

Assignment 3

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Download all python codes from

https://github.com/Dishank422/AI1103-Probability-and-random-variables/blob/main/Assignment_3/codes

and latex-tikz codes from

https://github.com/Dishank422/AI1103-Probability-and-random-variables/blob/main/Assignment_3/main.tex

1 PROBLEM

(Gate IN - 2021 Q.37) Consider that X and Y are independent continuous valued random variables with uniform PDF given by $X \sim U(2, 3)$ and $Y \sim U(1, 4)$. Then $\Pr(Y \leq X)$ is equal to

2 SOLUTION

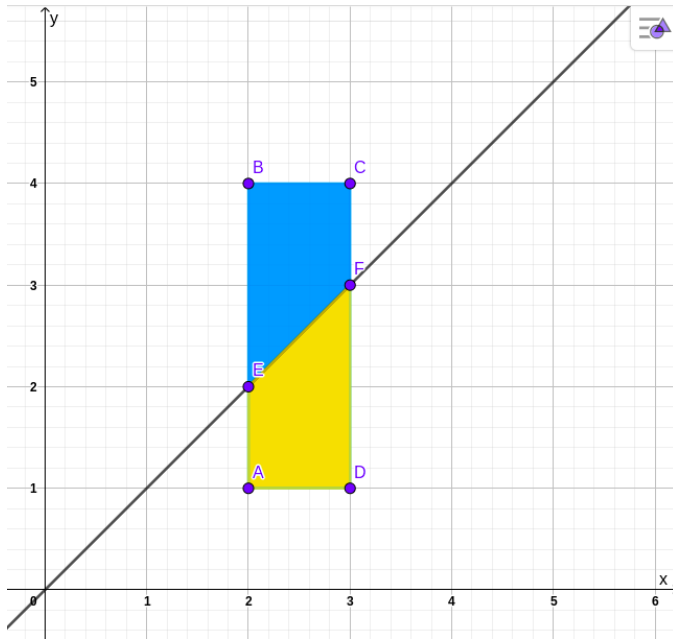


Fig. 0: Probability Distribution of (X, Y)

In figure 0, rectangle ABCD represents sample space of (X, Y) . $Y \leq X$ for any point (X, Y) if and only if the point lies on or below line EF. Therefore

$$\Pr(Y \leq X) = \frac{\text{Area of } AEFB}{\text{Area of } ABCD} \quad (2.0.1)$$

$$= \frac{1}{2} \quad (2.0.2)$$

Alternately, we can write

$$\Pr(Y \leq X) = \int_2^3 \Pr(Y \leq x | X = x) \quad (2.0.3)$$

Since X and Y are independent, from (2.0.3), we have

$$\Pr(Y \leq X) = \int_2^3 F_Y(x) dx \quad (2.0.4)$$

$$= \int_2^3 \frac{x-1}{3} dx \quad (2.0.5)$$

$$= \frac{1}{2} \quad (2.0.6)$$