Assignment 9

Dishank Jain - AI20BTECH11011

Download all python codes from

```
https://github.com/Dishank422/AI1103−Probability
    −and−random−variables/blob/main/
    Assignment_9/codes
```

and latex-tikz codes from

```
https://github.com/Dishank422/AI1103−Probability
    −and−random−variables/blob/main/
    Assignment_9/main.tex
```

1 Problem

(CSIR UGC NET, June 2016, Q.107)Suppose X and Y are independent and identically distributed random variables and let $Z = X + Y$. Then the distribution of Z is in the same family as that of X and Y if X is

1) Normal
2) Exponential
3) Uniform
4) Binomial

2 Solution

1) Let X and Y be independent and identically distributed normal random variables. Then the characteristic function of X and Y is given by

$$\Phi_X(\omega) = e^{j\eta\omega-\sigma^2\omega^2/2} \tag{2.0.1}$$

The characteristic function of Z is given by

$$\Phi_Z(\omega) = \Phi_X^2(\omega) \tag{2.0.2}$$
$$= e^{2j\eta\omega-\sigma^2\omega^2} \tag{2.0.3}$$

Thus Z is a normal random variable with parameters 2η and $2\sigma^2$. Thus option (1) is correct.

2) Let X and Y be independent and identically distributed exponential random variables. Then the characteristic function of X and Y is given by

$$\Phi_X(\omega) = \frac{\lambda}{1-j\omega} \tag{2.0.4}$$

The characteristic function of Z is given by

$$\Phi_Z(\omega) = \Phi_X^2(\omega) \tag{2.0.5}$$
$$= \frac{\lambda^2}{(1-j\omega)^2} \tag{2.0.6}$$

Thus Z is not an exponential random variable. Therefore option (2) is wrong.

3) Let X and Y be independent and identically distributed uniform random variables such that X, Y ∼ U(a,b). Then the characteristic function of X and Y is given by

$$\Phi_X(\omega) = \frac{e^{jb\omega}-e^{ja\omega}}{j\omega(b-a)} \tag{2.0.7}$$

The characteristic function of Z is given by

$$\Phi_Z(\omega) = \Phi_X^2(\omega) \tag{2.0.8}$$
$$= -\frac{(e^{jb\omega}-e^{ja\omega})^2}{\omega^2(b-a)^2} \tag{2.0.9}$$

Thus Z is not a uniform random variable. Thus option (3) is wrong.

4) Let X and Y be independent and identically distributed binomial random variables. Then the characteristic function of X and Y is given by

$$\Phi_X(\omega) = (pe^{j\omega}+q)^n \tag{2.0.10}$$

The characteristic function of Z is given by

$$\Phi_Z(\omega) = \Phi_X^2(\omega) \tag{2.0.11}$$
$$= (pe^{j\omega}+q)^{2n} \tag{2.0.12}$$

Thus Z is a binomial random variable with parameter 2n. Thus option (4) is correct.

The following figures show the experimental distributions for Z in each case. The simulation length was kept one million.

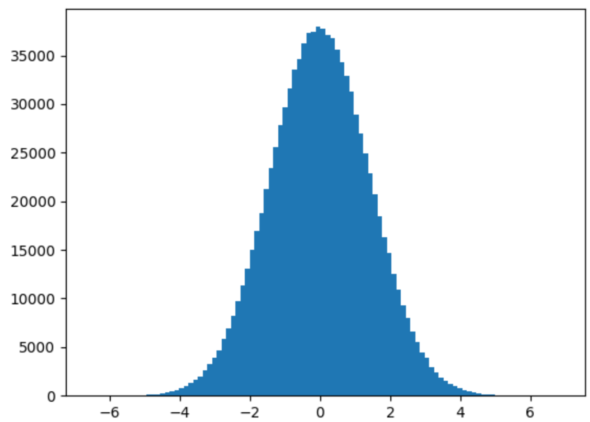

Fig. 4: Z when X is standard normal

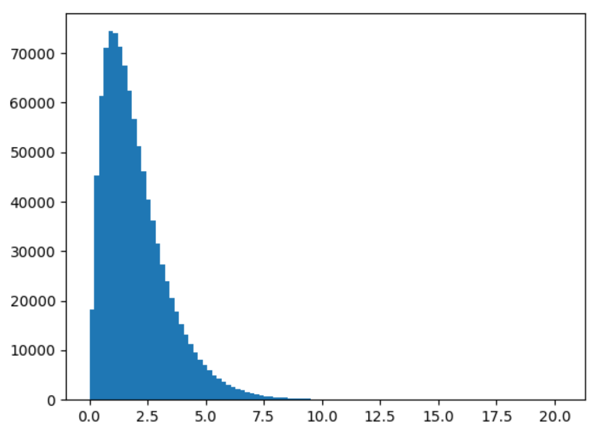

Fig. 4: Z when X is exponential with $\lambda = 1$

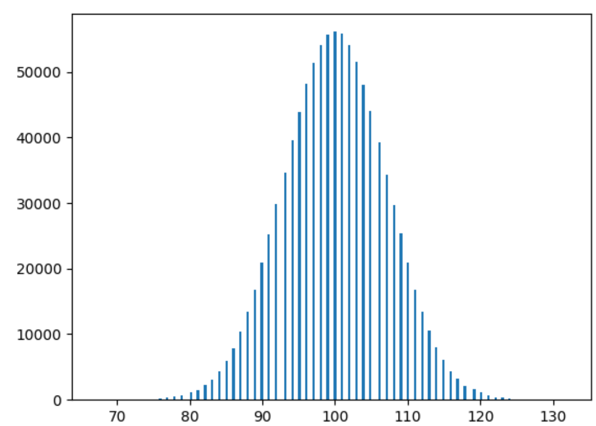

Fig. 4: Z when X $\sim$ B(100,0.5)

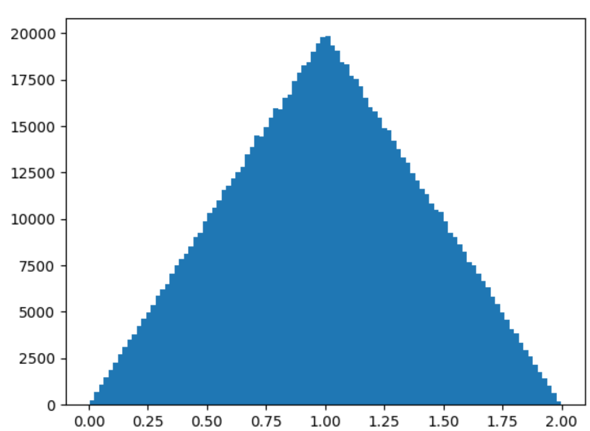

Fig. 4: Z when X $\sim$ U(0,1)