

ASSIGNMENT 4

Dishank Jain
AI20BTECH11011

Download all python codes from

<https://github.com/Dishank422/EE3900/blob/main/assignment4/codes>

and latex-tikz codes from

<https://github.com/Dishank422/EE3900/blob/main/assignment4/Assignment4.tex>

1 RAMSEY 1.2 LOCI Q 4

A point moves so that it's distance from the y-axis is equal to the distance from the point $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Find the equation of the locus.

2 SOLUTION

Let $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ be the point. The equation of y-axis is given by

$$\mathbf{R} = \lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.1)$$

xR is perpendicular to y-axis.

$$\Rightarrow (\mathbf{R} - \mathbf{x})^T \mathbf{R} = 0 \quad (2.0.2)$$

$$\Rightarrow \mathbf{x}^T \mathbf{R} = \|\mathbf{R}\|^2 \quad (2.0.3)$$

$$\Rightarrow \mathbf{x}^T \begin{pmatrix} 0 \\ 1 \end{pmatrix} \|\mathbf{R}\| = \|\mathbf{R}\|^2 \quad (2.0.4)$$

$$\Rightarrow \|\mathbf{R}\| = \mathbf{x}^T \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.5)$$

Let $\mathbf{C} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Then

$$xC = \|\mathbf{x} - \mathbf{C}\| \quad (2.0.6)$$

$$xR = \|\mathbf{x} - \mathbf{R}\| \quad (2.0.7)$$

We are given $xR = xC$.

$$\Rightarrow \|\mathbf{x} - \mathbf{C}\|^2 = \|\mathbf{x} - \mathbf{R}\|^2 \quad (2.0.8)$$

$$\Rightarrow \|\mathbf{x}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{x}^T \mathbf{C} = \|\mathbf{x}\|^2 + \|\mathbf{R}\|^2 - 2\mathbf{x}^T \mathbf{R} \quad (2.0.9)$$

Subtracting $\|\mathbf{x}\|^2$ on both sides and using 2.0.3,

$$\|\mathbf{C}\|^2 - 2\mathbf{x}^T \mathbf{C} = \|\mathbf{R}\|^2 - 2\|\mathbf{R}\|^2 \quad (2.0.10)$$

$$\Rightarrow 2\mathbf{x}^T \mathbf{C} = \|\mathbf{C}\|^2 + \|\mathbf{R}\|^2 \quad (2.0.11)$$

$$\Rightarrow 2\mathbf{x}^T \mathbf{C} = \|\mathbf{C}\|^2 + \left(\mathbf{x}^T \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)^2 \quad (2.0.12)$$

$$\Rightarrow \mathbf{x}^T \begin{pmatrix} 0 \\ 1 \end{pmatrix} (0 \ 1) \mathbf{x} - 2\mathbf{x}^T \mathbf{C} + \|\mathbf{C}\|^2 = 0 \quad (2.0.13)$$

$$\Rightarrow \mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2\mathbf{x}^T (-\mathbf{C}) + \|\mathbf{C}\|^2 = 0 \quad (2.0.14)$$

$$\Rightarrow \mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2\mathbf{x}^T \begin{pmatrix} -2 \\ -1 \end{pmatrix} + 5 = 0 \quad (2.0.15)$$

Therefore 2.0.15 is the required locus.

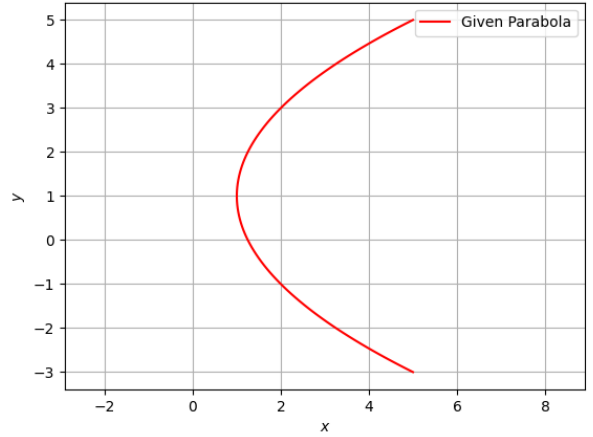


Fig. 0: Plot of the locus