

ASSIGNMENT 1

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Download all latex-tikz codes from

https://github.com/Dishank422/EE3900/blob/main/Gate-Assignment1/latex_code.tex

1 EC 2019 Q.33

Let the state-space representation on an LTI system be $\dot{x}(t) = Ax(t) + Bu(t)$, $y(t) = Cx(t) + du(t)$ where A,B,C are matrices, d is a scalar, u(t) is the input to the system, and y(t) is its output. Let $B = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^T$ and $d = 0$. Which one of the following options for A and C will ensure that the transfer function of this LTI system is

$$H(s) = \frac{1}{s^3 + 3s^2 + 2s + 1} \quad (1.0.1)$$

$$(A) \ A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} \text{ and } C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$

$$(B) \ A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{pmatrix} \text{ and } C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$

$$(C) \ A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} \text{ and } C = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$$

$$(D) \ A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{pmatrix} \text{ and } C = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$$

2 SOLUTION

By definition, the transfer function is given by

$$H(s) = \frac{Y(s)}{U(s)} \quad (2.0.1)$$

where Y(s) and U(s) are the Laplace transforms of y(t) and u(t) respectively, i.e.

$$Y(s) = \mathcal{L}(y(t)); \ U(s) = \mathcal{L}(u(t)) \quad (2.0.2)$$

$$\text{Let } Y(s) = Z(s) \quad (2.0.3)$$

$$\Rightarrow U(s) = (s^3 + 3s^2 + 2s + 1)Z(s) \quad (2.0.4)$$

$$\text{Let } \mathcal{L}^{-1}(Z(s)) = z(t) \quad (2.0.5)$$

$$\Rightarrow \mathcal{L}^{-1}(U(s)) = \mathcal{L}^{-1}(s^3 Z(s)) + \mathcal{L}^{-1}(3s^2 Z(s)) + \mathcal{L}^{-1}(2s Z(s)) + \mathcal{L}^{-1}(Z(s)) \quad (2.0.6)$$

$$\Rightarrow u(t) = \ddot{z}(t) + 3\ddot{z}(t) + 2\dot{z}(t) + z(t) \quad (2.0.7)$$

Equation 2.0.7 is a linear third degree ordinary differential equation. We can convert this into a system of first order differential equations as follows.

$$\text{Let } x_1 = z \quad (2.0.8)$$

$$x_2 = \dot{x}_1 = \dot{z} \quad (2.0.9)$$

$$x_3 = \dot{x}_2 = \ddot{z} \quad (2.0.10)$$

$$\Rightarrow \dot{x}_3 = \ddot{z} = u - 3x_3 - 2x_2 - x_1 \quad (2.0.11)$$

$$\Rightarrow \dot{x}(t) = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} \quad (2.0.12)$$

$$= \begin{pmatrix} x_2 \\ x_3 \\ u - 3x_3 - 2x_2 - x_1 \end{pmatrix} \quad (2.0.13)$$

$$= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u \quad (2.0.14)$$

$$= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u(t) \quad (2.0.15)$$

We know that $\dot{x}(t) = Ax(t) + Bu(t)$ and $B = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^T$. Hence on comparison

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} \quad (2.0.16)$$

Also, since $d = 0$,

$$y(t) = Cx(t) + du(t) = Cx(t) \quad (2.0.17)$$

But from 2.0.3,

$$\mathcal{L}^{-1}(Y(s)) = \mathcal{L}^{-1}(Z(s)) \quad (2.0.18)$$

$$\implies y(t) = z(t) \quad (2.0.19)$$

$$= x_1 \quad (2.0.20)$$

$$= \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad (2.0.21)$$

$$= \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} x(t) \quad (2.0.22)$$

$$\implies C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \quad (2.0.23)$$

Hence option (A) is the correct option.