ASSIGNMENT 1

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1 VECTORS 2.13

Show that the points $\mathbf{A} = \begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix}$ and

$\mathbf{C} = \begin{pmatrix} 11 \\ 3 \\ 7 \end{pmatrix}$ are collinear and find the ratio in which $\mathbf{B}$ divides AC.

2 SOLUTION

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 4 \\ 2 \\ 6 \end{pmatrix}, \mathbf{C} - \mathbf{A} = \begin{pmatrix} 10 \\ 5 \\ 15 \end{pmatrix} \quad (2.0.1)$$

Forming the matrix M,

$$\mathbf{M} = \begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{C} - \mathbf{A} \end{pmatrix}^\top \quad (2.0.2)$$

$$= \begin{pmatrix} 4 & 2 & 6 \\ 10 & 5 & 15 \end{pmatrix} \quad (2.0.3)$$

Using matrix transformation,

$$\mathbf{M} = \begin{pmatrix} 4 & 2 & 6 \\ 10 & 5 & 15 \end{pmatrix} \xleftarrow{R_2 \to R_2 - \frac{5}{2}R_1} \begin{pmatrix} 4 & 2 & 6 \\ 0 & 0 & 0 \end{pmatrix} \quad (2.0.4)$$

$$\implies rank(\mathbf{M}) = 1 \quad (2.0.5)$$

Thus $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are collinear.
Let $\mathbf{B}$ divide AC in the ratio $\lambda : 1$.

$$\implies \frac{\lambda}{1} = \frac{AB}{BC} \quad (2.0.6)$$

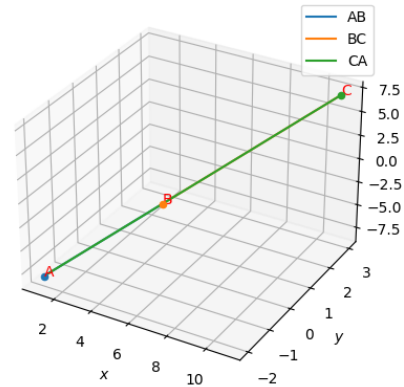$$\implies \|\mathbf{B} - \mathbf{A}\| = \lambda \|\mathbf{C} - \mathbf{B}\| \quad (2.0.7)$$

$$\implies \lambda = \frac{2}{3} \quad (2.0.8)$$

Note: For the above computations see codes/computations.py

Thus $\mathbf{B}$ divides AC in the ratio 2:3.

Fig. 0: Plot of the line