ASSIGNMENT 5

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Download all python codes from

https://github.com/Dishank422/EE3900/blob/main/assignment5/codes

and latex-tikz codes from

https://github.com/Dishank422/EE3900/blob/main/assignment5/Assignment5.tex

1 Quadratic forms Q 2.68

Find the normal at the point $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ on the curve $2y + x^2 = 3$.

2 Solution

The given curve can be expressed as

$$x^2 + 2y - 3 = 0 \tag{2.0.1}$$

$$\implies \mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \ \mathbf{u} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \ \mathbf{f} = -3 \tag{2.0.2}$$

Since $|\mathbf{V}| = 0$, the given curve represents a parabola. The eigenvalues are given by

$$\lambda_1 = 0, \ \lambda_2 = 1 \tag{2.0.3}$$

with corresponding eigenvectors

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} = 0 \implies \mathbf{p}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.4}$$

$$\begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x} = 0 \implies \mathbf{p}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.5}$$

To find the vertex of the parabola,

$$\begin{pmatrix} \mathbf{u}^\top + \kappa \mathbf{p}_1^\top \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -\mathbf{f} \\ \kappa \mathbf{p}_1 - \mathbf{u} \end{pmatrix} \tag{2.0.6}$$

where, $\kappa = \mathbf{u}^\top \mathbf{p}_1 = 1$

$$\implies \begin{pmatrix} 0 & 2 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.7}$$

$$\implies \mathbf{c} = \begin{pmatrix} 0 \\ 1.5 \end{pmatrix} \tag{2.0.8}$$

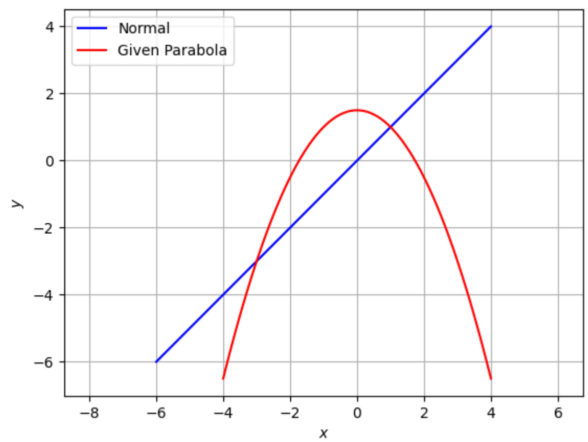

Fig. 0: Plot of the normal

Now to evaluate the direction vector $\mathbf{m}$,

$$\mathbf{m}^\top (\mathbf{V}\mathbf{q} + \mathbf{u}) = 0 \tag{2.0.9}$$

$$\implies \mathbf{m}^\top \left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) = 0 \tag{2.0.10}$$

$$\implies \mathbf{m}^\top \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 0 \tag{2.0.11}$$

$$\implies \mathbf{m} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{2.0.12}$$

The normal is obtained as

$$\mathbf{m}^\top (\mathbf{x} - \mathbf{q}) = 0 \tag{2.0.13}$$

$$\implies \begin{pmatrix} -1 & 1 \end{pmatrix} \left(\mathbf{x} - \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right) = 0 \tag{2.0.14}$$

$$\implies \begin{pmatrix} -1 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{2.0.15}$$

The above results are verified in Fig. 0.