ASSIGNMENT 3

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Download all python codes from

https://github.com/Dishank422/EE3900/blob/main/assignment3_elegant/codes

and latex-tikz codes from

https://github.com/Dishank422/EE3900/blob/main/assignment3_elegant/Assignment3.tex

1 Constructions Q2.4

Construct a quadrilateral ABCD such that BC = 4.5, AC = 5.5, CD = 5, BD = 7 and AD = 5.5.

2 Solution

Lemma 2.1. *In triangle XYZ, if the 2.0.1-2.0.5 hold,*

$$XY = z \tag{2.0.1}$$
$$YZ = x \tag{2.0.2}$$
$$XZ = y \tag{2.0.3}$$
$$\mathbf{X} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.4}$$
$$\mathbf{Y} = \begin{pmatrix} z \\ 0 \end{pmatrix} \tag{2.0.5}$$

then

$$\mathbf{Z} = \begin{pmatrix} m \\ \pm n \end{pmatrix} \tag{2.0.6}$$
$$\text{where } m = \frac{XY^2 + XZ^2 - YZ^2}{2 \times XY} \tag{2.0.7}$$
$$n = \sqrt{(XZ^2 - m^2)} \tag{2.0.8}$$

Note: The proof for above can be found in the manual (problem 1.3). Using AC = 5.5,

$$\text{Let } \mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5.5 \\ 0 \end{pmatrix} \tag{2.0.9}$$
$$\mathbf{D} = \begin{pmatrix} h \\ k \end{pmatrix} \tag{2.0.10}$$

We know

$$\|\mathbf{D} - \mathbf{A}\| = 5.5 \tag{2.0.11}$$
$$\|\mathbf{D} - \mathbf{C}\| = 5 \tag{2.0.12}$$
$$\|\mathbf{B} - \mathbf{C}\| = 4.5 \tag{2.0.13}$$
$$\|\mathbf{B} - \mathbf{D}\| = 7 \tag{2.0.14}$$

$$h = \frac{AC^2 + AD^2 - CD^2}{2AC} \tag{2.0.15}$$
$$= \frac{5.5^2 + 5.5^2 - 5^2}{2 \times 5.5} \tag{2.0.16}$$
$$= 3.23 \tag{2.0.17}$$
$$k = \sqrt{(AD^2 - h^2)} \tag{2.0.18}$$
$$= \sqrt{(5.5^2 - h^2)} \tag{2.0.19}$$
$$= 4.45 \tag{2.0.20}$$
$$\implies \mathbf{D} = \begin{pmatrix} 3.23 \\ 4.45 \end{pmatrix} \tag{2.0.21}$$

Note: Above computations can be found in codes/finding_D.py.

Now we shall transform co-ordinates of $\mathbf{C}, \mathbf{D}$ such that C will become the new origin and $\mathbf{D}$ comes along the positive x-axis. We shall first shift origin to C. For this, we have to subtract $\begin{pmatrix} 5.5 \\ 0 \end{pmatrix}$ from co-ordinates of $\mathbf{C}, \mathbf{D}$.

$$\implies \mathbf{C}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.22}$$
$$\mathbf{D}_1 = \begin{pmatrix} -2.27 \\ 4.45 \end{pmatrix} \tag{2.0.23}$$

Next we shall rotate the co-ordinates such that $\mathbf{D}_1$ is on the x-axis. For this, we have to multiply the co-ordinates with the clockwise rotator matrix

$\mathbf{R}$, given as

$$\mathbf{R} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{2.0.24}$$

$$where\ \theta = \arccos \frac{\mathbf{D}_1 \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}^\top}{\|\mathbf{D}_1\|} \tag{2.0.25}$$

$$= \arccos \frac{-2.27}{5} \tag{2.0.26}$$

$$= 2.04\ radians \tag{2.0.27}$$

$$\implies \mathbf{R} = \begin{pmatrix} -0.45 & 0.89 \\ -0.89 & -0.45 \end{pmatrix} \tag{2.0.28}$$

$$\implies \mathbf{D}_2 = \mathbf{R}\mathbf{D}_1 = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \tag{2.0.29}$$

Note: norm of $\mathbf{D}$ is calculated using codes/norm_D1.py, arccos is calculated using codes/finding_theta.py, R is calculated using codes/R.py and $\mathbf{D}_2$ is calculated using D2.py.

Since $\mathbf{C}_1$ is origin

$$\mathbf{C}_2 = \mathbf{C}_1 \tag{2.0.30}$$

$$Let\ \mathbf{B}_2 = \begin{pmatrix} p \\ q \end{pmatrix} \tag{2.0.31}$$

Using 2.0.13 and 2.0.14,

$$\|\mathbf{B}_2 - \mathbf{C}_2\| = 4.5 \tag{2.0.32}$$

$$\|\mathbf{B}_2 - \mathbf{D}_2\| = 7 \tag{2.0.33}$$

$$p = \frac{B_2C_2^2 + C_2D_2^2 - B_2D_2^2}{2C_2D_2} \tag{2.0.34}$$

$$= \frac{4.5^2 + 5^2 - 7^2}{2 \times 5} \tag{2.0.35}$$

$$= -0.38 \tag{2.0.36}$$

$$q = \pm\sqrt{(B_2C_2^2 - p^2)} \tag{2.0.37}$$

$$= \pm 4.48 \tag{2.0.38}$$

$$\implies \mathbf{B}_2 = \begin{pmatrix} -0.38 \\ 4.48 \end{pmatrix}, \begin{pmatrix} -0.38 \\ -4.48 \end{pmatrix} \tag{2.0.39}$$

Note: q is found using codes/q.py.

To get $\mathbf{B}_1$, we operate on $\mathbf{B}_2$ with inverse of $\mathbf{R}$. Inverse of $\mathbf{R}$ is nothing but $\mathbf{R}(-\theta)$. Therefore,

$$\mathbf{R}^{-1} = \begin{pmatrix} \cos-\theta & \sin-\theta \\ -\sin-\theta & \cos-\theta \end{pmatrix} \tag{2.0.40}$$

$$= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{2.0.41}$$

$$= \begin{pmatrix} -0.45 & -0.89 \\ 0.89 & -0.45 \end{pmatrix} \tag{2.0.42}$$

$$\implies \mathbf{B}_1 = \mathbf{R}^{-1}\mathbf{B}_2 = \begin{pmatrix} -3.82 \\ -2.35 \end{pmatrix}, \begin{pmatrix} 4.16 \\ 1.68 \end{pmatrix} \tag{2.0.43}$$

To finally get $\mathbf{B}$, we add $\begin{pmatrix} 5.5 \\ 0 \end{pmatrix}$ to $\mathbf{B}_1$.

$$\implies \mathbf{B} = \begin{pmatrix} 1.68 \\ -2.35 \end{pmatrix}, \begin{pmatrix} 9.66 \\ 1.68 \end{pmatrix} \tag{2.0.44}$$

We want $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ in clockwise or anti-clockwise order. Therefore only the first value of $\mathbf{B}$ is possible.

$$\implies \mathbf{B} = \begin{pmatrix} 1.68 \\ -2.35 \end{pmatrix} \tag{2.0.45}$$

Using the co-ordinates of the vertices as found, the following quadrilateral is plotted.