ASSIGNMENT 4

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Download all python codes from

and latex-tikz codes from

1 RAMSEY 1.2 LOCI Q 4

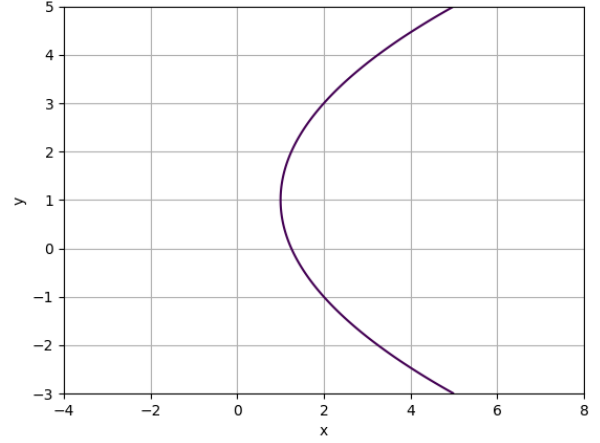A point moves so that it's distance from the y-axis is equal to the distance from the point $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Find the equation of the locus.

Fig. 0: Plot of the locus

2 SOLUTION

Let $\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix}$ be the point. The equation of y-axis is given by

$$\mathbf{R} = \lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.1}$$

XR is perpendicular to y-axis.

$$\implies (\mathbf{R} - \mathbf{X})^\top \mathbf{R} = 0 \tag{2.0.2}$$

$$\implies \mathbf{X}^\top \mathbf{R} = \|\mathbf{R}\|^2 \tag{2.0.3}$$

$$\implies \mathbf{X}^\top \begin{pmatrix} 0 \\ 1 \end{pmatrix} \|\mathbf{R}\| = \|\mathbf{R}\|^2 \tag{2.0.4}$$

$$\implies \|\mathbf{R}\| = \mathbf{X}^\top \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.5}$$

Let $\mathbf{C} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Then

$$XC = \|\mathbf{X} - \mathbf{C}\| \tag{2.0.6}$$

$$XR = \|\mathbf{X} - \mathbf{R}\| \tag{2.0.7}$$

We are given XR = XC.

$$\implies \|\mathbf{X} - \mathbf{C}\|^2 = \|\mathbf{X} - \mathbf{R}\|^2 \tag{2.0.8}$$

$$\implies \|\mathbf{X}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{X}^\top \mathbf{C} = \|\mathbf{X}\|^2 + \|\mathbf{R}\|^2 - 2\mathbf{X}^\top \mathbf{R} \tag{2.0.9}$$

Subtracting $\|\mathbf{X}\|^2$ on both sides and using 2.0.3,

$$\|\mathbf{C}\|^2 - 2\mathbf{X}^\top \mathbf{C} = \|\mathbf{R}\|^2 - 2\|\mathbf{R}\|^2 \tag{2.0.10}$$

$$\implies 2\mathbf{X}^\top \mathbf{C} = \|\mathbf{C}\|^2 + \|\mathbf{R}\|^2 \tag{2.0.11}$$

$$\implies 2\mathbf{X}^\top \mathbf{C} = \|\mathbf{C}\|^2 + \left(\mathbf{X}^\top \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right)^2 \tag{2.0.12}$$

$$\implies 2(2x + y) = 5 + y^2 \tag{2.0.13}$$

$$\implies y^2 = 4x + 2y - 5 \tag{2.0.14}$$

Therefore 2.0.14 is the required locus.