Assignment 5

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1 Problem

(Gate EC - 2018 Q. 23) Let X_1, X_2, X_3 and X_4 be independent normal random variables with zero mean and unit variance. The probability that X_4 is the smallest among the four is.....

2 Solution

$$\Pr\left(X_4 = min(X_1, X_2, X_3, X_4)\right)$$
$$= \Pr\left(X_1, X_2, X_3 > x | X_4 = x\right) \quad (2.0.1)$$

Thus required probability

$$= \int_{-\infty}^{\infty} (1 - F_{X_1}(x))(1 - F_{X_2}(x))(1 - F_{X_3}(x))f_{X_4}(x)dx \quad (2.0.2)$$

$$= \int_{-\infty}^{\infty} (\Phi(-x))^3 \phi(x)dx \quad (2.0.3)$$

$$= \int_{-\infty}^{\infty} (\Phi(-x))^3 \phi(-x)dx \quad (2.0.4)$$

Substituting $\Phi(-x)$ by u and $\phi(-x)dx$ by du, we get required probability

$$= \int_0^1 u^3 du \quad (2.0.5)$$

$$= \frac{1}{4} \quad (2.0.6)$$

Here $\phi(x)$ and $\Phi(x)$ represent the pdf and cdf of standard normal random variable respectively.