Assignment 5

Dishank Jain - AI20BTECH11011

Download all python codes from

```
https://github.com/Dishank422/AI1103−Probability
    −and−random−variables/blob/main/
    Assignment_5/codes
```

and latex-tikz codes from

```
https://github.com/Dishank422/AI1103−Probability
    −and−random−variables/blob/main/
    Assignment_5/main.tex
```

1 Problem

(Gate EC - 2018 Q. 23) Let X_1, X_2, X_3 and X_4 be independent normal random variables with zero mean and unit variance. The probability that X_4 is the smallest among the four is.....

2 Solution

$$\begin{aligned}\Pr(X_4 = min(X_1, X_2, X_3, X_4)) \\ = \Pr(X_1, X_2, X_3 > x | X_4 = x)\end{aligned} \tag{2.0.1}$$

Since X_1, X_2, X_3 and X_4 are independent, required probability

$$= \int_{-\infty}^{\infty} (1 - F_{X_1}(x))(1 - F_{X_2}(x))(1 - F_{X_3}(x)) f_{X_4}(x) dx \tag{2.0.2}$$

$$= \int_{-\infty}^{\infty} (1 - \Phi(x))^3 \phi(x) dx \tag{2.0.3}$$

Substituting

$$u = 1 - \Phi(x) \tag{2.0.4}$$

$$du = -\phi(x) dx \tag{2.0.5}$$

we get required probability

$$= -\int_{1}^{0} u^3 du \tag{2.0.6}$$

$$= \frac{1}{4} \tag{2.0.7}$$

Note that in eq. (2.0.6) the integral is from 1 to 0 because

$$1 - \Phi(-\infty) = 0 \tag{2.0.8}$$

$$1 - \Phi(\infty) = 1 \tag{2.0.9}$$

Here $\phi(x)$ and $\Phi(x)$ represent the pdf and cdf of standard normal random variable respectively.