#### 1

# **ASSIGNMENT 4**

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## Download all python codes from

https://github.com/Dishank422/EE3900/blob/main/assignment4/codes

and latex-tikz codes from

https://github.com/Dishank422/EE3900/blob/main/assignment4/Assignment4.tex

### 1 Ramsey 1.2 Loci Q 4

A point moves so that it's distance from the y-axis is equal to the distance from the point  $\binom{2}{1}$ . Find the equation of the locus.

#### 2 Solution

Let  $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$  be the point. The equation of y-axis is given by

$$\mathbf{R} = \lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.1}$$

xR is perpendicular to y-axis.

$$\Longrightarrow (\mathbf{R} - \mathbf{x})^{\mathsf{T}} \mathbf{R} = 0 \tag{2.0.2}$$

$$\Longrightarrow \mathbf{X}^{\mathsf{T}}\mathbf{R} = ||\mathbf{R}||^2 \tag{2.0.3}$$

$$\Longrightarrow \mathbf{x}^{\mathsf{T}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} ||\mathbf{R}|| = ||\mathbf{R}||^2 \tag{2.0.4}$$

$$\Longrightarrow \|\mathbf{R}\| = \mathbf{x}^{\mathsf{T}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.5}$$

Let  $C = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ . Then

$$xC = ||\mathbf{x} - \mathbf{C}|| \tag{2.0.6}$$

$$xR = ||\mathbf{x} - \mathbf{R}|| \tag{2.0.7}$$

We are given xR = xC.

$$\Longrightarrow ||\mathbf{x} - \mathbf{C}||^2 = ||\mathbf{x} - \mathbf{R}||^2 \tag{2.0.8}$$

$$\Longrightarrow ||\mathbf{x}||^2 + ||\mathbf{C}||^2 - 2\mathbf{x}^{\mathsf{T}}\mathbf{C} = ||\mathbf{x}||^2 + ||\mathbf{R}||^2 - 2\mathbf{x}^{\mathsf{T}}\mathbf{R}$$
(2.0.9)

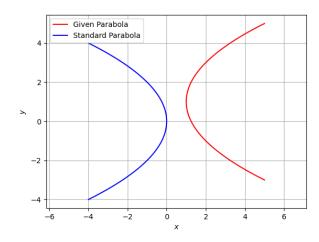


Fig. 0: Plot of the locus

Subtracting  $||\mathbf{x}||^2$  on both sides and using 2.0.3,

$$\|\mathbf{C}\|^2 - 2\mathbf{x}^{\mathsf{T}}\mathbf{C} = \|\mathbf{R}\|^2 - 2\|\mathbf{R}\|^2$$
 (2.0.10)

$$\Longrightarrow 2\mathbf{x}^{\mathsf{T}}\mathbf{C} = \|\mathbf{C}\|^2 + \|\mathbf{R}\|^2 \tag{2.0.11}$$

$$\Longrightarrow 2\mathbf{C}^{\top}\mathbf{x} = \|\mathbf{C}\|^2 + \left(\mathbf{x}^{\top} \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right)^2 \tag{2.0.12}$$

$$\Longrightarrow \mathbf{x}^{\top} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} - 2\mathbf{C}^{\top} \mathbf{x} + ||\mathbf{C}||^2 = 0 \quad (2.0.13)$$

$$\Longrightarrow \mathbf{x}^{\mathsf{T}} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2(-\mathbf{C})^{\mathsf{T}} \mathbf{x} + ||\mathbf{C}||^2 = 0 \quad (2.0.14)$$

$$\Longrightarrow \mathbf{x}^{\mathsf{T}} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -2 & -1 \end{pmatrix} \mathbf{x} + 5 = 0 \quad (2.0.15)$$

Therefore 2.0.15 is the required locus.