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Assignment 3

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Download all python codes from

https://github.com/Dishank422/AI1103-Probability -and-random-variables/blob/main/ Assignment 3/codes

and latex-tikz codes from

https://github.com/Dishank422/AI1103-Probability -and-random-variables/blob/main/ Assignment_3/main.tex

1 Problem

(Gate IN - 2021 Q.37) Consider that X and Y are independent continuous valued random variables with uniform PDF given by $X \sim U(2,3)$ and $Y \sim U(1,4)$. Then $\Pr(Y \leq X)$ is equal to

2 Solution

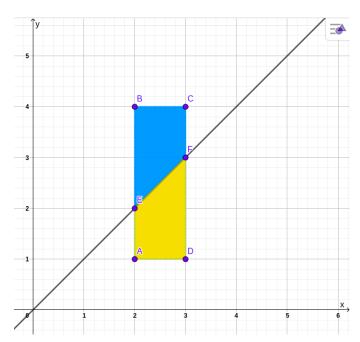


Fig. 0: Probability Distribution of (X, Y)

In figure 0, rectangle ABCD represents sample space of (X, Y). $Y \le X$ for any point (X, Y) if and only if the point lies on or below line EF. Therefore

$$Pr(Y \le X) = \frac{Area\ of\ AEFD}{Area\ of\ ABCD}$$
 (2.0.1)

$$=\frac{1}{2}$$
 (2.0.2)

Alternately, we can write

$$\Pr(Y \le X) = \int_{2}^{3} \Pr(Y \le X | X = x)$$
 (2.0.3)

$$= \int_{2}^{3} F_{Y}(x)dx \tag{2.0.4}$$

$$= \int_{2}^{3} \frac{x-1}{3} dx \tag{2.0.5}$$

$$=\frac{1}{2}$$
 (2.0.6)