BIA652-HW2

Instruction:

- Type your answers using Word (equation editor / MathType) or LaTeX. You may also use Jupyter Notebook for typing the document with math equations (see [1] and [2]). Handwritten solution will not be accepted.
- Save or print your submission as a PDF file and submit on Canvas.
- You may discuss among yourselves and ask hints from the TAs. However, you may not share your typed solution with others. The final work must be individual. Do not post the HW assignment or your solution to other websites such as Course Hero or GitHub.

Q1. [Normal to Lognormal Transformation] Show that if a random variable X ~ Normal(μ, σ), then exp(X) has the following pdf:

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left[-\frac{(\ln x - \mu)^2}{2\sigma^2}\right],\ x > 0.$$

Q2. In the bank example in page 44-45 of the slides, the joint pdf of X, Y is given by

$$f_{X,Y}(x, y) = \frac{6}{5}(x + y^2), \qquad 0 \le x \le 1, 0 \le y \le 1$$

a) What is the marginal pdf of $f_Y(y)$?
b) What is the conditional pdf of $f_{X|Y}(x|y = 0.5)$?

Q3. Exponential Distribution has a *memoryless* property. Intuitively, it means that the probability of customer service answering you call (assuming waiting time is exponential) in the next 10 mins is the same, no matter if you have waited an hour on the line or just picked up the phone. Formally, if X ∼ exponential(λ), f(x) = λ exp(-λx), and *t* and *s* are two positive numbers, use the definition of conditional probability to show that

$$P(X > t + s \mid X > t) = P(X > s).$$

Hint: Find the cdf of X first, and note that $P(X > t + s \cap X > t) = P(X > t + s)$

Q4. Roll a fair die (uniform 1,2,3,4,5,6) repeatedly. You and Peter are betting on the number shown on each roll. If the number is 4 or less, you win \$1; otherwise, you pay Peter \$2.5.

a) What is the expected value of the payoff for you?
b) What is the variance of your payoff?

Q5. Let X be uniformly distributed (continuous) on the interval [1,2]. Find E(1/X).

Q6. Let us assume that the lifetime of light bulbs follows an exponential distribution with the pdf:

$$f(x) = \lambda \exp(-\lambda x)$$

We test 10 bulbs and their lifetimes are 5, 3, 5, 1, 7, 3, 4, 8, 2, 3 years, respectively. What is the MLE for λ? What is the method of moments estimator for λ?

Q7. Let $x_1, x_2, \dots x_n$ be an iid sample with pdf

$$f(x|\theta) = \theta x^{\theta - 1}, 0 \le x \le 1, \theta > 0$$

Find the MLE of θ.