H.W - 2

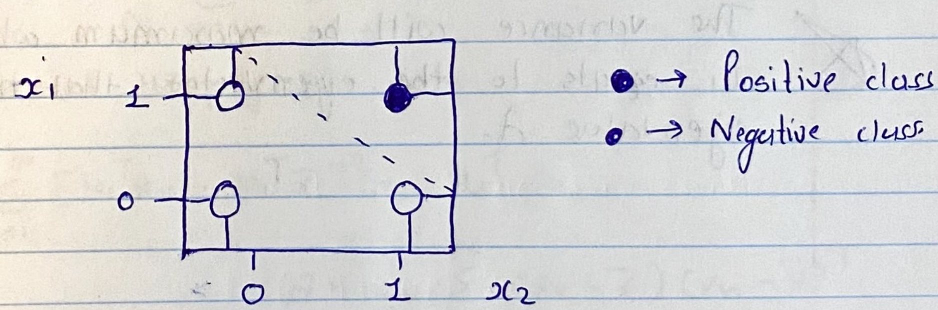
(1) Perceptron Algorithm

⇒ Let's first define the truth table for the NAND function

→ Truth table:

x_1	x_2	Class	Label
0	1	Negative	1
1	1	Positive	0
1	0	Negative	1
0	0	Negative	1

→ From the truth table we can build following



(1) From above figure we can say that for boolen NAND function, negative and positive class are linearly separable.

(2) As we know boolen function NAND is linearly separable we can use Perceptron Algorithm to obtain the linear decision boundary.

Let's assume our decision boundry is initially as follows.

$$x_1 + x_2 - 1/2 = 0$$

And our intial weights are as follows.

$w_0 = -1/2 = -0.5$

$w_1 = 1$

$w_2 = 1$

$\Rightarrow$ Applying Perceptron Algoritham

ITERATION: 1

for $f(0,1)$, Target $= -1$

$y = w_0 + w_1 x_1 + w_2 x_2$

$= -0.5 + (1)(0) + (1)(1)$

$= -0.5 + 1$

$= 0.5$

Here $y > 0$ and target $= -1$ so we have misclassification.

Hence we updat our weights.

$w_0 = w_0^* + \text{Target}$

$= -0.5 + (-1)$

$= -1.5$

$w_1 = w_1^* + \text{Target} * x_1$

$= 1 + (-1)(0)$

$= 1$

$w_2 = w_2^* + \text{Target} * x_2$

$= 1 + (-1)(1)$

$= 1 - 1 = 0$

for $f(1,1)$, Target $= +1$

$y = w_0 + w_1 x_1 + w_2 x_2$

$= -1.5 + (1)(1) + (0)(1)$

$= -1.5 + 1$

$= -0.5$

Here $y < 0$ and target $= +1$ So we have misclassification.

Hence we update our weights.

$w_0 = w_0^* + \text{Target}$
$= -1.5 + 1$
$= -0.5$

$w_1 = w_1^* + \text{Target} \times x_1$
$= 1 + (1)(1)$
$= 2$

$w_2 = w_2^* + \text{Target} * x_2$
$= 0 + (1)(1)$
$= 1$

for f(1, 0) Target = -1

$y = w_0 + w_1 x_1 + w_2 x_2$
$= -0.5 + (2)(1) + (1)(0)$
$= 1.5$

Here $y > 0$ and target = -1 so we have misclassification.
Hence we update our weights

$w_0 = w_0^* + \text{Target}$
$= -0.5 - 1$
$= -1.5$

$w_1 = w_1^* + \text{Target} * x_1$
$= 2 + (-1)(1)$
$= 1$

$w_2 = w_2^* + \text{Target} * x_2$
$= 1 + (-1)(0)$
$= 1$

for f(0, 0) Target = -1

$y = w_0 + w_1 x_1 + w_2 x_2$
$= -1.5 + (1)(0) + (1)(0)$
$= -1.5$

Here $y < 0$ and target = -1 so have correct classification

ITERATION-2

for f(0, 1) Target = -1

$y = w_0 + w_1 x_1 + w_2 x_2$
$= -1.5 + (1)(0) + (1)(1)$
$= -1.5 + 1$
$= -0.5$

Here $y < 0$ and target = -1 so we have correct classification.

for $f(1,1)$ Target $= +1$

$$y = w_0 + w_1 x_1 + w_2 x_2$$
$$= -1.5 + (1)(1) + (1)(1)$$
$$= -1.5 + 2$$
$$= 0.5$$

Here $y > 0$ and target $= +1$ so we have correct classification.

for $f(1,0)$ Target $= -1$

$$y = w_0 + w_1 x_1 + w_2 x_2$$
$$= -1.5 + (1)(1) + (1)(0)$$
$$= -1.5 + 1$$
$$= -0.5$$

Here $y < 0$ and target $= -1$ so we have correct classificatio

for $f(0,0)$ Target $= -1$

$$y = w_0 + w_1 x_1 + w_2 x_2$$
$$= -1.5 + (1)(0) + (1)(0)$$
$$= -1.5$$

Here $y < 0$ and target $= -1$ so we have correct classificatio

Since we have successfully classified all the points correctly our final weights for boolen NAND function is

$w_0 = -1.5$ $w_1 = 1$ and $w_2 = 1$

And our decision boundry will be

$$-1.5 + x_1 + x_2 = 0$$

$$\boxed{x_1 + x_2 = 1.5}$$