

Assignment 3

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Download all python codes from

[https://github.com/Dishank422/AI1103-Probability
-and-random-variables/blob/main/
Assignment_3/codes](https://github.com/Dishank422/AI1103-Probability-and-random-variables/blob/main/Assignment_3/codes)

and latex-tikz codes from

[https://github.com/Dishank422/AI1103-Probability
-and-random-variables/blob/main/
Assignment_3/main.tex](https://github.com/Dishank422/AI1103-Probability-and-random-variables/blob/main/Assignment_3/main.tex)

In figure 0, rectangle ABCD represents sample space of (X, Y). $Y \leq X$ for any point (X, Y) if and only if the point lies on or below line EF. Therefore

$$\Pr(Y \leq X) = \frac{\text{Area of AEFD}}{\text{Area of ABCD}} \quad (2.0.1)$$

$$= \frac{1}{2} \quad (2.0.2)$$

Alternately, we have PDF and CDF of X and Y given by

$$f_X(x) = \begin{cases} 1 & 2 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.3)$$

$$F_X(x) = \begin{cases} 0 & x < 2 \\ x - 2 & 2 \leq x \leq 3 \\ 1 & x > 3 \end{cases} \quad (2.0.4)$$

$$f_Y(y) = \begin{cases} 1 & 1 \leq y \leq 4 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.5)$$

$$F_Y(y) = \begin{cases} 0 & y < 1 \\ \frac{y - 1}{3} & 1 \leq y \leq 4 \\ 1 & y > 4 \end{cases} \quad (2.0.6)$$

Thus

$$\Pr(Y \leq X) = \int_{-\infty}^{\infty} F_Y(x) f_X(x) dx \quad (2.0.7)$$

$$= \int_2^3 \frac{x - 1}{3} dx \quad (2.0.8)$$

$$= \frac{1}{2} \quad (2.0.9)$$

1 PROBLEM

(Gate IN - 2021 Q.37) Consider that X and Y are independent continuous valued random variables with uniform PDF given by $X \sim U(2, 3)$ and $Y \sim U(1, 4)$. Then $\Pr(Y \leq X)$ is equal to

2 SOLUTION

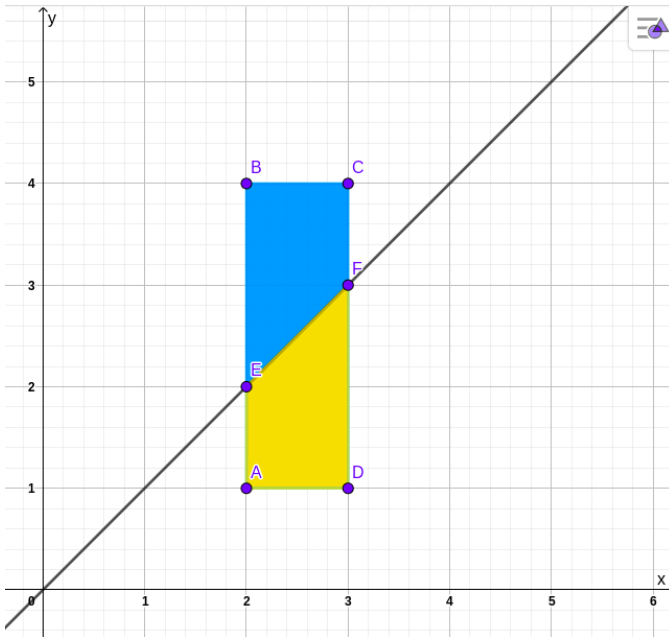


Fig. 0: Probability Distribution of (X, Y)