Answer

max $z = 120x + 100y$
subject to $2x + 2y \le 8$, $5x + 3y \le 15$, $x \ge 0$, $y \ge 0$.

Now using S_1 and S_2 slack variable the standard form is

max $z = 120x + 100y + 0.S_1 + 0.S_2$

s.t $2x + 2y + S_1 + 0.S_2 = 8$
$5x + 3y + 0.S_1 + S_2 = 15$

simplex table

	x_1	x_2	S_1	S_2	BFS
Z	-120	-100	0	0	0
S_1	2	2	1	0	8
S_2	5	3	0	1	15

	x_1	x_2	S_1	S_2	BFS
Z	0	-28	0	24	360
S_1	0	4/5	1	-2/5	2
x_1	1	3/5	0	1/5	3

	x_1	x_2	S_2	S_2	BFS
Z	0	0	35	10	430
x_2	0	1	5/4	-1/2	5/2
x_1	1	0	-3/4	1/2	3/2

Now all $Z \ge 0$ so optimal value is attend.
so the solution max $z = 430$, $x_1 = 3/2$, $x_2 = 5/2$.

(B) Dual problem min $w = 8w_1 + 15w_2$
s.t $2w_1 + 5w_2 \ge 120$, $2w_1 + 3w_2 \ge 100$
$w_1, w_2 \ge 0$.

from the above table the solution of dual problem is

$w_1 = 35, \; w_2 = 10, \quad \min w = 45$

(c) complementary slackness condition: x^0, y^0 be the feasible solution of primal and dual problem will be optimal ~~[illegible]~~ if $(y^0)^T(b - Ax^0) = 0$ — (i)

and $(x^0)^T(A^T y^0 - c^T) = 0$. — (ii)

Here $x^{0T} = (3/2, 5/2)$ $A^T = \begin{bmatrix} 2 & 5 \\ 2 & 3 \end{bmatrix}$, $y^0 = \begin{pmatrix} 35 \\ 10 \end{pmatrix}$, $c^T = (120, 100)$

$b = \begin{pmatrix} 8 \\ 15 \end{pmatrix}$.

from (i) $(35, 10)\left\{\begin{pmatrix} 8 \\ 15 \end{pmatrix} - \begin{pmatrix} 2 & 2 \\ 5 & 3 \end{pmatrix}\begin{pmatrix} 3/2 \\ 5/2 \end{pmatrix}\right\} = 0$.

from (ii) $(3/2, 5/2)\left\{\begin{pmatrix} 2 & 5 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 35 \\ 10 \end{pmatrix} - \begin{pmatrix} 120 \\ 100 \end{pmatrix}\right\} = 0$.

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