**HW 1**

**Name: Dishant Naik**

**CWID: 10454341**

**Question 1**

**[Normal to Lognormal Transformation] Show that if a random variable X ~ Normal(𝜇, 𝜎), then exp(X) has**

**the following pdf:**

A picture containing text

Description automatically generated

If X is normally distributed with μ mean and σ variance then exp(X) said to be in Lognormal distribution.

Which means that it is not symmetric, and it cannot take negative values.

Let’s define few things first

And we also know that

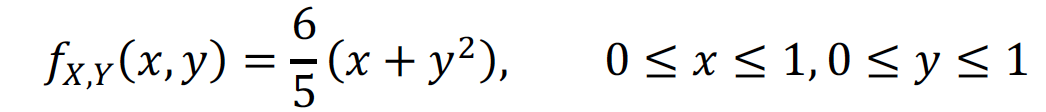
So, exp()

then,

**exp()**

**Question 2**

**In the bank example in page 44-45 of the slides, the joint pdf of X, Y is given by**

****

**a) What is the marginal pdf of 𝑓y(𝑦)?**

= +

= +

= + |10

= +

= +

**=**

**b) What is the conditional pdf of 𝑓X|Y (𝑥|𝑦 = 0.5)?**

=

=

So,

**=**

**Question 3**

**Exponential Distribution has a memoryless property. Intuitively, it means that the probability of customer**

**service answering you call (assuming waiting time is exponential) in the next 10 mins is the same, no matter if**

**you have waited an hour on the line or just picked up the phone. Formally, if X ∼ exponential(λ), f(x) = λ exp(-**

**λx), and t and s are two positive numbers, use the definition of conditional probability to show that**

**P(X > t + s | X > t) = P(X > s).**

**Hint: Find the cdf of X first, and note that P(X > t + s Ç X > t) = P(X > t + s)**

Now we know that CDF of is,

Now according to conditional probability rule,

------ [1]

Since . If then . So,

And also, s and t both are positive, and

So [1] can be written as

Hence proved.

**Question 4**

**Roll a fair die (uniform 1,2,3,4,5,6) repeatedly. You and Peter are betting on the number shown on each roll.**

**If the number is 4 or less, you win $1; otherwise, you pay Peter $2.5.**

**a) What is the expected value of the payoff for you?**

**b) What is the variance of your payoff?**

**Question 5**

**Let X be uniformly distributed (continuous) on the interval [1,2]. Find E(1/X).**

We know that,

So,

Here

= |x| |21

=

= **0.6931**

**Question 6**

**Let us assume that the lifetime of light bulbs follows an exponential distribution with the pdf:**

**f(x) = λ exp(-λx)**

**We test 10 bulbs and their lifetimes are 5, 3, 5, 1, 7, 3, 4, 8, 2, 3 years, respectively. What is the MLE for λ? What**

**is the method of moments estimator for λ?**

Here following information is given,

Now,

=

=

Taking log on both sides,

Let’s take MLE,

**Question 7**

**Let x1, x2,….,xn be an iid samples with pdf**

**Find MLE of**

Taking ln

Taking derivative,

Setting ,