HW 1

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Question 1

What is the number of possible English words that consist of four different letters? For example: abcd, dxgy, nhau...

Here we have to find the number of possibilities a word consist of 4 different letters can have using 26 alphabets.
So, we can approach this question by multiplying number of possible inputs we can have up to 4 spaces.
At first space we have 26 unique options
At second space we have 25 unique options as we already have used 1 unique letter for 1^{st} space
At third space we have 24 unique options as we already have used 2 unique letters for 1^{st} and 2^{nd} spaces
At fourth space we have 23 unique as we already have used 3 unique letters for 1^{st} 2^{nd} and 3^{rd} spaces

Finally answer = 26 * 25 * 24 * 23 = **358,800 unique words**

Question 2

Two events are independent, and their probability are 0.75 and 0.25 respectively. What is the probability that exactly one of the two events, not both, occur?

Suppose two independent events are A and B.
And we are given that p(A) = 0.75 and p(B) = 0.25

First let's find out probability of both events occurring
As we already know both events are independent
p(AB) = p(A) * p(B) =0.75 * 0.25 = 0.1875

Now we know that probability of any event occurring is 1 so we can subtract p(AB) from 1 to find probability of exactly one of A and B occurring.
p(A or B) = 1 – p(AB) = 1 – 0.1875 = **0.8125**

Question 3

The probability that visiting a particular doctor results in neither blood draw nor X-ray is 35%. Of all patients visiting that doctor, 30% took X-ray and 40% require blood draw. Calculate the probability that a visit needs both X-ray and blood draw.

Let's first assign an identifier to our events
A = X-ray
B = Blood draw

Now we are given following probabilities
p(No A nor B) = 0.35
p(A) = 0.30
p(B) = 0.40

We can derive following probability from given ones
p(Both A and B happened) = $p(A \cup B)$ = 1 – p(No A nor B)
= 1 – 0.35
= 0.65

Now to find probability that a visit need both X-ray and blood draw which is $p(A \cap B)$, we can use probability rule
$p(A \cup B) = p(A) + p(B) - p(A \cap B)$
Let's plugin the values

0.65 = 0.30 + 0.40 – p(A ∩ B)
0.65 = 0.70 – p(A ∩ B)
p(A ∩ B) = 0.70 – 0.65 = 0.05

So there is 5% or 0.05 probability that a visit needs both x-ray and Blood draw.

Question 4

A paper bag contains three dice. Dice A's faces are numbered 1, 2, 3, 4, 5, 6; Dice B's faces are numbered 2, 2, 4, 6, 6, and 6; and Dice C has all six faces numbered 6. You draw a dice from the bag and roll it twice. What is the probability that you get 6 both times?

Here we will have to calculate 3 probability for 3 different dices
Also, here event of rolling dice one time and then rolling it again are independent events so we can multiply the probity as well

Now let's calculate probability picking dice A from the bag and rolling it twice and getting 6 both time
For dice A we only have 1 '6' on dice so probability of rolling it and getting 6 is 1/6

$$\begin{aligned} p(Drawing\ A\ from\ the\ bag\ and\ rolling\ it\ twice\ and\ getting\ 6) &= 1/6 * 1/6 \\ &= 0.0277 \end{aligned}$$

Now let's calculate probability picking dice B from the bag and rolling it twice and getting 6 both time
For dice B we have 3 '6s' on dice so probability of rolling it and getting 6 is 3/6 = 1/2

$$\begin{aligned} p(Drawing\ B\ from\ the\ bag\ and\ rolling\ it\ twice\ and\ getting\ 6) &= 1/2 * 1/2 \\ &= 0.25 \end{aligned}$$

Now let's calculate probability picking dice C from the bag and rolling it twice and getting 6 both time
Here dice C have 6 on all of its faces so rolling it and getting 6 is always 1
Hence,

$$p(Drawing\ C\ from\ the\ bag\ and\ rolling\ it\ twice\ and\ getting\ 6) = 1$$

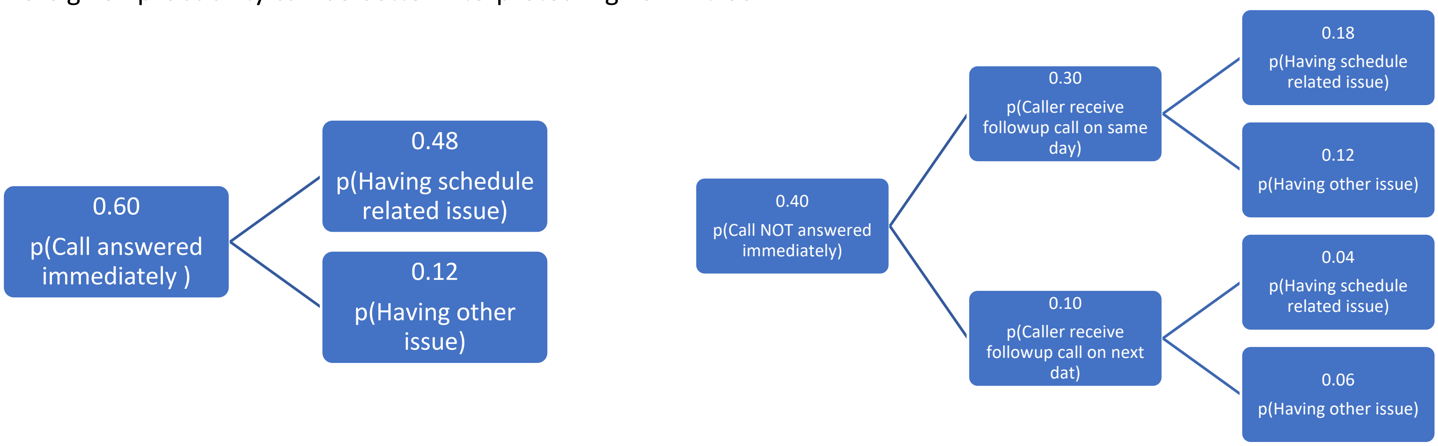
Finally, we can calculate probability of drawing a dice from the bag and rolling it twice and getting 6 both times

$$\begin{aligned} &p(getting\ 6\ both\ time) \\ &\quad = p(Drawing\ A\ from\ the\ bag\ and\ rolling\ it\ twice\ and\ getting\ 6) \\ &\quad * p(Drawing\ B\ from\ the\ bag\ and\ rolling\ it\ twice\ and\ getting\ 6) \\ &\quad * p(Drawing\ B\ from\ the\ bag\ and\ rolling\ it\ twice\ and\ getting\ 6) \\ &\quad = 0.0277 * 0.25 * 1 \\ &\quad = \mathbf{0.0069} \end{aligned}$$

Question 5

Stevens Registrar's Office answers 60% of the calls from students immediately. Of the 40% calls that are not answered immediately, 75% or the caller receive a follow up call the same day, and 25% receive a follow up call the next day. Of those students whose calls are answered immediately, 80% have schedule related issues. Of those students who were followed up the same day, 60% have schedule related issues. Of those students who were followed up the next day, 40% have schedule related issues. Given that a student has schedule related issues, what is the probability that the call is answered immediately?

Here given probability can be better interpreted if given in tree



Now we can use Bayes formula to answer our question

Suppose A = Call answered immediately

B = Schedule related issue

p(A|B) = $\frac{p(B|A) * p(A)}{p(B)}$ ------------------------[1]

From the tree we can derive all the required probability for [1]

p(B|A) = 0.48

p(A) = 0.60

p(B) = 0.48 + 0.18 + 0.04 = 0.7

Finally plug in all the values in [1]

p(A|B) = $\frac{0.48 * 0.60}{0.70}$

= **0.4114**

Question 6

A university has found there is a 3% probability that an application contains an error. Assume applications are mutually independent. An officer randomly selects 100 applications, what is the probability that 95% or less of the selected applications have no error?

We can solve this problem using Bernoulli Trials formula
p(success) = ${}^{n}C_{k}$ * $(p)^{k}$ * $(1 - p)^{n-k}$
n: total choice
k: picked choice
p: prob of success

Here we want probability of 95% or less application being errorless
So we can first find p(96 % applications being errorless) + p(97 % applications being errorless) + p(98 % applications being errorless) + p(99 % applications being errorless) + p(100 % applications being errorless) using Bernoulli trials and finally subtract it from 1 to find desired probability

First find individual probability for 96%, 97%, 98%, 99%, 100%

p(96 % applications being errorless) = ${}^{100}C_{96}$ * $(0.97)^{96}$ * $(0.03)^{4}$ = 0.1706056
p(97 % applications being errorless) = ${}^{100}C_{97}$ * $(0.97)^{97}$ * $(0.03)^{3}$ = 0.22747413
p(98 % applications being errorless) = ${}^{100}C_{98}$ * $(0.97)^{98}$ * $(0.03)^{2}$ = 0.22515296
p(99 % applications being errorless) = ${}^{100}C_{99}$ * $(0.97)^{99}$ * $(0.03)^{1}$ = 0.14706961
p(100 % applications being errorless) = ${}^{100}C_{100}$ * $(0.97)^{100}$ * $(0.03)^{0}$ = 0.04755251
[Calculation source MS Excel]

p(96% or more application being errorless)
= 0.04755251 + 0.22747413 + 0.22515296 + 0.14706961 + 0.04755251
= 0.81785481

Finally,
p(95% or less applications being errorless) = 1 - p(96% or more application being errorless)
= 1 - 0.81785481
= **0.18214519**

NOTE- Table created in excel to calculates probabilities:

0.97 0.03

nCk	p^k	(1-p)^n-k	nCk * p^k * (1-p)^n-k
3921225	0.05371388	0.00000081	0.170605596
161700	0.05210246	0.000027	0.227474127
4950	0.05053939	0.0009	0.225152963
100	0.0490232	0.03	0.147069612
1	0.04755251	1	0.047552508
		Sum	0.817854806
		1 - Sum	0.182145194

Question 7

Suppose that in a city, the number of homicides can be approximated by a Poisson process with λ = 1.5 per month. What is the probability of two or more than two homicides in one week?

Here we know that data follows Poisson distribution
And its λ = 1.5 per month

Now using Poisson formula lets fine p(x=0) where x reprints homicides
p(x = 0) = $\lambda^0/0! * e^{-\lambda}$
= 0.2231

Now same for p(x=1)
p(x = 1) = $\lambda^1/1! * e^{-\lambda}$
= 0.3346

We know the p(x = 1) and p(x = 0) and total probability is always 1
So p(x >= 2) = $\sum_{x=2}^{\infty} \lambda^x/x! * e^{-\lambda}$
= 1 – p(x <= 1)
= 1 – p(x = 0) – p(x = 1)
= 1 – 0.2231 – 0.3346
= **0.4423**

Question 8

Suppose that the lifetime of an electronic component follows an exponential distribution with λ = .1. Find the probability that the lifetime is between 5 and 15.

Here we know that lifetime of an electronic component follows an exponential distribution
And λ = 0.1

Let's first find F(15) and F(5) using $\lambda * e^{-\lambda x}$ formula

F(15) = 0.7768 (Source Excel : =EXPON.DIST(15,0.1,TRUE))
F(5) = 0.3934 (Source Excel: =EXPON.DIST(15,0.1,TRUE))

Now
p(Lifetime between 5 and 15) = $\int_5^{15} f(x)\,dx$
= F(15) – F(5)
= 0.7768 – 0.3934
= **0.3834**

Question 9

Suppose that X has the density function f(x) = cx^2 for 0 ≤ x ≤ 1 and f(x) = 0 otherwise.
a. Find c.
b. Find the cdf.
c. What is P(0.1≤ X <.5)?

(a)
Here we know that X is a density function and it holds $\int_{-\infty}^{\infty} f(x) = 1$

So, we can use this to find c

$$\int_0^1 f(x) = 1$$

$$\int_0^1 \mathbf{cx^2} = 1$$

c * x^3 / 3 lim(0 – 1)

(c / 3) – 0 = 1

c = 3

(b)

CDF = p(X <= x) =

$$= \int_{-\infty}^{x} \mathbf{3x^2}$$

$$= \int_0^x \mathbf{3x^2}$$

= X^3 lim (0 - x)
= x^3 – 0
= x^3

$$\mathbf{F(x)} = \begin{cases} \mathbf{0} & \mathbf{x <= 0} \\ \mathbf{x^3} & \mathbf{0 < x < 1} \\ \mathbf{1} & \mathbf{x >= 1} \end{cases}$$

(c)
P(0.1 < X < 0.5) = $F(0.5)^3$ - $F(0.1)^3$
= 0.125 – 0.001
= **0.124**

Question 10

Based on historical data, on average, Hoboken experiences one flooding event every 2.4 month. Assume that the time between flooding events can be modeled using an exponential distribution. What is the median number of months from now until Hoboken experiences its next flooding event?

Here we know that flooding events can be modeled using exponential distribution
And it is also given that Hoboken experiences flooding events every 2.4 months which is our λ
So $\lambda = 2.4$

Hence

Median = $(\ln 2) / \lambda$

$= 0.6931 / 2.4$

$= 0.2887$