Assignment 4

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Download all python codes from

```
https://github.com/Dishank422/AI1103-Probability
    -and-random-variables/blob/main/
    Assignment_4/codes
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and latex-tikz codes from

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https://github.com/Dishank422/AI1103-Probability
    -and-random-variables/blob/main/
    Assignment_4/main.tex
```

1 Problem

(Gate MA - 2016 Q.49) Let X be a standard normal random variable. Then $\Pr\left(X < 0 \middle| \, |\lfloor X \rfloor| = 1\right)$ is equal to

a) $\dfrac{\Phi(1) - \frac{1}{2}}{\Phi(2) - \frac{1}{2}}$

b) $\dfrac{\Phi(1) + \frac{1}{2}}{\Phi(2) + \frac{1}{2}}$

c) $\dfrac{\Phi(1) - \frac{1}{2}}{\Phi(2) + \frac{1}{2}}$

d) $\dfrac{\Phi(1) + 1}{\Phi(2) + 1}$

2 Solution

$$|\lfloor X \rfloor| = 1 \tag{2.0.1}$$
$$\implies \lfloor X \rfloor = 1 \textit{ or } -1 \tag{2.0.2}$$
$$\implies X \in [1, 2) \cup [-1, 0) \tag{2.0.3}$$

Here

$$\lfloor X \rfloor = \textit{greatest integer less than or equal to } X$$

Thus required probability

$$= \frac{\Pr\left(X \in [-1, 0)\right)}{\Pr\left(X \in [1, 2) \cup [-1, 0)\right)} \tag{2.0.4}$$

Using symmetry of standard normal random variable about y = 0, we have required probability

$$= \frac{\Pr\left(X \in (0, 1]\right)}{\Pr\left(X \in [1, 2) \cup (0, 1]\right)} \tag{2.0.5}$$
$$= \frac{\Pr\left(X \in (0, 1]\right)}{\Pr\left(X \in (0, 2)\right)} \tag{2.0.6}$$
$$= \frac{\Pr\left(X < 1\right) - \Pr\left(X < 0\right)}{\Pr\left(X < 2\right) - \Pr\left(X < 0\right)} \tag{2.0.7}$$
$$= \frac{\Phi(1) - \Phi(0)}{\Phi(2) - \Phi(0)} \tag{2.0.8}$$
$$= \frac{\Phi(1) - \frac{1}{2}}{\Phi(2) - \frac{1}{2}} \tag{2.0.9}$$
$$= \frac{0.841 - 0.5}{0.977 - 0.5} \tag{2.0.10}$$
$$= 0.715 \tag{2.0.11}$$

Here $\Phi(x)$ represents the standard normal cumulative density function. Thus

$$X \sim N(0, 1) \tag{2.0.12}$$

and

$$\Phi(x) = \int_{-\infty}^{x} f(X) dx \tag{2.0.13}$$

It can easily be seen that $\Phi(0) = \frac{1}{2}$, which has been used to obtain (2.0.9). (2.0.10) was obtained by consulting tables for $\Phi(x)$