Assignment 1

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Download all python codes from

https://github.com/Dishank422/AI1103−Probability−and−random−variables/blob/main/Assignment_2/codes

and latex-tikz codes from

https://github.com/Dishank422/AI1103−Probability−and−random−variables/blob/main/Assignment_2/main.tex

1 Problem

(Gate 11)The probability that a given positive number lying between 1 and 100 (both inclusive) is NOT divisible by 2, 3 or 5 is......

2 Solution

Let $X \in \{1, 2, ..., 100\}$ be the random variable representing the outcome for random selection of a number in $\{1, ..., 100\}$.
Since X has a uniform distribution, the probability mass function (pmf) is represented as

$$\Pr(X = n) = \begin{cases} \frac{1}{100} & 1 \leq n \leq 100 \\ 0 & otherwise \end{cases} \tag{2.0.1}$$

Let A represent the event that the number is divisible by 2. Let B represent the event that the number is divisible by 3. Let C represent the event that the number is divisible by 5.
We need to find the probability that the number is not divisible by 2, 3 or 5. Thus we need to find $1 - \Pr(A + B + C)$
We know

$$\Pr(A + B + C) = \Pr(A) + \Pr(B) + \Pr(C) - \Pr(AB) - \Pr(BC) - \Pr(AC) + \Pr(ABC) \tag{2.0.2}$$

$$\Pr(A) = \sum_n \Pr(X = n | n\ is\ divisible\ by\ 2) \tag{2.0.3}$$
$$= \frac{50}{100} \tag{2.0.4}$$

$$\Pr(B) = \sum_n \Pr(X = n | n\ is\ divisible\ by\ 3) \tag{2.0.5}$$
$$= \frac{33}{100} \tag{2.0.6}$$

$$\Pr(C) = \sum_n \Pr(X = n | n\ is\ divisible\ by\ 5) \tag{2.0.7}$$
$$= \frac{20}{100} \tag{2.0.8}$$

$$\Pr(AB) = \sum_n \Pr(X = n | n\ is\ divisible\ by\ 6) \tag{2.0.9}$$
$$= \frac{16}{100} \tag{2.0.10}$$

$$\Pr(BC) = \sum_n \Pr(X = n | n\ is\ divisible\ by\ 15) \tag{2.0.11}$$
$$= \frac{6}{100} \tag{2.0.12}$$

$$\Pr(AC) = \sum_n \Pr(X = n | n\ is\ divisible\ by\ 10) \tag{2.0.13}$$
$$= \frac{10}{100} \tag{2.0.14}$$

$$\Pr(ABC) = \sum_n \Pr(X = n | n\ is\ divisible\ by\ 30) \tag{2.0.15}$$
$$= \frac{3}{100} \tag{2.0.16}$$

Substituting in (2.0.2), we get

$$\Pr(A + B + C) = \frac{50}{100} + \frac{33}{100} + \frac{20}{100} - \frac{16}{100} - \frac{6}{100} - \frac{10}{100} + \frac{3}{100} \tag{2.0.17}$$

Thus,

$$\Pr(A + B + C) = \frac{74}{100} \tag{2.0.18}$$

Thus required probability =

$$1 - \Pr(A + B + C) = \frac{26}{100} \tag{2.0.19}$$