ASSIGNMENT 1

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Download all python codes from

https://github.com/Dishank422/EE3900/blob/main/assignment1/codes/codes.py

and latex-tikz codes from

https://github.com/Dishank422/EE3900/blob/main/assignment1/Assignment1.tex

1 Vectors 2.13

Show that the points $\mathbf{A} = \begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 11 \\ 3 \\ 7 \end{pmatrix}$ are collinear and find the ratio in which $\mathbf{B}$ divides AC.

2 Solution

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix}, \mathbf{C} - \mathbf{A} = \begin{pmatrix} 10 \\ 5 \\ 15 \end{pmatrix} \tag{2.0.1}$$

Forming the matrix M,

$$\mathbf{M} = \begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{C} - \mathbf{A} \end{pmatrix}^{\top} \tag{2.0.2}$$

$$= \begin{pmatrix} 4 & 2 & 6 \\ 10 & 5 & 15 \end{pmatrix} \tag{2.0.3}$$

Using matrix transformation,

$$\mathbf{M} = \begin{pmatrix} 4 & 2 & 6 \\ 10 & 5 & 15 \end{pmatrix} \xleftrightarrow{R_2 \to R_2 - \frac{5}{2}R_1} \begin{pmatrix} 4 & 2 & 6 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.4}$$

$$\implies rank(\mathbf{M}) = 1 \tag{2.0.5}$$

Thus $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are collinear.
Let $\mathbf{B}$ divide AC in the ratio $\lambda : 1$.

$$\implies \frac{\lambda}{1} = \frac{AB}{BC} \tag{2.0.6}$$

$$\implies \|\mathbf{B} - \mathbf{A}\| = \lambda \|\mathbf{C} - \mathbf{B}\| \tag{2.0.7}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix}, \mathbf{C} - \mathbf{B} = \begin{pmatrix} 6 \\ 3 \\ 9 \end{pmatrix} \tag{2.0.8}$$

$$\implies 2\sqrt{14} = \lambda \times 3\sqrt{14} \tag{2.0.9}$$

$$\implies \lambda = \frac{2}{3} \tag{2.0.10}$$

Thus $\mathbf{B}$ divides AC in the ratio 2:3.

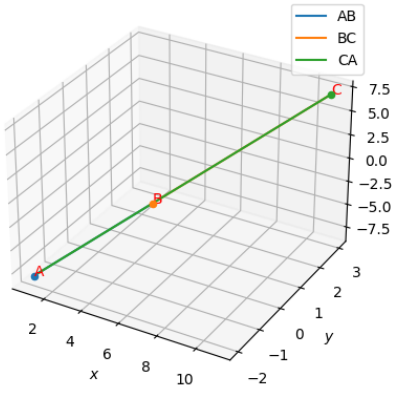

Fig. 0: Plot of the line