ASSIGNMENT 1

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Download all python codes from

https://github.com/Dishank422/EE3900/blob/main/Gate−Assignment1/codes/codes.py

and latex-tikz codes from

https://github.com/Dishank422/EE3900/blob/main/Gate−Assignment1/latex_code.tex

1 EC 2019 Q.33

Let the state-space representation on an LTI system be $\dot{x}(t) = Ax(t)+Bu(t)$, $y(t) = Cx(t)+du(t)$ where A,B,C are matrices, d is a scalar, u(t) is the input to the system, and y(t) is its output. Let $B = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^\top$ and $d = 0$. Which one of the following options for A and C will ensure that the transfer function of this LTI system is

$$H(s) = \frac{1}{s^3 + 3s^2 + 2s + 1} \tag{1.0.1}$$

(A) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$

(B) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$

(C) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$

(D) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$

2 Solution

By definition, the transfer function is given by

$$H(s) = \frac{Y(s)}{U(s)} \tag{2.0.1}$$

where Y(s) and U(s) are the Laplace transforms of y(t) and u(t) respectively, i.e.

$$Y(s) = \mathcal{L}(y(t));\; U(s) = \mathcal{L}(u(t)) \tag{2.0.2}$$

$$Let\ Y(s) = Z(s) \tag{2.0.3}$$

$$\implies U(s) = (s^3 + 3s^2 + 2s + 1)Z(s) \tag{2.0.4}$$

$$Let\ \mathcal{L}^{-1}(Z(s)) = z(t) \tag{2.0.5}$$

$$\implies \mathcal{L}^{-1}(U(s)) = \mathcal{L}^{-1}(s^3Z(s)) + \mathcal{L}^{-1}(3s^2Z(s)) + \mathcal{L}^{-1}(2sZ(s)) + \mathcal{L}^{-1}(Z(s)) \tag{2.0.6}$$

$$\implies u(t) = \dddot{z}(t) + 3\ddot{z}(t) + 2\dot{z}(t) + z(t) \tag{2.0.7}$$

Equation 2.0.7 is a linear third degree ordinary differential equation. We can convert this into a system of first order differential equations as follows.

$$Let\ x_1 = z \tag{2.0.8}$$

$$x_2 = \dot{x}_1 = \dot{z} \tag{2.0.9}$$

$$x_3 = \dot{x}_2 = \ddot{z} \tag{2.0.10}$$

$$\implies \dot{x}_3 = \dddot{z} = u - 3x_3 - 2x_2 - x_1 \tag{2.0.11}$$

$$\implies \dot{x}(t) = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} \tag{2.0.12}$$

$$= \begin{pmatrix} x_2 \\ x_3 \\ u - 3x_3 - 2x_2 - x1 \end{pmatrix} \tag{2.0.13}$$

$$= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u \tag{2.0.14}$$

$$= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u(t) \tag{2.0.15}$$

We know that $\dot{x}(t) = Ax(t) + Bu(t)$ and $B = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^\top$. Hence on comparison

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} \tag{2.0.16}$$

Also, since $d = 0$,

$$y(t) = Cx(t) + du(t) = Cx(t) \tag{2.0.17}$$

But from 2.0.3,

$$\mathcal{L}^{-1}(Y(s)) = \mathcal{L}^{-1}(Z(s)) \tag{2.0.18}$$

$$\implies y(t) = z(t) \tag{2.0.19}$$

$$= x_1 \tag{2.0.20}$$

$$= \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \tag{2.0.21}$$

$$= \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} x(t) \tag{2.0.22}$$

$$\implies C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \tag{2.0.23}$$

Hence option (A) is the correct option.