

ASSIGNMENT 3

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Download all python codes from

<https://github.com/Dishank422/EE3900/blob/main/assignment3/codes>

and latex-tikz codes from

<https://github.com/Dishank422/EE3900/blob/main/assignment3/Assignment3.tex>

Note: Above computations can be found in codes/finding_D.py.

Now we shall transform co-ordinates of **C**, **D** such that **C** will become the new origin and **D** comes along the positive x-axis. We shall first shift origin to **C**. For this, we have to subtract $\begin{pmatrix} 5.5 \\ 0 \end{pmatrix}$ from co-ordinates of **C**, **D**.

$$\Rightarrow \mathbf{C}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.14)$$

$$\mathbf{D}_1 = \begin{pmatrix} -2.27 \\ 4.45 \end{pmatrix} \quad (2.0.15)$$

1 CONSTRUCTIONS Q2.4

Construct a quadrilateral ABCD such that BC = 4.5, AC = 5.5, CD = 5, BD = 7 and AD = 5.5.

2 SOLUTION

Using AC = 5.5,

$$\text{Let } \mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5.5 \\ 0 \end{pmatrix} \quad (2.0.1)$$

$$\mathbf{D} = \begin{pmatrix} h \\ k \end{pmatrix} \quad (2.0.2)$$

We know

$$\|\mathbf{D} - \mathbf{A}\| = 5.5 \quad (2.0.3)$$

$$\|\mathbf{D} - \mathbf{C}\| = 5 \quad (2.0.4)$$

$$\|\mathbf{B} - \mathbf{C}\| = 4.5 \quad (2.0.5)$$

$$\|\mathbf{B} - \mathbf{D}\| = 7 \quad (2.0.6)$$

$$h = \frac{AC^2 + AD^2 - CD^2}{2AC} \quad (2.0.7)$$

$$= \frac{5.5^2 + 5.5^2 - 5^2}{2 \times 5.5} \quad (2.0.8)$$

$$= 3.23 \quad (2.0.9)$$

$$k = \sqrt{AD^2 - h^2} \quad (2.0.10)$$

$$= \sqrt{5.5^2 - h^2} \quad (2.0.11)$$

$$= 4.45 \quad (2.0.12)$$

$$\Rightarrow \mathbf{D} = \begin{pmatrix} 3.23 \\ 4.45 \end{pmatrix} \quad (2.0.13)$$

Next we shall rotate the co-ordinates such that **D**₁ is on the x-axis. For this, we have to multiply the co-ordinates with the clockwise rotator matrix **R**, given as

$$\mathbf{R} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \quad (2.0.16)$$

$$\mathbf{D}_1 \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}^\top \quad (2.0.17)$$

$$\text{where } \theta = \arccos \frac{\|\mathbf{D}_1\|}{5} \quad (2.0.18)$$

$$= \arccos \frac{-2.27}{5} \quad (2.0.19)$$

$$= 2.04 \text{ radians} \quad (2.0.19)$$

$$\Rightarrow \mathbf{R} = \begin{pmatrix} -0.45 & 0.89 \\ -0.89 & -0.45 \end{pmatrix} \quad (2.0.20)$$

$$\Rightarrow \mathbf{D}_2 = \mathbf{R}\mathbf{D}_1 = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \quad (2.0.21)$$

Note: norm of **D** is calculated using codes/norm_D1.py, arccos is calculated using codes/finding_theta.py, **R** is calculated using codes/R.py and **D**₂ is calculated using D2.py.

Since **C**₁ is origin

$$\mathbf{C}_2 = \mathbf{C}_1 \quad (2.0.22)$$

$$\text{Let } \mathbf{B}_2 = \begin{pmatrix} p \\ q \end{pmatrix} \quad (2.0.23)$$

Using 2.0.5 and 2.0.6,

$$\|\mathbf{B}_2 - \mathbf{C}_2\| = 4.5 \quad (2.0.24)$$

$$\|\mathbf{B}_2 - \mathbf{D}_2\| = 7 \quad (2.0.25)$$

$$p = \frac{B_2 C_2^2 + C_2 D_2^2 - B_2 D_2^2}{2 C_2 D_2} \quad (2.0.26)$$

$$= \frac{4.5^2 + 5^2 - 7^2}{2 \times 5} \quad (2.0.27)$$

$$= -0.38 \quad (2.0.28)$$

$$q = \pm \sqrt{(B_2 C_2^2 - p^2)} \quad (2.0.29)$$

$$= \pm 4.48 \quad (2.0.30)$$

$$\Rightarrow \mathbf{B}_2 = \begin{pmatrix} -0.38 \\ 4.48 \end{pmatrix}, \begin{pmatrix} -0.38 \\ -4.48 \end{pmatrix} \quad (2.0.31)$$

Note: q is found using codes/q.py.

To get \mathbf{B}_1 , we operate on \mathbf{B}_2 with inverse of \mathbf{R} . Inverse of \mathbf{R} is nothing but $\mathbf{R}(-\theta)$. Therefore,

$$\mathbf{R}^{-1} = \begin{pmatrix} \cos -\theta & \sin -\theta \\ -\sin -\theta & \cos -\theta \end{pmatrix} \quad (2.0.32)$$

$$= \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \quad (2.0.33)$$

$$= \begin{pmatrix} -0.45 & -0.89 \\ 0.89 & -0.45 \end{pmatrix} \quad (2.0.34)$$

$$\Rightarrow \mathbf{B}_1 = \mathbf{R}^{-1} \mathbf{B}_2 = \begin{pmatrix} -3.82 \\ -2.35 \end{pmatrix}, \begin{pmatrix} 4.16 \\ 1.68 \end{pmatrix} \quad (2.0.35)$$

To finally get \mathbf{B} , we add $\begin{pmatrix} 5.5 \\ 0 \end{pmatrix}$ to \mathbf{B}_1 .

$$\Rightarrow \mathbf{B} = \begin{pmatrix} 1.68 \\ -2.35 \end{pmatrix}, \begin{pmatrix} 9.66 \\ 1.68 \end{pmatrix} \quad (2.0.36)$$

We want $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ in clockwise or anti-clockwise order. Therefore only the first value of \mathbf{B} is possible.

$$\Rightarrow \mathbf{B} = \begin{pmatrix} 1.68 \\ -2.35 \end{pmatrix} \quad (2.0.37)$$

Using the co-ordinates of the vertices as found, the following quadrilateral is plotted.

