ASSIGNMENT 1

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Download all latex-tikz codes from

https://github.com/Dishank422/EE3900/blob/main/Gate−Assignment1/latex_code.tex

1 EC 2019 Q.33

Let the state-space representation on an LTI system be $\dot{x}(t) = Ax(t)+Bu(t)$, $y(t) = Cx(t)+du(t)$ where A,B,C are matrices, d is a scalar, u(t) is the input to the system, and y(t) is its output. Let $B = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^\top$ and $d = 0$. Which one of the following options for A and C will ensure that the transfer function of this LTI system is

$$H(s) = \frac{1}{s^3 + 3s^2 + 2s + 1} \tag{1.0.1}$$

(A) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$

(B) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$

(C) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$

(D) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{pmatrix}$ and $C = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$

2 Solution

We are given

$$\begin{pmatrix} \dot{x}(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} A & B \\ C & d \end{pmatrix} \begin{pmatrix} x(t) \\ u(t) \end{pmatrix} \tag{2.0.1}$$

Taking Laplace transform on both sides,

$$\begin{pmatrix} sX(s) \\ Y(s) \end{pmatrix} = \begin{pmatrix} A & B \\ C & d \end{pmatrix} \begin{pmatrix} X(s) \\ U(s) \end{pmatrix} \tag{2.0.2}$$

$$\implies sX(s) = AX(s) + BU(s) \tag{2.0.3}$$

$$\implies X(s) = (sI - A)^{-1}BU(s) \tag{2.0.4}$$

$$\implies Y(s) = CX(s) + dU(s) \tag{2.0.5}$$

$$= C(sI - A)^{-1}BU(s) + dU(s) \tag{2.0.6}$$

Since, $d = 0$,

$$Y(s) = C(sI - A)^{-1}BU(s) \tag{2.0.7}$$

By definition,

$$Y(s) = H(s)U(s) \tag{2.0.8}$$

$$\implies H(s) = C(sI - A)^{-1}B \tag{2.0.9}$$

$$= \frac{1}{s^3 + 3s^2 + 2s + 1} \tag{2.0.10}$$

$$\implies C(sI - A)^{-1}B = \frac{1}{s^3 + 3s^2 + 2s + 1} \tag{2.0.11}$$

Now we substitute the options into eq 2.0.11.

(A)

$$C(sI - A)^{-1}B = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1 & 2 & s+3 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{2.0.12}$$

$$= \frac{1}{s^3 + 3s^2 + 2s + 1} \tag{2.0.13}$$

(B)

$$C(sI - A)^{-1}B = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 3 & 2 & s+1 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{2.0.14}$$

$$= \frac{1}{s^3 + s^2 + 2s + 3} \tag{2.0.15}$$

(C)

$$C(sI - A)^{-1}B = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1 & 2 & s+3 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{2.0.16}$$

$$= \frac{s^2}{s^3 + 3s^2 + 2s + 3} \tag{2.0.17}$$

(D)

$$C(sI - A)^{-1}B = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 3 & 2 & s+1 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{2.0.18}$$

$$= \frac{s^2}{s^3 + s^2 + 2s + 3} \tag{2.0.19}$$

Hence only option A is the correct option.