Assignment 3

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Download all python codes from

```
https://github.com/Dishank422/AI1103−Probability
    −and−random−variables/blob/main/
    Assignment_3/codes
```

and latex-tikz codes from

```
https://github.com/Dishank422/AI1103−Probability
    −and−random−variables/blob/main/
    Assignment_3/main.tex
```

1 Problem

(Gate IN - 2021 Q.37) Consider that X and Y are independent continuous valued random variables with uniform PDF given by $X \sim U(2,3)$ and $Y \sim U(1,4)$. Then $\Pr(Y \le X)$ is equal to .....

2 Solution

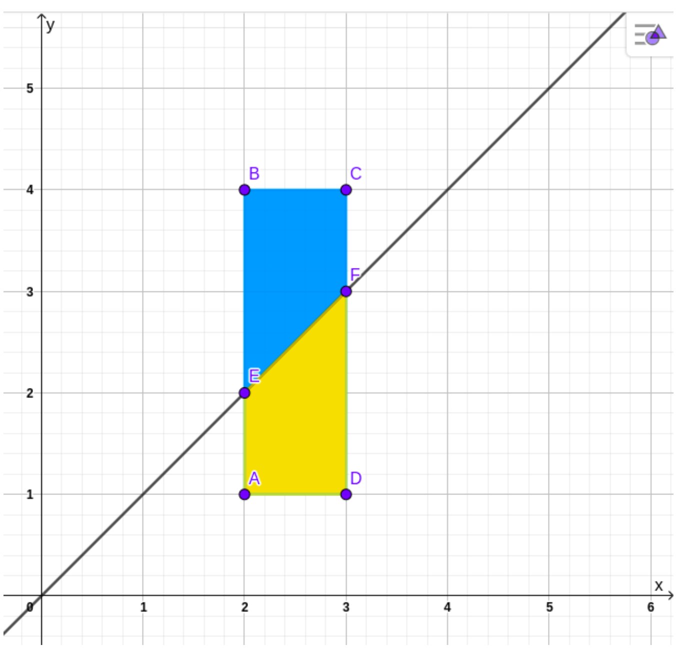

Fig. 0: Probability Distribution of (X, Y)

In figure 0, rectangle ABCD represents sample space of (X, Y). $Y \le X$ for any point (X, Y) if and only if the point lies on or below line EF. Therefore

$$\Pr(Y \le X) = \frac{Area\ of\ AEFD}{Area\ of\ ABCD} \tag{2.0.1}$$

$$= \frac{1}{2} \tag{2.0.2}$$

Alternately, we have

$$F_Y(x) = \frac{x-1}{3} \tag{2.0.3}$$

$$\implies F_Y(X) = \frac{X-1}{3} \tag{2.0.4}$$

PDF of X is given by

$$f(X) = \begin{cases} 1 & 2 \le X \le 3 \\ 0 & otherwise \end{cases} \tag{2.0.5}$$

Thus

$$\Pr(Y \le X) = \int_{-\infty}^{\infty} F_Y(X) f(X) dX \tag{2.0.6}$$

$$= \int_{-\infty}^{\infty} \frac{X-1}{3} dX \tag{2.0.7}$$

$$= \frac{1}{2} \tag{2.0.8}$$