Assignment 4

Dishank Jain - AI20BTECH11011

Download all python codes from

```
https://github.com/Dishank422/AI1103-Probability
    -and-random-variables/blob/main/
    Assignment_4/codes
```

and latex-tikz codes from

```
https://github.com/Dishank422/AI1103-Probability
    -and-random-variables/blob/main/
    Assignment_4/main.tex
```

1 Problem

(Gate MA - 2016 Q.49) Let X be a standard normal random variable. Then $\Pr(X < 0| \, \|\lfloor X \rfloor\| = 1)$ is equal to

a) $\dfrac{\Phi(1)-\frac{1}{2}}{\Phi(2)-\frac{1}{2}}$
b) $\dfrac{\Phi(1)+\frac{1}{2}}{\Phi(2)+\frac{1}{2}}$
c) $\dfrac{\Phi(1)-\frac{1}{2}}{\Phi(2)+\frac{1}{2}}$
d) $\dfrac{\Phi(1)+1}{\Phi(2)+1}$

2 Solution

Since X is standard normal random variable,

$$p_X(x) = \frac{1}{\sqrt{2\pi}} exp(\frac{-x^2}{2}) \tag{2.0.1}$$

$$\|\lfloor X \rfloor\| = 1 \tag{2.0.2}$$
$$\implies \lfloor X \rfloor = 1 \; or \; -1 \tag{2.0.3}$$
$$\implies X \in [1,2) \cup [-1,0) \tag{2.0.4}$$

Here

$$\lfloor X \rfloor = greatest\ integer\ less\ than\ or\ equal\ to\ X$$

Thus required probability

$$= \frac{\Pr(X \in [-1,0))}{\Pr(X \in [1,2) \cup [-1,0))} \tag{2.0.5}$$
$$= \frac{\int_{-1}^{0} \frac{1}{\sqrt{2\pi}} exp(\frac{-x^2}{2}) dx}{\int_{-1}^{0} \frac{1}{\sqrt{2\pi}} exp(\frac{-x^2}{2}) dx + \int_{1}^{2} \frac{1}{\sqrt{2\pi}} exp(\frac{-x^2}{2}) dx} \tag{2.0.6}$$
$$= \frac{\int_{0}^{1} \frac{1}{\sqrt{2\pi}} exp(\frac{-x^2}{2}) dx}{\int_{0}^{1} \frac{1}{\sqrt{2\pi}} exp(\frac{-x^2}{2}) dx + \int_{1}^{2} \frac{1}{\sqrt{2\pi}} exp(\frac{-x^2}{2}) dx} \tag{2.0.7}$$
$$= \frac{\Phi(1)-\Phi(0)}{\Phi(1)-\Phi(0)+\Phi(2)-\Phi(1)} \tag{2.0.8}$$
$$= \frac{\Phi(1)-\frac{1}{2}}{\Phi(2)-\frac{1}{2}} \tag{2.0.9}$$
$$= \frac{0.841-0.5}{0.977-0.5} \tag{2.0.10}$$
$$= 0.715 \tag{2.0.11}$$

Here $\Phi(x)$ represents the standard normal cumulative density function given by

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} exp(\frac{-x^2}{2}) dx \tag{2.0.12}$$

It can easily be seen that $\Phi(0) = \frac{1}{2}$, which has been used to obtain (2.0.9). (2.0.10) was obtained by consulting tables for $\Phi(x)$