0 = 9 \rightarrow \text{same}$$

Similarly when checked for remaining vertices, the condition not satisfied. No updates.

Final matrix D^{CH} :

$$\begin{bmatrix} 0 & 5 & 8 & 9 \\ \infty & 0 & 3 & 4 \\ \infty & \infty & 0 & 1 \\ \infty & \infty & \infty & 0 \end{bmatrix}.$$

$$\frac{6}{6}$$

~~2h~~
17/5/25

LEETCODE 5

NUMBER OF WAYS TO ARRIVE AT DESTINATION

ALGORITHM

```
# Number of ways to arrive at a destination
Pseudo code:

Initialize graph as adjacency list
Initialize minTime[n] = ∞ for all nodes
Initialize ways[n] = 0 for all nodes

Set minTime[0] = 0
Set ways[0] = 1

Create min-heap priority queue pq & insert (0,0) → (time, node)
while pq is not empty:
    time, u = pq.pop()
    If time > minTime[u]:
        continue
    For each neighbor v of u with edge time t:
        newTime = time + t
        If newTime < minTime[v]:
            minTime[v] = newTime
            ways[v] = ways[u]
            pq.push((newTime, v))
        Else If newTime == minTime[v]:
            ways[v] = (ways[v] + ways[u]) mod (1e9 + 7)

Return ways[n - 1]
```

CODE

```
#include <stdio.h>
```

```
#include <stdlib.h>
```

```
#include <limits.h>
```

```
#define MOD 1000000007
```

```
typedef struct {
```

```
    int to, weight;
```

```
} Edge;
```

```
typedef struct {  
    Edge* edges;  
    int size, capacity;  
} AdjList;
```

```
typedef struct {  
    long long time;  
    int node;  
} HeapNode;
```

```
typedef struct {  
    HeapNode* arr;  
    int size;  
} MinHeap;
```

```
void swap(HeapNode* a, HeapNode* b) {  
    HeapNode tmp = *a;  
    *a = *b;  
    *b = tmp;  
}
```

```
void heapify_up(MinHeap* h, int i) {  
    while (i > 0 && h->arr[i].time < h->arr[(i - 1) / 2].time) {  
        swap(&h->arr[i], &h->arr[(i - 1) / 2]);  
        i = (i - 1) / 2;  
    }  
}
```

```
void heapify_down(MinHeap* h, int i) {  
    int left, right, smallest;
```

```

while (2 * i + 1 < h->size) {
    left = 2 * i + 1;
    right = 2 * i + 2;
    smallest = i;

    if (left < h->size && h->arr[left].time < h->arr[smallest].time)
        smallest = left;
    if (right < h->size && h->arr[right].time < h->arr[smallest].time)
        smallest = right;

    if (smallest == i) break;
    swap(&h->arr[i], &h->arr[smallest]);
    i = smallest;
}
}

void push(MinHeap* h, long long time, int node) {
    h->arr[h->size++] = (HeapNode){time, node};
    heapify_up(h, h->size - 1);
}

HeapNode pop(MinHeap* h) {
    HeapNode top = h->arr[0];
    h->arr[0] = h->arr[--h->size];
    heapify_down(h, 0);
    return top;
}

int countPaths(int n, int** roads, int roadsSize, int* roadsColSize) {
    // Step 1: Build graph

```



```

AdjList* graph = malloc(n * sizeof(AdjList));
for (int i = 0; i < n; ++i) {
    graph[i].edges = malloc(100 * sizeof(Edge));
    graph[i].size = 0;
    graph[i].capacity = 100;
}

for (int i = 0; i < roadsSize; ++i) {
    int u = roads[i][0], v = roads[i][1], w = roads[i][2];
    graph[u].edges[graph[u].size++] = (Edge){v, w};
    graph[v].edges[graph[v].size++] = (Edge){u, w};
}

// Step 2: Initialize minTime and ways
long long* minTime = malloc(n * sizeof(long long));
int* ways = malloc(n * sizeof(int));
for (int i = 0; i < n; ++i) {
    minTime[i] = LLONG_MAX;
    ways[i] = 0;
}
minTime[0] = 0;
ways[0] = 1;

// Step 3: Min-heap priority queue
MinHeap heap;
heap.arr = malloc(100000 * sizeof(HeapNode));
heap.size = 0;
push(&heap, 0, 0); // (time, node)

while (heap.size > 0) {

```

```

HeapNode curr = pop(&heap);
long long time = curr.time;
int u = curr.node;

if (time > minTime[u]) continue;

for (int i = 0; i < graph[u].size; ++i) {
    int v = graph[u].edges[i].to;
    int t = graph[u].edges[i].weight;
    long long newTime = time + t;

    if (newTime < minTime[v]) {
        minTime[v] = newTime;
        ways[v] = ways[u];
        push(&heap, newTime, v);
    } else if (newTime == minTime[v]) {
        ways[v] = (ways[v] + ways[u]) % MOD;
    }
}

int result = ways[n - 1];

// Clean up
for (int i = 0; i < n; ++i) free(graph[i].edges);
free(graph);
free(minTime);
free(ways);
free(heap.arr);

```

```

return result;
}

```

OUTPUT

Accepted Runtime: 0 ms

• Case 1 • Case 2

Input

n =
7

roads =
[[0,6,7],[0,1,2],[1,2,3],[1,3,3],[6,3,1],[6,5,1],[2,5,1],[0,4,5],[4,6,2]]

Output

4

Expected

4

Accepted Runtime: 0 ms

• Case 1 • Case 2

Input

n =
2

roads =
[[1,0,10]]

Output

1

Expected

1

TRACING

Tracing

Ex: n=4
roads = [[0,1,1], [1,2,1], [0,2,2], [2,3,1], [1,3,2]]

dist = [0, ∞, ∞, ∞]
ways = [1, 0, 0, 0]
heap = [(0,0)]

1) Pop(0,0)
→ neighbours: 1 (wt 1), 2 (wt 2)
check n1:
dist[1] = ∞ > 0 + 1 → update dist[1] = 1, ways[1] = ways[0] = 1
push(1,1)

check n2:
dist[2] = ∞ > 0 + 2 → update dist[2] = 2, ways[2] = ways[0] = 1
push(2,2)

now: dist = [0, 1, 2, ∞]
ways = [1, 1, 1, 0]
heap = [(1,1), (2,2)]

2) Pop(1,1)
n: 0, 2, 3
n0: dist[0] = 0 < 1 + 1(1) → x
n2: dist[2] = 2 == 1 + 1(1) → equal dist → ways[2] += ways[1] = 2
n3: dist[3] = ∞ > 1 + 2(1) → update dist[3] = 3, ways[3] = 1
push(3,3)
dist = [0, 1, 2, 3]
ways = [1, 2, 1, 1]
heap = [(2,2), (3,3)]

3) Pop(2,2)
n: 0 (wt 2), 1 (wt 1), 3 (wt 1)
check n0:
dist[0] = 0 < 2 + 2(1) → no update
check n1:
dist[1] = 1 < 2 + 3(3) → no update
check n3:
dist[3] = 3 = 2 + 1(1) → equal distances → ways[3] += ways[2] = 4

Now: dist = [0, 1, 2, 3]
ways = [1, 1, 2, 3]
heap = (3, 3)

4) Pop (3, 3)

* n = 2 (wt 1) + 1 (wt 2)

n 2: $d[2] = 2 < 3 + 1 (4) \rightarrow$ no update

n 3: $d[3] = 1 < 3 + 2 (5) \rightarrow$ no update

Heap \rightarrow empty \rightarrow end.

Result:

Shortest distance from 0 to 3 is 3

Number of shortest paths ways [3] = 3