

Assignment 5

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Download all python codes from

[https://github.com/Dishank422/AI1103-Probability
-and-random-variables/blob/main/
Assignment_5/codes](https://github.com/Dishank422/AI1103-Probability-and-random-variables/blob/main/Assignment_5/codes)

and latex-tikz codes from

[https://github.com/Dishank422/AI1103-Probability
-and-random-variables/blob/main/
Assignment_5/main.tex](https://github.com/Dishank422/AI1103-Probability-and-random-variables/blob/main/Assignment_5/main.tex)

Note that in eq. (2.0.6) the integral is from 1 to 0 because

$$1 - \Phi(-\infty) = 1 \quad (2.0.8)$$

$$1 - \Phi(\infty) = 0 \quad (2.0.9)$$

Here $\phi(x)$ and $\Phi(x)$ represent the pdf and cdf of standard normal random variable respectively.

1 PROBLEM

(Gate EC - 2018 Q. 23) Let X_1, X_2, X_3 and X_4 be independent normal random variables with zero mean and unit variance. The probability that X_4 is the smallest among the four is.....

2 SOLUTION

$$\begin{aligned} \Pr(X_4 = \min(X_1, X_2, X_3, X_4)) \\ = \Pr(X_1, X_2, X_3 > x | X_4 = x) \end{aligned} \quad (2.0.1)$$

Since X_1, X_2, X_3 and X_4 are independent, required probability

$$= \int_{-\infty}^{\infty} (1 - F_{X_1}(x))(1 - F_{X_2}(x))(1 - F_{X_3}(x))f_{X_4}(x)dx \quad (2.0.2)$$

$$= \int_{-\infty}^{\infty} (1 - \Phi(x))^3 \phi(x)dx \quad (2.0.3)$$

Substituting

$$u = 1 - \Phi(x) \quad (2.0.4)$$

$$du = -\phi(x)dx \quad (2.0.5)$$

we get required probability

$$= - \int_1^0 u^3 du \quad (2.0.6)$$

$$= \frac{1}{4} \quad (2.0.7)$$