ASSIGNMENT 4

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Download all python codes from

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https://github.com/Dishank422/EE3900/blob/main/assignment4/codes
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and latex-tikz codes from

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https://github.com/Dishank422/EE3900/blob/main/assignment4/Assignment4.tex
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1 Ramsey 1.2 Loci Q 4

A point moves so that it's distance from the y-axis is equal to the distance from the point $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Find the equation of the locus.

2 Solution

Let $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ be the point. The equation of y-axis is given by

$$\mathbf{R} = \lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.1}$$

xR is perpendicular to y-axis.

$$\implies (\mathbf{R} - \mathbf{x})^\top \mathbf{R} = 0 \tag{2.0.2}$$
$$\implies \mathbf{x}^\top \mathbf{R} = \|\mathbf{R}\|^2 \tag{2.0.3}$$
$$\implies \mathbf{x}^\top \begin{pmatrix} 0 \\ 1 \end{pmatrix} \|\mathbf{R}\| = \|\mathbf{R}\|^2 \tag{2.0.4}$$
$$\implies \|\mathbf{R}\| = \mathbf{x}^\top \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.5}$$

Let $\mathbf{C} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Then

$$xC = \|\mathbf{x} - \mathbf{C}\| \tag{2.0.6}$$
$$xR = \|\mathbf{x} - \mathbf{R}\| \tag{2.0.7}$$

We are given $xR = xC$.

$$\implies \|\mathbf{x} - \mathbf{C}\|^2 = \|\mathbf{x} - \mathbf{R}\|^2 \tag{2.0.8}$$
$$\implies \|\mathbf{x}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{x}^\top \mathbf{C} = \|\mathbf{x}\|^2 + \|\mathbf{R}\|^2 - 2\mathbf{x}^\top \mathbf{R} \tag{2.0.9}$$

Subtracting $\|\mathbf{x}\|^2$ on both sides and using 2.0.3,

$$\|\mathbf{C}\|^2 - 2\mathbf{x}^\top \mathbf{C} = \|\mathbf{R}\|^2 - 2\|\mathbf{R}\|^2 \tag{2.0.10}$$
$$\implies 2\mathbf{x}^\top \mathbf{C} = \|\mathbf{C}\|^2 + \|\mathbf{R}\|^2 \tag{2.0.11}$$
$$\implies 2\mathbf{C}^\top \mathbf{x} = \|\mathbf{C}\|^2 + \left(\mathbf{x}^\top \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right)^2 \tag{2.0.12}$$
$$\implies \mathbf{x}^\top \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} - 2\mathbf{C}^\top \mathbf{x} + \|\mathbf{C}\|^2 = 0 \tag{2.0.13}$$
$$\implies \mathbf{x}^\top \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2(-\mathbf{C})^\top \mathbf{x} + \|\mathbf{C}\|^2 = 0 \tag{2.0.14}$$
$$\implies \mathbf{x}^\top \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2\begin{pmatrix} -2 & -1 \end{pmatrix} \mathbf{x} + 5 = 0 \tag{2.0.15}$$
$$\implies \mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \ \mathbf{u} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}, \ f = 5 \tag{2.0.16}$$

For obtaining the affine transformation, we use

$$\mathbf{x} = \mathbf{P}\mathbf{y} + \mathbf{c} \tag{2.0.17}$$

The corresponding eigenvalues of $\mathbf{V}$ are

$$\lambda_1 = 0, \ \lambda_2 = 1 \tag{2.0.18}$$
$$\implies \mathbf{D} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.19}$$

The corresponding eigenvectors are

$$\mathbf{p}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ \mathbf{p}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.20}$$
$$\implies \mathbf{P} = \begin{pmatrix} \mathbf{p}_1 & \mathbf{p}_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.21}$$

Since $|\mathbf{V}| = 0$,

$$\begin{pmatrix} \mathbf{u}^\top + \eta \mathbf{p}_1^\top \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \eta \mathbf{p}_1 - \mathbf{u} \end{pmatrix} \tag{2.0.22}$$

$$\eta = \mathbf{u}^\top \mathbf{p}_1 \tag{2.0.23}$$

$$\implies \eta = -2 \tag{2.0.24}$$

$$\implies \begin{pmatrix} -4 & -1 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -5 \\ 0 \\ 1 \end{pmatrix} \tag{2.0.25}$$

$$\implies \mathbf{c} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2.0.26}$$

Therefore, the locus of $\mathbf{x}$ is given by

$$\mathbf{y}^\top \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{y} = 4\mathbf{y} \tag{2.0.27}$$

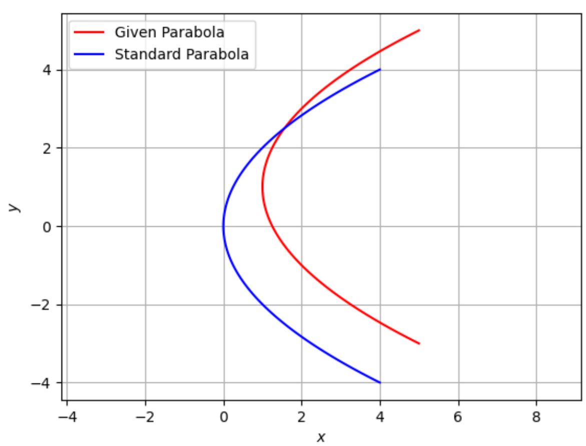

Fig. 0: Plot of the locus