Assignment 3

Dishank Jain - AI20BTECH11011

Download all python codes from

```
https://github.com/Dishank422/AI1103−Probability
    −and−random−variables/blob/main/
    Assignment_3/codes
```

and latex-tikz codes from

```
https://github.com/Dishank422/AI1103−Probability
    −and−random−variables/blob/main/
    Assignment_3/main.tex
```

1 Problem

(Gate IN - 2021 Q.37) Consider that X and Y are independent continuous valued random variables with uniform PDF given by $X \sim U(2,3)$ and $Y \sim U(1,4)$. Then $\Pr(Y \leq X)$ is equal to .....

2 Solution

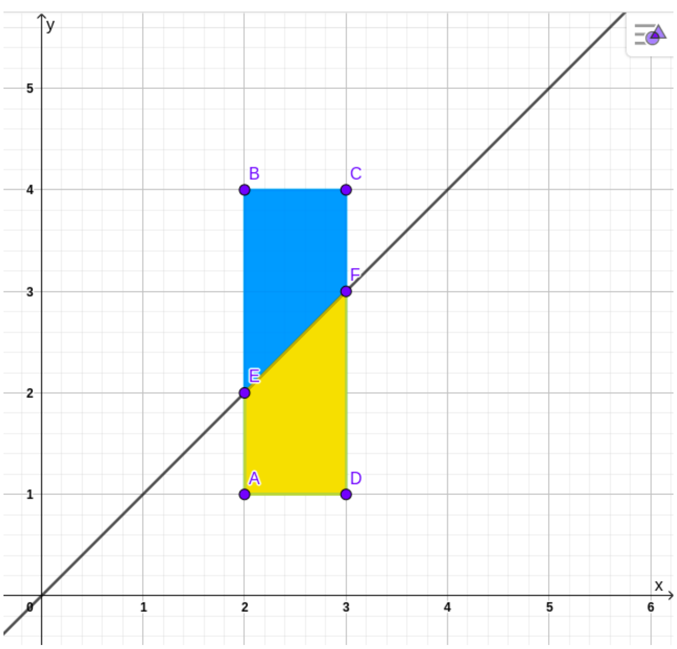

Fig. 0: Probability Distribution of (X, Y)

In figure 0, rectangle ABCD represents sample space of (X, Y). $Y \leq X$ for any point (X, Y) if and only if the point lies on or below line EF. Therefore

$$\Pr(Y \leq X) = \frac{Area\ of\ AEFD}{Area\ of\ ABCD} \tag{2.0.1}$$

$$= \frac{1}{2} \tag{2.0.2}$$

Alternately, we have PDF and CDF of X and Y given by

$$f(X) = \begin{cases} 1 & 2 \leq X \leq 3 \\ 0 & otherwise \end{cases} \tag{2.0.3}$$

$$F_X(x) = \begin{cases} 0 & x < 2 \\ x-2 & 2 \leq x \leq 3 \\ 1 & x > 3 \end{cases} \tag{2.0.4}$$

$$f(Y) = \begin{cases} 1 & 1 \leq Y \leq 4 \\ 0 & otherwise \end{cases} \tag{2.0.5}$$

$$F_Y(x) = \begin{cases} 0 & x < 1 \\ \frac{x-1}{3} & 1 \leq x \leq 4 \\ 1 & x > 4 \end{cases} \tag{2.0.6}$$

Thus

$$\Pr(Y \leq X) = \int_{-\infty}^{\infty} F_Y(x) f(x) dx \tag{2.0.7}$$

$$= \int_2^3 \frac{x-1}{3} dx \tag{2.0.8}$$

$$= \frac{1}{2} \tag{2.0.9}$$