

GATE ASSIGNMENT 4

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Download latex-tikz codes from

https://github.com/Dishank422/EE3900/blob/main/Gate-Assignment4/latex_code.tex

1 EC 1999 Q.2.1

The Fourier representation of an impulse train represented by $s(t) = \sum_{n=-\infty}^{\infty} d(t - nT_0)$ is given by

- (a) $\frac{1}{T_0} \sum_{n=-\infty}^{\infty} \exp - \frac{j2\pi nt}{T_0}$
- (b) $\frac{1}{T_0} \sum_{n=-\infty}^{\infty} \exp - \frac{j\pi nt}{T_0}$
- (c) $\frac{1}{T_0} \sum_{n=-\infty}^{\infty} \exp \frac{j\pi nt}{T_0}$
- (d) $\frac{1}{T_0} \sum_{n=-\infty}^{\infty} \exp \frac{j2\pi nt}{T_0}$

2 SOLUTION

Lemma 2.1. Any periodic signal $x(t)$ with period T_0 can be written as

$$x(t) = \sum_{n=-\infty}^{\infty} a_n \exp \frac{j2\pi nt}{T_0} \quad (2.0.1)$$

where, a_n is given by

$$a_n = \frac{1}{T_0} \int_{T_0} x(t) \exp - \frac{j2\pi nt}{T_0} dt \quad (2.0.2)$$

Proof. We shall verify equation 2.0.2.

$$a_n = \frac{1}{T_0} \int_{T_0} x(t) \exp - \frac{j2\pi nt}{T_0} dt \quad (2.0.3)$$

$$= \frac{1}{T_0} \int_{T_0} \left(\sum_{m=-\infty}^{\infty} a_m \exp \frac{j2\pi mt}{T_0} \right) \exp - \frac{j2\pi nt}{T_0} dt \quad (2.0.4)$$

$$= \frac{1}{T_0} \sum_{m=-\infty}^{\infty} \int_{T_0} a_m \exp \frac{j2\pi mt}{T_0} \exp - \frac{j2\pi nt}{T_0} dt \quad (2.0.5)$$

When $m = n$,

$$\int_{T_0} a_m \exp \frac{j2\pi mt}{T_0} \exp - \frac{j2\pi nt}{T_0} dt = \int_{T_0} a_n \quad (2.0.6)$$

$$= T_0 a_n \quad (2.0.7)$$

When $m \neq n$,

$$\int_{T_0} a_m \exp \frac{j2\pi mt}{T_0} \exp - \frac{j2\pi nt}{T_0} dt \quad (2.0.8)$$

$$= \frac{T_0 a_m}{j2\pi(m-n)} \exp \frac{j2\pi(m-n)t}{T_0} \Big|_{T_0} \quad (2.0.9)$$

Since $\exp \frac{j2\pi(m-n)t}{T_0}$ is periodic with period $\frac{T_0}{m-n}$, it's integral over any time interval of length $\frac{T_0}{m-n}$ or any integral multiple of $\frac{T_0}{m-n}$ will be 0. Therefore, when $m \neq n$,

$$\int_{T_0} a_m \exp \frac{j2\pi mt}{T_0} \exp - \frac{j2\pi nt}{T_0} dt = 0 \quad (2.0.10)$$

Continuing from equation 2.0.5,

$$a_n = \frac{1}{T_0} \sum_{m=-\infty}^{\infty} \int_{T_0} a_m \exp \frac{j2\pi mt}{T_0} \exp - \frac{j2\pi nt}{T_0} dt \quad (2.0.11)$$

$$= \frac{1}{T_0} (T_0 a_n) = a_n \quad (2.0.12)$$

□

We observe that $s(t)$ is periodic with period T_0 . Thus it's Fourier representation as a sum of complex exponents is given by

$$s(t) = \sum_{n=-\infty}^{\infty} a_n \exp \frac{j2\pi nt}{T_0} \quad (2.0.13)$$

where, a_n can be calculated as

$$a_n = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} s(t) \exp - \frac{j2\pi nt}{T_0} dt \quad (2.0.14)$$

Between $-\frac{T_0}{2}$ and $\frac{T_0}{2}$, we can say that $s(t) = d(t)$.

$$\Rightarrow a_n = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} d(t) \exp -\frac{j2\pi nt}{T_0} dt \quad (2.0.15)$$

$$= \frac{1}{T_0} \quad (2.0.16)$$

$$\Rightarrow s(t) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \exp \frac{j2\pi nt}{T_0} \quad (2.0.17)$$

Therefore option (d) is the correct option.