# Gate Assignment 1

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# Question

Let the state-space representation on an LTI system be $\dot{x}(t) = Ax(t) + Bu(t)$, $y(t) = Cx(t) + du(t)$ where A,B,C are matrices, d is a scalar, u(t) is the input to the system, and y(t) is its output. Let $B = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^\top$ and $d = 0$. Which one of the following options for A and C will ensure that the transfer function of this LTI system is

$$H(s) = \frac{1}{s^3 + 3s^2 + 2s + 1} \tag{1}$$

- A $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$
- B $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$
- C $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$
- D $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$

# Laplace transform

$$Assumption:\ x(t) = 0\ at\ t = 0 \tag{2}$$

$$\mathcal{L}(x(t)) = X(s) = \int_0^\infty x(t)e^{-st}dt \tag{3}$$

$$\implies \mathcal{L}(\dot{x}(t)) = \int_0^\infty \dot{x}(t)e^{-st}dt \tag{4}$$

$$= e^{-st}x(t)\Big|_0^\infty + \int_0^\infty sx(t)e^{-st}dt \tag{5}$$

$$= -x(0) + sX(s) \tag{6}$$

$$= sX(s) \tag{7}$$

## Solution

We are given

$$\begin{pmatrix} \dot{x}(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} A & B \\ C & d \end{pmatrix} \begin{pmatrix} x(t) \\ u(t) \end{pmatrix} \tag{8}$$

Taking Laplace transform on both sides,

$$\begin{pmatrix} sX(s) \\ Y(s) \end{pmatrix} = \begin{pmatrix} A & B \\ C & d \end{pmatrix} \begin{pmatrix} X(s) \\ U(s) \end{pmatrix} \tag{9}$$

$$\implies sX(s) = AX(s) + BU(s) \tag{10}$$

$$\implies X(s) = (sI - A)^{-1}BU(s) \tag{11}$$

$$\implies Y(s) = CX(s) + dU(s) \tag{12}$$

$$= C(sI - A)^{-1}BU(s) + dU(s) \tag{13}$$

By definition,

$$Y(s) = H(s)U(s) \tag{14}$$

$$\implies H(s) = C(sI - A)^{-1}B + d \tag{15}$$

$$= \frac{1}{s^3 + 3s^2 + 2s + 1} \tag{16}$$

$$\implies C(sI - A)^{-1}B + d = \frac{1}{s^3 + 3s^2 + 2s + 1} \tag{17}$$

# Solution Contd.

Now we substitute A and C from each option into eq. 17 and verify if H(s) is the same as we require.

A.

$$C(sI - A)^{-1}B + d = \frac{1}{s^3 + 3s^2 + 2s + 1} \tag{18}$$

B.

$$C(sI - A)^{-1}B + d = \frac{1}{s^3 + 1s^2 + 2s + 3} \tag{19}$$

C.

$$C(sI - A)^{-1}B + d = \frac{s^2}{s^3 + 3s^2 + 2s + 1} \tag{20}$$

D.

$$C(sI - A)^{-1}B + d = \frac{s^2}{s^3 + 1s^2 + 2s + 3} \tag{21}$$

Hence only option A is the correct option. For above calculations, refer
https://github.com/Dishank422/EE3900/blob/main/Gate-Assignment1/codes/codes.py