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1 Algorithms (57)

1.1 Algorithm Design (1) top

1.1.1 Algorithm Design: TIFR CSE 2011 | Part B | Question: 29 top

You are given ten rings numbered from 1 to 10, and three pegs labeled A, B, and C. Initially all the rings are on peg A, arranged from top to bottom in ascending order of their numbers. The goal is to move all the rings to peg B in the minimum number of moves obeying the following constraints:

i. In one move, only one ring can be moved.
ii. A ring can only be moved from the top of its peg to the top of a new peg.
iii. At no point can a ring be placed on top of another ring with a lower number.

How many moves are required?

A. 501 B. 1023 C. 2011 D. 10079 E. None of the above.

tifr2011 algorithms algorithm-design

Answer key

1.2 Asymptotic Notations (9) top

1.2.1 Asymptotic Notations: TIFR CSE 2011 | Part B | Question: 27 top

Let n be a large integer. Which of the following statements is **TRUE**?

A. $n^{\frac{1}{\sqrt{\log_2 n}}} < \sqrt{\log_2 n} < n^{\frac{1}{100}}$
B. $n^{\frac{1}{100}} < n^{\frac{1}{\sqrt{\log_2 n}}} < \sqrt{\log_2 n}$
C. $n^{\frac{1}{\sqrt{\log_2 n}}} < n^{\frac{1}{100}} < \sqrt{\log_2 n}$
D. $\sqrt{\log_2 n} < n^{\frac{1}{\sqrt{\log_2 n}}} < n^{\frac{1}{100}}$
E. $\sqrt{\log_2 n} < n^{\frac{1}{100}} < n^{\frac{1}{\sqrt{\log_2 n}}}$

tifr2011 asymptotic-notations

Answer key

1.2.2 Asymptotic Notations: TIFR CSE 2012 | Part B | Question: 6 top

Let n be a large integer. Which of the following statements is **TRUE**?

A. $2^{\sqrt{2\log n}} < \frac{n}{\log n} < n^{1/3}$
B. $\frac{n}{\log n} < n^{1/3} < 2^{\sqrt{2\log n}}$
C. $2^{\sqrt{2\log n}} < n^{1/3} < \frac{n}{\log n}$
D. $n^{1/3} < 2^{\sqrt{2\log n}} < \frac{n}{\log n}$
E. $\frac{n}{\log n} < 2^{\sqrt{2\log n}} < n^{1/3}$

tifr2012 algorithms asymptotic-notations

Answer key

1.2.3 Asymptotic Notations: TIFR CSE 2014 | Part B | Question: 8 top

Which of these functions grows fastest with n?

A. e^n/n.
B. $e^{n-0.9\log n}$.

C. 2^n.

D. $(\log n)^{n-1}$.

E. None of the above.

tifr2014 algorithms asymptotic-notations

Answer key

1.2.4 Asymptotic Notations: TIFR CSE 2016 | Part B | Question: 7 top

Let $n = m!$. Which of the following is **TRUE**?

A. $m = \Theta(\log n / \log \log n)$
B. $m = \Omega(\log n / \log \log n)$ but not $m = O(\log n / \log \log n)$
C. $m = \Theta(\log^2 n)$
D. $m = \Omega(\log^2 n)$ but not $m = \text{O}(\log^2 n)$
E. $m = \Theta(\log^{1.5} n)$

tifr2016 asymptotic-notations

Answer key

1.2.5 Asymptotic Notations: TIFR CSE 2017 | Part A | Question: 4 top

Which of the following functions asymptotically grows the fastest as n goes to infinity?

A. $(\log \log n)!$

B. $(\log \log n)^{\log n}$

C. $(\log \log n)^{\log \log \log n}$

D. $(\log n)^{\log \log n}$

E. $2^{\sqrt{\log \log n}}$

tifr2017 algorithms asymptotic-notations

Answer key

1.2.6 Asymptotic Notations: TIFR CSE 2018 | Part A | Question: 3 top

Which of the following statements is TRUE for all sufficiently large integers n ?

A. $2^{2^{\sqrt{\log \log n}}} < 2^{\sqrt{\log n}} < n$
B. $2^{\sqrt{\log n}} < n < 2^{2^{\sqrt{\log \log n}}}$
C. $n < 2^{\sqrt{\log n}} < 2^{2^{\sqrt{\log \log n}}}$
D. $n < 2^{2^{\sqrt{\log \log n}}} < 2^{\sqrt{\log n}}$
E. $2^{\sqrt{\log n}} < 2^{2^{\sqrt{\log \log n}}} < n$

tifr2018 algorithms asymptotic-notations

Answer key

1.2.7 Asymptotic Notations: TIFR CSE 2018 | Part B | Question: 5 top

Which of the following functions, given by there recurrence, grows the fastest asymptotically?

A. $T(n) = 4T\left(\frac{n}{2}\right) + 10n$
B. $T(n) = 8T\left(\frac{n}{3}\right) + 24n^2$
C. $T(n) = 16T\left(\frac{n}{4}\right) + 10n^2$
D. $T(n) = 25T\left(\frac{n}{5}\right) + 20(n \log n)^{1.99}$
E. They all are asymptotically the same

tifr2018 asymptotic-notations recurrence-relation

Answer key

1.2.8 Asymptotic Notations: TIFR CSE 2019 | Part B | Question: 5 top

Stirling's approximation for $n!$ states for some constants c_1, c_2

$$c_1 n^{n+\frac{1}{2}} e^{-n} \leq n! \leq c_2 n^{n+\frac{1}{2}} e^{-n}.$$

What are the tightest asymptotic bounds that can be placed on $n!$?

A. $n! = \Omega(n^n)$ and $n! = \mathcal{O}(n^{n+\frac{1}{2}})$
B. $n! = \Theta(n^{n+\frac{1}{2}})$
C. $n! = \Theta((\frac{n}{e})^n)$
D. $n! = \Theta((\frac{n}{e})^{n+\frac{1}{2}})$
E. $n! = \Theta(n^{n+\frac{1}{2}} 2^{-n})$

tifr2019 algorithms asymptotic-notations

Answer key

1.2.9 Asymptotic Notations: TIFR CSE 2020 | Part B | Question: 10 top

Among the following asymptotic expressions, which of these functions grows the slowest (as a function of n) asymptotically?

A. $2^{\log n}$
B. n^{10}
C. $(\sqrt{\log n})^{\log^2 n}$
D. $(\log n)^{\sqrt{\log n}}$
E. $2^{2^{\sqrt{\log \log n}}}$

tifr2020 algorithms asymptotic-notations time-complexity

Answer key

1.3 Binary Search (1) top

1.3.1 Binary Search: TIFR CSE 2019 | Part A | Question: 5 top

Asha and Lata play a game in which Lata first thinks of a natural number between 1 and 1000. Asha must find out that number by asking Lata questions, but Lata can only reply by saying "Yes" or "no". Assume that Lata always tells the truth. What is the least number of questions that Asha needs to ask within which she can always find out the number Lata has thought of?

A. 10
B. 32
C. 100
D. 999
E. None of the above

tifr2019 algorithm-design binary-search

Answer key

1.4 Graph Algorithms (3) top

1.4.1 Graph Algorithms: TIFR CSE 2013 | Part B | Question: 15 top

Let G be an undirected graph with n vertices. For any subset S of vertices, the set of neighbours of S consists of the union of S and the set of vertices S' that are connected to some vertex in S by an edge of G. The graph G has the nice property that every subset of vertices S of size at most $n/2$ has at least $1.5|S|$-many neighbours. What is the length of a longest path in G?

A. $O(1)$
B. $O(\log \log n)$ but not $O(1)$
C. $O(\log n)$ but not $O(\log \log n)$
D. $O(\sqrt{n})$ but not $O(\log n)$
E. $O(n)$ but not $O(\sqrt{n})$

tifr2013 graph-algorithms

Answer key☟

1.4.2 Graph Algorithms: TIFR CSE 2013 | Part B | Question: 5 top☝

Given a weighted directed graph with n vertices where edge weights are integers (positive, zero, or negative), determining whether there are paths of arbitrarily large weight can be performed in time

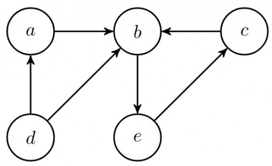
A. $O(n)$
B. $O(n.\log(n))$ but not $O(n)$
C. $O(n^{1.5})$ but not $O(n \log n)$
D. $O(n^3)$ but not $O(n^{1.5})$
E. $O(2^n)$ but not $O(n^3)$

tifr2013 algorithms graph-algorithms

Answer key☟

1.4.3 Graph Algorithms: TIFR CSE 2014 | Part B | Question: 3 top☝

Consider the following directed graph.



Suppose a depth-first traversal of this graph is performed, assuming that whenever there is a choice, the vertex earlier in the alphabetical order is to be chosen. Suppose the number of tree edges is T, the number of back edges is B and the number of cross edges is C. Then

A. $B = 1$, $C = 1$, and $T = 4$.
B. $B = 0$, $C = 2$, and $T = 4$.
C. $B = 2$, $C = 1$, and $T = 3$.
D. $B = 1$, $C = 2$, and $T = 3$.
E. $B = 2$, $C = 2$, and $T = 1$.

tifr2014 algorithms graph-algorithms

Answer key☟

1.5 Identify Function (4) top☝

1.5.1 Identify Function: TIFR CSE 2010 | Part B | Question: 24 top☝

Consider the following program operating on four variables u, v, x, y, and two constants X and Y.

```
x, y, u, v:= X, Y, Y, X;
While (x ≠ y)
do
   if (x > y) then x, v := x - y, v + u;
   else if (y > x) then y, u:= y - x, u + v;
od;
print ((x + y) / 2);  print ((u + v) / 2);
```

Given $X > 0 \land Y > 0$, pick the true statement out of the following:

A. The program prints $\gcd(X, Y)$ and the first prime larger than both X and Y.
B. The program prints $\gcd(X, Y)$ followed by $\text{lcm}(X, Y)$.
C. The program prints $\gcd(X, Y)$ followed by $\frac{1}{2} \times \text{lcm}(X, Y)$.
D. The program prints $\frac{1}{2} \times \gcd(X, Y)$ followed by $\frac{1}{2} \times \text{lcm}(X, Y)$.
E. The program does none of the above.

tifr2010 algorithms identify-function

Answer key☟

1.5.2 Identify Function: TIFR CSE 2014 | Part B | Question: 2 top☝

Consider the following code.

```
def brian(n):
   count = 0

   while ( n ! = 0 )
        n = n & ( n-1 )
       count = count + 1

  return count
```

Here n is meant to be an unsigned integer. The operator & considers its arguments in binary and computes their bit wise AND. For example, $22 \ \& 15$ gives 6, because the binary (say 8-bit) representation of 22 is 00010110 and the binary representation of 15 is 00001111, and the bit-wise AND of these binary strings is 00000110, which is the binary representation of 6. What does the function **brian** return?

A. The highest power of 2 dividing n, but zero if n is zero.
B. The number obtained by complementing the binary representation of n.
C. The number of ones in the binary representation of n.
D. The code might go into an infinite loop for some n.
E. The result depends on the number of bits used to store unsigned integers.

tifr2014 algorithms identify-function

Answer key☟

1.5.3 Identify Function: TIFR CSE 2014 | Part B | Question: 20 top☝

Consider the following game. There is a list of distinct numbers. At any round, a player arbitrarily chooses two numbers a, b from the list and generates a new number c by subtracting the smaller number from the larger one. The numbers a and b are put back in the list. If the number c is non-zero and is not yet in the list, c is added to the list. The player is allowed to play as many rounds as the player wants. The score of a player at the end is the size of the final list.

Suppose at the beginning of the game the list contains the following numbers: $48, 99, 120, 165$ and 273. What is the score of the best player for this game?

A. 40 B. 16 C. 33 D. 91 E. 123

tifr2014 algorithms identify-function

Answer key☟

1.5.4 Identify Function: TIFR CSE 2017 | Part A | Question: 12 top☝

Consider the following program modifying an $n \times n$ square matrix A:

```
for i=1 to n:
  for j=1 to n:
     temp=A[i][j]+10
     A[i][j]=A[j][i]
     A[j][i]=temp-10
  end for
end for
```

Which of the following statements about the contents of matrix A at the end of this program must be TRUE?

A. the new A is the transpose of the old A
B. all elements above the diagonal have their values increased by 10 and all the values below have their values decreased by 10
C. all elements above the diagonal have their values decreased by 10 and all the values below have their values increased by 10
D. the new matrix A is symmetric, that is, $A[i][j] = A[j][i]$ for all $1 \leq i, j \leq n$
E. A remains unchanged

tifr2017 algorithms identify-function

Answer key☟

1.6 Maximum Minimum (3) top☝

1.6.1 Maximum Minimum: TIFR CSE 2014 | Part B | Question: 10 top☝

Given a set of n distinct numbers, we would like to determine both the smallest and the largest number. Which of the following statements is TRUE?

A. These two elements can be determined using $O\left(\log^{100} n\right)$ comparisons.
B. $O\left(\log^{100} n\right)$ comparisons do not suffice, however these two elements can be determined using $n + O(\log n)$ comparisons.
C. $n + O(\log n)$ comparisons do not suffice, however these two elements can be determined using $3\lceil n/2 \rceil$ comparisons.
D. $3\lceil n/2 \rceil$ comparisons do not suffice, however these two elements can be determined using $2(n-1)$ comparisons.
E. None of the above.

tifr2014 algorithms maximum-minimum

Answer key☟

1.6.2 Maximum Minimum: TIFR CSE 2014 | Part B | Question: 6 top☝

Consider the problem of computing the minimum of a set of n distinct numbers. We choose a permutation uniformly at random (i.e., each of the n! permutations of $\langle 1, \ldots, n \rangle$ is chosen with probability $(1/n!)$ and we inspect the numbers in the order given by this permutation. We maintain a variable MIN that holds the minimum value seen so far. MIN is initialized to ∞ and if we see a value smaller than MIN during our inspection, then MIN is updated. For example, in the inspection given by the following sequence, MIN is updated four times.

$5\ 9\ 4\ 2\ 6\ 8\ 0\ 3\ 1\ 7$

What is the expected number of times MIN is updated?

A. $O(1)$ B. $H_n = \sum_{i=1}^{n} 1/i$ C. $\sqrt{n}$ D. $n/2$ E. n

tifr2014 algorithms maximum-minimum

Answer key☟

1.6.3 Maximum Minimum: TIFR CSE 2014 | Part B | Question: 9 top☝

Given a set of n distinct numbers, we would like to determine the smallest three numbers in this set using comparisons. Which of the following statements is TRUE?

A. These three elements can be determined using $O\left(\log^2 n\right)$ comparisons.
B. $O\left(\log^2 n\right)$ comparisons do not suffice, however these three elements can be determined using

$n + O(1)$ comparisons.
C. $n + O(1)$ comparisons do not suffice, however these three elements can be determined using $n + O(\log n)$ comparisons.
D. $n + O(\log n)$ comparisons do not suffice, however these three elements can be determined using $O(n)$ comparisons.
E. None of the above.

tifr2014 algorithms maximum-minimum

Answer key☟

1.7 Minimum Spanning Tree (2) top☝

1.7.1 Minimum Spanning Tree: TIFR CSE 2018 | Part B | Question: 13 top☝

Let $n \geq 3$, and let G be a simple, connected, undirected graph with the same number n of vertices and edges. Each edge of G has a distinct real weight associated with it. Let T be the minimum weight spanning tree of G. Which of the following statements is NOT ALWAYS TRUE ?

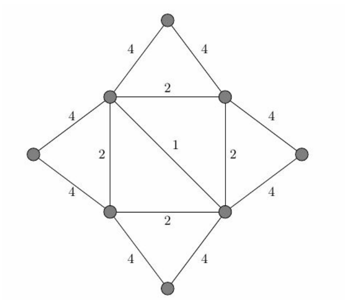
A. The minimum weight edge of G is in T.
B. The maximum weight edge of G is not in T.
C. G has a unique cycle C and the minimum weight edge of C is also in T.
D. G has a unique cycle C and the maximum weight edge of C is not in T.
E. T can be found in $O(n)$ time from the adjacency list representation of G.

tifr2018 graph-algorithms minimum-spanning-tree

Answer key☟

1.7.2 Minimum Spanning Tree: TIFR CSE 2019 | Part B | Question: 2 top☝

How many distinct minimum weight spanning trees does the following undirected, weighted graph have ?



A. 8 B. 16 C. 32 D. 64 E. None of the above

tifr2019 algorithms minimum-spanning-tree

Answer key☟

1.8 P Np Npc Nph (9) top☝

1.8.1 P Np Npc Nph: TIFR CSE 2010 | Part B | Question: 39 top☝

Suppose a language L is $\mathbf{NP}$ complete. Then which of the following is FALSE?

A. $L \in \mathbf{NP}$
B. Every problem in $\mathbf{P}$ is polynomial time reducible to L.

C. Every problem in $\mathbf{NP}$ is polynomial time reducible to L.
D. The Hamilton cycle problem is polynomial time reducible to L.
E. $\mathbf{P} \neq \mathbf{NP}$ and $L \in \mathbf{P}$.

tifr2010 algorithms p-np-npc-nph

Answer key☟

1.8.2 P Np Npc Nph: TIFR CSE 2011 | Part B | Question: 37 top☝

Given an integer $n \geq 3$, consider the problem of determining if there exist integers $a, b \geq 2$ such that $n = a^b$. Call this the forward problem. The reverse problem is: given a and b, compute a^b (mod b). Note that the input length for the forward problem is $\lfloor \log n \rfloor + 1$, while the input length for the reverse problem is $\lfloor \log a \rfloor + \lfloor \log b \rfloor + 2$. Which of the following statements is TRUE?

A. Both the forward and reverse problems can be solved in time polynomial in the lengths of their respective inputs.
B. The forward problem can be solved in polynomial time, however the reverse problem is NP-hard.
C. The reverse problem can be solved in polynomial time, however the forward problem is NP-hard.
D. Both the forward and reverse problem are NP-hard.
E. None of the above.

tifr2011 algorithms p-np-npc-nph

Answer key☟

1.8.3 P Np Npc Nph: TIFR CSE 2012 | Part B | Question: 20 top☝

This question concerns the classes P and NP. If you are familiar with them, you may skip the definitions and go directly to the question.
Let L be a set. We say that L is in P if there is some algorithm which given input x decides if x is in L or not in time bounded by a polynomial in the length of x. For example, the set of all connected graphs is in P, because there is an algorithm which, given a graph graph, can decide if it is connected or not in time roughly proportional to the number of edges of the graph.

The class NP is a superset of class P. It contains those sets that have membership witnesses that can be verified in polynomial time. For example, the set of composite numbers is in NP. To see this take the witness for a composite number to be one of its divisors. Then the verification process consists of performing just one division using two reasonable size numbers. Similarly, the set of those graphs that have a Hamilton cycle, i.e. a cycle containing all the vertices of the graph, is in in NP. To verify that the graph has a Hamilton cycle we just check if the witnessing sequence of vertices indeed a cycle of the graph that passes through all the vertices of the graph. This can be done in time that is polynomial in the size of the graph.

More precisely, if L is a set in P consisting of elements of the form (x, w), then the set

$$M = \{x : \exists w, |w| \leq |x|^k \text{ and } (x, w) \in L\},$$

is in N P .
Let $G = (V, E)$ be a graph. G is said to have perfect matching if there is a subset M of the edges of G so that

i. No two edges in M intersect (have a vertex in common); and
ii. Every vertex of G has an edge in M.

Let MATCH be the set of all graphs that have a perfect matching. Let $\overline{\text{MATCH}}$ be the set of graphs that do not have a perfect matching. Let $o(G)$ be the number of components of G that have an odd number of vertices.

Tutte's Theorem: $G \in \text{MATCH}$ if and only if for all subsets S of V, the number of components in $G - S$ (the graph formed by deleting the vertices in S) with an odd number of vertices is at most $|S|$. That is,

$$G \in \text{MATCH} \leftrightarrow \forall S \subseteq V o(G - S) \leq |S|.$$

Which of the following is true?

A. $\text{MATCH} \in NP$ and $\overline{\text{MATCH}} \notin NP$
B. $\overline{\text{MATCH}} \in NP$ and $\text{MATCH} \notin NP$
C. $\text{MATCH} \in NP$ and $\overline{\text{MATCH}} \in NP$
D. $\text{MATCH} \notin P$ and $\overline{\text{MATCH}} \notin P$
E. none of the above

tifr2012 algorithms p-np-npc-nph

Answer key☟

1.8.4 P Np Npc Nph: TIFR CSE 2013 | Part B | Question: 7 top☝

Which of the following is not implied by $P = NP$?

A. 3SAT can be solved in polynomial time.
B. Halting problem can be solved in polynomial time.
C. Factoring can be solved in polynomial time.
D. Graph isomorphism can be solved in polynomial time.
E. Travelling salesman problem can be solved in polynomial time.

tifr2013 algorithms p-np-npc-nph

Answer key☟

1.8.5 P Np Npc Nph: TIFR CSE 2015 | Part B | Question: 13 top☝

Two undirected graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are said to be isomorphic if there exist a bijection $\pi : V_1 \rightarrow V_2$ such that for all $u, v \in V_1, (u, v) \in E_1$ if and only $(\pi(u), \pi(v)) \in E_2$. Consider the following language.

L={(G, H)| G and H are undirected graphs such that a subgraph of G is isomorphic to H}

Then which of the following are true?

(i) $L \in NP$.

(ii) L is NP- hard.

(iii) L is undecidable.

A. Only (i)
B. Only (ii)
C. Only (iii)
D. (i) and (ii)
E. (ii) and (iii)

tifr2015 p-np-npc-nph non-gate

Answer key☟

1.8.6 P Np Npc Nph: TIFR CSE 2017 | Part B | Question: 15 top☝

A multivariate polynomial in n variables with integer coefficients has a binary root if it is possible to assign each variable either 0 or 1, so that the polynomial evaluates to 0. For example, the multivariate polynomial $-2x_1^3 - x_1x_2 + 2$ has the binary root $(x_1 = 1, x_2 = 0)$. Then determining whether a multivariate polynomial, given as the sum of monimials, has a binary root:

A. is trivial: every polynomial has a binary root
B. can be done in polynomial time

C. is NP-hard, but not in NP

D. is in NP, but not in P and not NP-hard

E. is both in NP and NP-hard

tifr2017 algorithms p-np-npc-nph

Answer key

1.8.7 P Np Npc Nph: TIFR CSE 2017 | Part B | Question: 2 top

Consider the following statements:

i. Checking if a given $undirected$ graph has a cycle is in P
ii. Checking if a given $undirected$ graph has a cycle is in NP
iii. Checking if a given $directed$ graph has a cycle is in P
iv. Checking if a given $directed$ graph has a cycle is in NP

Which of the above statements is/are TRUE? Choose from the following options.

A. Only i and ii
B. Only ii and iv
C. Only ii, iii, and iv
D. Only i, ii and iv
E. All of them

tifr2017 algorithms p-np-npc-nph

Answer key

1.8.8 P Np Npc Nph: TIFR CSE 2018 | Part B | Question: 15 top

G respresents an undirected graph and a cycle refers to a simple cycle (no repeated edges or vertices).
Define the following two languages.

$SCYCLE = \{(G, k) \mid G \text{ contains a cycle of length at most k}\}$
and
$LCYCLE = \{(G, k) \mid G \text{ contains a cycle of length at least k}\}$

Which of the following is NOT known to be TRUE (to the best of our current knowledge) ?

A. $SCYCLE \in P$
B. $LCYCLE \in NP$.
C. $LCYCLE \leq_p SCYCLE$ (i.e, there is a polynomial time many-to -one reduction from $LCYCLE$ to $SCYCLE$).
D. $LCYCLE$ is NP-complete.
E. $SCYCLE \leq_p LCYCLE$ (i.e, there is a polynomial time many-to-one reduction from $SCYCLE$ to $LCYCLE$).

tifr2018 algorithms p-np-npc-nph non-gate

Answer key

1.8.9 P Np Npc Nph: TIFR CSE 2019 | Part B | Question: 7 top

A formula is said to be a 3-CF-formula if it is a conjunction (i.e., an AND) of clauses, and each clause has at most 3 literals. Analogously, a formula is said to be a 3-DF-formula if it is a disjunction (i.e., an OR) of clauses of at most 3 literals each.

Define the languages 3-CF-SAT and 3-DF-SAT as follows:

$$\text{3-CF-SAT} = \{\Phi \mid \Phi \text{ is a } \textit{satisfiable} \text{ 3-CF formula}\}$$

$$\text{3-DF-SAT} = \{\Phi \mid \Phi \text{ is a } \textit{satisfiable} \text{ 3-DF formula}\}$$

Which of the following best represents our current knowledge of these languages ?

A. Both 3-CF-SAT and 3-DF-SAT are in NP but only 3-CF-SAT is NP-complete
B. Both 3-CF-SAT and 3-DF-SAT are in NP-complete
C. Both 3-CF-SAT and 3-DF-SAT are in P
D. Both 3-CF-SAT and 3-DF-SAT are in NP but only 3-DF-SAT is NP-complete
E. Neither 3-CF-SAT nor 3-DF-SAT are in P

tifr2019 algorithms p-np-npc-nph

Answer key

1.9 Quick Sort (1) top

1.9.1 Quick Sort: TIFR CSE 2018 | Part B | Question: 7 top

Consider the recursive quicksort algorithm with "random pivoting". That is, in each recursive call, a pivot is chosen uniformly at random from the sub-array being sorted.When this randomized algorithm is applied to an array of size n all whose elements are distinct, what is the probability that the smallest and the largest elements in the array are compared during a run of the algorithm ?

A. $\left(\frac{1}{n}\right)$
B. $\left(\frac{2}{n}\right)$
C. $\Theta\left(\frac{1}{n\log n}\right)$
D. $O\left(\frac{1}{n^2}\right)$
E. $\Theta\left(\frac{1}{n\log^2 n}\right)$

tifr2018 algorithms sorting quick-sort

Answer key

1.10 Recurrence Relation (3) top

1.10.1 Recurrence Relation: TIFR CSE 2014 | Part B | Question: 11 top

Consider the following recurrence relation:

$$T\left(n\right)=\begin{cases} T\left(\frac{n}{k}\right)+T\left(\frac{3n}{4}\right)+n & \text{if } n\geq 2\\ 1 & \text{if } n=1 \end{cases}$$

Which of the following statements is FALSE?

A. $T(n)$ is $O(n^{3/2})$ when $k=3$.
B. $T(n)$ is $O(n\log n)$ when $k=3$.
C. $T(n)$ is $O(n\log n)$ when $k=4$.
D. $T(n)$ is $O(n\log n)$ when $k=5$.
E. $T(n)$ is $O(n)$ when $k=5$.

tifr2014 algorithms recurrence-relation

Answer key

1.10.2 Recurrence Relation: TIFR CSE 2015 | Part B | Question: 1 top

Consider the following recurrence relation:

$$T(n)=\begin{cases} 2T(\lfloor\sqrt{n}\rfloor)+\log n & \text{if } n\geq 2\\ 1 & \text{if } n=1 \end{cases}$$

Which of the following statements is TRUE?

A. $T(n)$ is $O(\log n)$.
B. $T(n)$ is $O(\log n\cdot\log\log n)$ but not $O(\log n)$.
C. $T(n)$ is $O(\log^{3/2} n)$ but not $O(\log n\cdot\log\log n)$.

D. $T(n)$ is $O(\log^2 n)$ but not $O(\log^{3/2} n)$.
E. $T(n)$ is $O(\log^2 n \cdot \log\log n)$ but not $O(\log^2 n)$.

tifr2015 algorithms recurrence-relation time-complexity

Answer key

1.10.3 Recurrence Relation: TIFR CSE 2017 | Part A | Question: 15 top

Let $T(a,b)$ be the function with two arguments (both nonnegative integral powers of 2) defined by the following recurrence:

- $T(a,b) = T\left(\frac{a}{2}, b\right) + T\left(a, \frac{b}{2}\right)$ if $a, b \geq 2$;
- $T(a,1) = T\left(\frac{a}{2}, 1\right)$ if $a \geq 2$;
- $T(1,b) = T\left(1, \frac{b}{2}\right)$ if $b \geq 2$;
- $T(1,1) = 1$.

What is $T(2^r, 2^s)$?

A. rs
B. $r+s$
C. $\begin{pmatrix} 2^r + 2^s \\ 2^r \end{pmatrix}$
D. $\begin{pmatrix} r+s \\ r \end{pmatrix}$
E. 2^{r-s} if $r \geq s$, otherwise 2^{s-r}

tifr2017 algorithms recurrence-relation

Answer key

1.11 Searching (2) top

1.11.1 Searching: TIFR CSE 2010 | Part B | Question: 29 top

Suppose you are given an array A with $2n$ numbers.

The numbers in odd positions are sorted in ascending order, that is, $A[1] \leq A[3] \leq \ldots \leq A[2n-1]$.

The numbers in even positions are sorted in descending order, that is, $A[2] \geq A[4] \geq \ldots \geq A[2n]$.

What is the method you would recommend for determining if a given number is in the array?

A. Sort the array using quick-sort and then use binary search.
B. Merge the sorted lists and perform binary search.
C. Perform a single binary search on the entire array.
D. Perform separate binary searches on the odd positions and the even positions.
E. Search sequentially from the end of the array.

tifr2010 searching

Answer key

1.11.2 Searching: TIFR CSE 2012 | Part B | Question: 11 top

Consider the following three version of the binary search program. Assume that the elements of type T can be compared with each other; also assume that the array is sorted.

```
i, j, k : integer;
a : array [1....N] of T;
x : T;

Program 1 :   i := 1; j := N;
```

```
         repeat
             k := (i + j) div 2;
             if a[k] < x then i := k else j := k
         until (a[k] = x) or (i > j)
Program 2 :   i := 1; j := N;
          repeat
             k := (i + j) div 2;
             if x < a[k] then j := k - 1;
             if a[k] < x then i := k + 1;
          until i > j
Program 3 :=  i := 1; j := N
          repeat
             k := (i + j) div 2;
             if x < a[k] then j := k else i := k + 1
          until i > j
```

A binary search program is called correct provided it terminates with $a[k] = x$ whenever such an element exists, or it terminates with $a\,[k] \neq x$ if there exists no array element with value x. Which of the following statements is correct?

A. Only Program 1 is correct
B. Only Program 2 is correct
C. Only Program 1 and 2 are correct.
D. Both Program 2 and 3 are correct
E. All the three programs are wrong

tifr2012 algorithms searching

Answer key☟

1.12 Shortest Path (1) top☝

1.12.1 Shortest Path: TIFR CSE 2018 | Part B | Question: 9 top☝

Let $G = (V, E)$ be a DIRECTED graph, where each edge e has a positive weight $\omega(e)$, and all vertices can be reached from vertex s. For each vertex v, let $\phi(v)$ be the length of the shortest path from s to v. Let $G' = (V, E)$ be a new weighted graph with the same vertices and edges, but with the edge weight of every edge $e = (u \to v)$ changed to $\omega'(e) = \omega(e) + \phi(v) - \phi(u)$. Let P be a path from s to a vertex v, and let $\omega(P) = \sum_{e \in P} \omega_e$, and $\omega'(P) = \sum_{e \in P} \omega'_e$.
Which of the following options is NOT NECESSARILY TRUE ?

A. If P is a shortest path in G, then P is a shortest path in G'.
B. If P is a shortest path in F', then P is a shortest path in G.
C. If P is a shortest path in G, then $\omega'(P) = 2 \times \omega(P)$.
D. If P is NOT a shortest path in G, then $\omega'(P) < 2 \times \omega(P)$.
E. All of the above options are necessarily TRUE.

tifr2018 graph-algorithms shortest-path

Answer key☟

1.13 Sorting (9) top☝

1.13.1 Sorting: TIFR CSE 2010 | Part B | Question: 23 top☝

Suppose you are given n numbers and you sort them in descending order as follows:
First find the maximum. Remove this element from the list and find the maximum of the remaining elements, remove this element, and so on, until all elements are exhausted. How many comparisons does this method require in the worst case?

A. Linear in n.
B. $O\left(n^2\right)$ but not better.

C. $O(n \log n)$

D. Same as heap sort.

E. $O(n^{1.5})$ but not better.

tifr2010 algorithms time-complexity sorting

Answer key

1.13.2 Sorting: TIFR CSE 2010 | Part B | Question: 27 top

Consider the Insertion Sort procedure given below, which sorts an array L of size $n (\geq 2)$ in ascending order:

```
begin
  for xindex:= 2 to n do
    x := L [xindex];
    j:= xindex - 1;
    while j > 0 and L[j] > x do
      L[j + 1]:= L[j];
      j:= j - 1;
    end {while}
    L [j + 1]:=X;
  end{for}
end
```

It is known that insertion sort makes at most $n(n-1)/2$ comparisons. Which of the following is true?

A. There is no input on which insertion Sort makes $n(n-1)/2$ comparisons.
B. Insertion Sort makes $n(n-1)/2$ comparisons when the input is already sorted in ascending order.
C. Insertion Sort makes $n(n-1)/2$ comparisons only when the input is sorted in descending order.
D. There are more than one input orderings where insertion sort makes $n(n-1)/2$ comparisons.
E. Insertion Sort makes $n(n-1)/2$ comparisons whenever all the elements of L are not distinct.

tifr2010 algorithms sorting

Answer key

1.13.3 Sorting: TIFR CSE 2011 | Part B | Question: 21 top

Let $S = \{x_1, \ldots, x_n\}$ be a set of n numbers. Consider the problem of storing the elements of S in an array $A[1 \ldots n]$ such that the following min-heap property is maintained for all $2 \leq i \leq n: A[\lfloor i/2 \rfloor] \leq A[i]$. (Note that $\lfloor x \rfloor$ is the largest integer that is at most x). Which of the following statements is TRUE?

A. This problem can be solved in $O(\log n)$ time.
B. This problem can be solved in $O(n)$ time but not in $O(\log n)$ time.
C. This problem can be solved in $O(n \log n)$ time but not in $O(n)$ time.
D. This problem can be solved in $O\left(n^2\right)$ time but not in $O(n \log n)$ time.
E. None of the above.

tifr2011 algorithms sorting

Answer key

1.13.4 Sorting: TIFR CSE 2011 | Part B | Question: 31 top

Given a set of $n = 2^k$ distinct numbers, we would like to determine the smallest and the second smallest using comparisons. Which of the following statements is TRUE?

A. Both these elements can be determined using $2k$ comparisons.
B. Both these elements can be determined using $n-2$ comparisons.
C. Both these elements can be determined using $n+k-2$ comparisons.

D. $2n-3$ comparisons are necessary to determine these two elements.
E. nk comparisons are necessary to determine these two elements.

tifr2011 algorithms sorting

Answer key☟

1.13.5 Sorting: TIFR CSE 2011 | Part B | Question: 39 top☝

The first n cells of an array L contain positive integers sorted in decreasing order, and the remaining $m-n$ cells all contain 0. Then, given an integer x, in how many comparisons can one find the position of x in L?

A. At least n comparisons are necessary in the worst case.
B. At least $\log m$ comparisons are necessary in the worst case.
C. $O(\log(m-n))$ comparisons suffice.
D. $O(\log n)$ comparisons suffice.
E. $O(\log(m/n))$ comparisons suffice.

tifr2011 algorithms sorting

Answer key☟

1.13.6 Sorting: TIFR CSE 2012 | Part B | Question: 13 top☝

An array A contains n integers. We wish to sort A in ascending order. We are told that initially no element of A is more than a distance k away from its final position in the sorted list. Assume that n and k are large and k is much smaller than n. Which of the following is true for the worst case complexity of sorting A?

A. A can be sorted with constant $\cdot kn$ comparison but not with fewer comparisons.
B. A cannot be sorted with less than constant $\cdot n \log n$ comparisons.
C. A can be sorted with constant $\cdot n$ comparisons.
D. A can be sorted with constant $\cdot n \log k$ comparisons but not with fewer comparisons.
E. A can be sorted with constant $\cdot k^2 n$ comparisons but not fewer.

tifr2012 algorithms sorting

Answer key☟

1.13.7 Sorting: TIFR CSE 2012 | Part B | Question: 14 top☝

Consider the quick sort algorithm on a set of n numbers, where in every recursive subroutine of the algorithm, the algorithm chooses the median of that set as the pivot. Then which of the following statements is TRUE?

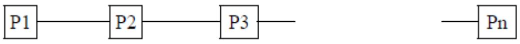
A. The running time of the algorithm is $\Theta(n)$.
B. The running time of the algorithm is $\Theta(n \log n)$.
C. The running time of the algorithm is $\Theta(n^{1.5})$.
D. The running time of the algorithm is $\Theta(n^2)$.
E. None of the above.

tifr2012 algorithms sorting

Answer key☟

1.13.8 Sorting: TIFR CSE 2013 | Part B | Question: 20 top

Suppose n processors are connected in a linear array as shown below. Each processor has a number. The processors need to exchange numbers so that the numbers eventually appear in ascending order (the processor P1 should have the minimum value and the the processor Pn should have the maximum value).



The algorithm to be employed is the following. Odd numbered processors and even numbered processors are activated alternate steps; assume that in the first step all the even numbered processors are activated. When a processor is activated, the number it holds is compared with the number held by its right-hand neighbour (if one exists) and the smaller of the two numbers is retained by the activated processor and the bigger stored in its right hand neighbour.
How long does it take for the processors to sort the values?

A. $n \log n$ steps
B. n^2 steps
C. n steps
D. $n^{1.5}$ steps
E. The algorithm is not guaranteed to sort

tifr2013 algorithms sorting

Answer key

1.13.9 Sorting: TIFR CSE 2017 | Part B | Question: 7 top

An array of n distinct elements is said to be un-sorted if for every index i such that $2 \leq i \leq n-1$, either $A[i] > \max\{A[i-1], A[i+1]\}$, or $A[i] < \min\{A[i-1], A[i+1]\}$. What is the time-complexity of the fastest algorithm that takes as input a sorted array A with n distinct elements, and un-sorts A?

A. $O(n \text{ log } n)$ but not $O(n)$
B. $O(n)$ but not $O(\sqrt{n})$
C. $O(\sqrt{n})$ but not $O(\log n)$
D. $O(\log n)$ but not $O(1)$
E. $O(1)$

tifr2017 algorithms sorting

Answer key

1.14 Spanning Tree (5) top

1.14.1 Spanning Tree: TIFR CSE 2011 | Part B | Question: 35 top

Let G be a connected simple graph (no self-loops or parallel edges) on $n \geq 3$ vertices, with distinct edge weights. Let $e_1, e_2, \ldots, e_m$ be an ordering of the edges in decreasing order of weight. Which of the following statements is FALSE?

A. The edge e_1 has to be present in every maximum weight spanning tree.
B. Both e_1 and e_2 have to be present in every maximum weight spanning tree.
C. The edge e_m has to be present in every minimum weight spanning tree.
D. The edge e_m is never present in any maximum weight spanning tree.
E. G has a unique maximum weight spanning tree.

tifr2011 algorithms graph-algorithms spanning-tree

Answer key

1.14.2 Spanning Tree: TIFR CSE 2013 | Part B | Question: 17 top

In a connected weighted graph with n vertices, all the edges have distinct positive integer weights. Then, the maximum number of minimum weight spanning trees in the graph is

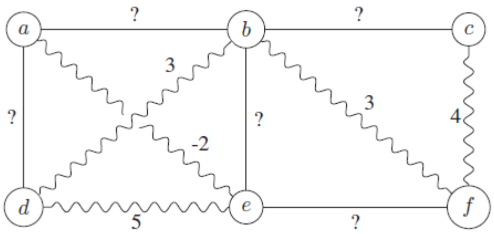
A. 1
B. n
C. equal to number of edges in the graph.
D. equal to maximum weight of an edge of the graph.
E. n^{n-2}

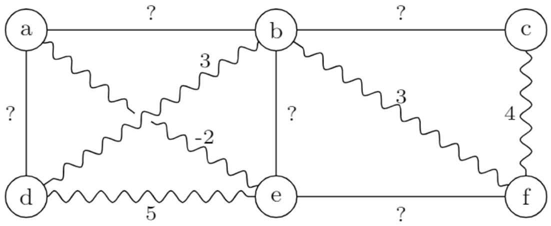
tifr2013 spanning-tree

Answer key☟

1.14.3 Spanning Tree: TIFR CSE 2014 | Part B | Question: 4 top☝

Consider the following undirected graph with some edge costs missing.





Suppose the wavy edges form a Minimum Cost Spanning Tree for G. Then, which of the following inequalities NEED NOT hold?

A. cost$(a, b) \geq 6$.
B. cost$(b, e) \geq 5$.
C. cost$(e, f) \geq 5$.
D. cost$(a, d) \geq 4$.
E. cost$(b, c) \geq 4$.

tifr2014 algorithms graph-algorithms spanning-tree

Answer key☟

1.14.4 Spanning Tree: TIFR CSE 2014 | Part B | Question: 5 top☝

Let $G = (V, E)$ be an undirected connected simple (i.e., no parallel edges or self-loops) graph with the weight function $w : E \to \mathbb{R}$ on its edge set. Let $w(e_1) < w(e_2) < \cdots < w(e_m)$, where $E = \{e_1, e_2, \ldots, e_m\}$. Suppose T is a minimum spanning tree of G. Which of the following statements is FALSE?

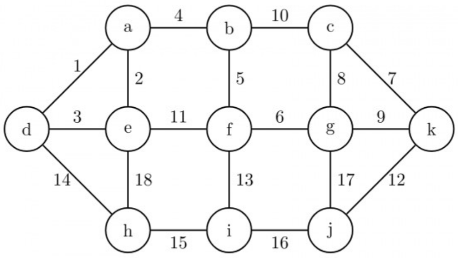
A. The tree T has to contain the edge e_1.
B. The tree T has to contain the edge e_2.
C. The minimum weight edge incident on each vertex has to be present in T.
D. T is the unique minimum spanning tree in G.
E. If we replace each edge weight $w_i = w(e_i)$ by its square w_i^2 , then T must still be a minimum spanning tree of this new instance.

tifr2014 algorithms spanning-tree

Answer key☟

1.14.5 Spanning Tree: TIFR CSE 2015 | Part B | Question: 2 top☝

Consider the following undirected connected graph G with weights on its edges as given in the figure below. A minimum spanning tree is a spanning tree of least weight and a maximum spanning tree is one with largest weight. A second best minimum spanning tree whose weight is the smallest among all spanning trees that are not minimum spanning trees in G.



Which of the following statements is TRUE in the above graph? (Note that all the edge weights are distinct in the above graph)

A. There is more than one minimum spanning tree and similarly, there is more than one maximum spanning tree here.
B. There is a unique minimum spanning tree, however there is more than one maximum spanning tree here.
C. There is more than one minimum spanning tree, however there is a unique maximum spanning tree here.
D. There is more than one minimum spanning tree and similarly, there is more than one second-best minimum spanning tree here.
E. There is unique minimum spanning tree, however there is more than one second-best minimum spanning tree here.

tifr2015 spanning-tree algorithms graph-algorithms

Answer key☟

1.15 Time Complexity (4) top☝

1.15.1 Time Complexity: TIFR CSE 2013 | Part B | Question: 12 top☝

It takes $O(n)$ time to find the median in a list of n elements, which are not necessarily in sorted order while it takes only $O(1)$ time to find the median in a list of n sorted elements. How much time does it take to find the median of $2n$ elements which are given as two lists of n sorted elements each?

A. $O(1)$
B. $O(\log n)$ but not $O(1)$
C. $O(\sqrt{n})$ but not $O(\log n)$
D. $O(n)$ but not $O(\sqrt{n})$
E. $O(n \log n)$ but not $O(n)$

tifr2013 algorithms time-complexity

Answer key☟

1.15.2 Time Complexity: TIFR CSE 2013 | Part B | Question: 18 top☝

Let S be a set of numbers. For $x \in S$, the rank of x is the number of elements in S that are less than or equal to x. The procedure $\text{Select}(S, r)$ takes a set S of numbers and a rank $r\ (1 \leq r \leq |S|)$ and returns the element in S of rank r. The procedure $\text{MultiSelect}(S, R)$ takes a set of numbers S and a list of ranks $R = \{r_1 < r_2 < \ldots < r_k\}$, and returns the list $\{x_1 < x_2 < \ldots < x_k\}$ of elements of S, such that the rank of x_i is r_i. Suppose there is an implementation for $\text{Select}(S, r)$ that uses at most (constant $\cdot |S|$) binary comparisons between elements of S. The minimum number of comparisons needed to implement MultiSelect(S, R) is

A. constant $\cdot$ $|S|\log|S|$
B. constant $\cdot$ $|S|$
C. constant $\cdot$ $|S||R|$
D. constant $\cdot$ $|R|\log|S|$
E. constant $\cdot$ $|S|(1+\log|R|)$

tifr2013 algorithms time-complexity

Answer key

1.15.3 Time Complexity: TIFR CSE 2014 | Part B | Question: 7 top

Which of the following statements is TRUE for all sufficiently large n?

A. $(\log n)^{\log\log n} < 2^{\sqrt{\log n}} < n^{1/4}$

B. $2^{\sqrt{\log n}} < n^{1/4} < (\log n)^{\log\log n}$

C. $n^{1/4} < (\log n)^{\log\log n} < 2^{\sqrt{\log n}}$

D. $(\log n)^{\log\log n} < n^{1/4} < 2^{\sqrt{\log n}}$

E. $2^{\sqrt{\log n}} < (\log n)^{\log\log n} < n^{1/4}$

tifr2014 algorithms time-complexity

Answer key

1.15.4 Time Complexity: TIFR CSE 2015 | Part B | Question: 3 top

Consider the following code fragment in the C programming language when run on a non-negative integer n.

```
int f (int n)
{
    if (n==0 || n==1)
      return 1;
    else
      return f (n - 1) + f(n - 2);
}
```

Assuming a typical implementation of the language, what is the running time of this algorithm and how does it compare to the optimal running time for this problem?

A. This algorithm runs in polynomial time in n but the optimal running time is exponential in n.
B. This algorithm runs in exponential time in n and the optimal running time is exponential in n.
C. This algorithm runs in exponential time in n but the optimal running time is polynomial in n.
D. This algorithm runs in polynomial time in n and the optimal running time is polynomial in n.
E. The algorithm does not terminate.

tifr2015 time-complexity

Answer key

Answer Keys

1.1.1	B	1.2.1	D	1.2.2	C	1.2.3	D	1.2.4	A

1.2.5	B
1.3.1	A
1.5.2	C
1.6.3	C
1.8.3	C
1.8.8	C
1.10.3	D
1.13.2	D
1.13.7	B
1.14.3	A
1.15.3	A

1.2.6	A
1.4.1	C
1.5.3	D
1.7.1	B
1.8.4	B
1.8.9	A
1.11.1	D
1.13.3	B
1.13.8	C
1.14.4	E
1.15.4	C

1.2.7	C
1.4.2	D
1.5.4	E
1.7.2	D
1.8.5	D
1.9.1	B
1.11.2	E
1.13.4	C
1.13.9	B
1.14.5	E

1.2.8	D
1.4.3	D
1.6.1	C
1.8.1	E
1.8.6	E
1.10.1	B
1.12.1	E
1.13.5	D
1.14.1	D
1.15.1	B

1.2.9	E
1.5.1	B
1.6.2	B
1.8.2	A
1.8.7	E
1.10.2	B
1.13.1	B
1.13.6	D
1.14.2	A
1.15.2	E

2 Compiler Design (3)

2.1 Parsing (3) top☝

2.1.1 Parsing: TIFR CSE 2012 | Part B | Question: 17 top☝

Which of the following correctly describes $LR(k)$ parsing?

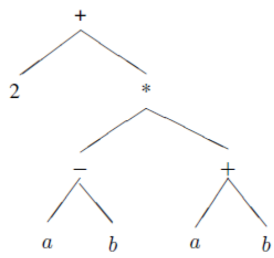
A. The input string is alternately scanned left to right and right to left with k reversals.
B. Input string is scanned once left to right with rightmost derivation and k symbol look-ahead.
C. $LR(k)$ grammers are expressively as powerful as context-free grammers.
D. Parser makes k left-to-right passes over input string.
E. Input string is scanned from left to right once with k symbol to the right as look-ahead to give left-most derivation.

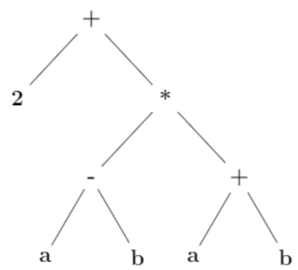
tifr2012 compiler-design parsing

Answer key☟

2.1.2 Parsing: TIFR CSE 2012 | Part B | Question: 8 top☝

Consider the parse tree





Assume that $*$ has higher precedence than $+$, $-$ and operators associate right to left (i.e $(a+b+c = (a+(b+c)))$. Consider

i. $2+a-b$
ii. $2+a-b*a+b$
iii. $(2+((a-b)*(a+b)))$
iv. $2+(a-b)*(a+b)$

The parse tree corresponds to

A. Expression (i)
B. Expression (ii)
C. Expression (iv) only
D. Expression (ii), (iii), and (iv)
E. Expression (iii) and (iv) only

tifr2012 compiler-design parsing

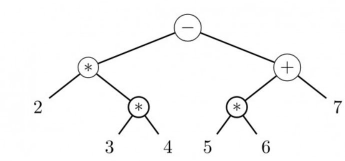
Answer key☟

2.1.3 Parsing: TIFR CSE 2015 | Part B | Question: 15 top☝

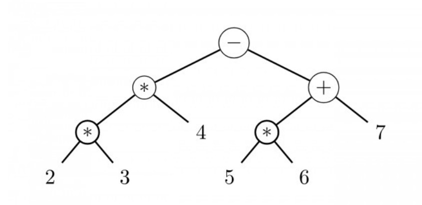
Consider the following grammar (the start symbol is E) for generating expressions.

- $E \to T - E \mid T + E \mid T$
- $T \to T * F \mid F$
- $F \to 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$

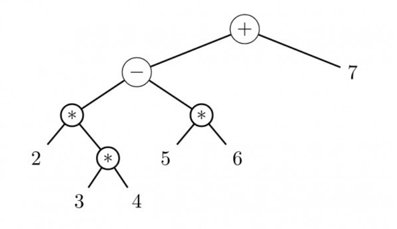
With respect to this grammar, which of the following trees is the valid evaluation tree for the expression $2 * 3 * 4 - 5 * 6 + 7$?



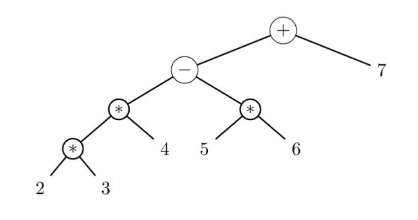
A.



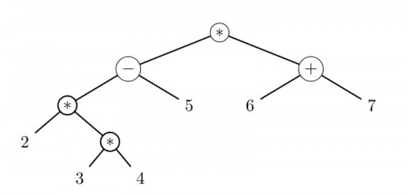
B.



C.



D.



E.

tifr2015 parsing

Answer key☟

Answer Keys

2.1.1	B	2.1.2	E	2.1.3	B

3 Computer Networks (1)

3.1 Network Security (1) top☝

3.1.1 Network Security: TIFR CSE 2011 | Part B | Question: 36 top☝

Consider malware programs. Which of the following is true?

A. A worm is a parasite.
B. A virus cannot affect a linux operating system.
C. A trojan can be in the payload of only a worm.
D. A worm and virus are self replicating programs.
E. There is no difference between a virus and a worm.

tifr2011 computer-networks network-security

Answer key☟

Answer Keys

3.1.1	D

4 Databases (2)

4.1 Relational Algebra (2) top

4.1.1 Relational Algebra: TIFR CSE 2010 | Part B | Question: 33 top

In a relational database there are three relations:

- $Customers = C(\text{CName})$
- $Shops = S(\text{SName})$
- $Buys = B(\text{CName, SName})$

Then the Relational Algebra expression (Π is the projection operator).

$$C - \Pi_{\text{CName}}((C \times S) - B)$$

returns the names of

A. Customers who buy from at least one shop.
B. Customers who buy from at least two shops.
C. Customers who buy from all shops.
D. Customers who do not buy buy anything at all.
E. None of the above.

tifr2010 databases relational-algebra

Answer key

4.1.2 Relational Algebra: TIFR CSE 2013 | Part B | Question: 19 top

In a relational database there are three relations:

- $Customers = C(\text{CName})$,
- $Shops = S(\text{SName})$,
- $Buys = B(\text{CName, SName})$.

Which of the following relational algebra expressions returns the names of shops that have no customers at all? [Here Π is the projection operator.]

A. $\Pi_{\text{SName}} B$
B. $S - B$
C. $S - \Pi_{\text{SName}} B$
D. $S - \Pi_{\text{SName}}((C \times S) - B)$
E. None of the above

tifr2013 databases relational-algebra

Answer key

Answer Keys

4.1.1	C	4.1.2	C

5 Digital Logic (11)

5.0.1 TIFR CSE 2020 | Part B | Question: 1 top

Consider the following Boolean valued function on n Boolean variables: $f(x_1,\ldots,x_n)=x_1+\cdots+x_n (\text{mod } 2)$, where addition is over integers, mapping $\textbf{'FALSE'}$ to 0 and $\textbf{'TRUE'}$ to 1. Consider Boolean circuits (with no feedback) that use only logical $\textbf{AND}$ and $\textbf{OR}$ gates, and where each gate has two input bits, each of which is either an input bit of f or the output bit of some other gate of the circuit. The circuit has a distinguished gate whose value is the output of the circuit. The minimum size of such a circuit computing f (asymptotically in n) is :

A. $2^{o(\log n)}$
B. n^c, for some fixed constant c
C. $n^{\omega(1)}$, but $n^{O(\log n)}$
D. $2^{\Theta(n)}$
E. None of the others

tifr2020

5.1 Boolean Algebra (4) top

5.1.1 Boolean Algebra: TIFR CSE 2010 | Part B | Question: 21 top

For $x \in \{0,1\}$, let $\neg x$ denote the negation of x, that is

$$\neg\, x = \begin{cases} 1 & \text{iff } x = 0 \\ 0 & \text{iff } x = 1 \end{cases}$$

.

If $x \in \{0,1\}^n$, then $\neg\, x$ denotes the component wise negation of x; that is:

$$(\neg\, x)_i = (\neg\, x_i \mid i \in [1..n])$$

Consider a circuit C, computing a function $f : \{0,1\}^n \rightarrow \{0,1\}$ using **AND** $(\wedge)$, **OR** $(\vee)$,and **NOT** $(\neg)$ gates. Let D be the circuit obtained from C by replacing each **AND** gate by an **OR** gate and replacing each **OR** gate by an **AND**. Suppose D computes the function g. Which of the following is true for all inputs x?

A. $g(x) = \neg\, f(x)$
B. $g(x) = f(x) \wedge f(\neg x)$
C. $g(x) = f(x) \vee f(\neg x)$
D. $g(x) = \neg f(\neg x)$
E. None of the above.

tifr2010 digital-logic boolean-algebra

Answer key

5.1.2 Boolean Algebra: TIFR CSE 2014 | Part B | Question: 17 top

Let $f : \{0,1\}^n \rightarrow \{0,1\}$ be a boolean function computed by a logical circuit comprising just binary AND and binary OR gates (assume that the circuit does not have any feedback). Let PARITY : $\{0,1\}^n \rightarrow \{0,1\}$ be the boolean function that outputs 1 if the total number of input bits set to 1 is odd. Similarly, let MAJORITY be the boolean function that outputs 1 if the number of input bits that are set to 1 is at least as large as the number of input bits that are set to 0. Then, which of the following is NOT possible?

A. $f(0,0,\ldots,0) = f(1,1,\ldots,1) = 0$.
B. $f(0,0,\ldots,0) = f(1,1\ldots,1) = 1$
C. f is the MAJORITY function.

D. f is the PARITY function.
E. f outputs 1 at exactly one assignment of the input bits.

tifr2014 boolean-algebra

Answer key☟

5.1.3 Boolean Algebra: TIFR CSE 2016 | Part B | Question: 1 top☝

A Boolean formula is said to be a *tautology* if it evaluates to TRUE for all assignments to its variables. Which one of the following is NOT a tautology?

A. $((p \vee q) \wedge (r \vee s)) \Rightarrow ((p \wedge r) \vee q \vee s)$
B. $((p \vee q) \wedge (r \vee s)) \Rightarrow (q \vee s)$
C. $((p \vee q) \wedge (r \vee s)) \Rightarrow (r \vee q \vee s)$
D. $((p \vee q) \wedge (r \vee s)) \Rightarrow (p \vee q \vee s)$
E. $((p \vee q) \wedge (r \vee s)) \Rightarrow (p \vee q)$

tifr2016 boolean-algebra

Answer key☟

5.1.4 Boolean Algebra: TIFR CSE 2020 | Part B | Question: 15 top☝

Suppose $X_{1a}, X_{1b}, X_{2a}, X_{2b}, \dots, X_{5a}, X_{5b}$ are ten Boolean variables each of which can take the value TRUE or FLASE. Recall the Boolean XOR $X \oplus Y := (X \wedge \neg Y) \vee (\neg X \wedge Y)$. Define the Boolean logic formulas

$$F := (X_{1a} \vee X_{1b}) \wedge (X_{2a} \vee X_{2b}) \wedge (X_{3a} \vee X_{3b}) \wedge (X_{4a} \vee X_{4b}) \wedge (X_{5a} \vee X_{5b}),$$

$$G_i := (X_{i,a} \oplus X_{i+1,a}) \vee (X_{i,b} \oplus X_{i+1,b}), \quad 1 \leq i \leq 4$$

$$G_5 := (X_{5a} \oplus X_{1a}) \vee (X_{5b} \oplus X_{1b}),$$

$$H := F \wedge G_1 \wedge G_2 \wedge G_3 \wedge G_4 \wedge G_5.$$

A truth assignment to the ten Boolean variables $X_{ia}, X_{ib}, \ 1 \leq i \leq 5$ is said to be a satisfying assignment if H takes the value TRUE for example,

$$(X_{1a}, X_{1b}, X_{2a}, X_{2b}, \dots, X_{5a}, X_{5b}) = (F, T, T, F, F, T, T, T, T, F)$$

is a satisfying assignment,

$$(X_{1a}, X_{1b}, X_{2a}, X_{2b}, \dots, X_{5a}, X_{5b}) = (F, T, T, T, F, T, T, T, T, F)$$

is another satisfying assignment, while

$$(X_{1a}, X_{1b}, X_{2a}, X_{2b}, \dots, X_{5a}, X_{5b}) = (F, T, T, F, F, T, T, F, T, F)$$

is not a satisfying assignment.

How many satisfying assignments does H have?

A. 20 B. 30 C. 32 D. 160 E. 1024

tifr2020 digital-logic boolean-algebra

Answer key☟

5.2 Canonical Normal Form (1) top

5.2.1 Canonical Normal Form: TIFR CSE 2015 | Part B | Question: 9 top

A Boolean expression is an expression made out of propositional letters (such as p, q, r) and operators $\wedge$, $\vee$ and $\neg$; e.g. $p \wedge \neg(q \vee \neg r)$. An expression is said to be in sum of product form (also called disjunctive normal form) if all $\neg$ occur just before letters and no $\vee$ occurs in scope of $\wedge$; e.g. $(p \wedge \neg q) \vee (\neg p \wedge q)$. The expression is said to be in product of sum form (also called conjunctive normal form) if all negations occur just before letters and no $\wedge$ occurs in the scope of $\vee$; e.g. $(p \vee \neg q) \wedge (\neg p \vee q)$. Which of the following is not correct?

A. Every Boolean expression is equivalent to an expression in sum of product form.
B. Every Boolean expression is equivalent to an expression in product of sum form.
C. Every Boolean expression is equivalent to an expression without $\vee$ operator.
D. Every Boolean expression is equivalent to an expression without $\wedge$ operator.
E. Every Boolean expression is equivalent to an expression without $\neg$ operator.

tifr2015 canonical-normal-form

Answer key

5.3 Digital Circuits (1) top

5.3.1 Digital Circuits: TIFR CSE 2015 | Part A | Question: 4 top

The Boolean function obtained by adding an inverter to each and every input of an AND gate is:

A. OR B. XOR C. $NAND$ D. NOR E. None of the above

tifr2015 digital-logic digital-circuits

Answer key

5.4 Gray Code (1) top

5.4.1 Gray Code: TIFR CSE 2017 | Part B | Question: 8 top

For any natural number n, an ordering of all binary strings of length n is a Gray code if it starts with 0^n, and any successive strings in the ordering differ in exactly one bit (the first and last string must also differ by one bit). Thus, for $n = 3$, the ordering (000, 100, 101, 111, 110, 010, 011, 001) is a Gray code. Which of the following must be TRUE for all Gray codes over strings of length n?

A. the number of possible Gray codes is even
B. the number of possible Gray codes is odd
C. In any Gray code, if two strings are separated by k other strings in the ordering, then they must differ in exactly $k + 1$ bits
D. In any Gray code, if two strings are separated by k other strings in the ordering, then they must differ in exactly k bits
E. none of the above

tifr2017 digital-logic binary-codes gray-code

Answer key

5.5 Number Representation (2) top

5.5.1 Number Representation: TIFR CSE 2011 | Part A | Question: 16 top

A variable that takes thirteen possible values can be communicated using?

A. Thirteen bits.
B. Three bits.
C. $\log_2 13$ bits.
D. Four bits.
E. None of the above.

tifr2011 number-representation

Answer key

5.5.2 Number Representation: TIFR CSE 2019 | Part B | Question: 1 top

Which of the following decimal numbers can be exactly represented in binary notation with a finite number of bits ?

A. 0.1
B. 0.2
C. 0.4
D. 0.5
E. All the above

tifr2019 digital-logic number-representation

Answer key

5.6 Number System (1) top

5.6.1 Number System: TIFR CSE 2020 | Part B | Question: 3 top

Consider the (decimal) number 182, whose binary representation is 10110110. How many positive integers are there in the following set?

$$\{n \in \mathbb{N} : n \leq 182 \text{ and n has } \textit{exactly four} \text{ ones in its binary representation}\}$$

A. 91
B. 70
C. 54
D. 35
E. 27

tifr2020 digital-logic number-system number-representation

Answer key

Answer Keys

5.0.1	E	5.1.1	D	5.1.2	D	5.1.3	B	5.1.4	B
5.2.1	E	5.3.1	D	5.4.1	A	5.5.1	D	5.5.2	D
5.6.1	C								

6 Discrete Mathematics: Combinatory (16)

6.0.1 TIFR CSE 2011 | Part A | Question: 2 top

In how many ways can the letters of the word ABACUS be rearranged such that the vowels always appear together?

A. $\frac{(6+3)!}{2!}$ B. $\frac{6!}{2!}$ C. $\frac{3!3!}{2!}$ D. $\frac{4!3!}{2!}$ E. None of the above

tifr2011 combinatory

Answer key

6.0.2 TIFR CSE 2012 | Part A | Question: 10 top

In how many different ways can r elements be picked from a set of n elements if

i. Repetition is not allowed and the order of picking matters?
ii. Repetition is allowed and the order of picking does not matter?

A. $\frac{n!}{(n-r)!}$ and $\frac{(n+r-1)!}{r!(n-1)!}$, respectively.
B. $\frac{n!}{(n-r)!}$ and $\frac{n!}{r!(n-1)!}$, respectively.
C. $\frac{n!}{r!(n-r)!}$ and $\frac{(n-r+1)!}{r!(n-1)!}$, respectively.
D. $\frac{n!}{r!(n-r)!}$ and $\frac{n!}{(n-r)!}$, respectively.
E. $\frac{n!}{r!}$ and $\frac{r!}{n!}$, respectively.

tifr2012 combinatory discrete-mathematics normal

Answer key

6.0.3 TIFR CSE 2015 | Part A | Question: 7 top

A 1×1 chessboard has one square, a 2×2 chessboard has five squares. Continuing along this fashion, what is the number of squares on the regular 8×8 chessboard?

A. 64 B. 65 C. 204 D. 144 E. 256

tifr2015 combinatory

Answer key

6.0.4 TIFR CSE 2017 | Part A | Question: 6 top

How many distinct words can be formed by permuting the letters of the word $ABRACADABRA$?

A. $\frac{11!}{5!\,2!\,2!}$ B. $\frac{11!}{5!\,4!}$ C. $11!\,5!\,2!\,2!$ D. $11!\,5!\,4!$ E. $11!$

tifr2017 combinatory discrete-mathematics easy

Answer key

6.0.5 TIFR CSE 2016 | Part A | Question: 15 top

In a tournament with 7 teams, each team plays one match with every other team. For each match, the team earns two points if it wins, one point if it ties, and no points if it loses. At the end of all matches, the teams are ordered in the descending order of their total points (the order among the teams with the same total are determined by a whimsical tournament referee). The first three teams in this ordering are then chosen to play in the next round. What is the minimum total number of points a team must earn in order to be guaranteed a place in the next round?

A. 13　B. 12　C. 11　D. 10　E. 9

tifr2016 combinatory discrete-mathematics normal

Answer key

6.0.6 TIFR CSE 2019 | Part B | Question: 13 top

A row of 10 houses has to be painted using the colours red, blue, and green so that each house is a single colour, and any house that is immediately to the right of a red or a blue house must be green. How many ways are there to paint the houses?

A. 199　B. 683　C. 1365　D. $3^{10} - 2^{10}$　E. 3^{10}

tifr2019 engineering-mathematics discrete-mathematics combinatory

Answer key

6.1 Balls In Bins (4) top

6.1.1 Balls In Bins: TIFR CSE 2012 | Part A | Question: 7 top

It is required to divide the $2n$ members of a club into n disjoint teams of 2 members each. The teams are not labelled. The number of ways in which this can be done is:

A. $\frac{(2n)!}{2^n}$　B. $\frac{(2n)!}{n!}$　C. $\frac{(2n)!}{2^n.n!}$　D. $\frac{n!}{2}$　E. None of the above

tifr2012 combinatory balls-in-bins

Answer key

6.1.2 Balls In Bins: TIFR CSE 2013 | Part A | Question: 9 top

There are n kingdoms and $2n$ champions. Each kingdom gets 2 champions. The number of ways in which this can be done is:

A. $\frac{(2n)!}{2^n}$　B. $\frac{(2n)!}{n!}$　C. $\frac{(2n)!}{2^n.n!}$　D. $\frac{n!}{2}$　E. None of the above

tifr2013 combinatory discrete-mathematics normal balls-in-bins

Answer key

6.1.3 Balls In Bins: TIFR CSE 2015 | Part A | Question: 8 top

There is a set of $2n$ people: n male and n female. A good party is one with equal number of males and females (including the one where none are invited). The total number of good parties is.

A. 2^n　B. n^2　C. $\binom{n}{\lfloor n/2 \rfloor}^2$　D. $\binom{2n}{n}$　E. None of the above

tifr2015 combinatory discrete-mathematics normal balls-in-bins

Answer key

6.1.4 Balls In Bins: TIFR CSE 2017 | Part A | Question: 5 top

How many distinct ways are there to split 50 identical coins among three people so that each person gets at least 5 coins?

A. 3^{35}　B. $3^{50} - 2^{50}$

C. $\binom{35}{2}$　D. $\binom{50}{15} \cdot 3^{35}$

E. $\binom{37}{2}$

tifr2017 combinatory discrete-mathematics normal balls-in-bins

Answer key

6.2 Generating Functions (1) top

6.2.1 Generating Functions: TIFR CSE 2010 | Part A | Question: 12 top

The coefficient of x^3 in the expansion of $(1+x)^3(2+x^2)^{10}$ is.

A. 2^{14}
B. 31
C. $\binom{3}{3}+\binom{10}{1}$
D. $\binom{3}{3}+2\binom{10}{1}$
E. $\binom{3}{3}\binom{10}{1}2^9$

tifr2010 generating-functions

Answer key

6.3 Modular Arithmetic (1) top

6.3.1 Modular Arithmetic: TIFR CSE 2018 | Part B | Question: 1 top

What is the remainder when 4444^{4444} is divided by 9?

A. 1 B. 2 C. 5 D. 7 E. 8

tifr2018 modular-arithmetic combinatory

Answer key

6.4 Pigeonhole Principle (2) top

6.4.1 Pigeonhole Principle: TIFR CSE 2014 | Part A | Question: 5 top

The rules for the University of Bombay five-a-side cricket competition specify that the members of each team must have birthdays in the same month. What is the minimum number of mathematics students needed to be enrolled in the department to guarantee that they can raise a team of students?

A. 23 B. 91 C. 60 D. 49 E. None of the above

tifr2014 combinatory discrete-mathematics normal pigeonhole-principle

Answer key

6.4.2 Pigeonhole Principle: TIFR CSE 2018 | Part A | Question: 6 top

What is the minimum number of students needed in a class to guarantee that there are at least 6 students whose birthdays fall in the same month ?

A. 6 B. 23 C. 61 D. 72 E. 91

tifr2018 pigeonhole-principle combinatory

Answer key

6.5 Recurrence Relation (2) top

6.5.1 Recurrence Relation: TIFR CSE 2014 | Part A | Question: 3 top

The Fibonacci sequence is defined as follows: $F_0=0, F_1=1,$ and for all integers $n\geq 2, F_n=F_{n-1}+F_{n-2}$. Then which of the following statements is FALSE?

A. $F_{n+2}=1+\sum_{i=0}^{n}F_i$ for any integer $n\geq 0$
B. $F_{n+2}\geq \emptyset^n$ for any integer $n\geq 0$, where $\emptyset=\left(\sqrt{5}+1\right)/2$ is the positive root of $x^2-x-1=0$.

C. F_{3n} is even, for every integer $n \geq 0$.
D. F_{4n} is a multiple of 3, for every integer $n \geq 0$.
E. F_{5n} is a multiple of 4, for every integer $n \geq 0$.

tifr2014 recurrence-relation easy

Answer key

6.5.2 Recurrence Relation: TIFR CSE 2017 | Part A | Question: 7 top

Consider the sequence $S_0, S_1, S_2, \dots$ defined as follows: $S_0 = 0,\ S_1 = 1$ and $S_n = 2S_{n-1} + S_{n-2}$ for $n \geq 2$. Which of the following statements is FALSE?

A. for every $n \geq 1$, S_{2n} is even
B. for every $n \geq 1$, S_{2n+1} is odd
C. for every $n \geq 1$, S_{3n} is multiple of 3
D. for every $n \geq 1$, S_{4n} is multiple of 6
E. none of the above

tifr2017 recurrence-relation

Answer key

Answer Keys

6.0.1	D	6.0.2	A	6.0.3	C	6.0.4	A	6.0.5	D
6.0.6	C	6.1.1	C	6.1.2	A	6.1.3	D	6.1.4	E
6.2.1	A	6.3.1	D	6.4.1	D	6.4.2	C	6.5.1	E
6.5.2	C								

7 Discrete Mathematics: Graph Theory (17)

7.1 Counting (1) top

7.1.1 Counting: TIFR CSE 2017 | Part B | Question: 12 top

An undirected graph is complete if there is an edge between every pair of vertices. Given a complete undirected graph on n vertices, in how many ways can you choose a direction for the edges so that there are no directed cycles?

A. n
B. $\frac{n(n-1)}{2}$
C. $n!$
D. 2^n
E. $2^m, \text{ where } m = \frac{n(n-1)}{2}$

tifr2017 graph-theory counting

Answer key

7.2 Degree Of Graph (3) top

7.2.1 Degree Of Graph: TIFR CSE 2010 | Part B | Question: 36 top

In a directed graph, every vertex has exactly seven edges coming in. What can one always say about the number of edges going out of its vertices?

A. Exactly seven edges leave every vertex.
B. Exactly seven edges leave some vertex.
C. Some vertex has at least seven edges leaving it.
D. The number of edges coming out of vertex is odd.
E. None of the above.

tifr2010 graph-theory degree-of-graph

Answer key

7.2.2 Degree Of Graph: TIFR CSE 2012 | Part B | Question: 2 top

In a graph, the degree of a vertex is the number of edges incident (connected) on it. Which of the following is true for every graph G?

A. There are even number of vertices of even degree.
B. There are odd number of vertices of even degree.
C. There are even number of vertices of odd degree.
D. There are odd number of vertices of odd degree.
E. All the vertices are of even degree.

tifr2012 graph-theory degree-of-graph

Answer key

7.2.3 Degree Of Graph: TIFR CSE 2018 | Part B | Question: 8 top

In an undirected graph G with n vertices, vertex 1 has degree 1, while each vertex $2, \dots, n-1$ has degree 10 and the degree of vertex n is unknown, Which of the following statement must be TRUE on the graph G?

A. There is a path from vertex 1 to vertex n.
B. There is a path from vertex 1 to each vertex $2, \dots, n-1$.
C. Vertex n has degree 1.

D. The diameter of the graph is at most $\frac{n}{10}$
E. All of the above choices must be TRUE

tifr2018 graph-theory degree-of-graph

Answer key☟

7.3 Graph Coloring (5) top☝

7.3.1 Graph Coloring: TIFR CSE 2013 | Part B | Question: 1 top☝

Let $G = (V, E)$ be a simple undirected graph on n vertices. A colouring of G is an assignment of colours to each vertex such that endpoints of every edge are given different colours. Let $\chi(G)$ denote the chromatic number of G, i.e. the minimum numbers of colours needed for a valid colouring of G. A set $B \subseteq V$ is an independent set if no pair of vertices in B is connected by an edge. Let $a(G)$ be the number of vertices in a largest possible independent set in G. In the absence of any further information about G we can conclude.

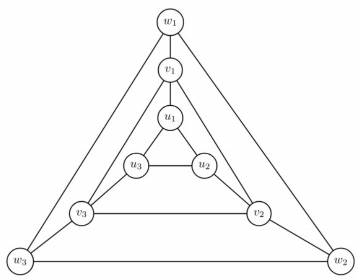
A. $\chi(G) \geq a(G)$
B. $\chi(G) \leq a(G)$
C. $a(G) \geq \frac{n}{\chi(G)}$
D. $a(G) \leq \frac{n}{\chi(G)}$
E. None of the above

tifr2013 graph-theory graph-coloring

Answer key☟

7.3.2 Graph Coloring: TIFR CSE 2017 | Part B | Question: 1 top☝

A vertex colouring with three colours of a graph $G = (V, E)$ is a mapping $c : V \rightarrow \{R, G, B\}$ so that adjacent vertices receive distinct colours. Consider the following undirected graph.



How many vertex colouring with three colours does this graph have?

A. 3^9
B. 6^3
C. 3×2^8
D. 27
E. 24

tifr2017 graph-theory graph-coloring

Answer key☟

7.3.3 Graph Coloring: TIFR CSE 2017 | Part B | Question: 10 top☝

A vertex colouring of a graph $G = (V, E)$ with k coulours is a mapping $c : V \rightarrow \{1, \dots, k\}$ such that $c(u) \neq c(v)$ for every $(u, v) \in E$. Consider the following statements:

i. If every vertex in G has degree at most d then G admits a vertex coulouring using $d + 1$ colours.
ii. Every cycle admits a vertex colouring using 2 colours
iii. Every tree admits a vertex colouring using 2 colours

Which of the above statements is/are TRUE? Choose from the following options:

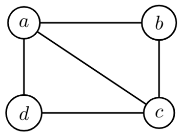
A. only i B. only i and ii C. only i and iii D. only ii and iii E. i, ii, and iii

tifr2017 graph-theory graph-coloring

Answer key

7.3.4 Graph Coloring: TIFR CSE 2018 | Part A | Question: 9 top

How many ways are there to assign colours from range $\{1,2,\ldots,r\}$ to vertices of the following graph so that adjacent vertices receive distinct colours?



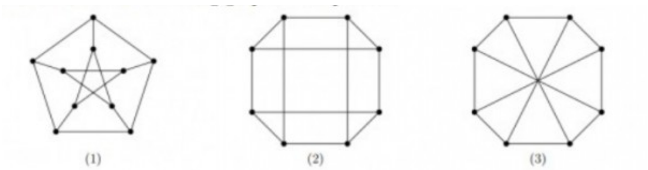
A. r^4
B. $r^4 - 4r^3$
C. $r^4 - 5r^3 + 8r^2 - 4r$
D. $r^4 - 4r^3 + 9r^2 - 3r$
E. $r^4 - 5r^3 + 10r^2 - 15r$

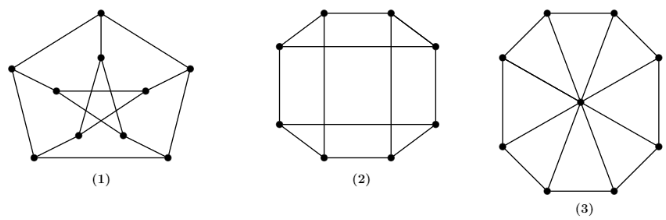
tifr2018 graph-theory graph-coloring

Answer key

7.3.5 Graph Coloring: TIFR CSE 2020 | Part B | Question: 11 top

Which of the following graphs are bipartite?





A. Only (1)
B. Only (2)
C. Only (2) and (3)
D. None of $(1),(2),(3)$
E. All of $(1),(2),(3)$

tifr2020 engineering-mathematics graph-theory graph-coloring

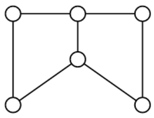
Answer key

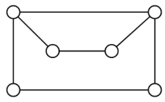
7.4 Graph Connectivity (4) top

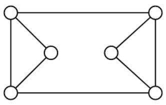
7.4.1 Graph Connectivity: TIFR CSE 2015 | Part B | Question: 5 top

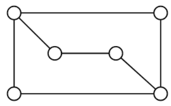
Suppose $\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$

is the adjacency matrix of an undirected graph with six vertices: that is, the rows and columns are indexed by vertices of the graph, and an entry is 1 if the corresponding vertices are connected by an edge and is 0 otherwise; the same order of vertices is used for the rows and columns. Which of the graphs below has the above adjacency matrix?

i. 

ii. 

iii. 

iv. 

A. Only (i)
B. Only (ii)
C. Only (iii)
D. Only (iv)
E. (i) and (ii)

tifr2015 graph-connectivity graph-theory

Answer key☟

7.4.2 Graph Connectivity: TIFR CSE 2019 | Part B | Question: 12 top☝

Let $G = (V, E)$ be a directed graph with $n (\geq 2)$ vertices, including a special vertex r. Each edge $e \in E$ has a strictly positive edge weight $w(e)$. An arborescence in G rooted at r is a subgraph H of G in which every vertex $u \in V \setminus \{r\}$ has a directed path to the special vertex r. The weight of an arborescence H is the sum of the weights of the edges in H.

Let H^* be a minimum arborescence rooted at r, and w^* the weight of H^*. Which of the following is NOT always true?

A. $w^* \geq \sum\limits_{u \in V \setminus \{r\}} \min\limits_{(u,v) \in E} w((u,v))$
B. $w^* \geq \sum\limits_{u \in V \setminus \{r\}} \min\limits_{(v,u) \in E} w((v,u))$
C. H^* has exactly $n-1$ edges
D. H^* is acyclic
E. w^* is less than the weight of the minimum weight directed Hamiltonian cycle in G, when G has a directed Hamiltonian cycle

tifr2019 graph-connectivity graph-theory difficult

Answer key☟

7.4.3 Graph Connectivity: TIFR CSE 2019 | Part B | Question: 15 top☝

Consider directed graphs on n labelled vertices $\{1, 2, \ldots, n\}$, where each vertex has exactly one edge coming in and exactly one edge going out. We allow self-loops. How many graphs have exactly two cycles ?

A. $\sum_{k=1}^{n-1} k!(n-k)!$

B. $\frac{n!}{2}\left[\sum_{k=1}^{n-1} \frac{1}{k(n-k)}\right]$

C. $n!\left[\sum_{k=1}^{n-1} \frac{1}{k}\right]$

D. $\frac{n!(n-1)}{2}$

E. None of the above

tifr2019 graph-connectivity graph-theory

Answer key

7.4.4 Graph Connectivity: TIFR CSE 2019 | Part B | Question: 3 top

A graph is d – regular if every vertex has degree d. For a d – regular graph on n vertices, which of the following must be TRUE?

A. d divides n

B. Both d and n are even

C. Both d and n are odd

D. At least one of d and n is odd

E. At least one of d and n is even

tifr2019 graph-connectivity graph-theory

Answer key

7.5 Line Graph (1) top

7.5.1 Line Graph: TIFR CSE 2017 | Part B | Question: 13 top

For an undirected graph $G = (V, E)$, the line graph $G' = (V', E')$ is obtained by replacing each edge in E by a vertex, and adding an edge between two vertices in V' if the corresponding edges in G are incident on the same vertex. Which of the following is TRUE of line graphs?

A. the line graph for a complete graph is complete

B. the line graph for a connected graph is connected

C. the line graph for a bipartite graph is bipartite

D. the maximum degree of any vertex in the line graph is at most the maximum degree in the original graph

E. each vertex in the line graph has degree one or two

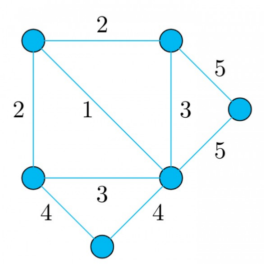
tifr2017 graph-theory line-graph

Answer key

7.6 Minimum Spanning Tree (1) top

7.6.1 Minimum Spanning Tree: TIFR CSE 2018 | Part B | Question: 3 top

How many distinct minimum weight spanning trees does the following undirected, weighted graph have ?



A. 1

B. 2

C. 4

D. 6

E. 8

tifr2018 graph-theory minimum-spanning-tree

Answer key

7.7 Spanning Tree (1) top

7.7.1 Spanning Tree: TIFR CSE 2015 | Part B | Question: 11 top

Let K_n be the complete graph on n vertices labeled $\{1, 2, \ldots, n\}$ with $m = \frac{n(n-1)}{2}$ edges. What is the number of spanning trees of K_n?

A. $\frac{m}{n-1}$

B. m^{n-1}

C. n^{n-2}

D. n^{n-1}

E. None of the above

tifr2015 graph-theory spanning-tree

Answer key

7.8 Tree (1) top

7.8.1 Tree: TIFR CSE 2011 | Part B | Question: 33 top

Which of the following is NOT a sufficient and necessary condition for an undirected graph G to be a tree?

A. G is connected and has $n-1$ edges.

B. G is acyclic and connected.

C. G is acyclic and has $n-1$ edges.

D. G is acyclic, connected and has $n-1$ edges.

E. G has $n-1$ edges.

tifr2011 graph-theory tree

Answer key

Answer Keys

7.1.1	C	7.2.1	C	7.2.2	C	7.2.3	A	7.3.1	C
7.3.2	E	7.3.3	C	7.3.4	C	7.3.5	B	7.4.1	E
7.4.2	B	7.4.3	B	7.4.4	E	7.5.1	B	7.6.1	C
7.7.1	C	7.8.1	E						

8 Discrete Mathematics: Mathematical Logic (15)

8.1 First Order Logic (7) top

8.1.1 First Order Logic: TIFR CSE 2010 | Part A | Question: 8 top

Which of the following is NOT necessarily true? { Notation: The symbol "¬"notes negation; $P(x,y)$ means that for given x and y, the property $P(x,y)$ is true }.

A. $(\forall x \forall y P(x,y)) \Rightarrow (\forall y \forall x P(x,y))$
B. $(\forall x \exists y \neg P(x,y)) \Rightarrow \neg(\exists x \forall y P(x,y))$
C. $(\exists x \exists y P(x,y)) \Rightarrow (\exists y \exists x P(x,y))$
D. $(\exists x \forall y P(x,y)) \Rightarrow (\forall y \exists x P(x,y))$
E. $(\forall x \exists y P(x,y)) \Rightarrow (\exists y \forall x P(x,y))$

tifr2010 mathematical-logic first-order-logic

Answer key

8.1.2 First Order Logic: TIFR CSE 2012 | Part A | Question: 2 top

If $Mr. M$ is guilty, then no witness is lying unless he is afraid. There is a witness who is afraid. Which of the following statements is true?

(Hint: Formulate the problem using the following predicates

- $G - Mr. M$ is guilty
- $W(x) - x$ is a witness
- $L(x) - x$ is lying
- $A(x) - x$ is afraid)

A. $Mr. M$ is guilty.
B. $Mr. M$ is not guilty.
C. From these facts one cannot conclude that $Mr. M$ is guilty.
D. There is a witness who is lying.
E. No witness is lying.

tifr2012 mathematical-logic first-order-logic

Answer key

8.1.3 First Order Logic: TIFR CSE 2012 | Part B | Question: 3 top

For a person p, let $w(p)$, $A(p,y)$, $L(p)$ and $J(p)$ denote that p is a woman, p admires y, p is a lawyer and p is a judge respectively. Which of the following is the correct translation in first order logic of the sentence: "All woman who are lawyers admire some judge"?

A. $\forall x : [(w(x) \Lambda L(x)) \Rightarrow (\exists y : (J(y) \Lambda w(y) \Lambda A(x,y)))]$
B. $\forall x : [(w(x) \Rightarrow L(x)) \Rightarrow (\exists y : (J(y) \Lambda A(x,y)))]$
C. $\forall x \forall y : [(w(x) \Lambda L(x)) \Rightarrow (J(y) \Lambda A(x,y))]$
D. $\exists y \forall x : [(w(x) \Lambda L(x)) \Rightarrow (J(y) \Lambda A(x,y))]$
E. $\forall x : [(w(x) \Lambda L(x)) \Rightarrow (\exists y : (J(y) \Lambda A(x,y)))]$

tifr2012 mathematical-logic first-order-logic

Answer key

8.1.4 First Order Logic: TIFR CSE 2016 | Part B | Question: 4 top☝

In the following, A stands for a set of apples, and $S(x, y)$ stands for "x is sweeter than y. Let

$$\Psi \equiv \exists x : x \in A$$

$$\Phi \equiv \forall x \in A : \exists y \in A : S(x, y).$$

Which of the following statements implies that there are infinitely many apples (i.e.,, A is an inifinite set)?

A. $\Psi \wedge \Phi \wedge [\forall x \in A : \neg S(x, x)]$
B. $\Psi \wedge \Phi \wedge [\forall x \in A : S(x, x)]$
C. $\Psi \wedge \Phi \wedge [\forall x, y \in A : S(x, x) \wedge S(x, y) \rightarrow S(y, y)]$
D. $\Psi \wedge \Phi \wedge [\forall x \in A : \neg S(x, x)] \wedge [\forall x, y, z \in A : S(x, y) \wedge S(y, z) \rightarrow s(y, x)]$
E. $\Psi \wedge \Phi \wedge [\forall x \in A : \neg S(x, x)] \wedge [\forall x, y, z \in A : S(x, y) \wedge S(y, z) \rightarrow s(x, z)]$

tifr2016 mathematical-logic first-order-logic

Answer key☟

8.1.5 First Order Logic: TIFR CSE 2017 | Part B | Question: 11 top☝

Given that

- $B(x)$ means "x is a bat",
- $F(x)$ means "x is a fly", and
- $E(x, y)$ means "x eats y",

what is the best English translation of

$$\forall x(F(x) \rightarrow \forall y(E(y, x) \rightarrow B(y)))?$$

A. all flies eat bats
B. every fly is eaten by some bat
C. bats eat only flies
D. every bat eats flies
E. only bats eat flies

tifr2017 first-order-logic

Answer key☟

8.1.6 First Order Logic: TIFR CSE 2017 | Part B | Question: 6 top☝

Consider the First Order Logic (FOL) with equality and suitable function and relation symbols. Which of the following is FALSE?

A. Partial orders cannot be axiomatized in FOL
B. FOL has a complete proof system
C. Natural numbers cannot be axiomatized in FOL
D. Real numbers cannot be axiomatized in FOL
E. Relational numbers cannot be axiomatized in FOL

tifr2017 first-order-logic normal

Answer key☟

8.1.7 First Order Logic: TIFR CSE 2019 | Part B | Question: 4 top☝

Let φ be a propositional formula on a set of variables A and φ be a propositional formula on a set of variables B , such that $\varphi \Rightarrow \psi$. A $Craig\,interpolant$ of φ and ψ is a propositional formula μ on variables $A \cap B$ such that $\varphi \Rightarrow \mu$ and $\mu \Rightarrow \psi$. Given propositional formula $\varphi = q \vee (r \wedge s)$ on the set of variables $A = \{q, r, s\}$ and propositional formula $\psi = \neg q \rightarrow (s \vee t)$ on the set of variables $B = \{q, s, t\}$, which of the following is a Craig interpolant for φ and ψ ?

A. q
B. φ itself
C. $q \vee s$
D. $q \vee r$
E. $\neg q \wedge s$

tifr2019 engineering-mathematics discrete-mathematics mathematical-logic first-order-logic

Answer key☟

8.2 Logical Reasoning (6) top☝

8.2.1 Logical Reasoning: TIFR CSE 2010 | Part A | Question: 4 top☝

- If the bank receipt is forged, then Mr. M is liable.
- If Mr. M is liable, he will go bankrupt.
- If the bank will loan him money, he will not go bankrupt.
- The bank will loan him money.

Which of the following can be concluded from the above statements?

A. Mr. M is liable
B. The receipt is not forged
C. Mr. M will go bankrupt
D. The bank will go bankrupt
E. None of the above

tifr2010 logical-reasoning mathematical-logic

Answer key☟

8.2.2 Logical Reasoning: TIFR CSE 2011 | Part A | Question: 1 top☝

- If either wages or prices are raised, there will be inflation.
- If there is inflation, then either the government must regulate it or the people will suffer.
- If the people suffer, the government will be unpopular.
- Government will not be unpopular.

Which of the following can be validly concluded from the above statements.

A. People will not suffer
B. If the inflation is not regulated, then wages are not raised
C. Prices are not raised
D. If the inflation is not regulated, then the prices are not raised
E. Wages are not raised

tifr2011 mathematical-logic normal logical-reasoning

Answer key☟

8.2.3 Logical Reasoning: TIFR CSE 2011 | Part A | Question: 12 top☝

The action for this problem takes place in an island of Knights and Knaves, where Knights always make true statements and Knaves always make false statements and everybody is either a Knight or a Knave. Two friends A and B lives in a house. The census taker (an outsider) knocks on the door and it is opened by A. The census taker says "I need information about you and your friend. Which if either is a Knight and which if either is a Knave?". "We are both Knaves" says A angrily and slams the door. What, if any thing can the census taker conclude?

A. A is a Knight and B is a Knave.
B. A is a Knave and B is a Knight.
C. Both are Knaves.
D. Both are Knights.
E. No conclusion can be drawn.

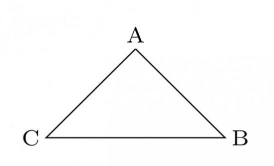
tifr2011 mathematical-logic logical-reasoning

Answer key

8.2.4 Logical Reasoning: TIFR CSE 2012 | Part A | Question: 3 top

Long ago,in a planet far far away, there lived three races of intelligent inhabitants: the blues (who always tell the truth), the whites (who always lie), and the pinks (who, when asked a series of questions, start with a lie and then tell the truth and lie alternately). To three creatures, chosen from the planet and seated facing each other at A, B, and C (see figure), the following three questions are put:

i. What race is your left-hand neighbour?
ii. What race is your right-hand neighbour?
iii. What race are you?



Here are their answers:

A. (i) White (ii) Pink (iii) Blue
B. (i) Pink (ii) Pink (iii) Blue
C. (i) White (ii) Blue (iii) Blue

What is the actual race of each of the three creatures?

A. A is Pink, B is White, C is Blue.
B. A is Blue, B is Pink, C is White.
C. A is Pink, B is Blue, C is Pink.
D. A is White, B is Pink, C is Blue.
E. Cannot be determined from the above data.

tifr2012 mathematical-logic logical-reasoning

Answer key

8.2.5 Logical Reasoning: TIFR CSE 2013 | Part A | Question: 3 top

Three candidates, Amar, Birendra and Chanchal stand for the local election. Opinion polls are conducted and show that fraction a of the voters prefer Amar to Birendra, fraction b prefer Birendra to Chanchal and fraction c prefer Chanchal to Amar. Which of the following is impossible?

A. $(a,b,c) = (0.51, 0.51, 0.51);$
B. $(a,b,c) = (0.61, 0.71, 0.67);$
C. $(a,b,c) = (0.68, 0.68, 0.68);$
D. $(a,b,c) = (0.49, 0.49, 0.49);$
E. None of the above.

tifr2013 set-theory&algebra logical-reasoning

Answer key

8.2.6 Logical Reasoning: TIFR CSE 2014 | Part A | Question: 8 top

All that glitters is gold. No gold is silver.

Claims:

1. No silver glitters.
2. Some gold glitters.

Then, which of the following is TRUE?

A. Only claim 1 follows.
B. Only claim 2 follows.
C. Either claim 1 or claim 2 follows
D. Neither claim 1 nor claim 2 follows.

but not both.
E. Both claim 1 and claim 2 follow.

tifr2014 mathematical-logic logical-reasoning

Answer key☟

8.3 Propositional Logic (2) top☝

8.3.1 Propositional Logic: TIFR CSE 2015 | Part A | Question: 5 top☝

What is logically equivalent to "If Kareena and Parineeti go to the shopping mall then it is raining":

A. If Kareena and Parineeti do not go to the shopping mall then it is not raining.
B. If Kareena and Parineeti do not go to the shopping mall then it is raining.
C. If it is raining then Kareena and Parineeti go to the shopping mall.
D. If it is not raining then Kareena and Parineeti do not go to the shopping mall.
E. None of the above.

tifr2015 mathematical-logic propositional-logic

Answer key☟

8.3.2 Propositional Logic: TIFR CSE 2018 | Part B | Question: 4 top☝

The notation "$\Rightarrow$" denotes "implies" and "$\wedge$" denotes "and" in the following formulae.

- Let X denote the formula: $(b \Rightarrow a) \Rightarrow (a \Rightarrow b)$
- Let Y denote the formula: $(a \Rightarrow b) \wedge b$

Which of the following is TRUE?

A. X is satisfiable and Y is not satisfiable.
B. X is satisfiable and Y is tautology.
C. X is not tautology and Y is not satisfiable.
D. X is not tautology and Y is satisfiable.
E. X is a tautology and Y is satisfiable,

tifr2018 mathematical-logic propositional-logic

Answer key☟

Answer Keys

8.1.1	E	8.1.2	C	8.1.3	E	8.1.4	E	8.1.5	E
8.1.6	A	8.1.7	C	8.2.1	B	8.2.2	X	8.2.3	B
8.2.4	C	8.2.5	C	8.2.6	E	8.3.1	D	8.3.2	D

9 Discrete Mathematics: Set Theory & Algebra (22)

9.1 Convex Sets Functions (1) top

9.1.1 Convex Sets Functions: TIFR CSE 2019 | Part A | Question: 6 top

A function $f:\mathbb{R}\to\mathbb{R}$ is said to be *convex* if for all $x,y\in\mathbb{R}$ and λ such that $0\leq\lambda\leq 1$,

$$f(\lambda x+(1-\lambda)y)\leq\lambda f(x)+(1-\lambda)f(y).$$

Let $f:\mathbb{R}\to\mathbb{R}$ be a convex function , and define the following functions:

$$p(x)=f(-x),\quad q(x)=-f(-x),\text{ and } r(x)=f(1-x).$$

Which of the functions p,q and r must be convex?

A. Only p B. Only q C. Only r D. Only p and r E. Only q and r

tifr2019 engineering-mathematics discrete-mathematics set-theory&algebra functions convex-sets-functions non-gate

Answer key

9.2 Functions (6) top

9.2.1 Functions: TIFR CSE 2012 | Part B | Question: 1 top

For $x,y\in\{0,1\}^n$, let $x\oplus y$ be the element of $\{0,1\}^n$ obtained by the component-wise exclusive-or of x and y. A Boolean function $F:\{0,1\}^n\to\{0,1\}$ is said to be linear if $F(x\oplus y)=F(x)\oplus F(y)$, for all x and y. The number of linear functions from $\{0,1\}^n$ to $\{0,1\}$ is.

A. 2^{2n} B. 2^{n+1} C. $2^{n-1}+1$ D. $n!$ E. 2^n

tifr2012 set-theory&algebra functions

Answer key

9.2.2 Functions: TIFR CSE 2013 | Part B | Question: 16 top

Consider a function $T_{k,n}:\{0,1\}^n\to\{0,1\}$ which returns 1 if at least k of its n inputs are 1. Formally, $T_{k,n}(x)=1$ if $\sum_1^n x_i\geq k$. Let $y\in\{0,1\}^n$ be such that y has exactly k ones. Then, the function $T_{k,n-1}\left(y_1,y_2,\dots .y_{i-1},y_{i+1},\dots,y_n\right)$ (where y_i is omitted) is equivalent to

A. $T_{k-1},n(y)$ B. $T_{k,n}(y)$ C. y_i D. $\neg y_i$ E. None of the above

tifr2013 set-theory&algebra functions

Answer key

9.2.3 Functions: TIFR CSE 2014 | Part B | Question: 18 top

Let k be an integer at least 4 and let $[k]=\{1,2,\dots,k\}$. Let $f:[k]^4\to\{0,1\}$ be defined as follows: $f(y_1,y_2,y_3,y_4)=1$ if an only if the $y_i's$ are all distinct. For each choice $z=(z_1,z_2,z_3)\in[k]^3$, let $g_z:[k]\to\{0,1\}$ be defined by $g_z(Y)=f(Y,z_1,z_2,z_3)$. Let N be the number of distinct functions g_z that are obtained as z varies in $\{1,2,\dots,k\}^3$, that is, $N=\left|\left\{g_z:z\in\{1,2,\dots,k\}^3\right\}\right|$. What is N?

A. k^3+1 B. $2^{\binom{k}{3}}$ C. $\binom{k}{3}$ D. $\binom{k}{3}+1$ E. $4\binom{k}{3}$

tifr2014 set-theory&algebra functions

Answer key

9.2.4 Functions: TIFR CSE 2017 | Part A | Question: 11 top☝

Let $f \circ g$ denote function composition such that $(f \circ g)(x) = f(g(x))$. Let $f : A \rightarrow B$ such that for all $g : B \rightarrow A$ and $h : B \rightarrow A$ we have $f \circ g = f \circ h \Rightarrow g = h$. Which of the following must be true?

A. f is onto (surjective)
B. f is one-to-one (injective)
C. f is both one-to-one and onto (bijective)
D. the range of f is finite
E. the domain of f is finite

tifr2017 set-theory&algebra functions

Answer key☟

9.2.5 Functions: TIFR CSE 2018 | Part B | Question: 10 top☝

For two n bit strings $x, y \in \{0,1\}^n$, define $z = x \oplus y$ to be the bitwise XOR of the two strings (that is, if x_i, y_i, z_i denote the i^{th} bits of x, y, z respectively, then $z_i = x_i + y_i \bmod 2$). A function $h : \{0,1\}^n \rightarrow \{0,1\}^n$ is called linear if $h(x \oplus y) = h(x) \oplus h(y)$, for every $x, y \in \{0,1\}^n$. The number of such linear functions for $n \geq 2$ is:

A. 2^n
B. 2^{n^2}
C. $2^{\frac{n}{2}}$
D. 2^{4n}
E. 2^{n^2+n}

tifr2018 functions

Answer key☟

9.2.6 Functions: TIFR CSE 2019 | Part A | Question: 12 top☝

Let f be a function with both input and output in the set $\{0,1,2,\dots,9\}$, and let the function g be defined as $g(x) = f(9-x)$. The function f is non-decreasing, so that $f(x) \geq f(y)$ for $x \geq y$. Consider the following statements:

i. There exists $x \in \{0,\dots,9\}$ so that $x = f(x)$
ii. There exists $x \in \{0,\dots,9\}$ so that $x = g(x)$
iii. There exists $x \in \{0,\dots,9\}$ so that $x = (f(x) + g(x)) \bmod 10$

Which of the above statements must be TRUE for ALL such functions f and g ?

A. Only (i)
B. Only (i) and (ii)
C. Only (iii)
D. None of them
E. All of them

tifr2019 engineering-mathematics discrete-mathematics set-theory&algebra functions

Answer key☟

9.3 Lattice (1) top☝

9.3.1 Lattice: TIFR CSE 2012 | Part B | Question: 4 top☝

Let $\wedge$, $\vee$ denote the meet and join operations of lattice. A lattice is called distributive if for all x, y, z,

$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$

It is called complete if meet and join exist for every subset. It is called modular if for all x, y, z

$z \leq x \Rightarrow x \wedge (y \vee z) = (x \wedge y) \vee z$

The positive integers under divisibility ordering i.e. $p \leq q$ if p divides q forms a.

A. Complete lattice.
B. Modular, but not distributive lattice.

C. Distributive lattice.
D. Lattice but not a complete lattice.
E. Under the give ordering positive integers do not form a lattice.

tifr2012 set-theory&algebra lattice

Answer key☟

9.4 Number Theory (1) top☝

9.4.1 Number Theory: TIFR CSE 2014 | Part A | Question: 14 top☝

Let m and n be any two positive integers. Then, which of the following is FALSE?

A. $m+1$ divides $m^{2n}-1$.
B. For any prime p, $m^p \equiv m(\mod p)$.
C. If one of m, n is prime, then there are integers x, y such that $mx+ny=1$.
D. If $m<n$, then $m!$ divides $n(n-1)(n-2)\dots(n-m+1)$.
E. If 2^n-1 is prime, then n is prime.

tifr2014 number-theory set-theory&algebra

Answer key☟

9.5 Partial Order (4) top☝

9.5.1 Partial Order: TIFR CSE 2012 | Part B | Question: 5 top☝

Let R be a binary relation over a set S. The binary relation R is called an equivalence relation if it is reflexive transitive and symmetric. The relation is called partial order if it is reflexive, transitive and anti symmetric. (Notation: Let aRb denote that order pair $(a,b) \in R$.) The relation R is called a well-order if R is a partial order and there does not exist an infinite descending chain (with respect to R) within S. An infinite sequence $x_1, x_2 \dots$ of elements of S is called an infinite descending chain if for all i we have $x_{i+1}Rx_i$ and $x_i \neq x_{i+1}$.

Take $S = \aleph \times \aleph$ and let the binary relation $\sqsubseteq$ over S be such that $(i_1, j_1) \sqsubseteq (i_2, j_2)$ if and only if either $(i_1 < i_2)$ or $((i_1 = i_2) \wedge (j_1 \leq j_2))$. Which statement is true of $\sqsubseteq$?

A. $\sqsubseteq$ is an equivalence relation but not a well order.
B. $\sqsubseteq$ is a partial order but not a well order.
C. $\sqsubseteq$ is a partial order and a well order.
D. $\sqsubseteq$ is an equivalence relation and a well order.
E. $\sqsubseteq$ is neither a partial order nor an equivalence relation.

tifr2012 set-theory&algebra partial-order

Answer key☟

9.5.2 Partial Order: TIFR CSE 2013 | Part B | Question: 4 top☝

A set S together with partial order $\ll$ is called a well order if it has no infinite descending chains, i.e. there is no infinite sequence $x_1, x_2, \dots$ of elements from S such that $x_{i+1} \ll x_i$ and $x_{i+1} \neq x_i$ for all i.

Consider the set of all words (finite sequence of letters $a-z$), denoted by W, in dictionary order.

A. Between "aa" and "az" there are only 24 words.

B. Between "aa" and "az" there are only 2^{24} words.

C. W is not a partial order.

D. W is a partial order but not a well order.

E. W is a well order.

tifr2013 set-theory&algebra partial-order

Answer key☟

9.5.3 Partial Order: TIFR CSE 2014 | Part B | Question: 15 top☝

Consider the set N^* of finite sequences of natural numbers with $x \leq_p y$ denoting that sequence x is a prefix of sequence y. Then, which of the following is true?

A. N^* is uncountable.
B. $\leq_p$ is a total order.
C. Every non-empty subset of N^* has a least upper bound.
D. Every non-empty subset of N^* has a greatest lower bound.
E. Every non-empty finite subset of N^* has a least upper bound.

tifr2014 set-theory&algebra partial-order

Answer key☟

9.5.4 Partial Order: TIFR CSE 2014 | Part B | Question: 16 top☝

Consider the ordering relation $x \mid y \subseteq N \times N$ over natural numbers N such that $x \mid y$ if there exists $z \in N$ such that $x \bullet z = y$. A set is called lattice if every finite subset has a least upper bound and greatest lower bound. It is called a complete lattice if every subset has a least upper bound and greatest lower bound. Then,

A. $\mid$ is an equivalence relation.

B. Every subset of N has an upper bound under $\mid$.

C. $\mid$ is a total order.

D. $(N, \mid)$ is a complete lattice.

E. $(N, \mid)$ is a lattice but not a complete lattice.

tifr2014 set-theory&algebra partial-order

Answer key☟

9.6 Polynomials (1) top☝

9.6.1 Polynomials: TIFR CSE 2012 | Part A | Question: 12 top☝

For the polynomial $p(x) = 8x^{10} - 7x^3 + x - 1$ consider the following statements (which may be true or false)

i. It has a root between $[0, 1]$.
ii. It has a root between $[0, -1]$.
iii. It has no roots outside $(-1, 1)$.

Which of the above statements are true?

A. Only (i).

B. Only (i) and (ii).

C. Only (i) and (iii).

D. Only (ii) and (iii).

E. All of (i), (ii) and (iii).

tifr2012 set-theory&algebra polynomials

Answer key☟

9.7 Set Theory (8) top☝

9.7.1 Set Theory: TIFR CSE 2010 | Part A | Question: 15 top

Let A, B be sets. Let $\bar{A}$ denote the compliment of set A (with respect to some fixed universe), and $(A - B)$ denote the set of elements in A which are not in B. Set $(A - (A - B))$ is equal to:

A. B
B. $A \cap \bar{B}$
C. $A - B$
D. $A \cap B$
E. $\bar{B}$

tifr2010 set-theory&algebra set-theory

Answer key

9.7.2 Set Theory: TIFR CSE 2010 | Part A | Question: 18 top

Let X be a set of size n. How many pairs of sets (A, B) are there that satisfy the condition $A \subseteq B \subseteq X$?

A. 2^{n+1}
B. 2^{2n}
C. 3^n
D. $2^n + 1$
E. 3^{n+1}

tifr2010 set-theory

Answer key

9.7.3 Set Theory: TIFR CSE 2011 | Part A | Question: 10 top

Let m, n denote two integers from the set $\{1, 2, \dots, 10\}$. The number of ordered pairs (m, n) such that $2^m + 2^n$ is divisible by 5 is.

A. 10
B. 14
C. 24
D. 8
E. None of the above

tifr2011 set-theory&algebra set-theory

Answer key

9.7.4 Set Theory: TIFR CSE 2011 | Part B | Question: 23 top

Suppose $(S_1, S_2, \dots, S_m)$ is a finite collection of non-empty subsets of a universe U. Note that the sets in this collection need not be distinct. Consider the following basic step to be performed on this sequence. While there exist sets S_i and S_j in the sequence, neither of which is a subset of the other, delete them from the sequence, and

i. If $S_i \cap S_j \neq \emptyset$, then add the sets $S_i \cup S_j$ and $S_i \cap S_j$ to the sequence;
ii. If $S_i \cap S_j = \emptyset$, then add only the set $S_i \cup S_j$ to the sequence.

In each step we delete two sets from the sequence and add at most two sets to the sequence. Also, note that empty sets are never added to the sequence. Which of the following statements is TRUE?

A. The size of the smallest set in the sequence decreases in every step
B. The size of the largest set in the sequence increases in every step
C. The process always terminates
D. The process terminates if U is finite but might not if U is infinite
E. There is a finite collection of subsets of a finite universe U and a choice of S_i and S_j in each step such that the process does not terminate

tifr2011 set-theory&algebra set-theory

Answer key

9.7.5 Set Theory: TIFR CSE 2012 | Part A | Question: 8 top

How many pairs of sets (A, B) are there that satisfy the condition $A, B \subseteq \{1, 2, \ldots, 5\}, A \cap B = \{\}$?

A. 125 B. 127 C. 130 D. 243 E. 257

tifr2012 set-theory&algebra set-theory

Answer key

9.7.6 Set Theory: TIFR CSE 2016 | Part A | Question: 8 top

Let A and B be finite sets such that $A \subseteq B$. Then, what is the value of the expression:

$$\sum_{C:A\subseteq C\subseteq B} (-1)^{|C\setminus A|},$$

Where $C \setminus A = \{x \in C : x \notin A\}$?

A. Always 0
B. Always 1
C. 0 if $A = B$ and 1 otherwise
D. 1 if $A = B$ and 0 otherwise
E. Depends on the size of the universe

tifr2016 set-theory&algebra set-theory

Answer key

9.7.7 Set Theory: TIFR CSE 2017 | Part A | Question: 10 top

For a set A define $P(A)$ to be the set of all subsets of A. For example, if $A = \{1, 2\}$ then $P(A) = \{\emptyset, \{1, 2\}, \{1\}, \{2\}\}$. Let $A \rightarrow P(A)$ be a function and A is not empty. Which of the following must be TRUE?

A. f cannot be one-to-one (injective)
B. f cannot be onto (surjective)
C. f is both one-to-one and onto (bijective)
D. there is no such f possible
E. if such a function f exists, then A is infinite

tifr2017 set-theory&algebra set-theory functions easy

Answer key

9.7.8 Set Theory: TIFR CSE 2019 | Part A | Question: 1 top

Let X be a set with n elements. How many subsets of X have odd cardinality?

A. n
B. 2^n
C. $2^{n/2}$
D. 2^{n-1}
E. Can not be determined without knowing whether n is odd or even

tifr2019 engineering-mathematics discrete-mathematics set-theory&algebra set-theory

Answer key

Answer Keys

9.1.1	D	9.2.2	D	9.2.3	D	9.2.4	B	9.2.5	B
9.2.6	B	9.3.1	C	9.4.1	C	9.5.1	C	9.5.2	E
9.5.3	D	9.5.4	E	9.6.1	E	9.7.1	D	9.7.2	C
9.7.3	C	9.7.4	C	9.7.5	D	9.7.6	D	9.7.7	B
9.7.8	D								

10 Engineering Mathematics: Calculus (21)

10.0.1 TIFR CSE 2020 | Part A | Question: 13 top

What is the area of the largest rectangle that can be inscribed in a circle of radius R?

A. $R^2/2$ B. $\pi \times R^2/2$ C. R^2 D. $2R^2$ E. None of the above

tifr2020

Answer key

10.1 Convergence (1) top

10.1.1 Convergence: TIFR CSE 2014 | Part A | Question: 15 top

Consider the following statements:

1. $b_1 = \sqrt{2}$, series with each $b_i = \sqrt{b_{i-1} + \sqrt{2}}, i \geq 2$, converges.
2. $\sum_{i=1}^{\infty} \frac{\cos(i)}{i^2}$ converges.
3. $\sum_{i=0}^{\infty} b_i$ converges if $\lim_{i \to \infty} \frac{|b_{i+1}|}{|b_i|} < 1$

Which of the following is TRUE?

A. Statements (1) and (2) but not (3).
B. Statements (2) and (3) but not (1).
C. Statements (1) and (3) but not (2).
D. All the three statements.
E. None of the three statements.

tifr2014 convergence non-gate

Answer key

10.2 Differentiation (1) top

10.2.1 Differentiation: TIFR CSE 2018 | Part A | Question: 5 top

Which of the following is the derivative of $f(x) = x^x$ when $x > 0$?

A. x^x
B. $x^x \ln\ x$
C. $x^x + x^x \ln\ x$
D. $(x^x)(x^x \ln\ x)$
E. None of the above; function is not differentiable for $x > 0$

tifr2018 calculus differentiation

Answer key

10.3 Integration (3) top

10.3.1 Integration: TIFR CSE 2011 | Part A | Question: 11 top

$$\int_0^1 \log_e(x)dx =$$

A. 1 B. -1 C. ∞ D. $-\infty$ E. None of the above

tifr2011 calculus integration

Answer key☟

10.3.2 Integration: TIFR CSE 2015 | Part A | Question: 10 top☝

Let $f(x), x \in [0,1]$, be any positive real valued continuous function. Then

$$\lim_{n\to\infty} (n+1) \int_0^1 x^n f(x) \mathrm{d}x$$

equals.

A. $\max_{x\in[0,1]} f(x)$
B. $\min_{x\in[0,1]} f(x)$
C. $f(0)$
D. $f(1)$
E. ∞

tifr2015 maxima-minima calculus non-gate integration

10.3.3 Integration: TIFR CSE 2019 | Part A | Question: 13 top☝

Consider the integral

$$\int_0^1 \frac{x^{300}}{1+x^2+x^3} dx$$

What is the value of this integral correct up to two decimal places?

A. 0.00 B. 0.02 C. 0.10 D. 0.33 E. 1.00

tifr2019 engineering-mathematics calculus integration

Answer key☟

10.4 Limits (7) top☝

10.4.1 Limits: TIFR CSE 2010 | Part A | Question: 7 top☝

The limit of $\dfrac{10^n}{n!}$ as $n \to \infty$ is.

A. 0 B. 1 C. e D. 10 E. ∞

tifr2010 calculus limits

Answer key☟

10.4.2 Limits: TIFR CSE 2011 | Part A | Question: 14 top☝

The limit

$$\lim_{x\to 0} \frac{d}{dx} \frac{\sin^2 x}{x}$$

is

A. 0 B. 2 C. 1 D. $\frac{1}{2}$ E. None of the above

tifr2011 calculus limits

Answer key

10.4.3 Limits: TIFR CSE 2011 | Part A | Question: 17 top

What is

$$\lim_{x \to 0} \frac{2^x - 1}{x}$$

A. 0 B. $\log_2(e)$ C. $\log_e(2)$ D. 1 E. None of the above

tifr2011 limits

Answer key

10.4.4 Limits: TIFR CSE 2012 | Part A | Question: 14 top

The limit $\lim_{n \to \infty} \left(\sqrt{n^2 + n} - n \right)$ equals.

A. ∞ B. 1 C. $1/2$ D. 0 E. None of the above

tifr2012 calculus limits

Answer key

10.4.5 Limits: TIFR CSE 2014 | Part A | Question: 16 top

Let $x_0 = 1$ and

$x_{n+1} = \frac{3+2x_n}{3+x_n}, n \geq 0.$

$x_\infty = \lim_{n \to \infty} x_n$ is

A. $\left(\sqrt{5} - 1\right)/2$
B. $\left(\sqrt{5} + 1\right)/2$
C. $\left(\sqrt{13} - 1\right)/2$
D. $\left(-\sqrt{13} - 1\right)/2$
E. None of the above

tifr2014 limits

Answer key

10.4.6 Limits: TIFR CSE 2014 | Part A | Question: 18 top

We are given a collection of real numbers where a real number $a_i \neq 0$ occurs n_i times. Let the collection be enumerated as $\{x_1, x_2, \dots x_n\}$ so that $x_1 = x_2 = \ldots = x_{n_1} = a_1$ and so on, and $n = \sum_i n_i$ is finite. What is

$$\lim_{k \to \infty} \left(\sum_{i=1}^{n} \frac{1}{|x_i|^k} \right)^{-1/k} ?$$

A. $\max_i \left(n_i |a_i|\right)$
B. $\min_i |a_i|$
C. $\min_i \left(n_i |a_i|\right)$
D. $\max_i |a_i|$
E. None of the above

tifr2014 limits

Answer key

10.4.7 Limits: TIFR CSE 2019 | Part A | Question: 15 top

Consider the matrix

$$A=\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{3}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

What is $\lim\limits_{n\to\infty} A^n$?

A. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

B. $\begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

C. $\begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$

D. $\begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

E. $\text{The limit exists, but it is none of the above}$

tifr2019 engineering-mathematics calculus limits

Answer key

10.5 Maxima Minima (8) top

10.5.1 Maxima Minima: TIFR CSE 2010 | Part A | Question: 3 top

The function $f(x)=2.5\log_e\left(2+\exp\left(x^2-4x+5\right)\right)$ attains a minimum at $x=$?

A. 0 B. 1 C. 2 D. 3 E. 4

tifr2010 calculus maxima-minima

Answer key

10.5.2 Maxima Minima: TIFR CSE 2011 | Part A | Question: 4 top

Consider the problem of maximizing x^2-2x+5 such that $0<x<2$. The value of x at which the maximum is achieved is:

A. 0.5 B. 1 C. 1.5 D. 1.75 E. None of the above

tifr2011 calculus maxima-minima

Answer key

10.5.3 Maxima Minima: TIFR CSE 2012 | Part A | Question: 13 top

The maximum value of the function

$f(x, y, z) = (x - 1/3)^2 + (y - 1/3)^2 + (z - 1/3)^2$

subject to the constraints

$x + y + z = 1, \quad x \geq 0, y \geq 0, z \geq 0$

is

A. $1/3$ B. $2/3$ C. 1 D. $4/3$ E. $4/9$

tifr2012 calculus maxima-minima

Answer key

10.5.4 Maxima Minima: TIFR CSE 2012 | Part A | Question: 15 top

Consider the differential equation $dx/dt = (1 - x)(2 - x)(3 - x)$. Which of its equilibria is unstable?

A. $x = 0$ B. $x = 1$ C. $x = 2$ D. $x = 3$ E. None of the above

tifr2012 calculus maxima-minima

Answer key

10.5.5 Maxima Minima: TIFR CSE 2013 | Part A | Question: 16 top

The minimum of the function $f(x) = x \log_e(x)$ over the interval $[\frac{1}{2}, \infty)$ is

A. 0 B. $-e$ C. $\frac{-\log_e(2)}{2}$ D. $\frac{-1}{e}$ E. None of the above

tifr2013 calculus maxima-minima

Answer key

10.5.6 Maxima Minima: TIFR CSE 2014 | Part A | Question: 9 top

Solve min $x^2 + y^2$ subject to

$$\begin{aligned} x + y &\geq 10, \\ 2x + 3y &\geq 20, \\ x &\geq 4, \\ y &\geq 4. \end{aligned}$$

A. 32 B. 50 C. 52 D. 100 E. None of the above

tifr2014 calculus maxima-minima

Answer key

10.5.7 Maxima Minima: TIFR CSE 2015 | Part A | Question: 11 top

Suppose that $f(x)$ is a continuous function such that $0.4 \leq f(x) \leq 0.6$ for $0 \leq x \leq 1$. Which of the following is always true?

A. $f(0.5) = 0.5$.
B. There exists x between 0 and 1 such that $f(x) = 0.8x$.
C. There exists x between 0 and 0.5 such that $f(x) = x$.
D. $f(0.5) > 0.5$.

E. None of the above statements are always true.

tifr2015 maxima-minima calculus

Answer key

10.5.8 Maxima Minima: TIFR CSE 2020 | Part A | Question: 8 top

Consider a function $f:[0,1] \to [0,1]$ which is twice differentiable in $(0,1)$. Suppose it has exactly one global maximum and exactly one global minimum inside $(0,1)$. What can you say about the behaviour of the first derivative f' and and second derivative f'' on $(0,1)$ (give the most precise answer)?

A. f' is zero at exactly two points, f'' need not be zero anywhere
B. f' is zero at exactly two points, f'' is zero at exactly one point
C. f' is zero at at least two points, f'' is zero at exactly one point
D. f' is zero at at least two points, f'' is zero at at least one point
E. f' is zero at at least two points, f'' is zero at at least two points

tifr2020 engineering-mathematics calculus maxima-minima

Answer key

Answer Keys

10.0.1	D	10.1.1	D	10.2.1	C	10.3.1	B	10.3.2	D
10.3.3	A	10.4.1	A	10.4.2	C	10.4.3	C	10.4.4	C
10.4.5	C	10.4.6	B	10.4.7	D	10.5.1	C	10.5.2	E
10.5.3	B	10.5.4	C	10.5.5	C	10.5.6	B	10.5.7	B
10.5.8	D								

11 Engineering Mathematics: Linear Algebra (14)

11.0.1 TIFR CSE 2020 | Part A | Question: 12 top

The hour needle of a clock is malfunctioning and travels in the anti-clockwise direction, i.e., opposite to the usual direction, at the same speed it would have if it was working correctly. The minute needle is working correctly. Suppose the two needles show the correct time at 12 noon, thus both needles are together at the 12 mark. After how much time do the two needles meet again?

A. $\frac{10}{11}$ hour
B. $\frac{11}{12}$ hour
C. $\frac{12}{13}$ hour
D. $\frac{19}{22}$ hour
E. One hour

tifr2020

Answer key

11.1 Eigen Value (1) top

11.1.1 Eigen Value: TIFR CSE 2019 | Part A | Question: 3 top

A is $n \times n$ square matrix for which the entries in every row sum to 1. Consider the following statements:

i. The column vector $[1, 1, \dots, 1]^T$ is an eigen vector of A.
ii. $\det(A - I) = 0$.
iii. $\det(A) = 0$.

Which of the above statements must be **TRUE**?

A. Only (i)
B. Only (ii)
C. Only (i) and (ii)
D. Only (i) and (iii)
E. $(i), (ii)$ and (iii)

tifr2019 engineering-mathematics linear-algebra eigen-value

Answer key

11.2 Matrix (8) top

11.2.1 Matrix: TIFR CSE 2010 | Part A | Question: 16 top

Let the characteristic equation of matrix M be $\lambda^2 - \lambda - 1 = 0$. Then.

A. M^{-1} does not exist.
B. M^{-1} exists but cannot be determined from the data.
C. $M^{-1} = M + I$
D. $M^{-1} = M - I$
E. M^{-1} exists and can be determined from the data but the choices (c) and (d) are incorrect.

tifr2010 linear-algebra matrix

Answer key

11.2.2 Matrix: TIFR CSE 2010 | Part A | Question: 5 top

A is symmetric positive definite matrix (i.e., $x^T Ax > 0$ for all non zero x). Which of the following statements is false?

A. At least one element is positive.
B. All eigen values are positive real.
C. Sum of the diagonal elements is
D. *det* (A) is positive.

positive.

E. None of the above.

tifr2010 linear-algebra matrix

Answer key

11.2.3 Matrix: TIFR CSE 2012 | Part B | Question: 12 top

Let A be a matrix such that $A^k = 0$. What is the inverse of $I - A$?

A. 0

B. I

C. A

D. $1 + A + A^2 + \ldots + A^{k-1}$

E. Inverse is not guaranteed to exist.

tifr2012 linear-algebra matrix

Answer key

11.2.4 Matrix: TIFR CSE 2013 | Part B | Question: 3 top

How many 4×4 matrices with entries from $0, 1$ have odd determinant?

Hint: Use modulo 2 arithmetic.

A. 20160 B. 32767 C. 49152 D. 57343 E. 65520

tifr2013 linear-algebra matrix

Answer key

11.2.5 Matrix: TIFR CSE 2015 | Part A | Question: 14 top

Consider the following 3×3 matrices.

$$M_1 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$M_2 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

How may $0 - 1$ column vectors of the form

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

are there such that $M_1 X = M_2 X$ (modulo 2)? (modulo 2 means all operations are done modulo 2, i.e, $3 = 1$ (modulo 2), $4 = 0$ (modulo 2)).

A. None B. Two C. Three D. Four E. Eight

tifr2015 matrix

Answer key

11.2.6 Matrix: TIFR CSE 2018 | Part A | Question: 12 top

A n $n \times n$ matrix M with real entries is said to be positive definite if for every non-zero n-dimensional vector x with real entries, we have $x^T M x > 0$. Let A and B be symmetric, positive definite matrices of size $n \times n$ with real entries.

Consider the following matrices, where I denotes the $n \times n$ identity matrix:

1. $A+B$
2. ABA
3. A^2+I

Which of the above matrices must be positive definite?

A. Only (2)
B. Only (3)
C. Only (1) and (3)
D. None of the above matrices are positive definite
E. All of the above matrices are positive definite

tifr2018 matrix linear-algebra

Answer key

11.2.7 Matrix: TIFR CSE 2018 | Part A | Question: 14 top

Let A be an $n \times n$ invertible matrix with real entries whose row sums are all equal to c. Consider the following statements:

1. Every row in the matrix $2A$ sums to $2c$.
2. Every row in the matrix A^2 sums to c^2.
3. Every row in the matrix A^{-1} sums to c^{-1}.

Which of the following is **TRUE**?

A. none of the statements $(1), (2), (3)$ is correct
B. statement (1) is correct but not necessarily statements (2) or (3)
C. statement (2) is correct but not necessarily statements (1) or (3)
D. statement (1) and (2) are correct but not necessarily statement (3)
E. all the three statements $(1), (2)$, and (3) are correct

tifr2018 matrix linear-algebra

Answer key

11.2.8 Matrix: TIFR CSE 2020 | Part A | Question: 5 top

Let A be am $n \times n$ invertible matrix with real entries whose column sums are all equal to 1. Consider the following statements:

1. Every column in the matrix A^2 sums to 2
2. Every column in the matrix A^3 sums to 3
3. Every column in the matrix A^{-1} sums to 1

Which of the following is TRUE?

A. none of the statements $(1), (2), (3)$ is correct
B. statement (1) is correct but not statements (2) or (3)
C. statement (2) is correct but not statements (1) or (3)
D. statement (3) is correct but not statements (1) or (2)
E. all the 3 statements $(1), (2)$, and (3) are correct

tifr2020 engineering-mathematics linear-algebra matrix

Answer key

11.3 Rank Of Matrix (1) top

11.3.1 Rank Of Matrix: TIFR CSE 2020 | Part A | Question: 2 top

Let M be a real $n \times n$ matrix such that for $every$ non-zero vector $x \in \mathbb{R}^n$, we have $x^T Mx > 0$. Then

A. Such an M cannot exist
B. Such Ms exist and their rank is always n
C. Such Ms exist, but their eigenvalues are always real
D. No eigenvalue of any such M can be real
E. None of the above

tifr2020 engineering-mathematics linear-algebra rank-of-matrix eigen-value

Answer key

11.4 Vector Space (3) top

11.4.1 Vector Space: TIFR CSE 2010 | Part A | Question: 11 top

The length of a vector $X = (x_1, \ldots, x_n)$ is defined as

$$||X|| = \sqrt{\sum_{i=1}^{n} x_i^2}$$

Given two vectors $X = (x_1, \ldots, x_n)$ and $Y = (y_1, \ldots, y_n)$, which of the following measures of discrepancy between X and Y is insensitive to the length of the vectors?

A. $||X - Y||$
B. $||X - Y|| \, / \, ||x|| \, ||y||$
C. $||X|| - ||Y||$
D. $\left|\left| \frac{X}{||X||} - \frac{Y}{||Y||} \right|\right|$
E. None of the above

tifr2010 linear-algebra vector-space

Answer key

11.4.2 Vector Space: TIFR CSE 2017 | Part A | Question: 2 top

For vectors x, y in $\mathbb{R}^n$, define the inner product $\langle x, y \rangle = \Sigma_{i=1}^{n} x_i y_i$, and the length of x to be $||x|| = \sqrt{\langle x, x \rangle}$. Let a, b be two vectors in $\mathbb{R}^n$ so that $||b|| = 1$. Consider the following statements:

i. $\langle a, b \rangle \leq ||b||$
ii. $\langle a, b \rangle \leq ||a||$
iii. $\langle a, b \rangle = ||a|| \, ||b||$
iv. $\langle a, b \rangle \geq ||b||$
v. $\langle a, b \rangle \geq ||a||$

Which of the above statements must be TRUE of a, b? Choose from the following options.

A. ii only
B. i and ii
C. iii only
D. iv only
E. iv and v

tifr2017 linear-algebra vector-space

Answer key

11.4.3 Vector Space: TIFR CSE 2020 | Part A | Question: 3 top

Let $d \geq 4$ and fix $w \in \mathbb{R}$. Let

$$S = \{a = (a_0, a_1, \dots, a_d) \in \mathbb{R}^{d+1} \mid f_a(w) = 0 \text{ and } f_a'(w) = 0\},$$

where the polynomial function $f_a(x)$ is defined as $f_a(x) := \sum_{i=0}^{d} a_i x^i$ and $f_a'(w)$ denotes the derivative of $f_a(x)$ with respect to x, evaluated at w. Then,

A. S is finite or infinite depending on the value of α
B. S is a 2-dimensional vector subspace of $\mathbb{R}^{d+1}$
C. S is a d-dimensional vector subspace of $\mathbb{R}^{d+1}$
D. S is a $(d-1)$-dimensional vector subspace of $\mathbb{R}^{d+1}$
E. None of the other options

tifr2020 engineering-mathematics linear-algebra vector-space

Answer Keys

11.0.1	C	11.1.1	C	11.2.1	D	11.2.2	E	11.2.3	D
11.2.4	A	11.2.5	B	11.2.6	E	11.2.7	E	11.2.8	D
11.3.1	B	11.4.1	D	11.4.2	A	11.4.3	D		

12.0.1 TIFR CSE 2020 | Part A | Question: 11 top

Suppose we toss $m = 3$ labelled balls into $n = 3$ numbered bins. Let A be the event that the first bin is empty while B be the event that the second bin is empty. $P(A)$ and $P(B)$ denote their respective probabilities. Which of the following is true?

A. $P(A) > P(B)$
B. $P(A) = \frac{1}{27}$
C. $P(A) > P(A \mid B)$
D. $P(A) < P(A \mid B)$
E. None of the above

tifr2020

Answer key

12.0.2 TIFR CSE 2020 | Part A | Question: 10 top

In a certain year, there were exactly four Fridays and exactly four Mondays in January. On what day of the week did the 20^{th} of the January fall that year (recall that January has 31 days)?

A. Sunday
B. Monday
C. Wednesday
D. Friday
E. None of the others

tifr2020 engineering-mathematics probability

Answer key

12.0.3 TIFR CSE 2013 | Part A | Question: 13 top

Doctors A and B perform surgery on patients in stages III and IV of a disease. Doctor A has performed a 100 surgeries (on 80 stage III and 20 stage IV patients) and 80 out of her 100 patients have survived (78 stage III and 2 stage IV survivors). Doctor B has also performed 100 surgeries (on 50 stage III and 50 stage IV patients). Her success rate is $\frac{600}{100}$ (49 stage III survivors and 11 stage IV survivors).A patient has been advised that she is equally likely to be suffering from stage III or stage IV of this disease. Which doctor would you recommend to this patient and why?

A. Doctor A since she has a higher success rate
B. Doctor A since she specializes in stage III patients and the success of surgery in stage IV patients is anyway too low
C. Doctor B since she has performed more stage IV surgeries
D. Doctor B since she appears to be more successful
E. There is not enough data since the choice depends on the stage of the disease the patient is suffering from.

tifr2013 probability

Answer key

12.0.4 TIFR CSE 2013 | Part A | Question: 4 top

A biased coin is tossed repeatedly. Assume that the outcomes of different tosses are independent and probability of heads is $\dfrac{2}{3}$ in each toss. What is the probability of obtaining an even number of heads in 5 tosses, zero being treated as an even number?

A. $\left(\frac{121}{243}\right)$ B. $\left(\frac{122}{243}\right)$ C. $\left(\frac{124}{243}\right)$ D. $\left(\frac{125}{243}\right)$ E. $\left(\frac{128}{243}\right)$

tifr2013 probability

Answer key

12.0.5 TIFR CSE 2012 | Part A | Question: 20 top

There are 1000 balls in a bag, of which 900 are black and 100 are white. I randomly draw 100 balls from the bag. What is the probability that the 101st ball will be black?

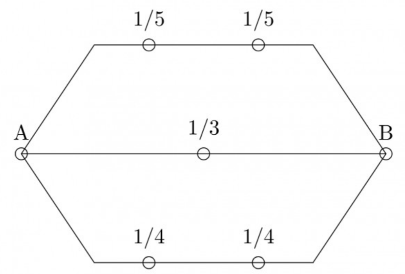
A. $9/10$
B. More than $9/10$ but less than 1.
C. Less than $9/10$ but more than 0.
D. 0
E. 1

tifr2012 probability

Answer key

12.0.6 TIFR CSE 2012 | Part A | Question: 19 top

An electric circuit between two terminals A and B is shown in the figure below, where the numbers indicate the probabilities of failure for the various links, which are all independent.



What is the probability that A and B are connected?

A. $\left(\frac{6}{25}\right)$ B. $\left(\frac{379}{400}\right)$ C. $\left(\frac{1}{1200}\right)$ D. $\left(\frac{1199}{1200}\right)$ E. $\left(\frac{59}{60}\right)$

tifr2012 probability

Answer key

12.0.7 TIFR CSE 2012 | Part A | Question: 17 top

A spider is at the bottom of a cliff, and is n inches from the top. Every step it takes brings it one inch closer to the top with probability $1/3$, and one inch away from the top with probability $2/3$, unless it is at the bottom in which case, it always gets one inch closer. What is the expected number of steps for the spider to reach the top as a function of n?

A. It will never reach the top.
B. Linear in n.
C. Polynomial in n.
D. Exponential in n.
E. Double exponential in n.

tifr2012 probability

Answer key

12.0.8 TIFR CSE 2012 | Part A | Question: 9 top

The probability of throwing six perfect dices and getting six different faces is

A. $1-\frac{6!}{6^6}$ B. $\frac{6!}{6^6}$ C. 6^{-6} D. $1-6^{-6}$ E. None of the above

tifr2012 probability

Answer key

12.0.9 TIFR CSE 2013 | Part A | Question: 14 top

An unbiased die is thrown n times. The probability that the product of numbers would be even is

A. $\dfrac{1}{(2n)}$ B. $\dfrac{1}{[(6n)!]}$ C. $1-6^{-n}$ D. 6^{-n} E. None of the above

tifr2013 probability

Answer key

12.0.10 TIFR CSE 2011 | Part A | Question: 9 top

You have to play three games with opponents A and B in a specified sequence. You win the series if you win two consecutive games. A is a stronger player than B. Which sequence maximizes your chance of winning the series?

A. AAB B. ABA C. BAB D. BAA E. All are the same.

tifr2011 probability

Answer key

12.0.11 TIFR CSE 2010 | Part A | Question: 13 top

A cube whose faces are colored is split into 1000 small cubes of equal size. The cubes thus obtained are mixed thoroughly. The probability that a cube drawn at random will have exactly two colored faces is:

A. 0.096 B. 0.12 C. 0.104 D. 0.24 E. None of the above

tifr2010 probability

Answer key

12.0.12 TIFR CSE 2011 | Part A | Question: 19 top

Three dice are rolled independently. What is the probability that the highest and the lowest value differ by 4?

A. $\left(\dfrac{1}{3}\right)$ B. $\left(\dfrac{1}{6}\right)$ C. $\left(\dfrac{1}{9}\right)$ D. $\left(\dfrac{5}{18}\right)$ E. $\left(\dfrac{2}{9}\right)$

tifr2011 probability

Answer key

12.0.13 TIFR CSE 2018 | Part A | Question: 10 top

Let C be a biased coin such that the probability of a head turning up is p. Let p_n denote the probability that an odd number of heads occurs after n tosses for $n \in \{0,1,2,\ldots\}$, Then which of the following is TRUE ?

A. $p_n = \dfrac{1}{2}$ for all $n \in \{0,1,2,\ldots\}$.

B. $p_n = (1-p)(1-p_{n-1}) + p.p_{n-1}$ for $n \geq 1$ and $p_0 = 0$.

C. $p_n = \sum_{i=1}^{n} p(1-2p)^{i-1}$ for $n \geq 1$.

D. If $p = \dfrac{1}{2}$, then $p_n = \dfrac{1}{2}$ for all $n \in \{0,1,2,\ldots\}$.

E. $p_n = 1$ if n is odd and 0 otherwise.

tifr2018 probability

Answer key

12.0.14 TIFR CSE 2013 | Part A | Question: 17 top

A stick of unit length is broken into two at a point chosen at random. Then, the larger part of the stick is further divided into two parts in the ratio $4:3$. What is the probability that the three sticks that are left CANNOT form a triangle?

A. $1/4$ B. $1/3$ C. $5/6$ D. $1/2$ E. $\log_e(2)/2$

tifr2013 probability

Answer key

12.0.15 TIFR CSE 2013 | Part B | Question: 10 top

Let m, n be positive integers with m a power of 2. Let $s = 100n^2 \log m$. Suppose $S_1, S_2, \ldots, S_m$ are subsets of $1, 2, \ldots, s$ such that $|S_i| = 10n \log m$ and $|S_i \cap S_j| \leq \log m$ for all $1 \leq i < j \leq m$. Such a collection of sets $S_1, \ldots, S_m$ is an example of a so-called Nisan-Wigderson design. We now consider the set membership problem, where we have to store an arbitrary subset $T \subseteq \{1, 2, \ldots, m\}, |T| = n$ as an array A of s bits so that given any integer $x, 1 \leq x \leq m$, we can discover whether $x \in T$ by reading only one bit of A. Consider the following strategy to solve this problem. Array A is initialized to all zeroes. Given the set T to be stored, we put a one in all the locations of A indexed by the union $\cup_{t \in T} S_t$. Now, given the integer x, we read a random location in A from S_x and declare that $x \in T$ if the bit in that location is one. This strategy gives the correct answer with probability

A. 1 if $x \in T$ and at most 0.1 if $x \notin T$.
B. At least 0.9 if $x \in T$ and at most 0.1 if $x \notin T$.
C. At least 0.9 if $x \in T$ and at least 0.9 if $x \notin T$.
D. 1 if $x \in T$ and at least 0.9 if $x \notin T$.
E. At least 0.9 if $x \in T$ and 1 if $x \notin T$.

tifr2013 probability

Answer key

12.0.16 TIFR CSE 2010 | Part A | Question: 10 top

A drawer contains 2 Blue, 4 Red and 2 Yellow balls. No two balls have the same radius. If two balls are randomly selected from the drawer, what is the probability that they will be of the same colour?

A. $\left(\frac{2}{7}\right)$ B. $\left(\frac{2}{5}\right)$
C. $\left(\frac{3}{7}\right)$ D. $\left(\frac{1}{2}\right)$
E. $\left(\frac{3}{5}\right)$

tifr2010 probability

Answer key

12.0.17 TIFR CSE 2018 | Part A | Question: 15 top

Suppose a box contains 20 balls: each ball has a distinct number in $\{1, \ldots, 20\}$ written on it. We pick 10 balls (without replacement) uniformly at random and throw them out of the box. Then we check if the ball with number "1" on it is present in the box. If it is present, then we throw it out of the box; else we pick a ball from the box uniformly at random and throw it out of the box.

What is the probability that the ball with number "2" on it is present in the box?

A. $9/20$ B. $9/19$ C. $1/2$ D. $10/19$ E. None of the above

tifr2018 probability

Answer key

12.0.18 TIFR CSE 2017 | Part A | Question: 9 top

Consider the *majority* function on three bits, $\mathbf{maj}: \{0,1\}^3 \to \{0,1\}$ where $\mathbf{maj}(x_1, x_2, x_3) = 1$ if and only if $x_1 + x_2 + x_3 \geq 2$. Let $p(\alpha)$ be the probability that the output is 1 when each input is set to 1 independently with probability α. What is $p'(\alpha) = \frac{d}{d\alpha}p(\alpha)$?

A. 3α
B. α^2
C. $6\alpha(1-\alpha)$
D. $3\alpha^2(1-\alpha)$
E. $6\alpha(1-\alpha)+\alpha^2$

tifr2017 probability

Answer key

12.0.19 TIFR CSE 2015 | Part A | Question: 1 top

Consider a 6-sided die with all sides not necessarily equally likely such that probability of an even number is $P(\{2,4,6\}) = \frac{1}{2}$, probability of a multiple of 3 is $P(\{3,6\}) = 1/3$ and probability of 1 is $P(\{1\}) = \frac{1}{6}$. Given the above conditions, choose the strongest (most stringent) condition of the following that must always hold about $P(\{5\})$, the probability of 5.

A. $P(\{5\}) = \frac{1}{6}$
B. $P(\{5\}) \geq \frac{1}{6}$
C. $P(\{5\}) \leq \frac{1}{6}$
D. $P(\{5\}) \leq \frac{1}{3}$
E. None of the above

tifr2015 probability

Answer key

12.0.20 TIFR CSE 2019 | Part A | Question: 4 top

What is the probability that a point $P = (\alpha, \beta)$ picked uniformly at random from the disk $x^2 + y^2 \leq 1$ satisfies $\alpha + \beta \leq 1$?

A. $\frac{1}{\pi}$
B. $\frac{3}{4} + \frac{1}{4} \cdot \frac{1}{\pi}$
C. $\frac{3}{4} + \frac{1}{4} \cdot \frac{2}{\pi}$
D. 1
E. $\frac{2}{\pi}$

tifr2019 engineering-mathematics discrete-mathematics probability

Answer key

12.0.21 TIFR CSE 2019 | Part A | Question: 14 top

A drawer contains 9 pens, of which 3 are red, 3 are blue, and 3 are green. The nine pens are drawn from the drawer one at at time (without replacement) such that each pen is drawn with equal probability from the remaining pens in the drawer. What is the probability that two red pens are drawn in succession ?

A. $7/12$ B. $1/6$ C. $1/12$ D. $1/81$ E. None of the above

tifr2019 engineering-mathematics probability

Answer key

12.0.22 TIFR CSE 2018 | Part A | Question: 13 top

A hacker knows that the password to the TIFR server is 10-letter string consisting of lower-case letters from the English alphabet. He guesses a set of 5 distinct 10-letter strings (with lower-case letters) uniformly at random. What is the probability that one of the guesses of the hacker is correct password?

A. $\frac{5}{(26)^{10}}$
B. $1 - \left(1 - \frac{1}{(26)^{10}}\right)^5$
C. $1 - \left\{\left(\frac{(26)^{10}-1}{(26)^{10}}\right)\left(\frac{(26)^{10}-2}{(26)^{10}}\right)\left(\frac{(26)^{10}-3}{(26)^{10}}\right)\left(\frac{(26)^{10}-4}{(26)^{10}}\right)\left(\frac{(26)^{10}-5}{(26)^{10}}\right)\right\}$
D. $\frac{1}{(26)^{10}}$
E. None of the above

tifr2018 probability

Answer key

12.0.23 TIFR CSE 2016 | Part A | Question: 12 top

There are two rocks A and B, located close to each other, in a lily pond. There is a frog that jumps randomly between the two rocks at time $t = 0, 1, 2, \ldots$. The location of the frog is determined as follows. Initially, at time $t = 0$, the frog is at A. From then on, the frog's location is determined as follows. If the frog is at A at time t, then at time $t + 1$, with probability $2/3$ it jumps to B and with probability $1/3$, it jumps on the spot and stays at A. If the frog is at B at time t, then at time $t + 1$, with probability $1/2$ it jumps to A and with probability $1/2$ it jumps on the spot and stays at B. What is the probability that the frog is at B at time 3 (just after its third jump)?

A. $\frac{1}{2}$ B. $\frac{31}{54}$ C. $\frac{14}{27}$ D. $\frac{61}{108}$ E. $\frac{2}{3}$

tifr2016 probability

Answer key

12.1 Binomial Distribution (3) top

12.1.1 Binomial Distribution: TIFR CSE 2010 | Part A | Question: 6 top

Given 10 tosses of a coin with probability of head = $.4$ = (1 - the probability of tail), the probability of at least one head is?

A. $(.4)^{10}$ B. $1 - (.4)^{10}$ C. $1 - (.6)^{10}$ D. $(.6)^{10}$ E. $10(.4)(.6)^9$

tifr2010 probability binomial-distribution

Answer key

12.1.2 Binomial Distribution: TIFR CSE 2010 | Part B | Question: 38 top

Suppose three coins are lying on a table, two of them with heads facing up and one with tails facing up. One coin is chosen at random and flipped. What is the probability that after the flip the majority of the coins(i.e., at least two of them) will have heads facing up?

A. $\left(\frac{1}{3}\right)$ B. $\left(\frac{1}{8}\right)$ C. $\left(\frac{1}{4}\right)$ D. $\left(\frac{1}{4}+\frac{1}{8}\right)$ E. $\left(\frac{2}{3}\right)$

tifr2010 probability binomial-distribution

Answer key

12.1.3 Binomial Distribution: TIFR CSE 2011 | Part A | Question: 3 top

The probability of three consecutive heads in four tosses of a fair coin is

A. $\left(\dfrac{1}{4}\right)$ B. $\left(\dfrac{1}{8}\right)$ C. $\left(\dfrac{1}{16}\right)$ D. $\left(\dfrac{3}{16}\right)$ E. None of the above

tifr2011 probability binomial-distribution

Answer key

12.2 Conditional Probability (3) top

12.2.1 Conditional Probability: TIFR CSE 2010 | Part A | Question: 19, TIFR CSE 2014 | Part A | Question: 6 top

Karan tells truth with probability $\dfrac{1}{3}$ and lies with probability $\dfrac{2}{3}$. Independently, Arjun tells truth with probability $\dfrac{3}{4}$ and lies with probability $\dfrac{1}{4}$. Both watch a cricket match. Arjun tells you that India won, Karan tells you that India lost. What probability will you assign to India's win?

A. $\left(\dfrac{1}{2}\right)$ B. $\left(\dfrac{2}{3}\right)$ C. $\left(\dfrac{3}{4}\right)$ D. $\left(\dfrac{5}{6}\right)$ E. $\left(\dfrac{6}{7}\right)$

tifr2010 probability conditional-probability tifr2014

Answer key

12.2.2 Conditional Probability: TIFR CSE 2012 | Part A | Question: 1 top

Amar and Akbar both tell the truth with probability $\dfrac{3}{4}$ and lie with probability $\dfrac{1}{4}$. Amar watches a test match and talks to Akbar about the outcome. Akbar, in turn, tells Anthony, "Amar told me that India won". What probability should Anthony assign to India's win?

A. $\left(\dfrac{9}{16}\right)$ B. $\left(\dfrac{6}{16}\right)$ C. $\left(\dfrac{7}{16}\right)$ D. $\left(\dfrac{10}{16}\right)$ E. None of the above

tifr2012 probability conditional-probability

Answer key

12.2.3 Conditional Probability: TIFR CSE 2013 | Part A | Question: 6 top

You are lost in the National park of Kabrastan. The park population consists of tourists and Kabrastanis. Tourists comprise two-thirds of the population the park and give a correct answer to requests for directions with probability $\dfrac{3}{4}$. The air of Kabrastan has an amnesaic quality, however, and so the answers to repeated questions to tourists are independent, even if the question and the person are the same. If you ask a Kabrastani for directions, the answer is always wrong.

Suppose you ask a randomly chosen passer-by whether the exit from the park is East or West. The answer is East. You then ask the same person again, and the reply is again East. What is the probability of East being correct?

A. $\left(\dfrac{1}{4}\right)$ B. $\left(\dfrac{1}{3}\right)$ C. $\left(\dfrac{1}{2}\right)$ D. $\left(\dfrac{2}{3}\right)$ E. $\left(\dfrac{3}{4}\right)$

tifr2013 probability conditional-probability

Answer key

12.3 Expectation (5) top

12.3.1 Expectation: TIFR CSE 2011 | Part A | Question: 6 top

Assume that you are flipping a fair coin, i.e. probability of heads or tails is equal. Then the expected number of coin flips required to obtain two consecutive heads for the first time is.

a. 4
b. 3
c. 6
d. 10
e. 5

tifr2011 probability expectation

Answer key

12.3.2 Expectation: TIFR CSE 2012 | Part B | Question: 7 top

A bag contains 16 balls of the following colors: 8 red, 4 blue, 2 green, 1 black, and 1 white. Anisha picks a ball randomly from the bag, and messages Babu its color using a string of zeros and ones. She replaces the ball in the bag, and repeats this experiment, many times. What is the minimum expected length of the message she has to convey to Babu per experiment?

A. $\frac{3}{2}$
B. $\log 5$
C. $\frac{15}{8}$
D. $\frac{31}{16}$
E. 2

tifr2012 probability expectation

Answer key

12.3.3 Expectation: TIFR CSE 2014 | Part A | Question: 17 top

A fair dice (with faces numbered $1, \dots, 6$) is independently rolled repeatedly. Let X denote the number of rolls till an even number is seen and let Y denote the number of rolls till 3 is seen. Evaluate $E(Y|X=2)$.

A. $6\frac{5}{6}$
B. 6
C. $5\frac{1}{2}$
D. $6\frac{1}{3}$
E. $5\frac{2}{3}$

tifr2014 expectation

Answer key

12.3.4 Expectation: TIFR CSE 2015 | Part A | Question: 6 top

Ram has a fair coin, i.e., a toss of the coin results in either head or tail and each event happens with probability exactly half $(1/2)$. He repeatedly tosses the coin until he gets heads in two consecutive tosses. The expected number of coin tosses that Ram does is.

A. 2
B. 4
C. 6
D. 8
E. None of the above

tifr2015 expectation

Answer key

12.3.5 Expectation: TIFR CSE 2020 | Part A | Question: 1 top

Two balls are drawn uniformly at random without replacement from a set of five balls numbered $1, 2, 3, 4, 5$. What is the expected value of the larger number on the balls drawn?

A. 2.5
B. 3
C. 3.5
D. 4
E. None of the above

tifr2020 engineering-mathematics probability expectation

Answer key☟

12.4 Independent Events (1) top☝

12.4.1 Independent Events: TIFR CSE 2020 | Part A | Question: 7 top☝

A lottery chooses four random winners. What is the probability that at least three of them are born on the same day of the week? Assume that the pool of candidates is so large that each winner is equally likely to be born on any of the seven days of the week independent of the other winners.

A. $\dfrac{17}{2401}$

B. $\dfrac{48}{2401}$

C. $\dfrac{105}{2401}$

D. $\dfrac{175}{2401}$

E. $\dfrac{294}{2401}$

tifr2020 engineering-mathematics probability independent-events

Answer key☟

12.5 Random Variable (2) top☝

12.5.1 Random Variable: TIFR CSE 2011 | Part A | Question: 7 top☝

Let X and Y be two independent and identically distributed random variables. Then $P\left(X>Y\right)$ is.

A. $\frac{1}{2}$

B. 1

C. 0

D. $\frac{1}{3}$

E. Information is insufficient.

tifr2011 probability random-variable

Answer key☟

12.5.2 Random Variable: TIFR CSE 2014 | Part A | Question: 19 top☝

Consider the following random function of x

$F(x)=1+Ux+Vx^2 \bmod 5,$

where U and V are independent random variables uniformly distributed over $\{0,1,2,3,4\}$. Which of the following is FALSE?

A. $F(1)$ is uniformly distributed over $\{0,1,2,3,4\}$.

B. $F(1), F(2)$ are independent random variables and both are uniformly distributed over $\{0,1,2,3,4\}$.

C. $F(1), F(2), F(3)$ are independent and identically distributed random variables.

D. All of the above.

E. None of the above.

tifr2014 probability random-variable

Answer key☟

12.6 Uniform Distribution (3) top☝

12.6.1 Uniform Distribution: TIFR CSE 2013 | Part A | Question: 18 top☝

Consider three independent uniformly distributed (taking values between 0 and 1) random variables. What is the probability that the middle of the three values (between the lowest and the highest value) lies between a and b where $0 \leq a < b \leq 1$?

A. $3(1-b)a(b-a)$
B. $3((b-a)-(b^2-a^2)/2)$
C. $6(1-b)a(b-a)$
D. $(1-b)a(b-a)$
E. $6((b^2-a^2)/2-(b^3-a^3)/3)$.

tifr2013 probability random-variable uniform-distribution

Answer key

12.6.2 Uniform Distribution: TIFR CSE 2015 | Part A | Question: 12 top

Consider two independent and identically distributed random variables X and Y uniformly distributed in $[0,1]$. For $\alpha \in [0,1]$, the probability that $\alpha \max(X,Y) < XY$ is

A. $1/(2\alpha)$
B. $\exp(1-\alpha)$
C. $1-\alpha$
D. $(1-\alpha)^2$
E. $1-\alpha^2$

tifr2015 probability random-variable uniform-distribution

Answer key

12.6.3 Uniform Distribution: TIFR CSE 2020 | Part A | Question: 4 top

Fix $n \geq 4$. Suppose there is a particle that moves randomly on the number line, but never leaves the set $\{1,2,\ldots,n\}$. Let the initial probability distribution of the particle be denoted by $\overrightarrow{\pi}$. In the first step, if the particle is at position i, it moves to one of the positions in $\{1,2,\ldots,i\}$ with uniform distribution; in the second step, if the particle is in location j, then it moves to one of the locations in $\{j,j+1,\ldots,n\}$ with uniform distribution.Suppose after two steps, the final ditribution of the particle is uniform. What is the initial distribution $\overrightarrow{\pi}$?

A. $\overrightarrow{\pi}$ is not unique
B. $\overrightarrow{\pi}$ is uniform
C. $\overrightarrow{\pi(i)}$ is non-zero for all even i and zero otherwise
D. $\overrightarrow{\pi}(1)=1$ and $\overrightarrow{\pi}(i)=0$ for $i \neq 1$
E. $\overrightarrow{\pi}(n)=1$ and $\overrightarrow{\pi}(i)=0$ for $i \neq n$

tifr2020 engineering-mathematics probability uniform-distribution

Answer Keys

12.0.1	C	12.0.2	A	12.0.3	D	12.0.4	B	12.0.5	A
12.0.6	B	12.0.7	D	12.0.8	B	12.0.9	E	12.0.10	B
12.0.11	A	12.0.12	E	12.0.13	C	12.0.14	A	12.0.15	D
12.0.16	A	12.0.17	B	12.0.18	C	12.0.19	D	12.0.20	C
12.0.21	A	12.0.22	A	12.0.23	B	12.1.1	C	12.1.2	E
12.1.3	D	12.2.1	E	12.2.2	D	12.2.3	C	12.3.1	C
12.3.2	C	12.3.3	E	12.3.4	C	12.3.5	D	12.5.1	E
12.5.2	C	12.6.1	E	12.6.2	D	12.6.3	D		

13 General Aptitude: Analytical Aptitude (10)

13.1 Logical Reasoning (10) top

13.1.1 Logical Reasoning: TIFR CSE 2010 | Part A | Question: 1 top

A box contains 731 black balls and 2000 white balls. The following process is to be repeated as long as possible. Arbitrarily select two balls from the box. If they are of the same color, throw them out and put a black ball into the box (enough extra black balls are available to do this). If they are of different color, place the white ball back into the box and throw the black ball away. Which of the following is correct?

A. The process can be applied indefinitely without any prior bound
B. The process will stop with a single white ball in the box
C. The process will stop with a single black ball in the box
D. The process will stop with the box empty
E. None of the above

tifr2010 analytical-aptitude logical-reasoning

Answer key

13.1.2 Logical Reasoning: TIFR CSE 2013 | Part A | Question: 10 top

Three men and three rakhsasas arrive together at a ferry crossing to find a boat with an oar, but no boatman. The boat can carry one or at the most two persons, for example, one man and one rakhsasas, and each man or rakhsasas can row. But if at any time, on any bank, (including those who maybe are in the boat as it touches the bank) rakhsasas outnumber men, the former will eat up the latter. If all have to go to the other side without any mishap, what is the minimum number of times that the boat must cross the river?

A. 7 B. 9 C. 11 D. 13 E. 15

tifr2013 analytical-aptitude logical-reasoning

Answer key

13.1.3 Logical Reasoning: TIFR CSE 2013 | Part A | Question: 11 top

Let there be a pack of 100 cards numbered 1 to 100. The i^{th} card states: "There are at most $i-1$ true cards in this pack". Then how many cards of the pack contain TRUE statements?

A. 0 B. 1 C. 100 D. 50 E. None of the above

tifr2013 logical-reasoning

Answer key

13.1.4 Logical Reasoning: TIFR CSE 2013 | Part A | Question: 2 top

Consider the following two types of elections to determine which of two parties A and B forms the next government in the 2014 Indian elections. Assume for simplicity an Indian population of size $545545 (= 545 * 1001)$. There are only two parties A and B and every citizen votes.

TYPE C: The country is divided into 545 constituencies and each constituency has 1001 voters. Elections are held for each constituency and a party is said to win a constituency if it receive a majority of the vote in that constituency. The party that wins the most constituencies forms the next government.

TYPE P: There are no constituencies in this model. Elections are held throughout the country and the party that wins the most votes (among 545545 voters forms the government.

Which of the following is true?

A. If the party forms the govt. by election TYPE C winning at least two-third of the constituencies, then it will also forms the govt. by election TYPE P.
B. If a party forms govt. by election TYPE C, then it will also form the govt. by election TYPE P.
C. If a party forms govt. by election TYPE P, then it will also form the govt. by election TYPE C.
D. All of the above
E. None of the above

tifr2013 logical-reasoning

Answer key

13.1.5 Logical Reasoning: TIFR CSE 2016 | Part A | Question: 1 top

Suppose the following statements about three persons in a room are true.

Chandni, Sooraj and Tara are in a room. Nobody else is in the room. Chandni is looking at Sooraj. Sooraj is looking at Tara. Chandni is married. Tara is not married. A married person in the room is looking at an unmarried person.

Then, Which of the following is necessarily true?

A. Sooraj is married
B. Sooraj is unmarried
C. The situation described is impossible
D. There is insufficient information to conclude if Sooraj is married or unmarried
E. None of the above

tifr2016 logical-reasoning

Answer key

13.1.6 Logical Reasoning: TIFR CSE 2017 | Part A | Question: 14 top

Consider the following game with two players, Aditi and Bharat. There are n tokens in a bag. The two players know n, and take turns removing tokens from the bag. In each turn, a player can either remove one token or two tokens. The player that removes the last token from the bag loses. Assume that Aditi always goes first. Further, we say that a player has a winning strategy if she or he can win the game, no matter what other player does. Which of the following statements is TRUE?

A. For $n = 3$, Bharath has a winning strategy. For $n = 4$, Aditi has a winning strategy.
B. For $n = 7$, Bharath has a winning strategy. For $n = 8$, Aditi has a winning strategy.
C. For both $n = 3$ and $n = 4$, Aditi has a winning strategy.
D. For both $n = 7$ and $n = 8$, Bharat has a winning strategy.
E. Bharat never has a winning strategy.

tifr2017 analytical-aptitude logical-reasoning

Answer key

13.1.7 Logical Reasoning: TIFR CSE 2018 | Part A | Question: 11 top

We are given a (possibly empty) set of objects. Each object in the set is colored either black or white, is shaped either circular or rectangular, and has a profile that is either fat or thin, Those properties obey the following principles:

1. Each white object is also circular.
2. Not all thin objects are black.

3. Each rectangular object is also either thin or white or both thin and white.

Consider the following statements:

i. If there is a thin object in the set, then there is also a white object.
ii. If there is a rectangular object in the set, then there are at least two objects.
iii. Every fat object in the set is circular.

Which of the above statements must be TRUE for the set?

A. (i) only
B. (i) and (ii) only
C. (i) and (iii) only
D. None of the statements must be TRUE
E. All of the statements must be TRUE

tifr2018 analytical-aptitude logical-reasoning

Answer key

13.1.8 Logical Reasoning: TIFR CSE 2018 | Part A | Question: 8 top

A crime has been committed with four people at the scene of the crime. You are responsible for finding out who did it. You have recorded the following statements from the four witnesses, and you know one of them has committed the crime.

1. Anuj says that Binky did it.
2. Binky says that Anuj did it.
3. Chacko says that Binky is telling the truth.
4. Desmond says that Chacko is not lying.

You know that exactly three of the statements recorded are FALSE. Who committed the crime?

A. Anuj
B. Binky
C. Chacko
D. Desmond
E. Either Anuj or Binky; the information is insufficient to pinpoint the criminal

tifr2018 logical-reasoning

Answer key

13.1.9 Logical Reasoning: TIFR CSE 2019 | Part A | Question: 10 top

Avni and Badal alternately choose numbers from the set $\{1,2,3,4,5,6,7,8,9\}$ without replacement (starting with Avni). The first person to choose numbers of which any 3 sum to 15 wins the game (for example, Avni wins if she chooses the numbers $8,3,5,2$ since $8+5+2=15$). A player is said to have a winning strategy if the player can always win the game, no matter what the other player does. Which of the following statements is TRUE?

As a hint, there are exactly 8 ways in which 3 numbers from the set $\{1,2,3,4,5,6,7,8,9\}$ can sum up to 15, shown as the three rows, the three columns, and the two diagonals in the following square:

$$\begin{array}{ccc} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{array}$$

A. Avni has a winning strategy
B. Badal has a winning strategy

C. Both of them have a winning strategy
D. Neither of them has a winning strategy
E. The Player that picks 9 has a winning strategy

tifr2019 general-aptitude analytical-aptitude logical-reasoning

Answer key☟

13.1.10 Logical Reasoning: TIFR CSE 2019 | Part A | Question: 11 top☝

Suppose there are n guests at a party (and no hosts). As the night progresses, the guests meet each other and shake hands. The same pair of guests might shake hands multiple times. for some parties stretch late into the night , and it is hard to keep track.Still, they don't shake hands with themselves. Let Odd be the set of guests who have shaken an odd number of hands, and let even be the set of guests who have shaken an even number of hands. Which of the following stays invariant throughout the night?

A. $|\text{Odd }| \mod 2$
B. $|\text{Even}|$
C. $|\text{Even }|-|\text{ Odd}|$
D. $2 \mid \text{Even }|-|\text{ Odd}|$
E. $2 \mid \text{Odd }|-|\text{ Even}|$

tifr2019 general-aptitude analytical-aptitude logical-reasoning

Answer key☟

Answer Keys

13.1.1	C	13.1.2	C	13.1.3	D	13.1.4	E	13.1.5	D
13.1.6	B	13.1.7	E	13.1.8	B	13.1.9	D	13.1.10	A

14 General Aptitude: Quantitative Aptitude (55)

14.0.1 TIFR CSE 2020 | Part A | Question: 14 top

A ball is thrown directly upwards from the ground at a speed of $10\,ms^{-1}$, on a planet where the gravitational acceleration is $10\,ms^{-2}$. Consider the following statements:

1. The ball reaches the ground exactly 2 seconds after it is thrown up
2. The ball travels a total distance of 10 metres before it reaches the ground
3. The velocity of the ball when it hits the ground is $10\,ms^{-1}$

What can you say now?

A. Only Statement 1 is correct
B. Only Statement 2 is correct
C. Only Statement 3 is correct
D. None of the Statements $1, 2$ or 3 is correct
E. All of the Statements $1, 2$ and 3 are correct

tifr2020

Answer key

14.1 Cartesian Coordinates (3) top

14.1.1 Cartesian Coordinates: TIFR CSE 2013 | Part B | Question: 9 top

Suppose n straight lines are drawn on a plane. When these lines are removed, the plane falls apart into several connected components called regions. A region R is said to be convex if it has the following property: whenever two points are in R, then the entire line segment joining them is in R. Suppose no two of the n lines are parallel. Which of the following is true?

A. $O(n)$ regions are produced, and each region is convex.
B. $O(n^2)$ regions are produced but they need not all be convex.
C. $O(n^2)$ regions are produced, and each region is convex.
D. $O(n \log n)$ regions are produced, but they need not all be convex.
E. All regions are convex but there may be exponentially many of them.

tifr2013 quantitative-aptitude geometry cartesian-coordinates

Answer key

14.1.2 Cartesian Coordinates: TIFR CSE 2014 | Part A | Question: 13 top

Let L be a line on the two dimensional plane. L's intercepts with the X and Y axes are respectively a and b. After rotating the co-ordinate system (and leaving L untouched), the new intercepts are a' and b' respectively. Which of the following is TRUE?

A. $\frac{1}{a}+\frac{1}{b}=\frac{1}{a}+\frac{1}{b}$.
B. $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{a'^2}+\frac{1}{b'^2}$.
C. $\frac{b}{a^2}+\frac{a}{b^2}=\frac{b'}{a'^2}+\frac{a}{b'^2}$.
D. $\frac{b}{a}+\frac{a}{b}=\frac{b'}{a'}+\frac{a'}{b'}$.
E. None of the above.

tifr2014 geometry cartesian-coordinates

Answer key

14.1.3 Cartesian Coordinates: TIFR CSE 2015 | Part A | Question: 13 top

Imagine the first quadrant of the real plane as consisting of unit squares. A typical square has 4 corners: $(i, j), (i+1, j), (i+1, j+1)$,and $(i, j+1)$, where (i, j) is a pair of non-negative integers. Suppose a line segment l connecting $(0, 0)$ to $(90, 1100)$ is drawn. We say that l passes through a unit square if it passes through a point in the interior of the square. How many unit squares does l pass through?

A. $98{,}990$ B. $9{,}900$ C. $1{,}190$ D. $1{,}180$ E. $1{,}010$

tifr2015 quantitative-aptitude cartesian-coordinates

Answer key

14.2 Circle (1) top

14.2.1 Circle: TIFR CSE 2011 | Part A | Question: 18 top

The equation of the tangent to the unit circle at point ($\cos\alpha, \sin\alpha$) is

A. $x\cos\alpha - y\sin\alpha = 1$
B. $x\sin\alpha - y\cos\alpha = 1$
C. $x\cos\alpha + y\sin\alpha = 1$
D. $x\sin\alpha - y\cos\alpha = 1$
E. None of the above

tifr2011 quantitative-aptitude geometry circle

Answer key

14.3 Clock Time (3) top

14.3.1 Clock Time: TIFR CSE 2010 | Part A | Question: 2 top

The hour hand and the minute hands of a clock meet at noon and again at mid-night. In between they meet N times, where N is.:

A. 6 B. 11 C. 12 D. 13 E. None of the above

tifr2010 quantitative-aptitude clock-time

Answer key

14.3.2 Clock Time: TIFR CSE 2013 | Part A | Question: 20 top

Consider a well functioning clock where the hour, minute and the seconds needles are exactly at zero. How much time later will the minutes needle be exactly one minute ahead ($1/60$ th of the circumference) of the hours needle and the seconds needle again exactly at zero?

Hint: When the desired event happens both the hour needle and the minute needle have moved an integer multiple of $1/60$ th of the circumference.

A. 144 minutes B. 66 minutes C. 96 minutes D. 72 minutes E. 132 minutes

tifr2013 quantitative-aptitude clock-time

Answer key

14.3.3 Clock Time: TIFR CSE 2014 | Part A | Question: 10 top

A person went out between 4pm and 5pm to chat with her friend and returned between 5pm and 6pm. On her return, she found that the hour-hand and the minute-hand of her (well-functioning) clock had just exchanged their positions with respect to their earlier positions at the time of her leaving. The person must have gone out to chat at

A. Twenty five minutes past 4pm.

B. Twenty six and $\frac{122}{143}$ minutes past 4pm.

C. Twenty seven and $\frac{1}{3}$ minutes past 4pm.

D. Twenty eight minutes past 4pm.

E. None of the above.

tifr2014 quantitative-aptitude clock-time

Answer key

14.4 Complex Number (3) top

14.4.1 Complex Number: TIFR CSE 2011 | Part A | Question: 13 top

If $z = \frac{\sqrt{3} - i}{2}$ and $\left(z^{95} + i^{67}\right)^{97} = z^n$, then the smallest value of n is

A. 1 B. 10 C. 11 D. 12 E. None of the above

tifr2011 quantitative-aptitude complex-number

Answer key

14.4.2 Complex Number: TIFR CSE 2011 | Part A | Question: 5 top

Three distinct points x, y, z lie on a unit circle of the complex plane and satisfy $x + y + z = 0$. Then x, y, z form the vertices of .

A. An isosceles but not equilateral triangle.

B. An equilateral triangle.

C. A triangle of any shape.

D. A triangle whose shape can't be determined.

E. None of the above.

tifr2011 quantitative-aptitude geometry complex-number non-gate

Answer key

14.4.3 Complex Number: TIFR CSE 2013 | Part A | Question: 7 top

For any complex number z, $arg\ z$ defines its phase, chosen to be in the interval $0 \leq argz < 360^\circ$. If z_1, z_2 and z_3 are three complex numbers with the same modulus but different phases ($argz_3 < argz_2 < argz_1 < 180^\circ$), then the quantity

$$\frac{arg\left(z_1/z_2\right)}{arg\left[(z_1 - z_3)/(z_2 - z_3)\right]}$$

is a constant, and has the value

A. 2 B. $\frac{1}{3}$ C. 1 D. 3 E. $\frac{1}{2}$

tifr2013 quantitative-aptitude complex-number non-gate

Answer key

14.5 Convex Sets Functions (1) top

14.5.1 Convex Sets Functions: TIFR CSE 2014 | Part A | Question: 12 top

Let $f(x) = 2^x$. Consider the following inequality for real numbers a, b and $0 < \lambda < 1$:

$f(\lambda a + b) \leq \lambda f(a) + (1 - \lambda) f(\frac{b}{1-\lambda})$.

Consider the following 3 conditions:

1. $\lambda = 0.5$
2. $0 < a \leq 2, b > 0$
3. $a/\lambda > 2, 0 < b \leq 1 - \lambda$

Which of the following statements is TRUE?

A. The above inequality holds under conditions (1) and (2) but not under condition (3).
B. The above inequality holds under conditions (2) and (3) but not under condition (1).
C. The above inequality holds under conditions (1) and (3) but not under condition (2).
D. The above inequality holds under all the three conditions.
E. The above inequality holds under none of the three conditions.

tifr2014 quantitative-aptitude convex-sets-functions non-gate

14.6 Cost Market Price (1) top☝

14.6.1 Cost Market Price: TIFR CSE 2012 | Part A | Question: 6 top☝

A certain pair of used shoes can be repaired for $Rs.1250$ and will last for 1 year. A pair of the same kind of shoes can be purchased new for $Rs.2800$ and will last for 2 years. The average cost per year of the new shoes is what percent greater than the cost of repairing the used shoes?

A. 5 B. 12 C. 15 D. 3 E. 24

tifr2012 cost-market-price

Answer key☟

14.7 Factors (3) top☝

14.7.1 Factors: TIFR CSE 2010 | Part A | Question: 20 top☝

How many integers from 1 to 1000 are divisible by 30 but not by 16?

A. 29 B. 31 C. 32 D. 33 E. 25

tifr2010 quantitative-aptitude factors

Answer key☟

14.7.2 Factors: TIFR CSE 2011 | Part A | Question: 15 top☝

The exponent of 3 in the product $100!$ is

A. 27 B. 33 C. 44 D. 48 E. None of the above

tifr2011 quantitative-aptitude factors tricky

Answer key☟

14.7.3 Factors: TIFR CSE 2013 | Part A | Question: 12 top☝

Among numbers 1 to 1000 how many are divisible by 3 or 7?

A. 333 B. 142 C. 475 D. 428 E. None of the above

tifr2013 quantitative-aptitude factors normal

Answer key☟

14.8 Fraction (2) top

14.8.1 Fraction: TIFR CSE 2014 | Part A | Question: 11 top

A large community practices birth control in the following peculiar fashion. Each set of parents continues having children until a son is born; then they stop. What is the ratio of boys to girls in the community if, in the absence of birth control, 51% of the babies are born male?

A. $51:49$
B. $1:1$
C. $49:51$
D. $51:98$
E. $98:51$

tifr2014 quantitative-aptitude fraction tricky

Answer key

14.8.2 Fraction: TIFR CSE 2017 | Part A | Question: 1 top

A suitcase weighs one kilogram plus half of its weight. How much does the suitcase weigh?

A. $1.3333...$ kilograms
B. 1.5 kilograms
C. $1.666...$ kilograms
D. 2 kilograms
E. cannot be determined from the given data

tifr2017 quantitative-aptitude fraction normal

Answer key

14.9 Geometry (9) top

14.9.1 Geometry: TIFR CSE 2010 | Part A | Question: 17 top

Suppose there is a sphere with diameter **at least** 6 inches. Through this sphere we drill a hole along a diameter. The part of the sphere lost in the process of drilling the hole looks like two caps joined to a cylinder, where the cylindrical part has length 6 inches. It turns out that the volume of the remaining portion of the sphere does not depend on the diameter of the sphere. Using this fact, determine the volume of the remaining part.

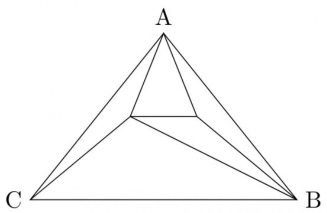
A. 24π cu. inches
B. 36π cu. inches
C. 27π cu. inches
D. 32π cu. inches
E. 35π cu. inches

tifr2010 quantitative-aptitude geometry

Answer key

14.9.2 Geometry: TIFR CSE 2012 | Part A | Question: 4 top

Let ABC be a triangle with n distinct points inside. A triangulation of ABC with respect to the n points is obtained by connecting as many points as possible, such that no more line segments can be added without intersecting other line segments. In other words ABC has been partitioned into triangles with end points at the n points or at the vertices A,B,C. For example, the following figure gives one possible triangulation of ABC with two points inside it.



Although there are many different ways to triangulate ABC with the n points inside, the number of triangles depends only on n. In the above figure it is five. How many triangles are there in a triangulation of ABC with n points inside it?

A. $3n-1$ B. n^2+1 C. $n+3$ D. $2n+1$ E. $4n-3$

tifr2012 quantitative-aptitude geometry

Answer key☟

14.9.3 Geometry: TIFR CSE 2012 | Part A | Question: 5 top☝

What is the maximum number of points of intersection between the diagonals of a convex octagon (8-vertex planar polygon)? Note that a polygon is said to be convex if the line segment joining any two points in its interior lies wholly in the interior of the polygon. Only points of intersection between diagonals that lie in the interior of the octagon are to be considered for this problem.

A. 55 B. 60 C. 65 D. 70 E. 75

tifr2012 quantitative-aptitude geometry

Answer key☟

14.9.4 Geometry: TIFR CSE 2013 | Part A | Question: 5 top☝

The late painter Maqbool Fida Husain once coloured the surface of a huge hollow steel sphere, of radius 1 metre, using just two colours, Red and Blue. As was his style however, both the red and blue areas were a bunch of highly irregular disconnected regions. The late sculptor Ramkinkar Baij then tried to fit in a cube inside the sphere, the eight vertices of the cube touching only red coloured parts of the surface of the sphere. Assume $\pi = 3.14$ for solving this problem. Which of the following is true?

A. Baij is bound to succeed if the area of the red part is $10 sq. metres$;
B. Baij is bound to fail if the area of the red part is $10 sq. metres$;
C. Baij is bound to fail if the area of the red part is $11 sq. metres$;
D. Baij is bound to succeed if the area of the red part is $11 sq. metres$;
E. None of the above.

tifr2013 geometry quantitative-aptitude

Answer key☟

14.9.5 Geometry: TIFR CSE 2015 | Part A | Question: 2 top☝

Consider a circle with a circumference of one unit length. Let $d < \frac{1}{6}$. Suppose that we independently throw two arcs, each of length d, randomly on this circumference so that each arc is uniformly distributed along the circle circumference. The arc attaches itself exactly to the circumference so that arc of length d exactly covers length d of the circumference. What can be said about the probability that the two arcs do not intersect each other?

A. It equals $(1-d)$
B. It equals $(1-3d)$
C. It equals $(1-2d)$
D. It equals 1
E. It equals $(1-d)(1-d)$

tifr2015 geometry

Answer key☟

14.9.6 Geometry: TIFR CSE 2015 | Part A | Question: 9 top☝

Consider a square of side length 2. We throw five points into the square. Consider the following statements:

i. There will always be three points that lie on a straight line.
ii. There will always be a line connecting a pair of points such that two points lie on one side of the line

and one point on the other.

iii. There will always be a pair of points which are at distance at most $\sqrt{2}$ from each other.

Which of the above is true:

A. (i) only B. (ii) only C. (iii) only D. (ii) and (iii) E. None of the above

tifr2015 geometry quantitative-aptitude easy

Answer key☟

14.9.7 Geometry: TIFR CSE 2017 | Part A | Question: 13 top☝

A set of points $S \subseteq \mathbb{R}^2$ is convex if for any points $x,\ y\ \in S$, every point on the straight line joining x and y is also in S. For two sets of points $S, T \subset \mathbb{R}^2$, define the sum $S+T$ as the set of points obtained by adding a point in S to a point T. That is, $S+T := \{(x_1,x_2) \in \mathbb{R}^2 : x_1 = y_1 + z_1,\ x_2 = y_2 + z_2, \quad (y_1,y_2) \in S, (z_1,z_2) \in T\}$. Similarly, $S-T := \{(x_1,x_2) \in \mathbb{R}^2 : x_1 = y_1 - z_1,\ x_2 = y_2 - z_2, \quad (y_1,y_2) \in S, (z_1,z_2) \in T\}$ is the set of points obtained by subtracting a point in T from a point in S. Which of the following statements is TRUE for all convex sets $S,\ T$?

A. $S+T$ is convex but not $S-T$
B. $S-T$ is convex but not $S+T$
C. exactle one of $S+T$ and $S-T$ is convex, but it depends on S and T which one
D. neither $S+T$ nor $S-T$ is convex
E. both $S+T$ and $S-T$ are convex

tifr2017 quantitative-aptitude geometry

Answer key☟

14.9.8 Geometry: TIFR CSE 2017 | Part A | Question: 8 top☝

In a tutorial on geometrical constructions, the teacher asks a student to construct a right-angled triangle ABC where the hypotenuse BC is 8 inches and the length of the perpendicular dropped from A onto the hypotenuse is h inches, and offers various choices for teh value of h. For which value of h can such a triangle NOT exist?

A. 3.90 inches
B. $2\sqrt{2}$ inches
C. $2\sqrt{3}$ inches
D. 4.1 inches
E. none of the above

tifr2017 quantitative-aptitude geometry

Answer key☟

14.9.9 Geometry: TIFR CSE 2018 | Part A | Question: 1 top☝

Consider a point A inside a circle C that is at distance 9 from the centre of a circle. Suppose you told that there is a chord of length 24 passing through A with A as its midpoint. How many distinct chords of C have integer length and pass through A?

A. 2 B. 6 C. 7 D. 12 E. 14

tifr2018 quantitative-aptitude geometry

Answer key☟

14.10 Logarithms (1) top☝

14.10.1 Logarithms: TIFR CSE 2010 | Part A | Question: 9 top

A table contains 287 entries. When any one of the entries is requested, it is encoded into a binary string and transmitted. The number of bits required is.

A. 8
B. 9
C. 10
D. Cannot be determined from the given information.
E. None of the above.

tifr2010 quantitative-aptitude theory-of-computation logarithms

Answer key

14.11 Modular Arithmetic (3) top

14.11.1 Modular Arithmetic: TIFR CSE 2019 | Part A | Question: 2 top

How many proper divisors (that is, divisors other than 1 or 7200) does 7200 have ?

A. 18
B. 20
C. 52
D. 54
E. 60

tifr2019 modular-arithmetic quantitative-aptitude

Answer key

14.11.2 Modular Arithmetic: TIFR CSE 2019 | Part A | Question: 7 top

What are the last two digits of $1!+2!+\cdots+100!$?

A. 00
B. 13
C. 30
D. 33
E. 73

tifr2019 modular-arithmetic quantitative-aptitude

Answer key

14.11.3 Modular Arithmetic: TIFR CSE 2019 | Part B | Question: 14 top

Let m and n be two positive integers. Which of the following is NOT always true?

A. If m and n are co-prime, there exist integers a and b such that $am+bn=1$
B. $m^{n-1} \equiv 1 (\bmod n)$
C. The rational number $\frac{n}{m} \cdot \frac{n-1}{m-1} \cdot \frac{n-2}{m-2} \cdots \frac{n-(m-2)}{m-(m-2)} \cdot \frac{n-(m-1)}{m-(m-1)}$ is an integer
D. $m+1$ is a factor of $m^{n(n+1)}-1$
E. If 2^n-1 is prime, then n is prime

tifr2019 general-aptitude quantitative-aptitude modular-arithmetic

Answer key

14.12 Number Representation (1) top

14.12.1 Number Representation: TIFR CSE 2012 | Part A | Question: 11 top

Let N be the sum of all numbers from 1 to 1023 except the five primes numbers: $2,3,11,17,31$. Suppose all numbers are represented using two bytes (sixteen bits). What is the value of the least significant byte (the least significant eight bits) of N?

A. 00000000
B. 10101110
C. 01000000
D. 10000000
E. 11000000

tifr2012 quantitative-aptitude number-representation

Answer key

14.13 Number Series (5) top

14.13.1 Number Series: TIFR CSE 2011 | Part A | Question: 8 top

The sum of the first n terms of the series $1, 11, 111, 1111, \ldots,$ is.

A. $\frac{1}{81}\left(10^{n+1} - 9n - 10\right)$
B. $\frac{1}{81}(10^n - 9n)$
C. $\frac{1}{9}\left(10^{n+1} - 1\right)$
D. $\frac{1}{9}\left(10^{n+1} - n10^n\right)$
E. None of the above

tifr2011 quantitative-aptitude number-series

Answer key

14.13.2 Number Series: TIFR CSE 2013 | Part A | Question: 15 top

Let $\operatorname{sgn}(x) = \begin{cases} +1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$

What is the value of the following summation?

$$\sum_{i=0}^{50} \operatorname{sgn}((2i-1)(2i-3)\ldots(2i-99))$$

A. 0 B. -1 C. $+1$ D. 25 E. 50

tifr2013 quantitative-aptitude number-series

Answer key

14.13.3 Number Series: TIFR CSE 2013 | Part A | Question: 8 top

Find the sum of the infinite series

$$\frac{1}{1 \times 3 \times 5} + \frac{1}{3 \times 5 \times 7} + \frac{1}{5 \times 7 \times 9} + \frac{1}{7 \times 9 \times 11} + \ldots\ldots$$

A. ∞ B. $\left(\frac{1}{2}\right)$ C. $\left(\frac{1}{6}\right)$ D. $\left(\frac{1}{12}\right)$ E. $\left(\frac{1}{14}\right)$

tifr2013 quantitative-aptitude number-series

Answer key

14.13.4 Number Series: TIFR CSE 2014 | Part A | Question: 7 top

Consider a sequence of non-negative numbers $x_n : n = 1, 2, \ldots$. Which of the following statements cannot be true?

A. $\sum_{n=1}^{\infty} x_n = \infty$ and $\sum_{n=1}^{\infty} x_n^2 = \infty$.
B. $\sum_{n=1}^{\infty} x_n = \infty$ and $\sum_{n=1}^{\infty} x_n^2 < \infty$.
C. $\sum_{n=1}^{\infty} x_n < \infty$ and $\sum_{n=1}^{\infty} x_n^2 < \infty$.
D. $\sum_{n=1}^{\infty} x_n \leq 5$ and $\sum_{n=1}^{\infty} x_n^2 \geq 25$.
E. $\sum_{n=1}^{\infty} x_n < \infty$ and $\sum_{n=1}^{\infty} x_n^2 = \infty$.

tifr2014 quantitative-aptitude number-series

Answer key

14.13.5 Number Series: TIFR CSE 2015 | Part A | Question: 3 top

Let $|z| < 1$. Define $M_n(z) = \sum_{i=1}^{10} z^{10^n(i-1)}$? what is

$$\prod_{i=0}^{\infty} M_i(z) = M_0(z) \times M_1(z) \times M_2(z) \times \dots ?$$

A. Can't be determined B. $1/(1-z)$ C. $1/(1+z)$ D. $1-z^9$ E. None of the above

tifr2015 quantitative-aptitude numerical-computation number-series

Answer key

14.14 Number System (2) top

14.14.1 Number System: TIFR CSE 2020 | Part A | Question: 15 top

The sequence $s_0, s_1, \dots, s_9$ is defined as follows:

- $s_0 = s_1 + 1$
- $2s_i = s_{i-1} + s_{i+1} + 2 \text{ for } 1 \leq i \leq 8$
- $2s_9 = s_8 + 2$

What is s_0?

A. 81 B. 95 C. 100 D. 121 E. 190

tifr2020 general-aptitude quantitative-aptitude number-system

Answer key

14.14.2 Number System: TIFR CSE 2020 | Part A | Question: 6 top

What is the maximum number of regions that the plane $\mathbb{R}^2$ can be partitioned into using 10 lines?

A. 25 B. 50 C. 55 D. 56 E. 1024

Hint: Let $A(n)$ be the maximum number of partitions that can be made by n lines. Observe that $A(0) = 1, A(2) = 2, A(2) = 4$ etc. Come up with a recurrence equation for $A(n)$.

tifr2020 general-aptitude quantitative-aptitude number-system

Answer key

14.15 Number Theory (1) top

14.15.1 Number Theory: TIFR CSE 2014 | Part A | Question: 20 top

Consider the equation $x^2 + y^2 - 3z^2 - 3t^2 = 0$. The total number of integral solutions of this equation in the range of the first 10000 numbers, i.e., $1 \leq x, y, z, t \leq 10000$, is

A. 200 B. 55 C. 100 D. 1 E. None of the above

tifr2014 number-theory quantitative-aptitude

Answer key

14.16 Numerical Computation (5) top

14.16.1 Numerical Computation: TIFR CSE 2010 | Part A | Question: 14 top

A marine biologist wanted to estimate the number of fish in a large lake. He threw a net and found 30 fish in the net. He marked all these fish and released them into the lake. The next morning he again threw the net and this time caught 40 fish, of which two were found to be marked. The (approximate) number of fish in the lake is:

A. 600 B. 1200 C. 68 D. 800 E. 120

tifr2010 quantitative-aptitude numerical-computation

Answer key

14.16.2 Numerical Computation: TIFR CSE 2014 | Part A | Question: 1 top

Consider the reactions

$$X + 2Y \rightarrow 3Z$$

$$2X + Z \rightarrow Y.$$

Let n_X, n_Y, n_Z denote the numbers of molecules of chemicals X, Y, Z in the reaction chamber. Then which of the following is conserved by both reactions?

A. $n_X + n_Y + n_Z$.
B. $n_X + 7n_Y + 5n_Z$.
C. $2n_X + 9n_Y - 3n_Z$.
D. $3n_X - 3n_Y + 13n_Z$.
E. None of the above.

tifr2014 quantitative-aptitude numerical-computation

Answer key

14.16.3 Numerical Computation: TIFR CSE 2014 | Part A | Question: 4 top

Consider numbers greater than one that satisfy the following properties:

i. They have no repeated prime factors;
ii. For all primes $p \geq 2$, p divides the number if and only if $p - 1$ divides the number.

The number of such numbers is

A. 0 B. 5 C. 100 D. Infinite E. None of the above

tifr2014 quantitative-aptitude difficult numerical-computation

Answer key

14.16.4 Numerical Computation: TIFR CSE 2015 | Part B | Question: 12 top

Let t_n be the sum of the first n natural numbers, for $n > 0$. A number is called triangular if it is equal to t_n for some n. Which of the following statements are true:

i. There exists three successive triangular numbers whose product is a perfect square.
ii. If the triangular number t_n is a perfect square, then so is $t_{4n(n+1)}$.
iii. The sum of the reciprocals of the first n triangular numbers is less than 2, i.e.

$$\frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \ldots + \frac{1}{t_n} < 2$$

A. (i) only
B. (ii) only
C. (iii) only
D. All of the above
E. None of the above

tifr2015 quantitative-aptitude normal numerical-computation

Answer key

14.16.5 Numerical Computation: TIFR CSE 2019 | Part A | Question: 9 top

Let A and B be two containers. Container A contains 50 litres of liquid X and container B contains 100 litres of liquid Y. Liquids X and Y are soluble in each other.

We now take 30 ml of liquid X from container A and put it into container B. The mixture in container B is then thoroughly mixed and 20 ml of the resulting mixture is put back into container A.At the end of this process let V_{AY} be the volume of liquid Y and V_{BX} be the volume of liquid X in container B. Which of the following must be TRUE ?

A. $V_{AY} < V_{BX}$
B. $V_{AY} > V_{BX}$
C. $V_{AY} = V_{BX}$
D. $V_{AY} + V_{BX} = 30$
E. $V_{AY} + V_{BX} = 20$

tifr2019 general-aptitude quantitative-aptitude numerical-computation

Answer key

14.17 Polynomials (1) top

14.17.1 Polynomials: TIFR CSE 2013 | Part B | Question: 2 top

Consider polynomials in a single variable x of degree d. Suppose $d < n/2$. For such a polynomial $p(x)$, let C_p denote the n-tuple $(P(i))_{1 \leq i \leq n}$. For any two such distinct polynomials p, q, the number of coordinates where the tuples C_p, C_q differ is.

A. At most d
B. At most $n - d$
C. Between d and $n - d$
D. At least $n - d$
E. None of the above.

tifr2013 polynomials non-gate

Answer key

14.18 Quantitative Aptitude (2) top

14.18.1 Quantitative Aptitude: TIFR CSE 2011 | Part A | Question: 20 top

Let $n > 1$ be an odd integer. The number of zeros at the end of the number $99^n + 1$ is

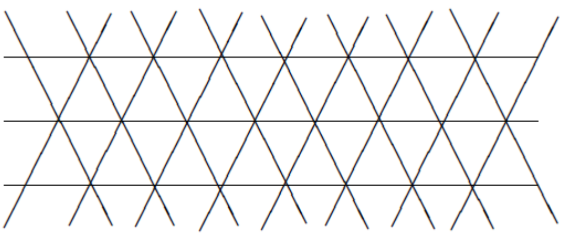
A. 1
B. 2
C. 3
D. 4
E. None of the above

tifr2011 quantitative-aptitude combinatory

Answer key

14.18.2 Quantitative Aptitude: TIFR CSE 2013 | Part A | Question: 1 top

An infinite two-dimensional pattern is indicated below.



The smallest closed figure made by the lines is called a unit triangle. Within every unit triangle, there is a mouse.

At every vertex there is a laddoo. What is the average number of laddoos per mouse?

A. 3 B. 2 C. 1 D. $\left(\frac{1}{2}\right)$ E. $\left(\frac{1}{3}\right)$

tifr2013 quantitative-aptitude combinatory

Answer key☟

14.19 Ratio Proportion (2) top☝

14.19.1 Ratio Proportion: TIFR CSE 2012 | Part A | Question: 18 top☝

A large community practices birth control in the following peculiar fashion. Each set of parents continues having children until a son is born; then they stop. What is the ratio of boys to girls in the community if, in the absence of birth control, 51% of the babies are born male?

A. $51 : 49$ B. $1 : 1$ C. $49 : 51$ D. $51 : 98$ E. $98 : 51$

tifr2012 quantitative-aptitude ratio-proportion

Answer key☟

14.19.2 Ratio Proportion: TIFR CSE 2014 | Part A | Question: 2 top☝

A body at a temperature of 30 Celsius is immersed into a heat bath at 0 Celsius at time $t = 0$. The body starts cooling at a rate proportional to the temperature difference. Assuming that the heat bath does not change in temperature throughout the process, calculate the ratio of the time taken for the body to reach 1 Celsius divided by the time taken for the body to reach 5 Celsius.

A. $\log 5$ B. $\frac{\log 29}{\log 25}$ C. e^5 D. $1 + \log_6 5$ E. None of the above

tifr2014 quantitative-aptitude ratio-proportion

Answer key☟

14.20 Sequence Series (1) top☝

14.20.1 Sequence Series: TIFR CSE 2013 | Part A | Question: 19 top☝

Consider a sequence of numbers $(\epsilon_n : n = 1, 2, \dots)$, such that $\epsilon_1 = 10$ and

$$\epsilon_{n+1} = \frac{20\epsilon_n}{20 + \epsilon_n}$$

for $n \geq 1$. Which of the following statements is true?

Hint: Consider the sequence of reciprocals.

A. The sequence $(\epsilon_n : n = 1, 2, \dots)$ converges to zero.
B. $\epsilon_n \geq 1$ for all n
C. The sequence $(\epsilon_n : n = 1, 2, \dots)$ is decreasing and converges to 1.
D. The sequence $(\epsilon_n : n = 1, 2, \dots)$ is decreasing and then increasing. Finally it converges to 1.
E. None of the above.

tifr2013 quantitative-aptitude sequence-series

Answer key☟

14.21 Speed Time Distance (2) top☝

14.21.1 Speed Time Distance: TIFR CSE 2012 | Part A | Question: 16 top

Walking at $4/5$ is normal speed a man is 10 minute too late. Find his usual time in minutes.

A. 81
B. 64
C. 52
D. 40
E. It is not possible to determine the usual time from given data.

tifr2012 quantitative-aptitude speed-time-distance

Answer key

14.21.2 Speed Time Distance: TIFR CSE 2017 | Part A | Question: 3 top

On planet TIFR, the acceleration of an object due to gravity is half that on planet earth. An object on planet earth dropped from a height h takes time t to reach the ground. On planet TIFR, how much time would an object dropped from height h take to reach the ground?

A. $\left(\frac{t}{\sqrt{2}}\right)$ B. $\sqrt{2}t$ C. $2t$ D. $\left(\frac{h}{t}\right)$ E. $\left(\frac{h}{2t}\right)$

tifr2017 quantitative-aptitude speed-time-distance

Answer key

14.22 Statistics (1) top

14.22.1 Statistics: TIFR CSE 2015 | Part A | Question: 15 top

Let A and B be non-empty disjoint sets of real numbers. Suppose that the average of the numbers in the first set is μ_A and the average of the numbers in the second set is μ_B; let the corresponding variances be v_A and v_B respectively. If the average of the elements in $A \cup B$ is $\mu = p.\,\mu_A + (1-p).\,\mu_B$, what is the variance of the elements in $A \cup B$?

A. $p.\,v_A + (1-p).\,v_B$
B. $(1-p).\,v_A + p.\,v_B$
C. $p.\,[v_A + (\mu_A - \mu)^2] + (1-p).\,[v_B + (\mu_B - \mu)^2]$
D. $(1-p).\,[v_A + (\mu_A - \mu)^2] + p.\,[v_B + (\mu_B - \mu)^2]$
E. $p.\,v_A + (1-p).\,v_B + (\mu_A - \mu_B)^2$

tifr2015 statistics

Answer key

14.23 Three Dimensional Geometry (1) top

14.23.1 Three Dimensional Geometry: TIFR CSE 2018 | Part A | Question: 2 top

Consider the following subset of $\mathbb{R}^3$ (the first two are cylinder, the third is a plane):

- $C_1 = \left\{(x,y,z) : y^2 + z^2 \leq 1\right\};$
- $C_2 = \left\{(x,y,z) : x^2 + z^2 \leq 1\right\};$
- $H = \{(x,y,z) : z = 0.2\};$

Let $A = C_1 \cap C_2 \cap H.$ Which of the following best describe the shape of set A?

A. Circle
B. Ellipse
C. Triangle
D. Square
E. An octagonal convex figure with curved sides

tifr2018 quantitative-aptitude geometry three-dimensional-geometry non-gate

Answer key

Answer Keys

14.0.1	E	14.1.1	C	14.1.2	B	14.1.3	D	14.2.1	C
14.3.1	E	14.3.2	E	14.3.3	B	14.4.1	A	14.4.2	B
14.4.3	A	14.5.1	D	14.6.1	B	14.7.1	A	14.7.2	D
14.7.3	D	14.8.1	A	14.8.2	D	14.9.1	B	14.9.2	D
14.9.3	D	14.9.4	D	14.9.5	C	14.9.6	C	14.9.7	E
14.9.8	D	14.9.9	D	14.10.1	D	14.11.1	C	14.11.2	B
14.11.3	B	14.12.1	E	14.13.1	A	14.13.2	C	14.13.3	D
14.13.4	E	14.13.5	B	14.14.1	C	14.14.2	D	14.15.1	E
14.16.1	A	14.16.2	B	14.16.3	E	14.16.4	D	14.16.5	A
14.17.1	D	14.18.1	B	14.18.2	D	14.19.1	A	14.19.2	D
14.20.1	A	14.21.1	D	14.21.2	B	14.22.1	C	14.23.1	D

15 General Aptitude: Verbal Aptitude (2)

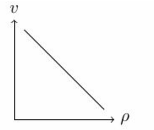
15.1 Quantitative Aptitude (2) top

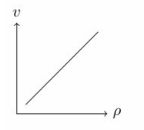
15.1.1 Quantitative Aptitude: TIFR CSE 2019 | Part A | Question: 8 top

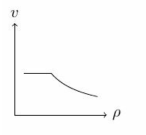
Consider the following toy model of traffic on a straight , single lane, highway. We think of cars as points, which move at the maximum speed v that satisfies the following constraints:

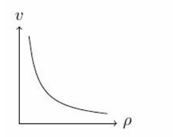
1. The speed is no more than the speed limit v_{max} mandated for the highway.
2. The speed is such that when traveling at this speed, it takes at least time t_0 (where t_0 is a fixed time representing the reaction time of drivers) to reach the car ahead, in case the car ahead stops suddenly.

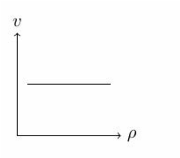
Let as assume that in the steady state, all cars on the highway move at the same speed v satisfying both the above constraints, and the distance between any two successive cars is the same. Let ρ denote the “density” , that is, the number of card per unit length of the highway. Which of the following graphs most accurately captures the relationship between the speed v and the density ρ in this model ?

A. 

B. 

C. 

D. 

E. 

tifr2019 general-aptitude quantitative-aptitude

Answer key

15.1.2 Quantitative Aptitude: TIFR CSE 2020 | Part A | Question: 9 top

A contiguous part, i.e., a set of adjacent sheets, is missing from Tharoor’s GRE preparation book. The number on the first missing page is 183, and it is known that the number on the last missing page has the same three digits, but in a different order. Note that every sheet has two pages, one at the front and one at the back. How many pages are missing from Tharoor's book?

A. 45 B. 135 C. 136 D. 198 E. 450

tifr2020 general-aptitude quantitative-aptitude

Answer key

Answer Keys

15.1.1	C		15.1.2	C

16 Non GATE: Object Oriented Programming (1)

16.1 Object Oriented Programming (1) top☝

16.1.1 Object Oriented Programming: TIFR CSE 2011 | Part B | Question: 40 top☝

Consider the class of object oriented languages. Which of the following is true?

A. Pascal is an object oriented language.
B. Object oriented languages require heap management.
C. Object oriented languages cannot be implemented in language C.
D. Object oriented languages are more powerful than declarative programming languages.
E. Parallelism cannot be realized in object oriented languages.

tifr2011 programming object-oriented-programming non-gate

Answer key☟

Answer Keys

16.1.1	B

17 Operating System (11)

17.1 Page Replacement (1) top

17.1.1 Page Replacement: TIFR CSE 2013 | Part B | Question: 14 top

Assume a demand paged memory system where ONLY THREE pages can reside in the memory at a time. The following sequence gives the order in which the program references the pages.

$1, 3, 1, 3, 4, 2, 2, 4$

Assume that least frequently used page is replaced when necessary. If there is more than one least frequently used pages then the least recently used page among them is replaced. During the program's execution, how many times will the pages $1, 2, 3$ and 4 be brought to the memory?

A. $2,2,2,2$ times, respectively
B. $1,1,1,2$ times, respectively
C. $1,1,1,1$ times, respectively
D. $2,1,2,2$ times, respectively
E. None of the above

tifr2013 operating-system page-replacement

Answer key

17.2 Process Synchronization (8) top

17.2.1 Process Synchronization: TIFR CSE 2010 | Part B | Question: 28 top

Consider the **concurrent** program:

```
x: 1;
  cobegin
    x:= x + 3 || x := x + x + 2
  coend
```

Reading and writing of variables is atomic, but the evaluation of an expression is not atomic.

The set of possible values of variable x at the end of the execution of the program is:

A. $\{4\}$
B. $\{8\}$
C. $\{4, 7, 10\}$
D. $\{4, 7, 8, 10\}$
E. $\{4, 7, 8\}$

tifr2010 process-synchronization

Answer key

17.2.2 Process Synchronization: TIFR CSE 2010 | Part B | Question: 32 top

Consider the following solution (expressed in Dijkstra's guarded command notation) to the mutual exclusion problem.

```
process P1 is
begin
  loop
    Non_critical_section;
    while not (Turn=1) do skip od;
    Critical_section_1;
    Turn:=2;
  end loop
end
```

```
process P2 is
begin
  loop
    Non_critical_section;
    while not (Turn=2) do skip od;
    Critical_section_2;
```

```
        Turn:=1;
    end loop
end
```

Initially, Turn $= 1$, Assume that the two process run forever and that no process stays in its critical and non-critical section infinitely. A mutual exclusion program is correct if it satisfies the following requirements.

1. Only one process can be in a critical region at a time.
2. Program is a dead-lock free, i.e., if both processes are trying to enter the critical region then at least one of them does enter the critical region.
3. Program is starvation-free; i.e, a process trying to enter the critical region eventually manages to do so.

The above mutual exclusion solution.

A. Does not satisfy the requirement (1).
B. Satisfy the requirement (1) but does not satisfy the requirement (2).
C. Satisfies the requirements (1) and (2), but does not satisfies the requirement (3).
D. Satisfies the requirement (1) and (3), but does not satisfies the requirement (2).
E. Satisfies all the requirement (1), (2), and (3).

tifr2010 operating-system process-synchronization

Answer key

17.2.3 Process Synchronization: TIFR CSE 2011 | Part B | Question: 22 top

Consider the program

```
P:: x:=1; y:=1; z:=1; u:=0
```

And the program

```
Q:: x, y, z, u := 1, 1, 1, 1; u:= 0
```

Which of the following is true?

A. P and Q are equivalent for sequential processors.
B. P and Q are equivalent for all multi-processor models.
C. P and Q are equivalent for all multi-core machines.
D. P and Q are equivalent for all networks of computers.
E. None of the above

tifr2011 operating-system process-synchronization

Answer key

17.2.4 Process Synchronization: TIFR CSE 2011 | Part B | Question: 26 top

Consider the following two scenarios in the dining philosophers problem:

i. First a philosopher has to enter a room with the table that restricts the number of philosophers to four.
ii. There is no restriction on the number of philosophers entering the room.

Which of the following is true?

A. Deadlock is possible in (i) and (ii).
B. Deadlock is possible in (i).
C. Starvation is possible in (i).
D. Deadlock is not possible in (ii).
E. Starvation is not possible in (ii)

tifr2011 operating-system process-synchronization

Answer key

17.2.5 Process Synchronization: TIFR CSE 2011 | Part B | Question: 28 top

Consider a basic block:

```
x:= a[i]; a[j]:= y; z:= a[j]
```

optimized by removing common sub expression a[i] as follows:

```
x:= a[i]; z:= x; a[j]:= y.
```

Which of the following is true?

A. Both are equivalent.
B. The values computed by both are exactly the same.
C. Both give exactly the same values only if i is not equal to j.
D. They will be equivalent in concurrent programming languages with shared memory.
E. None of the above.

tifr2011 process-synchronization operating-system normal

Answer key

17.2.6 Process Synchronization: TIFR CSE 2011 | Part B | Question: 34 top

Consider the class of synchronization primitives. Which of the following is false?

A. Test and set primitives are as powerful as semaphores.
B. There are various synchronizations that can be implemented using an array of semaphores but not by binary semaphores.
C. Split binary semaphores and binary semaphores are equivalent.
D. All statements a - c are false.
E. Petri nets with and without inhibitor arcs have the same power.

tifr2011 operating-system process-synchronization

Answer key

17.2.7 Process Synchronization: TIFR CSE 2012 | Part B | Question: 9 top

Consider the concurrent program

```
x := 1;
cobegin
   x := x + x + 1 || x := x + 2
coend;
```

Reading and writing of a variable is atomic, but evaluation of an expression is not atomic. The set of possible values of variable x at the end of execution of the program is

A. $\{3\}$ B. $\{7\}$ C. $\{3,5,7\}$ D. $\{3,7\}$ E. $\{3,5\}$

tifr2012 process-synchronization operating-system

Answer key

17.2.8 Process Synchronization: TIFR CSE 2015 | Part B | Question: 14 top

Consider the following concurrent program (where statements separated by | | with-in cobegin-coend are executed concurrently).

```
x:=1
cobegin
  x:= x + 1 ||   x:= x + 1  ||   x:= x + 1
coend
```

Reading and writing of variables is atomic but evaluation of expressions is not atomic. The set of possible values of x at the end of execution of the program is

A. $\{4\}$
B. $\{2,3,4\}$
C. $\{2,4\}$
D. $\{2,3\}$
E. $\{2\}$

tifr2015 process-synchronization operating-system normal

Answer key

17.3 Round Robin Scheduling (1) top

17.3.1 Round Robin Scheduling: TIFR CSE 2020 | Part B | Question: 8 top

Jobs keep arriving at a processor. A job can have an associated time length as well as a priority tag. New jobs may arrive while some earlier jobs are running. Some jobs may keep running indefinitely. A **starvation free** job-scheduling policy guarantees that no job waits indefinitely for service. Which of the following job-scheduling policies is starvation free?

A. Round – robin
B. Shortest job first
C. Priority queuing
D. Latest job first
E. None of the others

tifr2020 operating-system process-scheduling round-robin-scheduling

Answer key

17.4 Semaphore (1) top

17.4.1 Semaphore: TIFR CSE 2012 | Part B | Question: 10 top

Consider the blocked-set semaphore where the signaling process awakens any one of the suspended process; i.e.,

Wait (S): If $S > 0$ then $S \leftarrow S - 1$, else suspend the execution of this process.

Signal (S): If there are processes that have been suspended on semaphore S, then wake any one of them, else $S \leftarrow S + 1$

Consider the following solution of mutual exclusion problem using blocked-set semaphores.

```
s := 1;
cobegin
P(1)  ||   P(2)   ||   .....  ||    P(N)
coend
```

Where the task body P(i) is

```
begin
while true do
begin
 < non critical section >
 Wait (S)
<critical section>
 Signal (S)
end
end
```

Here N is the number of concurrent processors. Which of the following is true?

A. The program fails to achieve mutual exclusion of critical regions.
B. The program achieves mutual exclusion, but starvation freedom is ensured only for $N \leq 2$

C. The program does not ensure mutual exclusion if $N \geq 3$
D. The program achieves mutual exclusion, but allows starvation for any $N \geq 2$
E. The program achieves mutual exclusion and starvation freedom for any $N \geq 1$

tifr2012 operating-system process-synchronization semaphore

Answer key☟

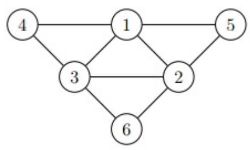
Answer Keys

17.1.1	B	17.2.1	C	17.2.2	E	17.2.3	D	17.2.4	C
17.2.5	E	17.2.6	Q-Q	17.2.7	C	17.2.8	B	17.3.1	A
17.4.1	B								

18 Others: Others (91)

18.0.1 TIFR CSE 2016 | Part A | Question: 2 top

Consider the graph shown below:



The following experiment is performed using this graph. First, an edge $e = \{i, j\}$ of the graph is chosen uniformly at random from the set of 9 possibilities. Next, a common neighbour k of i and j is chosen, again uniformly from the set of possibilities. (Note that the set of possibilities is always non-empty.) Thus, $\{i, j, k\}$ is a triangle in the graph. What is the probability that the triangle finally picked is $\{1, 2, 3\}$?

A. $\frac{1}{6}$ B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{2}{3}$ E. $\frac{5}{6}$

tifr2016 graph-theory probability

Answer key

18.0.2 TIFR CSE 2016 | Part B | Question: 6 top

A subset X of $\mathbb{R}^n$ is convex if for all $x, y \in X$ and all $\lambda \in (0, 1)$, we have $\lambda x + (1 - \lambda)y \in X$. If X is a convex set, which of the following statements is necessarily TRUE?

A. For every $x \in X$ there exist $y, z \in X - \{x\}$ and $\lambda \in (0, 1)$ so that $x = \lambda y + (1 - \lambda)z$
B. If $x, y \in X$ and $\lambda \geq 0$, then $\lambda x + (1 - \lambda)y \in X$
C. If $x_1, \dots, x_n \in X (n \geq 1)$, then $(x_1 + \dots + x_n)/n \in X$
D. If $x \in X$, then $\lambda x \in X$ for all scalars λ
E. If $x, y \in X$, then $x - y \in X$

tifr2016

18.0.3 TIFR CSE 2021 | Part B | Question: 3 top

What is the prefix expression corresponding to the expression:

$((9 + 8) * 7 + (6 * (5 + 4)) * 3) + 2?$

You may assume that $*$ has precedence over $+$?

A. $* + + 987 * * 6 + + 5432$
B. $* + + + 987 * * 6 + 5432$
C. $+ * + + 987 * * 6 + 5432$
D. $+ + * + 987 * * 6 + 5432$
E. $+ * + * 987 + + 6 * 5432$

tifr2021

Answer key

18.0.4 TIFR CSE 2016 | Part B | Question: 14 top

Consider a family $\mathcal{F}$ of subsets of $\{1, 2, \dots, n\}$ such that for any two distinct sets A and B in $\mathcal{F}$ we have: $A \subset B$ or $B \subset A$ or $A \cap B = \emptyset$. Which of the following statements is TRUE? (Hint:

what does the Venn diagram of this family look like?)

A. $|\mathcal{F}| \le 2n$ and there exists a family $\mathcal{F}$ such that $|\mathcal{F}| = 2n$
B. $|\mathcal{F}| \le n^2$ and there exists a family $\mathcal{F}$ such that $|\mathcal{F}| = n^2$
C. $|\mathcal{F}| \le 2n^2$ and there exists a family $\mathcal{F}$ such that $|\mathcal{F}| = 2n^2$
D. $|\mathcal{F}| \le 2^{n-1}$ and there exists a family $\mathcal{F}$ such that $|\mathcal{F}| = 2^{n-1}$
E. None of the above

tifr2016

Answer key

18.0.5 TIFR CSE 2016 | Part A | Question: 9 top

Suppose a rectangular farm has area 100 square meters. The lengths of its sides are not known. It is known, however, that all the edges are at least 2 meters in length. Which of the following statements about the rectangle's perimeter p (in meters) is FALSE?

A. p can take all values between 45 and 50
B. p can be 52 for some configuration
C. p can take all values between 55 and 60
D. p can be 70 for some configuration
E. p can be 39 for some configuration

tifr2016

Answer key

18.0.6 TIFR CSE 2016 | Part A | Question: 10 top

Consider the sequence $\langle s_n : n \ge 0 \rangle$ defined as follows: $s_0 = 0, s_1 = 1, s_2 = 1,$ and $s_n = s_{n-1} + s_{n-2} + s_{n-3}$, for $n \ge 3$. Which of the following statements is FALSE?

A. s_{4k} is even, for any $k \ge 0$
B. s_{4k+1} is odd, for any $k \ge 0$
C. s_{4k+2} is odd, for any $k \ge 0$
D. s_n is a multople of 3, for only finitely many values of n
E. s_{4k+3} is even, for any $k \ge 0$

tifr2016

Answer key

18.0.7 TIFR CSE 2016 | Part A | Question: 11 top

In one of the islands that his travels took him to, Gulliver noticed that the probability that a (uniformly) randomly chosen inhabitant has height at least 2 meters is 0.2. Also, 0.2 is the probability that a a (uniformly) randomly chosen inhabitant has height at most 1.5 meters. What can we conclude about the average height h in meters of the inhabitants of the island?

i. $1.5 \le h \le 2$
ii. $h \ge 1.3$
iii. $h \le 2.2$

Which of the above statements is necessarily true?

A. ii only
B. iii only
C. i, ii and iii
D. ii and iii only
E. None of the above

tifr2016 probability

Answer key

18.0.8 TIFR CSE 2016 | Part B | Question: 15 top

Let G be an undirected graph. For a pair (x, y) of distinct vertices of G, let **mincut**(x, y) be the least number of edges that should be delted from G so that the resulting graph has no $x - y$ path. Let a, b, c be three vertices in G such that **mincut**$(a, b) \leq$ **mincut**$(b, c) \leq mathsfmincut(c, a)$. Consider the following possibilities:

i. **mincut**$(a, b) <$ **mincut**$(b, c) <$ **mincut**(c, a)
ii. **mincut**$(a, b) =$ **mincut**$(b, c) <$ **mincut**(c, a)
iii. **mincut**$(a, b) <$ **mincut**$(b, c) =$ **mincut**(c, a)
iv. **mincut**$(a, b) =$ **mincut**$(b, c) =$ **mincut**(c, a)

Which of the following is TRUE?

A. All of i, ii iii, iv are possible
B. i, ii, iii are possible but not iv
C. i and iv are possible but neither ii nor iii
D. ii and iv are possible but neither i not iii
E. iii and iv are possible but neither i nor ii

tifr2016

Answer key

18.0.9 TIFR CSE 2016 | Part B | Question: 5 top

Consider the recursive function **mc91**.

```
int mc91(int n)
{
   print n
   if (n > 100) {
         return n-10;
   }
   else {
      return mc91(mc91(n+11));
   }
}
```

Let

$\text{Out} = \{n : \text{ there is an } x \in \{0, 1, \dots, 100\} \text{ such that } n \text{ is one of the integers printed by } \mathbf{mc91}(x)\}$

Then which of the following is **Out**?

A. $\{n : -\infty < n \leq 100\}$
B. $\{n : 0 \leq n \leq 101\}$
C. $\{n : 0 \leq n \leq 110\}$
D. $\{n : 0 \leq n \leq 111\}$
E. $\{n : 0 \leq n < +\infty\}$

tifr2016

Answer key

18.0.10 TIFR CSE 2016 | Part B | Question: 10 top

A *vertex cover* in an undirected graph G is a subset $C \subseteq V(G)$ such that every edge of G has an endpoint in C. An independent set in G is a subset $I \subseteq V(G)$ such that no edge has both its endpoints in I. Which of the following is TRUE of every graph G and every vertex cover C of G?

A. There exists an independent set of size $|C|$
B. $V(G) - C$ is an independent set

C. $|C| \geq |E(G)|/2$
D. $|C| \geq |V(G)|/2$
E. C intersects every independent set

tifr2016

Answer key

18.0.11 TIFR CSE 2021 | Part B | Question: 4 top

Consider the following two languages.

$$\begin{aligned} \text{PRIME} &= \{1^n \mid n \text{ is a prime number}\}, \\ \text{FACTOR} &= \{1^n 0 1^a 0 1^b \mid n \text{ has a factor in the range } [a, b]\} \end{aligned}.$$

What can you say about the languages PRIME and FACTOR?

A. PRIME is in P, but FACTOR is not in P.
B. Neither PRIME nor FACTOR are in P.
C. Both PRIME and FACTOR are in P.
D. PRIME is not in P, but FACTOR is in P.
E. None of the above since we can answer this question only if we resolve the status of the NP vs. P question.

tifr2021

Answer key

18.0.12 TIFR CSE 2016 | Part B | Question: 11 top

Let $n \geq 4$ be an integer. Regard the set $\mathbb{R}^n$ as a vector space over $\mathbb{R}$. Consider the following undirected graph H.

$$V(H) = \{S \subseteq \mathbb{R}^n : S \text{ is a basis for } \mathbb{R}^n\};$$

$$E(H) = \{\{S, T\} : |S \setminus T| = 1 \text{ and } |T \setminus S| = 1\},$$

where $S \setminus T = \{x \in S : x \notin T\}$. Which of the following statements is FALSE?

A. H has an inifinite number of vertices
B. The diameter of H is infinite
C. H is conneceted
D. H contains an infinite clique
E. H contains an infinite independent set

tifr2016

Answer key

18.0.13 TIFR CSE 2016 | Part B | Question: 12 top

A computer program computes a function $f\,\{0,1\}^* \times \{0,1\}^* \to \{0,1\}^*$. Suppose $f(a,b)$ ahs length $|b|^2$, where $|a|$ and $|b|$ are the lengths of a and b. Suppose, using this program, the following computation is performed.

```
x="01"
for i=1, ... , n do
  x=f("01", x)
```

Suppose at the end, the length of the string x is t. Which of the following is TRUE (assume $n \geq 10)?

A. $t \leq 2n$

B. $n < t \leq n^2$

C. $n^2 < t \leq n^{\log_2 n}$

D. $n^{\log_2 n} < t \leq 2^{(2n)}$

E. $2^{(2n)} < t$

tifr2016

Answer key☟

18.0.14 TIFR CSE 2016 | Part B | Question: 13 top☝

An undirected graph $G = (V, E)$ is said to be k-colourable if there exists a mapping $c : V \rightarrow \{1, 2, \dots k\}$ such that for every edge $\{u, v\} \in E$ we have $c(u) \neq c(v)$. Which of the following statements is FALSE?

A. G is $|V|$ -colourable
B. G is 2-colourable if there are no odd cycles in G
C. G is $(\Delta + 1)$-colourable where Δ is the maximum degree in G
D. There is a polynomial time algorithm to check if G is 2-colourable
E. If G has no triangle then it is 3-colourable

tifr2016

Answer key☟

18.0.15 TIFR CSE 2016 | Part A | Question: 3 top☝

Consider the following set of $3n$ linear equations in $3n$ variables:

$$\begin{array}{lll} x_1 - x_2 = 0 & x_4 - x_5 = 0 & \dots \quad x_{3n-2} - x_{3n-1} = 0 \\ x_2 - x_3 = 0 & x_5 - x_6 = 0 & \qquad x_{3n-1} - x_{3n} = 0 \\ x_1 - x_3 = 0 & x_4 - x_6 = 0 & \qquad x_{3n-2} = x_{3n} = 0 \end{array}$$

Let $S \subseteq \mathbb{R}^{3n}$ be the set of solutions to this set of equations. Then,

A. S is empty
B. S is a subspace of $\mathbb{R}^{3n}$ of dimension 1
C. S is a subspace of $\mathbb{R}^{3n}$ of dimension n
D. S is a subspace of $\mathbb{R}^{3n}$ of dimension $n - 1$
E. S has exactly n elements

tifr2016

Answer key☟

18.0.16 TIFR CSE 2021 | Part A | Question: 3 top☝

Let M be an $n \times m$ real matrix. Consider the following:

- Let k_1 be the smallest number such that M can be factorized as $A \cdot B$, where A is an $n \times k_1$ and B is a $k_1 \times m$ matrix.
- Let k_2 be the smallest number such that $M = \sum_{i=1}^{k_2} u_i v_i$, where each u_i is an $n \times 1$ matrix and each v_i is an $1 \times m$ matrix.
- Let k_3 be the column-rank of M.

Which of the following statements is TRUE?

A. $k_1 < k_2 < k_3$

B. $k_1 < k_3 < k_2$

C. $k_2 = k_3 < k_1$
D. $k_1 = k_2 = k_3$
E. No general relationship exists among k_1, k_2 and k_3

tifr2021

Answer key

18.0.17 TIFR CSE 2021 | Part A | Question: 14 top

Five married couples attended a party. In the party, each person shook hands with those they did not know. Everyone knows his or her spouse. At the end of the party, Shyamal, one of the attendees, listed the number of hands that other attendees including his spouse shook. He got every number from 0 to 8 once in the list. How many persons shook hands with Shyamal at the party?

A. 2
B. 4
C. 6
D. 8
E. Insufficient information

tifr2021

Answer key

18.0.18 TIFR CSE 2022 | Part A | Question: 5 top

Let $\mathcal{F}$ be the set of all functions mapping $\{1, \ldots, n\}$ to $\{1, \ldots, m\}$. Let f be a function that is chosen uniformly at random from $\mathcal{F}$. Let x, y be distinct elements from the set $\{1, \ldots, n\}$. Let p denote the probability that $f(x) = f(y)$. Then,

A. $p = 0$
B. $p = \frac{1}{n^m}$
C. $0 < p \leq \frac{1}{m^n}$
D. $p = \frac{1}{m}$
E. $p = \frac{1}{n}$

tifr2022

Answer key

18.0.19 TIFR CSE 2021 | Part B | Question: 12 top

Let G be an undirected graph. For any two vertices u, v in G, let $\text{cut}(u, v)$ be the minimum number of edges that should be deleted from G so that there is no path between u and v in the resulting graph. Let a, b, c, d be 4 vertices in G. Which of the following statements is impossible?

A. $\text{cut}(a, b) = 3$, $\text{cut}(a, c) = 2$ and $\text{cut}(a, d) = 1$
B. $\text{cut}(a, b) = 3$, $\text{cut}(b, c) = 1$ and $\text{cut}(b, d) = 1$
C. $\text{cut}(a, b) = 3$, $\text{cut}(a, c) = 2$ and $\text{cut}(b, c) = 2$
D. $\text{cut}(a, c) = 2$, $\text{cut}(b, c) = 2$ and $\text{cut}(c, d) = 2$
E. $\text{cut}(b, d) = 2$, $\text{cut}(b, c) = 2$ and $\text{cut}(c, d) = 1$

tifr2021

Answer key

18.0.20 TIFR CSE 2021 | Part B | Question: 5 top

For a language L over the alphabet $\{a, b\}$, let $\overline{L}$ denote the complement of L and let L^* denote the Kleene-closure of L. Consider the following sentences.

i. $\overline{L}$ and L^* are both context-free.
ii. $\overline{L}$ is not context-free but L^* is context-free.
iii. $\overline{L}$ is context-free but L^* is regular.

Which of the above sentence(s) is/are true if $L = \{a^n b^n \mid n \geq 0\}$?

A. Both (i) and (iii) B. Only (i) C. Only (iii) D. Only (ii) E. None of the above

tifr2021

Answer key

18.0.21 TIFR CSE 2022 | Part A | Question: 3 top

A binary string is a sequence of 0's and 1's. A binary string is *finite* if the sequence is finite, otherwise it is *infinite*. Examples of finite binary strings include 00010100, and $1111101010.$ Which of the following is TRUE about the set of all finite binary strings and the set of all infinite binary strings?

A. The set of all finite binary strings is countable while the set of all infinite binary strings is uncountable
B. The set of all finite binary strings is uncountable while the set of all infinite binary strings is countable
C. The set of all finite binary strings and the set of all infinite binary strings are both countable
D. The set of all finite binary strings and the set of all infinite binary strings are both uncountable
E. The set of all finite binary strings is countable while whether the set of all infinite binary strings is countable or not is not known

tifr2022

Answer key

18.0.22 TIFR CSE 2021 | Part B | Question: 15 top

Let $A[i] : i = 0, 1, 2, \ldots, n-1$ be an array of n distinct integers. We wish to sort A in ascending order. We are given that each element in the array is at a position that is at most k away from its position in the sorted array, that is, we are given that $A[i]$ will move to a position in $\{i-k, i-k+1, \ldots, i, \ldots, i+k-1, i+k\}$ after the array is sorted in ascending order. Suppose insertion sort is used to sort this array: that is, in the i th iteration, $A[i]$ is compared with the elements in positions $A[i-1], A[i-2], \ldots$ until one that is smaller is found and $A[i]$ is inserted after that element. Note that elements can be moved back when later insertions are made before them. Let $t(n)$ be the worst-case number of comparisons made by insertion sort for such inputs. Then,

A. $t(n) = \Theta\left(n^2\right)$
B. $t(n) = \Theta(n \log_2 n)$
C. $t(n) = \Theta(nk \log k)$
D. $t(n) = \Theta(n \log_2 k)$
E. $t(n) = \Theta(nk)$

tifr2021

Answer key

18.0.23 TIFR CSE 2020 | Part B | Question: 5 top

Let u be a point on the unit circle in the first quadrant (i.e., both coordinates of u are positive). Let θ be the angle subtended by u and the x axis at the origin. Let ℓ_u denote the infinite line passing through the origin and u. Consider the following operation O_u on points in the plane.

Operation O_u

INPUT: a point v on the plane

1. Reflect v in the x axis, obtaining $\tilde{v}$.
2. Reflect $\tilde{v}$ in ℓ_u, obtaining $\hat{v}$.

3. Output $\hat{v}$.

If $\hat{v}$ is the output of applying O_u on v, we write $O_u(v) = \hat{v}$. Further, we denote by O_u^k the iterates of O_u, i.e., $O_u^1(v) := O_u(v)$ and $O_u^k(v) := O_u(O_u^{k-1}(v))$ for all integers $k > 1$.
Consider a point v in the first quadrant such that v and the x-axis subtend an angle ϕ at the origin. Define $w = O_u^8(v)$. Assuming $\theta = 5^\circ$ and $\phi = 10^\circ$, what is the angle subtended by w and the x-axis at the origin?

A. 50° B. 85° C. 90° D. 145° E. 165°

tifr2020

Answer key

18.0.24 TIFR CSE 2020 | Part B | Question: 6 top

Consider the context-free grammar below (ϵ denotes the empty string, alphabet is $\{a, b\}$):

$$S \rightarrow \epsilon \mid aSb \mid bSa \mid SS.$$

What language does it generate?

A. $(ab)^* + (ba)^*$
B. $(abba)* + (baab)^*$
C. $(aabb)^* + (bbaa)^*$
D. Strings of the form $a^n b^n$ or $b^n a^n, n$ any positive integer
E. Strings with equal numbers of a and b

tifr2020

Answer key

18.0.25 TIFR CSE 2020 | Part B | Question: 7 top

Consider the following algorithm (Note: For positive integers, $p, q, p/q$ denotes the floor of the rational number $\frac{p}{q}$, assume that given $p, q, p/q$ can be computed in one step):

Input: Two positive integers $a, b, a \geq b$.

Output: A positive integers g.

```
while(b>0) {
        x = a – (a/b)*b;
        a = b;
        b = x;
    }

g = a;
```

Suppose K is an upper bound on a. How many iterations does the above algorithm take in the worst case?

A. $\Theta(\log K)$ B. $\Theta(K)$ C. $\Theta(K \log K)$ D. $\Theta(K^2)$ E. $\Theta(2^K)$

tifr2020

18.0.26 TIFR CSE 2020 | Part B | Question: 9 top

A particular Panini-Backus-Naur Form definition for a $<$ word $>$ is given by the following rules:

- $< \text{word} > ::= < \text{letter} > \mid < \text{letter} > < \text{pairlet} > \mid < \text{letter} > < \text{pairdig} >$
- $< \text{pairlet} > ::= < \text{letter} > < \text{letter} > \mid < \text{pairlet} > < \text{letter} > < \text{letter} >$

- $<$ pairdig $>::= <$ digit $><$ digit $>|<$ pairdig $><$ digit $><$ digit $>$
- $<$ letter $>::= a \mid b \mid c \mid \cdots \mid y \mid z$
- $<$ digit $>::= 0 \mid 1 \mid 2 \mid \cdots \mid 9$

Which of the following lexical entities can be derived from $<$ word $>$?

I. word
II. words
III. c22

A. None of I, II or III B. I and II only C. I and III only D. II and III only E. I, II and III

tifr2020

Answer key

18.0.27 TIFR CSE 2020 | Part B | Question: 12 top

Given the pseudocode below for the function **remains()**, which of the following statements is true about the output, if we pass it a positive integer $n > 2$?

```
int remains(int n)
{
    int x = n;
    for (i=(n-1);i>1;i--) {
      x = x % i ;
  }
    return x;
}
```

A. Output is always 0
B. Output is always 1
C. Output is 0 only if n is NOT a prime number
D. Output is 1 only if n is a prime number
E. None of the above

tifr2020

Answer key

18.0.28 TIFR CSE 2020 | Part B | Question: 13 top

Let G be an undirected graph. An Eulerian cycle of G is a cycle that traverses each edge of G exactly once. A Hamiltonian cycle of G is a cycle that traverses each vertex of G exactly once. Which of the following must be true?

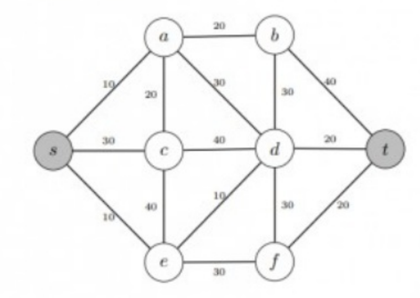
A. Checking if G has a Eulerian cycle can be done in polynomial time
B. Deciding if G has a Hamiltonian cycle is not NP-complete
C. If G has an Eulerian cycle, then it has a Hamiltonian cycle
D. A complete graph always has both an Eulerian cycle and a Hamiltonian cycle
E. All of the other statements are true

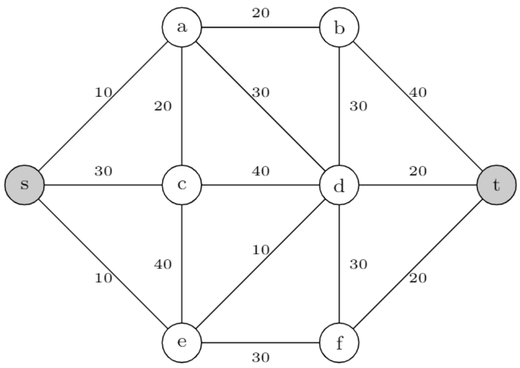
tifr2020

Answer key

18.0.29 TIFR CSE 2020 | Part B | Question: 14 top

The figure below describes the network of streets in a city where Motabhai sells pakoras from his cart. The number next to an edge is the time (in minutes) taken to traverse the corresponding street.





At present, the cart is required to start at point s and, after visiting each street at least once, reach point t. For example, Motabhai can visit the streets in the following order

$$s-a-c-s-e-c-d-a-b-d-f-e-d-b-t-f-d-t$$

in order to go from s to t. Note that the streets $\{b,d\}$ and $\{d,f\}$ are both visited twice in this strategy. The total time taken for this trip is 440 minutes [which is, 380 (the sum of traversal times of all streets in the network) $+60$ (the sum of the traversal times of streets $\{b,d\}$ and $\{d,f\}$)].

Motabhai now wants the cart to return to s at the end of the trip. So the previous strategy is not valid, and he must find a new strategy. How many minutes will Motabhai now take if he uses an optimal strategy?

Hint: s,t,b and f are the only odd degree nodes in the figure above.

A. 430 B. 440 C. 460 D. 470 E. 480

tifr2020

Answer key

18.0.30 TIFR CSE 2021 | Part B | Question: 14 top

Consider the following greedy algorithm for colouring an n-vertex undirected graph G with colours $c_1, c_2, \cdots$: consider the vertices of G in any sequence and assign the chosen vertex the first colour that has not already been assigned to any of its neighbours. Let $m(n,r)$ be the minimum number of edges in a graph that causes this greedy algorithm to use r colours. Which of the following is correct?

A. $m(n,r)=\Theta(r)$
B. $m(n,r)=\Theta(r\lceil\log_2 r\rceil)$
C. $m(n,r)=\binom{r}{2}$
D. $m(n,r)=nr$
E. $m(n,r)=n\binom{r}{2}$

tifr2021

Answer key

18.0.31 TIFR CSE 2021 | Part B | Question: 6 top

Consider the following pseudocode:

```
procedure HowManyDash(n)
 if n=0 then
  print '-'
 else if n=1 then
  print '-'
 else
  HowManyDash(n-1)
  HowManyDash(n-2)
 end if
end procedure
```

How many '-' does HowManyDash(10) print?

A. 9 B. 10 C. 55 D. 89 E. 1024

tifr2021

Answer key

18.0.32 TIFR CSE 2020 | Part B | Question: 4 top

A $clamp$ gate is an analog gate parametrized by two real numbers a and b, and denoted as $\text{clamp}_{a,b}$. It takes as input two non-negative real numbers x and y. Its output is defined as

$$clamp_{a,b}(x,y) = \begin{cases} ax+by & \text{when } ax+by \geq 0, \text{ and} \\ 0 & \text{when } ax+by < 0. \end{cases}$$

Consider circuits composed only of clamp gates, possibly parametrized by different pairs (a,b) of real numbers. How many clamp gates are needed to construct a circuit that on input non-negative reals x and y outputs the maximum of x and y?

A. 1
B. 2
C. 3
D. 4
E. No circuit composed only of clamp gates can compute the max function

tifr2020

Answer key

18.0.33 TIFR CSE 2021 | Part B | Question: 13 top

Let A be a 3×6 matrix with real-valued entries. Matrix A has rank 3. We construct a graph with 6 vertices where each vertex represents distinct column in A, and there is an edge between two vertices if the two columns represented by the vertices are linearly independent. Which of the following statements MUST be true of the graph constructed?

A. Each vertex has degree at most 2.
B. The graph is connected.
C. There is a clique of size 3.
D. The graph has a cycle of length 4.
E. The graph is 3-colourable.

tifr2021

Answer key

18.0.34 TIFR CSE 2021 | Part B | Question: 11 top

Suppose we toss a fair coin (i.e., both beads and tails have equal probability of appearing) repeatedly until the first time by which at least two heads and at least two tails have appeared in the sequence of tosses made. What is the expected number of coin tosses that we would have to make?

A. 8 B. 4 C. 5.5 D. 7.5 E. 4.5

tifr2021

Answer key

18.0.35 TIFR CSE 2021 | Part B | Question: 10 top

Let G be a connected bipartite simple graph (i.e., no parallel edges) with distinct edge weights. Which of the following statements on MST (minimum spanning tree) need NOT be true?

A. G has a unique MST.
B. Every MST in G contains the lightest edge.
C. Every MST in G contains the second lightest edge.
D. Every MST in G contains the third lightest edge.
E. No MST in G contains the heaviest edge.

tifr2021

Answer key

18.0.36 TIFR CSE 2021 | Part B | Question: 9 top

Let L be a context-free language generated by the context-free grammar $G = (V, \Sigma, R, S)$ where V is the finite set of variables, Σ the finite set of terminals (disjoints from V), R the finite set of rules and $S \in V$ the start variable. Consider the context-free grammar G' obtained by adding $S \to SS$ to the set of rules in G. What must be true for the language L' generated by G'?

A. $L' = LL$
B. $L' = L$
C. $L' = L^*$
D. $L' = \{xx \mid x \in L\}$
E. None of the above

tifr2021

Answer key

18.0.37 TIFR CSE 2021 | Part B | Question: 8 top

Let A and B be two matrices of size $n \times n$ and with real-valued entries. Consider the following statements.

1. If $AB = B$, then A must be the identity matrix.
2. If A is an idempotent (i.e. $A^2 = A$) nonsingular matrix, then A must be the identity matrix.
3. If $A^{-1} = A$, then A must be the identity matrix.

Which of the above statements MUST be true of A?

A. $1, 2$ and 3 B. Only 2 and 3 C. Only 1 and 2 D. Only 1 and 3 E. Only 2

tifr2021

Answer key

18.0.38 TIFR CSE 2021 | Part B | Question: 7 top

Which of the following regular expressions defines a language that is different from the other choices?

A. $b^*(a+b)^*a(a+b)^*ab^*(a+b)^*$
B. $a^*(a+b)^*ab^*(a+b)^*a(a+b)^*$
C. $(a+b)^*ab^*(a+b)^*a(a+b)^*b^*$
D. $(a+b)^*a(a+b)^*b^*a(a+b)^*a^*$
E. $(a+b)^*b^*a(a+b)^*b^*(a+b)^*$

tifr2021

Answer key

18.0.39 TIFR CSE 2022 | Part A | Question: 4 top

Consider the polynomial $p(x) = x^3 - x^2 + x - 1$. How many *symmetric* matrices with integer entries are there whose characteristic polynomial is p? (Recall that the *characteristic polynomial* of a square matrix A in the variable x is defined to be the determinant of the matrix $(A - xI)$ where I is the identity matrix.)

A. 0 B. 1 C. 2 D. 4 E. Infinitely many

tifr2022

Answer key

18.0.40 TIFR CSE 2021 | Part B | Question: 2 top

Let L be a singly-linked list X and Y be additional pointer variables such that X points to the first element of L and Y points to the last element of L. Which of the following operations cannot be done in time that is bound above by a constant?

A. Delete the first element of L.
B. Delete the last element of L.
C. Add an element after the last element of L.
D. Add an element before the first element of L.
E. Interchange the first two elements of L.

tifr2021

Answer key

18.0.41 TIFR CSE 2022 | Part A | Question: 2 top

We would like to invite a minimum number n of people (their birthdays are independent of each other) to a party such that the expected number of pairs of people that share the same birthday is at least 1. What should n be?

(Ignore leap years, so there are only 365 possible birthdays. Assume that birthdays fall with equal probability on each of the 365 days of the year.)

A. 23 B. 28 C. 92 D. 183 E. 366

tifr2022

Answer key

18.0.42 TIFR CSE 2021 | Part A | Question: 6 top☝

A matching in a graph is a set of edges such that no two edges in the set share a common vertex. Let G be a graph on $n\ vertices$ in which there is a subset M of $m\ edges$ which is a matching. Consider a random process where each vertex in the graph is independently selected with probability $0 < p < 1$ and let B be the set of vertices so obtained. What is the probability that there exists at least one edge from the matching M with both end points in the set B?

A. p^2
B. $1-(1-p^2)^m$
C. p^{2m}
D. $(1-p^2)^m$
E. $1-(1-p(1-p))^m$

tifr2021

Answer key☟

18.0.43 TIFR CSE 2022 | Part A | Question: 7 top☝

Initially, N white beads are arranged in a circle. A number k is chosen uniformly at random from $\{1,\ldots,N-1\}$. Then a set of k beads is chosen uniformly from the white beads, and these k beads are coloured black. The position of the beads remains unchanged. What is the probability that the black beads occur sequentially in the circle, i.e., at most two black beads have white beads next to them?

A. $\frac{2N}{2N+1}$
B. $\frac{N^2}{(N-1)(N-1)!}$
C. $\dfrac{N}{N-1}\displaystyle\sum_{k=1}^{N-1}\frac{1}{\binom{N}{k}}$
D. $\dfrac{1}{N}+\displaystyle\sum_{k=1}^{N-1}\frac{1}{\binom{N}{k}}$
E. None of the above

tifr2022

Answer key☟

18.0.44 TIFR CSE 2022 | Part A | Question: 9 top☝

You are given the following properties of sets $A, B, X,$ and Y. For notation, $|A|$ denotes the cardinality of set A (i.e., the number of elements in A), and $A \backslash B$ denotes the set of elements that are in A but not in B.

1. $A \cup B = X \cup Y$
2. $A \cap B = X \cap Y = \emptyset$
3. $|Y \backslash A| = 2$
4. $|A \backslash X| = 4$

Which of the following statements MUST then be FALSE?

A. $|X| = 5$
B. $|Y| = 5$
C. $|A \cup X| = |B \cup Y|$
D. $|A \cap X| = |B \cap Y|$
E. $|A| = |B|$

tifr2022

Answer key☟

18.0.45 TIFR CSE 2021 | Part A | Question: 13 top☝

What are the last two digits of 7^{2021}?

A. 67
B. 07
C. 27
D. 01
E. 77

tifr2021

Answer key

18.0.46 TIFR CSE 2021 | Part A | Question: 12 top

How many numbers in the range $0, 1, \ldots, 1365$ have exactly four 1's in their binary representation? (Hint: 1365_{10} is 10101010101_2, that is,

$$1365 = 2^{10} + 2^8 + 2^6 + 2^4 + 2^2 + 2^0.)$$

In the following, the binomial coefficient $\binom{n}{k}$ counts the number of k-element subsets of an n-element set.

A. $\binom{6}{4}$
B. $\binom{10}{4}$
C. $\binom{10}{4} + \binom{8}{3} + \binom{6}{2} + \binom{5}{1}$
D. $\binom{11}{4} + \binom{9}{3} + \binom{7}{2} + \binom{5}{1}$
E. 1024

tifr2021

Answer key

18.0.47 TIFR CSE 2021 | Part A | Question: 11 top

Find the following sum.

$$\frac{1}{2^2-1} + \frac{1}{4^2-1} + \frac{1}{6^2-1} + \cdots + \frac{1}{40^2-1}$$

A. $\frac{20}{41}$
B. $\frac{10}{41}$
C. $\frac{10}{21}$
D. $\frac{20}{21}$
E. 1

tifr2021

Answer key

18.0.48 TIFR CSE 2021 | Part A | Question: 10 top

Lavanya and Ketak each flip a fair coin (i.e., both heads and tails have equal probability of appearing) n times. What is the probability that Lavanya sees more heads than ketak?

In the following, the binomial coefficient $\binom{n}{k}$ counts the number of k-element subsets of an n-element set.

A. $\frac{1}{2}$
B. $\frac{1}{2}\left(1 - \sum_{i=0}^{n} \frac{\binom{n}{i}^2}{2^{2n}}\right)$
C. $\frac{1}{2}\left(1 - \sum_{i=0}^{n} \frac{\binom{n}{i}}{2^{2n}}\right)$

D. $\frac{1}{2}\left(1-\frac{1}{2^{2n}}\right)$

E. $\sum_{i=0}^{n} \frac{\binom{n}{i}}{2^n}$

tifr2021

Answer key

18.0.49 TIFR CSE 2021 | Part A | Question: 9 top

Fix $n \geq 6$. Consider the set $\mathcal{C}$ of binary strings $x_1, x_2 \dots x_n$ of length n such that the bits satisfy the following set of equalities, all modulo 2: $x_i + x_{i+1} + x_{i+2} = 0$ for all $1 \leq i \leq n-2$, $x_{n-1} + x_n + x_1 = 0$, and $x_n + x_1 + x_2 = 0$. What is the size of the set $\mathcal{C}$?

A. 1 for all $n \geq 6$
B. 4 for all $n \geq 6$
C. 0 for all $n \geq 6$
D. If $n \geq 6$ is divisible by 3 $|\mathcal{C}| = 1$. If $n \geq 6$ is not divisible by 3 then $|\mathcal{C}| = 4$.
E. If $n \geq 6$ is divisible by 3 $|\mathcal{C}| = 4$. If $n \geq 6$ is not divisible by 3 then $|\mathcal{C}| = 1$.

tifr2021

Answer key

18.0.50 TIFR CSE 2021 | Part A | Question: 8 top

Consider the sequence

$$y_n = \frac{1}{\int_1^n \frac{1}{(1+x/n)^3} dx}$$

for $\mathrm{n} = 2, 3, 4, \dots$ Which of the following is TRUE?

A. The sequence $\{y_n\}$ does not have a limit as $n \to \infty$.
B. $y_n \leq 1$ for all $\mathrm{n} = 2, 3, 4, \dots$
C. $\lim_{n\to\infty} y_n$ exists and is equal to $6/\pi^2$.
D. $\lim_{n\to\infty} y_n$ exists and is equal to 0.
E. The sequence $\{y_n\}$ first increases and then decreases as n takes values $2, 3, 4, \dots$

tifr2021

18.0.51 TIFR CSE 2021 | Part A | Question: 7 top

Let d be the positive square integers (that is, it is a square of some integer) that are factors of $20^5 \times 21^5$. Which of the following is true about d?

A. $50 \leq d < 100$
B. $100 \leq d < 150$
C. $150 \leq d < 200$
D. $200 \leq d < 300$
E. $300 \leq d$

tifr2021

Answer key

18.0.52 TIFR CSE 2021 | Part A | Question: 5 top

Let n, m and k be three positive integers such that $n \geq m \geq k$. Let S be a subset of $\{1, 2, \dots, n\}$ of size k. Consider sampling a function f uniformly at random from the set of all functions mapping $\{1,\ldots,n\}$ to $\{1,\ldots,\mathrm{m}\}$. What is the probability that f is not injective on the set S, i.e., there exist $i, j \in S$ such that $f(i)=f(j)$?

In the following, the binomial coefficient $\binom{n}{k}$ counts the number of k-element subsets of an n-element set.

A. $1-\frac{k!}{k^k}$
B. $1-\frac{m!}{m^k}$
C. $1-\frac{k!\binom{m}{k}}{m^k}$
D. $1-\frac{k!\binom{n}{k}}{n^k}$
E. $1-\frac{k!\binom{n}{k}}{m^k}$

tifr2021

Answer key☟

18.0.53 TIFR CSE 2022 | Part A | Question: 1 top☝

A snail crawls up a vertical pole 75 feet high, starting from the ground. Each day it crawls up 5 feet, and each night it slides down 4 feet. When will it first reach the top of the pole?

A. 75^{th} day
B. 74^{th} day
C. 73^{rd} day
D. 72^{nd} day
E. 71^{st} day

tifr2022

Answer key☟

18.0.54 TIFR CSE 2022 | Part A | Question: 8 top☝

Let A be the $(n+1)\times(n+1)$ matrix given below, where $n \geq 1$. For $i \leq n$, the i-th row of A has every entry equal to $2i-1$ and the last row, i.e., the $(n+1)$-th row of A has every entry equal to $-n^2$.

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 3 & 3 & \cdots & 3 \\ \vdots & \vdots & \vdots & \vdots \\ 2n-1 & 2n-1 & \cdots & 2n-1 \\ -n^2 & -n^2 & \cdots & -n^2 \end{bmatrix}$$

Which of the following statements is TRUE for all $n \geq 1$?

A. A has rank n
B. A^2 has rank 1
C. All the eigenvalues of A are distinct
D. All the eigenvalues of A are 0
E. None of the above

tifr2022

Answer key☟

18.0.55 TIFR CSE 2021 | Part A | Question: 4 top☝

What is the probability that at least two out of four people have their birthdays in the same month, assuming their birthdays are uniformly distributed over the twelve months?

A. $\frac{25}{48}$

B. $\frac{5}{8}$

C. $\frac{5}{12}$

D. $\frac{41}{96}$

E. $\frac{55}{96}$

tifr2021

Answer key

18.0.56 TIFR CSE 2021 | Part A | Question: 2 top

What is the area of a rectangle with the largest perimeter that can be inscribed in the unit circle (i.e., all the vertices of the rectangle are on the circle with radius 1)?

A. 1 B. 2 C. 3 D. 4 E. 5

tifr2021

Answer key

18.0.57 TIFR CSE 2021 | Part A | Question: 1 top

A box contains 5 red marbles, 8 green marbles, 11 blue marbles, and 15 yellow marbles. We draw marbles uniformly at random without replacement from the box. What is the minimum number of marbles to be drawn to ensure that out of the marbles drawn, at least 7 are of the same colour?

A. 7 B. 8 C. 23 D. 24 E. 39

tifr2021

Answer key

18.0.58 TIFR CSE 2022 | Part A | Question: 15 top

Fix $n \geq 4$. Suppose there is a particle that moves randomly on the number line, but never leaves the set $\{1, 2, \ldots, n\}$. The initial probability distribution of the particle is π i.e., the probability that particle is in location i is given by $\pi(i)$. In the first step, if the particle is at position i, it moves to one of the positions in $\{1, 2, \ldots, i\}$ with uniform distribution; in the second step, if the particle is in location j, then it moves to one of the locations in $\{j, j+1, \ldots, n\}$ with uniform distribution. Suppose after two steps, the final distribution of the particle is uniform. What is the initial distribution π?

A. π is not unique

B. π is uniform

C. $\pi(i)$ is non-zero for all even i and zero otherwise

D. $\pi(1) = 1$ and $\pi(i) = 0$ for $i \neq 1$

E. $\pi(n) = 1$ and $\pi(i) = 0$ for $i \neq n$

tifr2022

Answer key

18.0.59 TIFR CSE 2022 | Part A | Question: 14 top

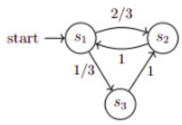
Suppose $w(t) = 4e^{it}$, $x(t) = 3e^{i(t+\pi/3)}$, $y(t) = 3e^{i(t-\pi/3)}$ and $z(t) = 3e^{i(t+\pi)}$ are points that move in the complex plane as the time t varies in $(-\infty, \infty)$. Let $c(t)$ be the point in the complex plane such that $|w(t) - c(t)|^2 + |x(t) - c(t)|^2 + |y(t) - c(t)|^2 + |z(t) - c(t)|^2$ is minimum. For each value of t, the point $c(t)$ is unique, but $c(t)$ moves at constant speed as t varies. At what speed? That is, what is $\left|\frac{\mathrm{d}}{\mathrm{d}t}c(t)\right|$?

A. $\frac{1}{2\pi}$ B. 2π C. $\sqrt{3}\pi$ D. $\frac{1}{\sqrt{3}\pi}$ E. 1

tifr2022

18.0.60 TIFR CSE 2022 | Part A | Question: 13 top

Consider the transition system shown in the figure below with the initial state s_1. A token is initially placed at s_1, and it moves to s_2 with probability $\frac{2}{3}$, and to s_3 with probability $\frac{1}{3}$. From s_2 and s_3, the token always moves to s_1 and s_2 respectively. A *run* of the system consists of an infinite sequence of states constructed by moving the token from one state to another following the transitions forever. Assuming such a run is chosen randomly, what is the fraction of times that the state s_2 is expected to appear in the run?



A. $\frac{1}{7}$ B. $\frac{2}{7}$ C. $\frac{3}{7}$ D. $\frac{5}{7}$ E. None of the above

tifr2022

18.0.61 TIFR CSE 2022 | Part A | Question: 12 top

Alice plays the following game on a math show. There are 7 boxes and identical prizes are hidden inside 3 of the boxes. Alice is asked to choose a box where a prize might be. She chooses a box uniformly at random. From the unchosen boxes which do not have a prize, the host opens an arbitrary box and shows Alice that there is no prize in it. The host then allows Alice to change her choice if she so wishes. Alice chooses a box uniformly at random from the other 5 boxes (other than the one she chose first and the one opened by the host). Her probability of winning the prize is

A. $3/7$ B. $1/2$ C. $17/30$ D. $18/35$ E. $9/19$

tifr2022

Answer key

18.0.62 TIFR CSE 2022 | Part A | Question: 11 top

Let X be a finite set. A family $\mathcal{F}$ of subsets of X is said to be *upward closed* if the following holds for all sets $A, B \subseteq X$:

$$A \in \mathcal{F} \text{ and } A \subseteq B \Rightarrow B \in \mathcal{F}.$$

For families $\mathcal{F}$ and $\mathcal{G}$ of subsets of X, let

$$\mathcal{F} \sqcup \mathcal{G} = \{A \cup B : A \in \mathcal{F} \text{ and } B \in \mathcal{G}\}.$$

Suppose $\mathcal{F}$ and $\mathcal{G}$ are upward closed families. Then which of the following is true?

A. $\mathcal{F} \sqcup \mathcal{G} = \mathcal{F} \cap \mathcal{G}$
B. $\mathcal{F} \sqcup \mathcal{G} = \mathcal{F} \cup \mathcal{G}$
C. $\mathcal{F} \sqcup \mathcal{G} = \mathcal{F} \backslash \mathcal{G}$
D. $\mathcal{F} \sqcup \mathcal{G} = \mathcal{G} \backslash \mathcal{F}$
E. None of the above

tifr2022

18.0.63 TIFR CSE 2022 | Part A | Question: 10 top

Consider a bag containing colored marbles. There are n marbles in the bag such that there is exactly one pair of marbles of color i for each $i \in \{1, \dots, m\}$ and the rest of the marbles are of distinct colors (different from colors $\{1, \dots, m\}$). You draw two marbles uniformly at random (without replacement). What is the probability that both marbles are of same color?

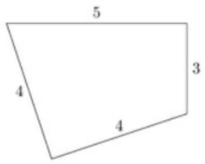
A. $\frac{m}{n}$
B. $\frac{2m}{n}$
C. $\frac{2m}{n(n-1)}$
D. $\frac{2m}{n^2}$
E. $\frac{m}{n(n-1)}$

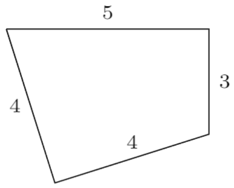
tifr2022

Answer key

18.0.64 TIFR CSE 2021 | Part A | Question: 15 top

Let P be a convex polygon with sides $5, 4, 4, 3$. For example, the following:





Consider the shape in the plane that consists of all points within distance 1 from some point in P. If ℓ is the perimeter of the shape, which of the following is always correct?

A. ℓ cannot be determined from the given information.
B. $20 \leq \ell < 21$
C. $21 \leq \ell < 22$
D. $22 \leq \ell < 23$
E. $23 \leq \ell < 24$

tifr2021

18.0.65 TIFR CSE 2022 | Part B | Question: 1 top

Which data structure is commonly used to implement breadth first search in a graph?

A. A queue
B. A stack
C. A heap
D. A hash table
E. A splay tree

tifr2022

Answer key

18.0.66 TIFR CSE 2022 | Part B | Question: 11 top

Consider the following function **count**, that takes as input a, an array of integers, and N, the size of the array.

```
int count(int a[], int N) {
  int i, j, count_FN;
  count_FN = 0;
  for (i=1 ; i<N ; i++) {
    j=i-1 ;
      while (a[j]>a[i]) {
        count_FN++;
```

```
            j--;
        }
    }
    return count_FN;
}
```

Further, let **count_IS** be the number of comparisons made by the insertion sort algorithm on the array a.

Which of the following statements is TRUE for some constant c?

A. For all $\mathsf{N} \geq c$, there exists an array of size N for which **count_IS** $\geq \mathsf{N}^2/c$, while **count_FN** $\leq c\mathsf{N}$
B. For all $\mathsf{N} \geq c$, there exists an array of size N for which **count_FN** $\geq \mathsf{N}^2/c$, while **count_IS** $\leq c\mathsf{N}$
C. For all $\mathsf{N} \geq c$, for all arrays of size N, **count_FN** $\leq$ **count_IS** $\leq c \times$ **count_FN**
D. For all $\mathsf{N} \geq c$, for all arrays of size N, **count_FN** $\geq \mathsf{N}^2/c$
E. None of the above

tifr2022

18.0.67 TIFR CSE 2022 | Part B | Question: 5 top

There is an unsorted list of n integers. You are given 3 distinct integers and you have to check if all 3 integers are present in the list or not. The only operation that you are allowed to perform is a comparison. Let A be an algorithm for this task that performs the least number of comparisons. Let c be the number of comparisons done by A. Then,

A. $c = 3n$ B. $c = 2n + 5$ C. $c \geq 3n - 1$ D. $c \leq n$ E. $c \leq 2n + 3$

tifr2022

Answer key

18.0.68 TIFR CSE 2022 | Part B | Question: 9 top

Let $n \geq 2$ be any integer. Which of the following statements is FALSE?

A. $n!$ divides the product of any n consecutive integers
B. $\sum_{i=0}^{n} \binom{n}{i} = 2^n$
C. $\binom{n}{i} = \binom{n-1}{i} + \binom{n-1}{i-1}$, where $1 \leq i \leq n-1$
D. If n is an odd prime, then n divides $2^{n-1} - 1$
E. n divides $\binom{2n}{n}$

tifr2022

Answer key

18.0.69 TIFR CSE 2022 | Part B | Question: 8 top

Let r_1 and r_2 be two regular expressions. They symbol $\equiv$ stands for equivalence of two regular expressions in the sense that if $r_1 \equiv r_2$, then both regular expressions describe the same language. Which of the following is/are FALSE?

i. $(r_1 r_2)^* r_1 \equiv r_1 (r_2 r_1)^*$
ii. $(r_1^* r_2)^* r_1^* \equiv (r_1 + r_2)^*$

iii. $(r_1^* r_2^*)^* \equiv (r_1 + r_2)^*$

A. Only (i) is false
B. Only (ii) is false
C. Only (iii) is false
D. Both (i) and (iii) are false
E. None of the above

tifr2022

18.0.70 TIFR CSE 2022 | Part B | Question: 7 top

Consider the following grammar: $\mathrm{P}, \mathrm{Q}, \mathrm{R}$ are non-terminals; c, d are terminals; P is the start symbol; and the production rules follow.

$\mathrm{P} ::= \mathrm{QR}$

$\mathrm{Q} ::= \mathrm{c}$

$\mathrm{Q} ::= \mathrm{RcR}$

$\mathrm{R} ::= \mathrm{ddQ}$

Which of the following is **False:**

A. The length of every string produced by the grammar is even
B. No string produced by the grammar has an odd number of consecutive d's
C. No string produced by the grammar has four consecutive d's
D. No string produced by the grammar has three consecutive c's
E. Every string produced by the grammar has at least has many d's as c's

tifr2022

Answer key

18.0.71 TIFR CSE 2022 | Part A | Question: 6 top

Let f be a polynomial of degree $n \geq 3$ all of whose roots are non-positive real numbers. Suppose that $f(1) = 1$. What is the maximum possible value of $f'(1)$?

A. 1
B. n
C. $n + 1$
D. $\frac{n(n+1)}{2}$
E. $f'(1)$ can be arbitrarily large given only the constraints in the question

tifr2022

18.0.72 TIFR CSE 2022 | Part B | Question: 12 top

Given an undirected graph G, an ordering σ of its vertices is called a *perfect ordering* if for every vertex v, the neighbours of v which precede v in σ form a clique in G.

Recall that given an undirected graph G, a *clique* in G is a subset of vertices every two of which are connected by an edge, while a *perfect colouring* of G *with* k colours is an assignment of labels from the set $\{1, 2, \ldots, k\}$ to the vertices of G such that no two vertices which are adjacent in G receive the same label.

Consider the following problems.

Problem SPECIAL-CLIQUE

INPUT: An undirected graph G, a positive integer k, and a perfect ordering σ of the vertices of G.

OUTPUT: Yes, if G has a clique of size at least k, No otherwise.

Problem SPECIAL-COLOURING

INPUT: An undirected graph G, a positive integer k, and a perfect ordering σ of the vertices of G.

OUTPUT: Yes, if G has a proper colouring with at most k colours, No otherwise.

Assume that $\mathrm{P} \neq NP$. Which of the following statements is true?

A. Both SPECIAL-CLIQUE and SPECIAL-COLOURING are undecidable
B. Only SPECIAL-CLIQUE is in P
C. Only SPECIAL-COLOURING is in P
D. Both SPECIAL-CLIQUE and SPECIAL-COLOURING are in P
E. Neither of SPECIAL-CLiQUE and SPECIAL-COLOURING is in P, but both are decidable

tifr2022

18.0.73 TIFR CSE 2022 | Part B | Question: 6 top

We are given a graph G along with a matching M and a vertex cover C in it such that $|M| = |C|$. Consider the following statements:

1. M is a maximum matching in G.
2. C is a minimum vertex cover in G.
3. G is a bipartite graph.

Which of the following is TRUE?

A. Only statement (1) is correct
B. Only statement (2) is correct
C. Only statement (3) is correct
D. Only statements (1) and (2) are correct
E. All the three statements $(1), (2)$, and (3) are correct

tifr2022

Answer key

18.0.74 TIFR CSE 2022 | Part B | Question: 4 top

Consider the following algorithm for computing the factorial of a positive integer n, specified in binary:

```
prod ← 1
for i from 1 to n
   prod ← prod × i
output prod
```

Assume that the number of bit operations required to multiply a k-bit positive integer with an ℓ-bit positive integer is at least $\Omega(k+l)$ and at most $O(kl)$. Then, the number of bit operations required by this algorithm is

A. $O(n)$
B. $O(n \log n)$ but $\omega(n)$
C. $O\left(n^2\right)$ but $\omega(n \log n)$
D. $O\left(n^3\right)$ but $\omega\left(n^2\right)$
E. None of the above

tifr2022

18.0.75 TIFR CSE 2022 | Part B | Question: 13 top

Consider a directed graph $G=(V, E)$, where each edge $e \in E$ has a positive edge weight c_e. Determine the appropriate choices for the blanks below so that the value of the following linear program is the length of the shortest directed path in G from s to t. (Assume that the graph has at least one path from s to t.)

$$\begin{array}{rrcll} \underline{\text{(blank 1)imize}} & X_t & & & \\ \text{s.t.} & X_s & = & 0 & \\ & X_w - X_v & \underline{\text{(blank 2)}} & c_e & \text{(for each edge } e = (v, w) \in E). \end{array}$$

A. $\text{blank 1 : max, blank 2 : } \leq$
B. $\text{blank 1 : max, blank 2 : } \geq$
C. $\text{blank 1 : min, blank 2 : } \leq$
D. $\text{blank 1 : min, blank 2 : } \geq$
E. $\text{blank 1 : min, blank 2 : } =$

tifr2022

18.0.76 TIFR CSE 2022 | Part B | Question: 3 top

Consider the problem of sorting n single digit integers (base 10). This problem can be solved in time

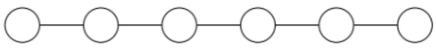
A. $O(n \log n)$ but not $O(n \log \log n)$
B. $O(n \log \log n)$ but not $O(n)$
C. $O(n)$ but not $O(n/ \log \log n)$
D. $O(n/ \log \log n)$
E. None of the above.

tifr2022

Answer key

18.0.77 TIFR CSE 2022 | Part B | Question: 2 top

Let $G = (V, E)$ be an undirected simple graph. A subset $M \subseteq E$ is a *matching* in G if distinct edges in M do not share a vertex. A matching is *maximal* if no strict superset of M is a matching. How many maximal matchings does the following graph have?



A. 1
B. 2
C. 3
D. 4
E. 5

tifr2022

Answer key

18.0.78 TIFR CSE 2022 | Part B | Question: 10 top

Consider the assertions

$(\mathrm{A1})$ Given a directed graph G with positive weights on the edges, two special vertices s and t, and an integer k - it is NP-complete to determine if G has an $s - t$ path of length at most k.

$(\mathrm{A2})\ \mathrm{P} = \mathrm{NP}$

Then, which of the following is true?

A. $\mathrm{A1}$ implies $\mathrm{A2}$ and $\mathrm{A2}$ implies $\mathrm{A1}$
B. $\mathrm{A1}$ implies $\mathrm{A2}$ and $\mathrm{A2}$ does not imply $\mathrm{A1}$
C. $\mathrm{A1}$ does not imply $\mathrm{A2}$ and $\mathrm{A2}$ implies $\mathrm{A1}$
D. $\mathrm{A1}$ does not imply $\mathrm{A2}$ and $\mathrm{A2}$ does not imply $\mathrm{A1}$
E. None of the above.

tifr2022

18.0.79 TIFR CSE 2022 | Part B | Question: 14 top

Let G be a directed graph (with no self-loops or parallel edges) with $n \geq 2$ vertices and m edges. Consider the $n \times m$ incidence matrix M of G, whose rows are indexed by the vertices of G and the columns by the edges of G. The entry $m_{v,e}$ is defined as follows.

$$m_{v,e} = \begin{cases} -1 & \text{if } e = (v, w) \text{ for some vertex } w, \\ +1 & \text{if } e = (u, v) \text{ for some vertex } u, \\ 0 & \text{otherwise.} \end{cases}$$

Suppose every vertex of G is reachable from a special source vertex of G. Then, what is the rank of M?

A. $m-1$ B. $m-n+1$ C. $\lceil m/2 \rceil$ D. $n-1$ E. $\lceil n/2 \rceil$

tifr2022

Answer key

18.0.80 TIFR CSE 2022 | Part B | Question: 15 top

Let $\mathbb{R}$ denote the set of real numbers. Let $d \geq 4$ and $\alpha \in \mathbb{R}$. Let

$$S = \left\{ (a_0, a_1, \ldots, a_d) \in \mathbb{R}^{d+1} : \sum_{i=0}^{d} a_i \alpha^i = 0 \text{ and } \sum_{i=0}^{d} i a_i \alpha^{i-1} = 0 \right\}.$$

Then,

A. S is finite or infinite depending on the value of α
B. S is a 2-dimensional vector subspace of $\mathbb{R}^{d+1}$
C. S is a d-dimensional vector subspace of $\mathbb{R}^{d+1}$
D. S is a $(d-1)$-dimensional vector subspace of $\mathbb{R}^{d+1}$
E. For each $(a_0, a_1, \ldots, a_d) \in S$, the function

$$x \mapsto \sum_{i=0}^{d} a_i x^i$$

has a local optimum at α

tifr2022

18.1 3 Sat (1) top

18.1.1 3 Sat: TIFR CSE 2016 | Part B | Question: 3 top

Assume $P \neq NP$. Which of the following is not TRUE?

A. 2-SAT in NP
B. 2-SAT in coNP
C. 3-SAT is polynmial-time reducible to 2-SAT
D. 4-SAT is polynmial-time reducible to 3-SAT
E. 2-SAT in P

tifr2016 p-np-npc-nph 3-sat 2-sat

Answer key

18.2 Binomial Theorem (1) top

18.2.1 Binomial Theorem: TIFR CSE 2016 | Part A | Question: 13 top

Let $n \geq 2$ be any integer. Which of the following statements is not necessarily true?

A. $\binom{n}{i} = \binom{n-1}{i} + \binom{n-1}{i-1}$, where $1 \leq i \leq n-1$
B. $n!$ divides the product of any n consecutive integers
C. $\Sigma_{i=0}^{n} \binom{n}{i} = 2^n$
D. n divides $\binom{n}{i}$, for all $i \in \{1, 2, \dots, n-1\}$
E. If n is an odd prime, then n divides $2^{n-1} - 1$

tifr2016 binomial-theorem

Answer key

18.3 Closure Property (1) top

18.3.1 Closure Property: TIFR CSE 2016 | Part B | Question: 2 top

Which language class has the following properties?

It is closed under union and intersection but not complement.

A. Regular language
B. Context-free language
C. Recursive language
D. Recursively enumerable language
E. Languages that are not recursively enumerable

tifr2016 theory-of-computation closure-property

Answer key

18.4 Complex Number (1) top

18.4.1 Complex Number: TIFR CSE 2016 | Part A | Question: 6 top

Which of the following statements about the eigen values of I_n, the $n \times n$ identity matrix (over complex numbers), is true?

A. The eigen values are $1, \omega, \omega^2, \dots, \omega^{n-1}$, where ω is a primitive n-th root of unity
B. The only eigen value is -1
C. Both 0 and 1 are eigen values, but there are no other eigen values
D. The eigen values are $1, 1/2, 1/3, \dots, 1/n$
E. The only eigen value is 1

tifr2016 matrix complex-number

Answer key

18.5 Divergence (1) top

18.5.1 Divergence: TIFR CSE 2016 | Part A | Question: 5 top

For a positive integer $N \geq 2$, let

$$A_N := \Sigma_{n=2}^{N} \frac{1}{n};$$

$$B_N := \int_{x=1}^{N} \frac{1}{x} dx$$

Which of the following statements is true?

A. As $N \to \infty$, A_N increases to infinity but B_N coverages to a finite number
B. $A_N < B_N$ and the difference decreases as $N \to \infty$
C. $A_N < B_N < A_N + 1$
D. $B_N < A_N < B_N + 1$
E. As $N \to \infty$, B_N increases to infinity but A_N coverages to a finite number

tifr2016 convergence divergence integration

Answer key☟

18.6 Dynamic Programming (1) top☝

18.6.1 Dynamic Programming: TIFR CSE 2016 | Part A | Question: 7 top☝

Let S be the 4×4 square grid $\{(x, y) : x, y \in \{0, 1, 2, 3\}\}$. A $monotone\ path$ in this grid starts at $(0, 0)$ and at each step either moves one unit up or one unit right. For example, fromthe point (x, y) one can in one step either move to $(x + 1, y) \in S$ or $(x, y + 1) \in S$, but never leave S. Let the number of distinct monotone paths to reach point $(2, 2)$ starting from $(0, 0)$ bt z. How many distinct monotone paths are there to reach point $(3, 3)$ starting from $(0, 0)$?

A. $2z + 6$ B. $3z + 6$ C. $2z + 8$ D. $3z + 8$ E. $3z + 4$

tifr2016 combinatory dynamic-programming

Answer key☟

18.7 Euler Graph (1) top☝

18.7.1 Euler Graph: TIFR CSE 2016 | Part B | Question: 9 top☝

Which of the following graphs DOES NOT have an Eulerian circuit? (Recall that an Eulerian circuit in an undirected graph is a walk in the graph that starts at a vertex ans returns to the vertex after tracelling on each edge exactly once.)

A. $K_{9,9}$
B. $K_{8,8}$
C. $K_{12,12}$
D. K_9
E. The graph G on vertex set $\{1, 2, \ldots, 9\}$ with edge set

$$E(G) = \{\{i, j\} : 1 \leq i < j \leq 5\, or\, 5 \leq i < j \leq 9\}.$$

tifr2016 discrete-mathematics graph-theory euler-graph normal

Answer key☟

18.8 Generalaptitude (1) top☝

18.8.1 Generalaptitude: TIFR CSE 2016 | Part A | Question: 14 top☝

A $diagonal$ in a polygon is a straight line segment that connects two non-adjacent vertices, and is contained in the interior of the polygon (except for its points). Two such diagonals are said to cross if they have a point in common in the interior of the polygon. In one such polygon with n vertices, a certain number (say k) of non-crossing diagonals were drawn to cut up the inside of the polygon into regions, each of which was a quadrilateral. how many diagonals were drawn, that is, what is k?

A. cannot be determined from the information given
B. $\frac{n}{2}-2$
C. $\frac{n}{4}-1$
D. $n-4$
E. $n^2-9.5n+22$

tifr2016 graph-theory generalaptitude

Answer key☟

18.9 P Np Npc Nph (1) top☝

18.9.1 P Np Npc Nph: TIFR CSE 2016 | Part B | Question: 8 top☝

Consider the following language

$$\text{PRIMES} = \left\{ \underbrace{111\ldots11}_{p \text{ times}} : p \text{ is prime} \right\}$$

Then, which of the following is TRUE?

A. PRIMES is regular
B. PRIMES is undecidable
C. PRIMES is decidable in polynomial time
D. PRIMES is context free but not regular
E. PRIMES is NP-complete and P $\neq$ NP

tifr2016 decidability p-np-npc-nph

Answer key☟

18.10 Uniform Hashing (1) top☝

18.10.1 Uniform Hashing: TIFR CSE 2016 | Part A | Question: 4 top☝

There are n balls $b_1, \ldots, b_n$ and n boxes. Each ball is placed in box chosen independently and uniformly at random. We say that (b_i, b_j) is a $colliding\ pair$ if $i < j$, and b_i and b_j are placed in the same box. WHat is the expected number of $colliding\ pairs$?

A. $\frac{n-1}{2}$
B. 0
C. 1
D. $\frac{n}{4}$
E. $\binom{n}{2}$

tifr2016 probability uniform-hashing

Answer key☟

18.11 Work Time (1) top☝

18.11.1 Work Time: TIFR CSE 2018 | Part A | Question: 4 top☝

The distance from your home to your office is 4 kilometers and your normal walking speed is 4 Km/hr. On the first day, you walk at your normal walking speed and take time T_1 to reach office.

On the second day, you walk at a speed of 3 Km/hr from 2 Kilometers, and at a speed of 5 Km/hr for the remaining 2 Kilometers and you take time T_2 to reach office.

On the third day, you walk at a speed of 3 Km/hr for 30 minutes, and at 5 Km/hr for the remaining time and take time T_3 to reach office.

What can you say about the ordering of T_1, T_2 and T_3

A. $T_1 > T_2$ and $T_1 < T_3$
B. $T_1 = T_2 = T_3$
C. $T_1 < T_2$ and $T_1 > T_3$
D. $T_1 = T_2$ and $T_1 < T_3$
E. $T_1 < T_2$ and $T_1 = T_3$

tifr2018 quantitative-aptitude work-time

Answer key☟

Answer Keys

18.0.1	A	18.0.2	C	18.0.3	D	18.0.4	A	18.0.5	E
18.0.6	D	18.0.7	A	18.0.8	D	18.0.9	D	18.0.10	B
18.0.11	C	18.0.12	B	18.0.13	D	18.0.14	E	18.0.15	C
18.0.16	D	18.0.17	B	18.0.18	D	18.0.19	E	18.0.20	B
18.0.21	A	18.0.22	E	18.0.23	C	18.0.24	E	18.0.25	A
18.0.26	D	18.0.27	B	18.0.28	A	18.0.29	A	18.0.30	C
18.0.31	D	18.0.32	B	18.0.33	C	18.0.34	C	18.0.35	E
18.0.36	E	18.0.37	E	18.0.38	E	18.0.39	A	18.0.40	B
18.0.41	B	18.0.42	B	18.0.43	C	18.0.44	B	18.0.45	B
18.0.46	C	18.0.47	A	18.0.48	B	18.0.49	E	18.0.50	D
18.0.51	C	18.0.52	C	18.0.53	E	18.0.54	D	18.0.55	D
18.0.56	B	18.0.57	D	18.0.58	D	18.0.59	E	18.0.60	C
18.0.61	D	18.0.62	A	18.0.63	C	18.0.64	D	18.0.65	A
18.0.66	A	18.0.67	E	18.0.68	E	18.0.69	E	18.0.70	C
18.0.71	B	18.0.72	D	18.0.73	D	18.0.74	D	18.0.75	A
18.0.76	C	18.0.77	D	18.0.78	A	18.0.79	D	18.0.80	D
18.1.1	C	18.2.1	D	18.3.1	D	18.4.1	E	18.5.1	C
18.6.1	C	18.7.1	A	18.8.1	B	18.9.1	C	18.10.1	A
18.11.1	E								

19 Programming and DS: DS (10)

19.1 Array (1) top☝

19.1.1 Array: TIFR CSE 2011 | Part B | Question: 30 top☝

Consider an array $A[1...n]$. It consists of a permutation of numbers $1....n$. Now compute another array $B[1...n]$ as follows: $B[A[i]] := i$ for all i. Which of the following is true?

A. B will be a sorted array.
B. B is a permutation of array A.
C. Doing the same transformation twice will not give the same array.
D. B is not a permutation of array A.
E. None of the above.

tifr2011 data-structures array

Answer key☟

19.2 Binary Search Tree (1) top☝

19.2.1 Binary Search Tree: TIFR CSE 2010 | Part B | Question: 26 top☝

Suppose there is a balanced binary search tree with n nodes, where at each node, in addition to the key, we store the number of elements in the sub tree rooted at that node.

Now, given two elements a and b, such that $a < b$, we want to find the number of elements x in the tree that lie between a and b, that is, $a \leq x \leq b$. This can be done with (choose the best solution).

A. $O(\log n)$ comparisons and $O(\log n)$ additions.
B. $O(\log n)$ comparisons but no further additions.
C. $O\left(\sqrt{n}\right)$ comparisons but $O(\log n)$ additions.
D. $O(\log n)$ comparisons but a constant number of additions.
E. $O(n)$ comparisons and $O(n)$ additions, using depth-first- search.

tifr2010 binary-search-tree

Answer key☟

19.3 Binary Tree (5) top☝

19.3.1 Binary Tree: TIFR CSE 2012 | Part B | Question: 16 top☝

Consider a complete binary tree of height n, where each edge is one Ohm resistor. Suppose all the leaves of the tree are tied together. Approximately how much is the effective resistance from the root to this bunch of leaves for very large n?

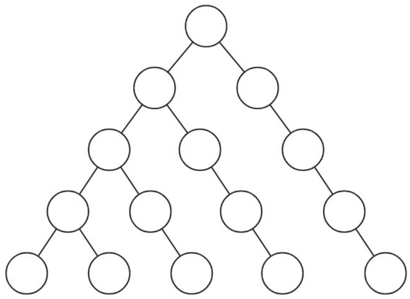
A. Exponential in n.
B. Cubic in n.
C. Linear in n.
D. Logarithmic in n.
E. Of the order square root of n.

tifr2012 binary-tree

Answer key☟

19.3.2 Binary Tree: TIFR CSE 2013 | Part B | Question: 13 top☝

Given a binary tree of the following form and having n nodes, the height of the tree is



A. $\Theta(\log n)$
B. $\Theta(n)$
C. $\Theta(\sqrt{n})$
D. $\Theta(n/\log n)$
E. None of the above.

tifr2013 binary-tree data-structures

Answer key

19.3.3 Binary Tree: TIFR CSE 2014 | Part B | Question: 1 top

Let T be a rooted binary tree whose vertices are labelled with symbols $a, b, c, d, e, f, g, h, i, j, k$. Suppose the in-order (visit left subtree, visit root, visit right subtree) and post-order (visit left subtree, visit right subtree, visit root) traversals of T produce the following sequences.

in-order:$a, b, c, d, e, f, g, h, i, j, k$

post-order:$a, c, b, e, f, h, j, k, i, g, d$

How many leaves does the tree have?

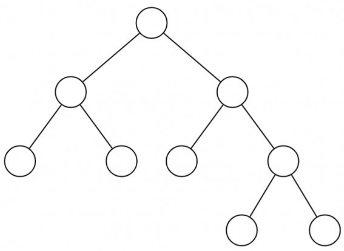
A. THREE.
B. FOUR.
C. FIVE.
D. SIX.
E. Cannot be determined uniquely from the given information.

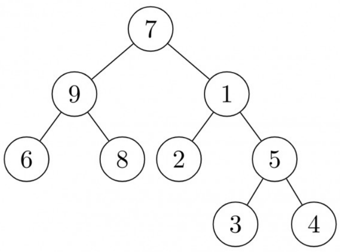
tifr2014 binary-tree data-structures easy

Answer key

19.3.4 Binary Tree: TIFR CSE 2015 | Part B | Question: 4 top

First, consider the tree on the left.





On the right, the nine nodes of the tree have been assigned numbers from the set $\{1, 2, \ldots, 9\}$ so that for every node, the numbers in its left subtree and right subtree lie in disjoint intervals (that is, all numbers in one subtree are less than all numbers in the other subtree). How many such assignments are possible? Hint: Fix a value for the root and ask what values can then appear in its left and right subtrees.

A. $2^9 = 512$
B. $2^4.3^2.5.9 = 6480$
C. $2^3.3.5.9 = 1080$
D. $2^4 = 16$
E. $2^3.3^3 = 216$

tifr2015 binary-tree combinatory

Answer key

19.3.5 Binary Tree: TIFR CSE 2018 | Part B | Question: 6 top

Consider the following implementation of a binary tree data strucrure. The operator $+$ denotes list-concatenation.

That is, $[a, b, c] + [d, e] = [a, b, c, d, e]$.

```
struct TreeNode:
 int value
 TreeNode leftChild
 TreeNode rightChild

function preOrder(T):
 if T == null:
  return []
 else:
  return [T.value] + preOrder(T.leftChild) + preOrder(T.rightChild)

function inOrder(T):
 if T == null:
  return []
 else:
  return inOrder(T.leftChild) + [T.value] + inOrder(T.rightChild)

function postOrder(T):
 if T == null:
  return []
 else:
  return postOrder(T.leftChild) + postOrder(T.rightChild) + [T.value]
```

For some T the functions inOrder(T) and preOrder(T) return the following:

$$\text{inOrder(T)} : \quad [12, 10, 6, 9, 7, 2, 15, 5, 1, 13, 4, 3, 8, 14, 11]$$

$$\text{preOrder(T)} : \quad [5, 2, 10, 12, 9, 6, 7, 15, 13, 1, 3, 4, 14, 8, 11]$$

What does postOrder(T) return ?

A. $[12, 6, 10, 7, 15, 2, 9, 1, 4, 13, 8, 11, 14, 3, 5]$
B. $[11, 8, 14, 4, 3, 1, 13, 15, 7, 6, 9, 12, 10, 2, 5]$
C. $[11, 14, 8, 3, 4, 13, 1, 5, 15, 2, 7, 9, 6, 10, 12]$
D. $[12, 6, 7, 9, 10, 15, 2, 1, 4, 8, 11, 14, 3, 13, 5]$
E. Cannot be uniquely determined from given information.

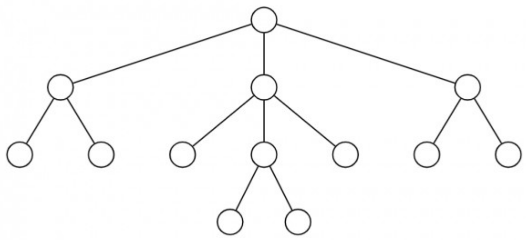
tifr2018 data-structures binary-tree

Answer key

19.4 Heap (1) top

19.4.1 Heap: TIFR CSE 2014 | Part B | Question: 19 top

Consider the following tree with 13 nodes.



Suppose the nodes of the tree are randomly assigned distinct labels from $\{1, 2, \dots, 13\}$, each permutation being equally likely. What is the probability that the labels form a min-heap (i.e., every node receives the minimum label in its subtree)?

A. $\left(\frac{1}{6!}\right)\left(\frac{1}{3!}\right)^2$
B. $\left(\frac{1}{3!}\right)^2\left(\frac{1}{2!}\right)^3$
C. $\left(\frac{1}{13}\right)\left(\frac{1}{6}\right)\left(\frac{1}{3}\right)^3$
D. $\frac{2}{13}$
E. $\frac{1}{2^{13}}$

tifr2014 heap

Answer key

19.5 Stack (1) top

19.5.1 Stack: TIFR CSE 2017 | Part B | Question: 3 top

We have an implementation that supports the following operations on a stack (in the instructions below, **s** is the name of the stack).

- **isempty(s)** : returns **True** if **s** is empty, and **False** otherwise.
- **top(s)** : returns the top element of the stack, but does not pop the stack; returns **null** if the stack is empty.
- **push(s, x)** : places **x** on top of the stack.
- **pop(s)** : pops the stack; does nothing if **s** is empty.

Consider the following code:

```
pop_ray_pop(x):
  s=empty
  for i=1 to length(x):
    if (x[i] == '('):
      push(s, x[i])
    else:
      while (top(s)=='('):
        pop(s)
      end while
      push(s, ')')
    end if
  end for
  while not isempty(s):
    print top(s)
    pop(s)
  end while
```

What is the output of this program when

```
pop_ray_pop("(((()((())((((")
```

is executed?

A. ((((
B.))) ((((
C.)))
D. (((()))
E. ()()

tifr2017 data-structures stack

Answer key

19.6 Tree (1) top

19.6.1 Tree: TIFR CSE 2012 | Part B | Question: 15 top

Let T be a tree of n nodes. Consider the following algorithm, that constructs a sequence of leaves $u_1, u_2 \dots$. Let u_1 be some leaf of tree. Let u_2 be a leaf that is farthest from u_1. Let u_3 be the leaf that is farthest from u_2, and, in general, let u_{i+1} be a leaf of T that is farthest from u_i (if there are many choices for u_{i+1}, pick one arbitrarily). The algorithm stops when some u_i is visited again. What can u say about the distance between u_i and u_{i+1}, as $i = 1, 2, \dots$?

A. For some trees, the distance strictly reduces in each step.
B. For some trees, the distance increases initially and then decreases.

C. For all trees, the path connecting u_2 and u_3 is a longest path in the tree.
D. For some trees, the distance reduces initially, but then stays constant.
E. For the same tree, the distance between the last two vertices visited can be different, based on the choice of the first leaf u_1.

tifr2012 data-structures tree

Answer key☟

Answer Keys

19.1.1	B	19.2.1	A	19.3.1	A	19.3.2	C	19.3.3	C
19.3.4	B	19.3.5	D	19.4.1	C	19.5.1	D	19.6.1	C

20 Programming and DS: Programming (11)

20.0.1 TIFR CSE 2011 | Part B | Question: 24 top

Consider the program

```
x:=0; y:=0; (r1:=x; r2:=x; y:= if r1 = r2 then 1 ‖ r3:= y; x:= r3)
```

Note that $\|$ denotes the parallel operator. In which of the following cases can the program possibly result in a final state with $r1 = 0; r2 = r3 = 1$.

A. Such a transformation is not possible in Java.
B. Such a program transformation is possible in Java.
C. Possible in Pascal when the compiler appropriately translates the $\|$ operator to interleaved pascal statements.
D. Possible in all sequential programming languages when the compiler appropriately translates the $\|$ operator to interleaved statements in the sequential language.
E. None of the above.

tifr2011 programming non-gate

20.1 Loop Invariants (4) top

20.1.1 Loop Invariants: TIFR CSE 2010 | Part B | Question: 30 top

Consider the following program for summing the entries of the array b: array $[0..N-1]$ of integers, where N is a positive integer. (The symbol '<>' denotes 'not equal to').

```
var
   i, s: integer;
Program
   i:= 0;
   s:= 0;
[*] while i <> N do
      s := s + b[i];
      i := i + 1;
   od
```

Which of the following gives the invariant that holds at the beginning of each loop, that is, each time the program arrives at point $[*]$?

A. $s = \sum_{j=0}^{N} b[j] \ \& \ 0 \leq i \leq N$
B. $s = \sum_{j=0}^{i=1} b[j] \ \& \ 0 \leq i < N$
C. $s = \sum_{j=0}^{i} b[j] \ \& \ 0 < i \leq N$
D. $s = \sum_{j=1}^{N} b[j] \ \& \ 0 \leq i < N$
E. $s = \sum_{j=0}^{i-1} b[j] \ \& \ 0 \leq i \leq N$

tifr2010 programming loop-invariants

Answer key

20.1.2 Loop Invariants: TIFR CSE 2010 | Part B | Question: 37 top

Consider the program where a, b are integers with $b > 0$.

```
x:=a; y:=b; z:=0;
while y > 0 do
   if odd (x) then
      z:= z + x;
      y:= y - 1;
   else y:= y % 2;
      x:= 2 * x;
   fi
```

Invariant of the loop is a condition which is true before and after every iteration of the loop. In the above program the loop invariant is given by

$$0 \leq y \text{ and } z + x * y = a * b$$

Which of the following is true of the program?

A. The program will not terminate for some values of a, b.
B. The program will terminate with $z = 2^b$
C. The program will terminate with $z = a * b$.
D. The program will not terminate for some values of a, b but when it does terminate, the condition $z = a * b$ will hold.
E. The program will terminate with $z = a^b$

tifr2010 programming loop-invariants

Answer key

20.1.3 Loop Invariants: TIFR CSE 2017 | Part B | Question: 5 top

Consider the following psuedocode fragment, where y is an integer that has been initialized.

```
int i=1
int j=1
while (i<10):
   j=j*i
   i=i+1
   if (i==y):
      break
   end if
end while
```

Consider the following statements:

i. $(i == 10)$ or $(i == y)$
ii. If $y > 10$, then $i == 10$
iii. If $j = 6$, then $y == 4$

Which of the above statements is/are TRUE at the end of the while loop? Choose from the following options.

A. i only
B. iii only
C. ii and iii only
D. i, ii, and iii
E. None of the above

tifr2017 programming loop-invariants

Answer key

20.1.4 Loop Invariants: TIFR CSE 2019 | Part B | Question: 9 top

Consider the following program fragment:

```
var x, y: integer;
x := 1; y := 0;
while y < x do
begin
  x := 2*x;
  y := y+1
end;
```

For the above fragment , which of the following is a loop invariant ?

A. $x = y + 1$
B. $x = (y + 1)^2$
C. $x = (y + 1)2^y$
D. $x = 2^y$
E. None of the above, since the loop does not terminate

tifr2019 programming loop-invariants

Answer key

20.2 Parameter Passing (2) top

20.2.1 Parameter Passing: TIFR CSE 2011 | Part B | Question: 32 top

Various parameter passing mechanisms have been in used in different programming languages. Which of the following statements is true?

A. Call by value result is used in language Ada.
B. Call by value result is the same as call by name.
C. Call by value is the most robust.
D. Call by reference is the same as call by name.
E. Call by name is the most efficient.

tifr2011 programming parameter-passing

Answer key

20.2.2 Parameter Passing: TIFR CSE 2019 | Part B | Question: 8 top

Consider the following program fragment:

```
var a,b : integer;
procedure G(c,d: integer);
begin
   c:=c-d;
   d:=c+d;
   c:=d-c
end;
a:=2;
b:=3;
G(a,b);
```

If both parameters to G are passed by reference, what are the values of a and b at the end of the above program fragment ?

A. $a = 0$ and $b = 2$ B. $a = 3$ and $b = 2$ C. $a = 2$ and $b = 3$ D. $a = 1$ and $b = 5$ E. None of the above

tifr2019 programming parameter-passing

Answer key

20.3 Programming In C (2) top

20.3.1 Programming In C: TIFR CSE 2018 | Part A | Question: 7 top

```
Consider the following function definition.
void greet(int n)
{
   if(n>0)
   {
      printf("hello");
      greet(n-1);
   }
   printf("world");
}
```

If you run greet(n) for some non-negative integer n, what would it print?

A. n times "hello", followed by n+1 times "world"
B. n times "hello", followed by n times "world"
C. n times "helloworld"
D. n+1 times "helloworld"
E. n times "helloworld", followed by "world"

tifr2018 programming-in-c

Answer key

20.3.2 Programming In C: TIFR CSE 2019 | Part B | Question: 6 top

Given the following pseudocode for function $\text{printx}()$ below, how many times is x printed if we execute $\text{printx}(5)$?

```
void printx(int n)  {
   if(n==0){
      printf("x");
}
for(int i=0;i<=n-1;++i){
   printx(n-1);
}
}
```

A. 625
B. 256
C. 120
D. 24
E. 5

tifr2019 programming programming-in-c

Answer key

20.4 Recursion (2) top

20.4.1 Recursion: TIFR CSE 2010 | Part B | Question: 31 top

Consider the following computation rules. **Parallel-outermost rule**: Replace all the outermost occurrences of F (i.e., all occurrences of F which do not occur as arguments of other F's) simultaneously. **Parallel - innermost rule**: Replace all the innermost occurrences of F (i.e.,all occurrences of F with all arguments free of F's) simultaneously. Now consider the evaluations of the recursive program over the integers.

```
F(x, y) <== if x = 0 then 0 else
        [ F(x + 1, F(x, y)) * F(x - 1, F(x, y))]
```

where the multiplication functions * is extended as follows:

```
0 * w & w * 0 are 0
a * w & w * a are w (for any non-zero integer a)
w * w is w
```

We say that $F(x, y) =$ w when the evaluation of $F(x, y)$ does not terminate. Computing $F(1, 0)$ using the parallel - innermost and parallel - outermost rule yields

A. w and 0 respectively
B. 0 and 0 respectively
C. w and w respectively
D. w and 1 respectively
E. none of the above

tifr2010 programming recursion

Answer key

20.4.2 Recursion: TIFR CSE 2011 | Part B | Question: 38 top

Consider the class of recursive and iterative programs. Which of the following is false?

A. Recursive programs are more powerful than iterative programs.
B. For every iterative program there is an equivalent recursive program.
C. Recursive programs require dynamic memory management.
D. Recursive programs do not terminate sometimes.
E. Iterative programs and recursive programs are equally expressive.

tifr2011 recursion programming

Answer key

Answer Keys

20.0.1	B	20.1.1	E	20.1.2	A	20.1.3	D	20.1.4	D
20.2.1	E	20.2.2	B	20.3.1	A	20.3.2	C	20.4.1	A
20.4.2	E								

21 Theory of Computation (28)

21.1 Closure Property (2) top

21.1.1 Closure Property: TIFR CSE 2013 | Part B | Question: 11 top

Which of the following statements is FALSE?

A. The intersection of a context free language with a regular language is context free.
B. The intersection of two regular languages is regular.
C. The intersection of two context free languages is context free
D. The intersection of a context free language and the complement of a regular language is context free.
E. The intersection of a regular language and the complement of a regular language is regular.

tifr2013 theory-of-computation closure-property

Answer key

21.1.2 Closure Property: TIFR CSE 2014 | Part B | Question: 14 top

Which the following is FALSE?

A. Complement of a recursive language is recursive.
B. A language recognized by a non-deterministic Turing machine can also be recognized by a deterministic Turing machine.
C. Complement of a context free language can be recognized by a Turing machine.
D. If a language and its complement are both recursively enumerable then it is recursive.
E. Complement of a non-recursive language can never be recognized by any Turing machine.

tifr2014 theory-of-computation closure-property

Answer key

21.2 Decidability (3) top

21.2.1 Decidability: TIFR CSE 2010 | Part B | Question: 25 top

Which of the following problems is decidable? (Here, CFG means context free grammar and CFL means context free language.)

A. Given a CFG G, find whether $L(G) = R$, where R is regular set.
B. Given a CFG G, find whether $L(G) = \{\}$.
C. Find whether the intersection of two CFLs is empty.
D. Find whether the complement of CFL is a CFL.
E. Find whether CFG G_1 and CFG G_2 generate the same language, i.e, $L(G_1) = L(G_2)$.

tifr2010 theory-of-computation context-free-language decidability

Answer key

21.2.2 Decidability: TIFR CSE 2011 | Part B | Question: 25 top

Let A_{TM} be defined as follows:

$$A_{TM} = \{\langle M, w \rangle \mid \text{The Turing machine } M \text{ accepts the word } w\}$$

And let L be some $\mathbf{NP}-$ complete language. Which of the following statements is FALSE?

A. $L \in \mathbf{NP}$
B. Every problem in $\mathbf{NP}$ is polynomial time reducible to L.
C. Every problem in $\mathbf{NP}$ is polynomial time reducible to A_{TM}.
D. Since L is $\mathbf{NP}-$ complete, A_{TM} is polynomial time reducible to L.
E. $A_{TM} \notin \mathbf{NP}$.

tifr2011 theory-of-computation decidability

Answer key

21.2.3 Decidability: TIFR CSE 2020 | Part B | Question: 2 top

Consider the following statements.

1. The intersection of two context-free languages is always context-free
2. The super-set of a context-free languages is never regular
3. The subset of a decidable language is always decidable
4. Let $\Sigma = \{a, b, c\}$. Let $L \subseteq \Sigma$ be the language of all strings in which either the number of occurrences of a is the same as the number of occurrences of b OR the number occurrences of b is the same as the number of occurrences of c. Then, L is not context-free.

Which of the above statements are true?

A. Only (1)
B. Only (1) and (2)
C. Only $(1), (2)$ and (3)
D. Only (4)
E. None of $(1), (2), (3), (4)$ are true.

tifr2020 theory-of-computation context-free-language decidability

Answer key

21.3 Identify Class Language (10) top

21.3.1 Identify Class Language: TIFR CSE 2010 | Part B | Question: 22 top

Let L consist of all binary strings beginning with a 1 such that its value when converted to decimal is divisible by 5. Which of the following is true?

A. L can be recognized by a deterministic finite state automaton.
B. L can be recognized by a non-deterministic finite state automaton but not by a deterministic finite state automaton.
C. L can be recognized by a deterministic push-down automaton but not by a non-deterministic finite state automaton.
D. L can be recognized by a non-deterministic push-down automaton but not by a deterministic push-down automaton.
E. L cannot be recognized by any push-down automaton.

tifr2010 theory-of-computation identify-class-language

Answer key

21.3.2 Identify Class Language: TIFR CSE 2010 | Part B | Question: 35 top

Consider the following languages over the alphabet $\{0, 1\}$.

$$L1 = \{x.x^R \mid x \in \{0,1\}^*\}$$
$$L2 = \{x.x \mid x \in \{0,1\}^*\}$$

Where x^R is the reverse of string x; e.g. $011^R = 110$. Which of the following is true?

A. Both $L1$ and $L2$ are regular.
B. $L1$ is context-free but not regular where as $L2$ is regualr.
C. Both $L1$ and $L2$ are context free and neither is regular.
D. $L1$ is context free but $L2$ is not context free.
E. Both $L1$ and $L2$ are not context free.

tifr2010 theory-of-computation identify-class-language

Answer key☟

21.3.3 Identify Class Language: TIFR CSE 2012 | Part B | Question: 18 top☝

Let a^i denote a sequence $a.a\dots a$ with i letters and let $\aleph$ be the set of natural numbers $1, 2, \dots$. Let $L_1 = \left\{a^i b^{2i} \mid i \in \aleph\right\}$ and $L_2 = \left\{a^i b^{i^2} \mid i \in \aleph\right\}$ be two languages. Which of the following is correct?

A. Both L_1 and L_2 are context-free languages.
B. L_1 is context-free and L_2 is recursive but not context-free.
C. Both L_1 and L_2 are recursive but not context-free.
D. L_1 is regular and L_2 is context-free.
E. Complement of L_2 is context-free.

tifr2012 theory-of-computation identify-class-language

Answer key☟

21.3.4 Identify Class Language: TIFR CSE 2014 | Part B | Question: 13 top☝

Let L be a given context-free language over the alphabet $\{a, b\}$. Construct L_1, L_2 as follows. Let $L_1 = L - \{xyx \mid x, y \in \{a, b\}^*\}$, and $L_2 = L \cdot L$. Then,

A. Both L_1 and L_2 are regular.
B. Both L_1 and L_2 are context free but not necessarily regular.
C. L_1 is regular and L_2 is context free.
D. L_1 and L_2 both may not be context free.
E. L_1 is regular but L_2 may not be context free.

tifr2014 theory-of-computation identify-class-language

Answer key☟

21.3.5 Identify Class Language: TIFR CSE 2015 | Part B | Question: 8 top☝

Let $\sum_1 = \{a\}$ be a one letter alphabet and $\sum_2 = \{a, b\}$ be a two letter alphabet. A language over an alphabet is a set of finite length words comprising letters of the alphabet. Let L_1 and L_2 be the set of languages over $\sum_1$ and $\sum_2$ respectively. Which of the following is true about L_1 and L_2:

A. Both are finite.
B. Both are countably infinite.
C. L_1 is countable but L_2 is not.
D. L_2 is countable but L_1 is not.
E. Neither of them is countable.

tifr2015 identify-class-language

Answer key☟

21.3.6 Identify Class Language: TIFR CSE 2017 | Part B | Question: 14 top☝

Consider the following grammar G with terminals $\{[,]\}$, start symbol S, and non-terminals

$\{A, B, C\}$:

$$S \rightarrow AC \mid SS \mid AB$$

$$C \rightarrow SB$$

$$A \rightarrow [$$

$$B \rightarrow]$$

A language L is called prefix-closed if for every $x \in L$, every prefix of x is also in L. Which of the following is FALSE?

A. $L(G)$ is context free
B. $L(G)$ is infinite
C. $L(G)$ can be recognized by a deterministic push down automaton
D. $L(G)$ is prefix-closed
E. $L(G)$ is recursive

tifr2017 theory-of-computation identify-class-language

Answer key☟

21.3.7 Identify Class Language: TIFR CSE 2017 | Part B | Question: 4 top☝

Let L be the language over the alphabet $\{1, 2, 3, (,)\}$ generated by the following grammar (with start symbol S, and non-terminals $\{A, B, C\}$):

$S \rightarrow A\,B\,C$
$A \rightarrow ($
$B \rightarrow 1\,B \mid 2\,B \mid 3\,B$
$B \rightarrow 1 \mid 2 \mid 3$
$C \rightarrow)$

Then, which of the following is TRUE?

A. L is finite
B. L is not recursively enumerable
C. L is regular
D. L contains only strings of even length
E. L is context-free but not regular

tifr2017 theory-of-computation identify-class-language

Answer key☟

21.3.8 Identify Class Language: TIFR CSE 2018 | Part B | Question: 11 top☝

Consider the language $L \subseteq \{a, b, c\}^*$ defined as

$$L = \{a^p b^q c^r : p = q \quad or \quad q = r \quad or \quad r = p\}.$$

Which of the following answer is TRUE about the complexity of this language?

A. L is regular but not context-free
B. L is context-free but not regular
C. L is decidable but not context free
D. The complement of L, defined as $\overline{L} = \{a, b, c\}^* \setminus L$, is regular.
E. L is regular, context-free and decidable

tifr2018 identify-class-language theory-of-computation

Answer key

21.3.9 Identify Class Language: TIFR CSE 2018 | Part B | Question: 14 top

Define the language $\text{INFINITE}_{DFA} \equiv \{(A) \mid A \text{ is a DFA and } L(A) \text{ is an infinite language}\},$ where (A) denotes the description of the deterministic finite automata (DFA).Then which of the following about INFINITE_{DFA} is TRUE:

A. It is regular.
B. It is context-free but not regular.
C. It is Turing decidable (recursive).
D. It is Turing recognizable but not decidable.
E. Its complement is Turing recognizable but it is not decidable.

tifr2018 identify-class-language

Answer key

21.3.10 Identify Class Language: TIFR CSE 2019 | Part B | Question: 10 top

Let the language D be defined in the binary alphabet $\{0, 1\}$ as follows:

$$D := \{w \in \{0,1\}^* \mid \text{ substrings 01 and 10 occur an equal number of times in w}\}$$

For example , $101 \in D$ while $1010 \notin D$. Which of the following must be TRUE of the language D ?

A. D is regular
B. D is context-free but not regular
C. D is decidable but not context-free
D. D is decidable but not in NP
E. D is undecidable

tifr2019 theory-of-computation identify-class-language

Answer key

21.4 Recursive And Recursively Enumerable Languages (2) top

21.4.1 Recursive And Recursively Enumerable Languages: TIFR CSE 2010 | Part B | Question: 40 top

Which of the following statement is FALSE?

A. All recursive sets are recursively enumerable.
B. The complement of every recursively enumerable sets is recursively enumerable.
C. Every Non-empty recursively enumerable set is the range of some totally recursive function.
D. All finite sets are recursive.
E. The complement of every recursive set is recursive.

tifr2010 theory-of-computation recursive-and-recursively-enumerable-languages

Answer key

21.4.2 Recursive And Recursively Enumerable Languages: TIFR CSE 2012 | Part B | Question: 19 top

Which of the following statements is TRUE?

A. Every turning machine recognizable language is recursive.
B. The complement of every recursively enumerable language is recursively enumerable.

C. The complement of a recursive language is recursively enumerable.
D. The complement of a context-free language is context-free.
E. The set of turning machines which do not halt on empty input forms a recursively enumerable set.

tifr2012 theory-of-computation recursive-and-recursively-enumerable-languages

Answer key

21.5 Regular Expression (5) top

21.5.1 Regular Expression: TIFR CSE 2010 | Part B | Question: 34 top

Let r, s, t be regular expressions. Which of the following identities is correct?

A. $(r+s)^* = r^*s^*$
B. $r(s+t) = rs+t$
C. $(r+s)^* = r^* + s^*$
D. $(rs+r)^*r = r(sr+r)^*$
E. $(r^*s)^* = (rs)^*$

tifr2010 theory-of-computation regular-expression

Answer key

21.5.2 Regular Expression: TIFR CSE 2015 | Part B | Question: 7 top

Let a, b, c be regular expressions. Which of the following identities is correct?

A. $(a+b)^* = a^*b^*$
B. $a(b+c) = ab+c$
C. $(a+b)^* = a^* + b^*$
D. $(ab+a)^*a = a(ba+a)^*$
E. None of the above.

tifr2015 theory-of-computation regular-expression

Answer key

21.5.3 Regular Expression: TIFR CSE 2017 | Part B | Question: 9 top

Which of the following regular expressions correctly accepts the set of all $0/1$-strings with an even (possibly zero) number of 1s?

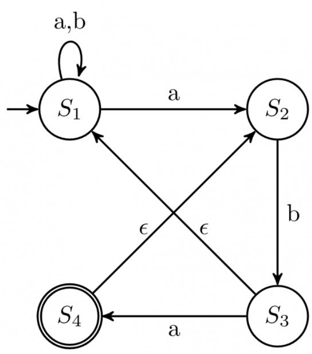
A. $(10^*10^*)^*$
B. $(0^*10^*1)^*$
C. $0^*1(10^*1)^*10^*$
D. $0^*1(0^*10^*10^*)^*10^*$
E. $(0^*10^*1)^*0^*$

tifr2017 theory-of-computation regular-expression

Answer key

21.5.4 Regular Expression: TIFR CSE 2018 | Part B | Question: 2 top

Consider the following non-deterministic automation, where s_1 is the start state and s_4 is the final (accepting) state. The alphabet is $\{a, b\}$. A transition with label ϵ can be taken without consuming any symbol from the input.



Which of the following regular expressions corresponds to the language accepted by this automation ?

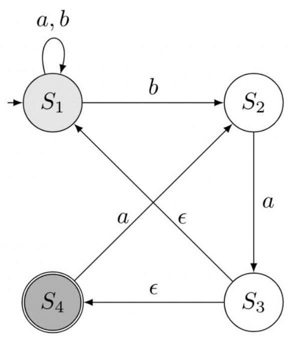
A. $(a+b)^*aba$ B. $aba(a+b)^*aba$ C. $(a+b)aba(b+a)^*$ D. $aba(a+b)^*$ E. $(ab)^*aba$

tifr2018 regular-expression finite-automata

Answer key

21.5.5 Regular Expression: TIFR CSE 2019 | Part B | Question: 11 top

Consider the following non-deterministic automaton,where s_1 is the start state and s_4 is the final (accepting) state. The alphabet is $\{a, b\}$. A transition with label ϵ can be taken without consuming any symbol from the input.



Which of the following regular expressions correspond to the language accepted by this automaton ?

A. $(a+b)^*aba$ B. $(a+b)^*ba^*$ C. $(a+b)^*ba(aa)^*$ D. $(a+b)^*$ E. $(a+b)^*baa^*$

tifr2019 theory-of-computation regular-expression

Answer key

21.6 Regular Language (6) top

21.6.1 Regular Language: TIFR CSE 2013 | Part B | Question: 6 top

Let L and L' be languages over the alphabet Σ. The left quotient of L by L' is

$L/L' \overset{def}{=} \{w \in \Sigma^* : wx \in L \text{ for some } x \in L'\}$

Which of the following is true?

A. If L/L' is regular then L' is regular.
B. If L is regular then L/L' is regular.
C. If L/L' is regular then L is regular.
D. L/L' is a subset of L.
E. If L/L' and L' are regular, then L is regular.

tifr2013 theory-of-computation regular-language

Answer key

21.6.2 Regular Language: TIFR CSE 2013 | Part B | Question: 8 top

Which one of the following languages over the alphabet $0, 1$ is regular?

A. The language of balanced parentheses where $0, 1$ are thought of as $(,)$ respectively.
B. The language of palindromes, i.e. bit strings x that read the same from left to right as well as right to left.
C. $L = \left\{0^{m^2} : 3 \leq m\right\}$

D. The Kleene closure L^*, where L is the language in (c) above.
E. $\{0^m 1^n | 1 \leq m \leq n\}$

tifr2013 theory-of-computation regular-language

Answer key☟

21.6.3 Regular Language: TIFR CSE 2014 | Part B | Question: 12 top☝

Consider the following three statements:

i. Intersection of infinitely many regular languages must be regular.
ii. Every subset of a regular language is regular.
iii. If L is regular and M is not regular then $L \bullet M$ is necessarily not regular.

Which of the following gives the correct true/false evaluation of the above?

A. true, false, true.
B. false, false, true.
C. true, false, true.
D. false, false, false.
E. true, true, true.

tifr2014 theory-of-computation regular-language

Answer key☟

21.6.4 Regular Language: TIFR CSE 2015 | Part B | Question: 10 top☝

Consider the languages

$L_1 = \{a^m b^n c^p \mid (m = n \vee n = p) \wedge m + n + p \geq 10\}$

$L_2 = \{a^m b^n c^p \mid (m = n \vee n = p) \wedge m + n + p \leq 10\}$

State which of the following is true?

A. L_1 and L_2 are both regular.
B. Neither L_1 nor L_2 is regular.
C. L_1 is regular and L_2 is not regular.
D. L_1 is not regular and L_2 is regular.
E. Both L_1 and L_2 are infinite.

tifr2015 regular-language

Answer key☟

21.6.5 Regular Language: TIFR CSE 2015 | Part B | Question: 6 top☝

Let B consist of all binary strings beginning with a 1 whose value when converted to decimal is divisible by 7.

A. B can be recognized by a deterministic finite state automaton.
B. B can be recognized by a non-deterministic finite state automaton but not by a deterministic finite state automaton.
C. B can be recognized by a deterministic push-down automaton but not by a non-deterministic finite state automaton.
D. B can be recognized by a non-deterministic push-down automaton but not by a deterministic push-down automaton.
E. B cannot be recognized by any push down automaton, deterministic or non-deterministic.

tifr2015 theory-of-computation regular-language

Answer key☟

21.6.6 Regular Language: TIFR CSE 2018 | Part B | Question: 12 top

Consider the following statements:

i. For every positive integer n, let $\#n$ be the product of all primes less than or equal to n. Then, $\#p+1$ is a prime, for every prime p.
ii. π is a universal constant with value $\frac{22}{7}$.
iii. No polynomial time algorithm exists that can find the greatest common divisor of two integers given as input in binary.
iv. Let $L \equiv \{x \in \{0,1\}^* \mid x \text{ is the binary encoding of an integer that is divisible by } 31\}$ Then, L is a regular language.

Then which of the following is TRUE ?

A. Only statement **(i)** is correct.
B. Only statement **(ii)** is correct.
C. Only statement **(iii)** is correct.
D. Only statement **(iv)** is correct.
E. None of the statements are correct.

tifr2018 regular-language

Answer key

Answer Keys

21.1.1	C	21.1.2	E	21.2.1	B	21.2.2	D	21.2.3	E
21.3.1	A	21.3.2	D	21.3.3	B	21.3.4	C	21.3.5	E
21.3.6	D	21.3.7	C	21.3.8	B	21.3.9	C	21.3.10	A
21.4.1	B	21.4.2	C	21.5.1	D	21.5.2	D	21.5.3	E
21.5.4	A	21.5.5	C	21.6.1	B	21.6.2	D	21.6.3	D
21.6.4	D	21.6.6	D						