Assignment 5

Dishank Jain - AI20BTECH11011

Download all python codes from

https://github.com/Dishank422/AI1103−Probability
    −and−random−variables/blob/main/
    Assignment_5/codes

and latex-tikz codes from

https://github.com/Dishank422/AI1103−Probability
    −and−random−variables/blob/main/
    Assignment_5/main.tex

1 PROBLEM

(Gate EC - 2018 Q. 23) Let X_1, X_2, X_3 and X_4 be independent normal random variables with zero mean and unit variance. The probability that X_4 is the smallest among the four is.....

2 SOLUTION

Required probability

$$= \Pr(X_4 < X_1, X_4 < X_2, X_4 < X_3) \quad (2.0.1)$$

$$= \int_{-\infty}^{\infty} \Pr(X_1, X_2, X_3 > x | X_4 = x) \quad (2.0.2)$$

Since X_1, X_2, X_3 and X_4 are independent, required probability

$$= \int_{-\infty}^{\infty} (1 - F_{X_1}(x))(1 - F_{X_2}(x))(1 - F_{X_3}(x)) f_{X_4}(x) dx$$
$$(2.0.3)$$

$$= \int_{-\infty}^{\infty} (1 - \Phi(x))^3 \phi(x) dx \quad (2.0.4)$$

Substituting

$$u = 1 - \Phi(x) \quad (2.0.5)$$

$$du = -\phi(x) dx \quad (2.0.6)$$

we get required probability

$$= -\int_1^0 u^3 du \quad (2.0.7)$$

$$= \frac{1}{4} \quad (2.0.8)$$

Note that in eq. (2.0.7) the integral is from 1 to 0 because

$$1 - \Phi(-\infty) = 1 \quad (2.0.9)$$

$$1 - \Phi(\infty) = 0 \quad (2.0.10)$$

Here $\phi(x)$ and $\Phi(x)$ represent the pdf and cdf of standard normal random variable respectively.