Assignment 4

Dishank Jain - AI20BTECH11011

Download all python codes from

```
https://github.com/Dishank422/AI1103−Probability
    −and−random−variables/blob/main/
    Assignment_4/codes
```

and latex-tikz codes from

```
https://github.com/Dishank422/AI1103−Probability
    −and−random−variables/blob/main/
    Assignment_4/main.tex
```

1 Problem

(Gate MA - 2016 Q.49) Let X be a standard normal random variable. Then $\Pr(X < 0| \, \|\lfloor X \rfloor\| = 1)$ is equal to

a) $\dfrac{\Phi(1) - \frac{1}{2}}{\Phi(2) - \frac{1}{2}}$
b) $\dfrac{\Phi(1) + \frac{1}{2}}{\Phi(2) + \frac{1}{2}}$
c) $\dfrac{\Phi(1) - \frac{1}{2}}{\Phi(2) + \frac{1}{2}}$
d) $\dfrac{\Phi(1) + 1}{\Phi(2) + 1}$

2 Solution

Since X is standard normal random variable,

$$p_X(x) = \frac{1}{\sqrt{2\pi}} exp(\frac{-x^2}{2}) \tag{2.0.1}$$

$$\|\lfloor X \rfloor\| = 1 \tag{2.0.2}$$

$$\implies \lfloor X \rfloor = 1 \ or \ -1 \tag{2.0.3}$$

$$\implies X \in [1, 2) \cup [-1, 0) \tag{2.0.4}$$

Thus required probability

$$= \frac{\Pr(X \in [-1, 0))}{\Pr(X \in [1, 2) \cup [-1, 0))} \tag{2.0.5}$$

$$= \frac{\int_{-1}^{0} \frac{1}{\sqrt{2\pi}} exp(\frac{-x^2}{2}) dx}{\int_{-1}^{0} \frac{1}{\sqrt{2\pi}} exp(\frac{-x^2}{2}) dx + \int_{1}^{2} \frac{1}{\sqrt{2\pi}} exp(\frac{-x^2}{2}) dx} \tag{2.0.6}$$

$$= \frac{\int_{0}^{1} \frac{1}{\sqrt{2\pi}} exp(\frac{-x^2}{2}) dx}{\int_{0}^{1} \frac{1}{\sqrt{2\pi}} exp(\frac{-x^2}{2}) dx + \int_{1}^{2} \frac{1}{\sqrt{2\pi}} exp(\frac{-x^2}{2}) dx} \tag{2.0.7}$$

$$= \frac{\Phi(1) - \Phi(0)}{\Phi(1) - \Phi(0) + \Phi(2) - \Phi(1)} \tag{2.0.8}$$

$$= \frac{\Phi(1) - \frac{1}{2}}{\Phi(2) - \frac{1}{2}} \tag{2.0.9}$$

$$= \frac{0.841 - 0.5}{0.977 - 0.5} \tag{2.0.10}$$

$$= 0.715 \tag{2.0.11}$$

Here $\Phi(x)$ represents the standard normal cumulative density function given by

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} exp(\frac{-x^2}{2}) dx \tag{2.0.12}$$

It can easily be seen that $\Phi(0) = \frac{1}{2}$, which has been used to obtain (2.0.9). (2.0.10) was obtained by consulting tables for $\Phi(x)$