Example:

- Training:
  - The cat sat on the mat.
  - The dog barked loudly.
  - A cat ran quickly
- Tagged Sentences:
  - The cat sat on the mat. : **The/DT cat/NN sat/VBD on/IN the/DT mat/NN.**
  - The dog barked loudly.: **The/DT dog/NN barked/VBD loudly/RB.**
  - A cat ran quickly. : **A/DT cat/NN ran/VBD quickly/RB.**

Step 1: Compute Probabilities from Training Corpus

- **1.1 Individual Tag Counts (C(tag)):**
  - **DT** (Determiner): 4 times (The, the, A)
  - **NN** (Noun): 4 times (cat, mat, dog, cat)
  - **VBD** (Verb, past tense): 3 times (sat, barked, ran)
  - **IN** (Preposition): 1 time (on)
  - **RB** (Adverb): 2 times (loudly, quickly)
  - **Total number of tags = 14**
- 1.2 Compute emission Probabilities P(wi|ti)
  - Word-Tag Pair Counts : Count how often each word occurs with a specific tag in the training data.
  - The/DT: 3
  - A/DT: 1
  - cat/NN: 2
  - dog/NN: 1
  - mat/NN: 1
  - sat/VBD: 1
  - barked/VBD: 1
  - ran/VBD: 1
  - on/IN: 1
  - loudly/RB: 1
  - quickly/RB: 1

Tag	Word	Frequency
DT	The	3
DT	A	1
NN	cat	2
NN	dog	1
NN	mat	1
VBD	sat	1
VBD	barked	1
VBD	ran	1
RB	loudly	1
RB	quickly	1

• Step 1.3: Calculate Emission Probabilities

- Calculate the emission probability using the formula:

$$\mathbf{P(word|tag)=}\frac{\textit{Total frequency of the tag}}{\textit{Frequency of the word with the tag}}$$

- P(“The”|DT)=C(“The”,DT) / C(DT)=3/4
- P(“A”|DT)=1/4
- P(“cat”|NN)=C(“cat”,NN) / C(NN)=2/4=0.5
- P(“dog”|NN)=1/4
- P(“mat” | NN) = 1/4 = 0.25
- P(“barked”|VBD)=C(“barked”|VBD)/C(VBD)=1/3
- P(“sat”|VBD)=1/3
- P(“ran”|VBD)=1/3
- P(“on”|IN)=1/1=1
- P(“loudly”|RB)=1/2
- P(“quickly”|RB)=1/2

Step 2: Calculate the Tag Transition Probabilities

- Transition Probability is the probability of a tag occurring after another tag
- calculate the probability of transitioning from one tag to another based on the training data.
- Computed using $\mathbf{P(t_n \mid t_{n-1})}$
- The formula for calculating the tag transition probability is:
  - $P(\text{tag2}|\text{tag1}) = \frac{\textbf{Number of times tag1 is followed by tag2}}{\textbf{Total number of times tag1 occurs}}$

• 2.1 Count Tag Transitions in the Training Data

- The/DT cat/NN sat/VBD on/IN the/DT mat/NN
  - → Tag sequence: DT → NN → VBD → IN → DT → NN
- The/DT dog/NN barked/VBD loudly/RB
  - → Tag sequence: DT → NN → VBD → RB
- A/DT cat/NN ran/VBD quickly/RB
  - → Tag sequence: DT → NN → VBD → RB

- 2.2 Count the Occurrences of Each Tag Pair
  - DT → NN - 4 (from all sentences)
  - NN → VBD - 3 (cat → sat, dog → barked, cat → ran)
  - VBD → IN - 1 (sat → on)
  - IN → DT - 1 (on → the)
  - VBD → RB - 2 (barked → loudly, ran → quickly)
- 2.3 Calculate the Total Number of Times Each Tag Occurs
  - DT 4 (DT → NN happens 4 times)
  - NN 3 (NN → VBD happens 3 times)
  - VBD 3 (VBD → IN once, VBD → RB twice)
  - IN 1 (IN → DT happens once)
- 2.4 Calculate Tag Transition Probabilities
  - Calculate Tag Transition Probabilities
  - P(NN|DT) = C(DT, NN) / C(DT) = 4 / 4
  - P(VBD|NN) = C(NN, VBD) / C(NN) = 3 / 4
  - P(IN|VBD) = C(VBD, IN) / C(VBD) = 1 / 3
  - P(DT|IN) = C(IN, DT) / C(IN) = 1 / 1 = 1
  - P(RB|VBD) = C(VBD, RB) / C(VBD) = 2 / 3

Test Sentence: "The dog ran quickly."

- Step 1: initialize the tag sequence for each word in the sentence
  - T =(DT,NN,VB,RB)
- Step 2: Compute Emission Probabilities for the Test Sentence
  - P("The"|DT)= 3/4
  - P("dog"|NN)=1/4
  - P("ran"|VBD)=1/3
  - P("quickly"|RB)=1/2
- Step 3: Compute Transition Probabilities for the Test Sentence
  - P(DT) = 1
  - P(NN|DT) = 1
  - P(VBD|NN) = 3/4
  - P(RB|VBD) = 2/3
- Step 4: Calculate Joint Probability

  $P(T|W)= P(DT) \times P(the|DT) \times P(NN|DT) \times P(dog|NN) \times P(VBD|NN) \times P(ran|VBD) \times P(RB|VBD) \times P(quickly|RB) \times P(T|W)$

  $= 1 \times (3/4) \times 1 \times (1/4) \times (3/4) \times (1/3) \times (2/3) \times (1/2) \times 1 = 1/48 \approx 0.015625$
- Given the probabilities computed from the training corpus, the most probable tag sequence for the test sentence "The dog ran quickly" is: **The/DT dog/NN ran/VB quickly/RB**.
- The probability of this tag sequence occurring with these words is approximately 0.015625.

- Trigram Tag Probabilities (P(t3|t2, t1))
  - This is the probability of a tag occurring given the previous two tags.
  - The trigram tag probabilities represent the likelihood of a tag $t3$ occurring after two preceding tags $t1$ and $t2$.
  - **$P(t3|t2,t1)=\frac{Count\ of\ the\ bigram\ (t1\ ,t2,t3\ )}{Count\ of\ the\ tag\ (t1,t2)}$**
- Trigram Tag Counts
  - Count of (DT, NN, VBD)=2
  - Count of (NN, VBD, RB)=2

Computing trigram probability

- **Compute Transition Probabilities (Trigram)**
  - **Trigram Tag Probabilities (P(t3|t2, t1))**
    - This is the probability of a tag occurring given the previous two tags.
    - The trigram tag probabilities represent the likelihood of a tag $t3$ occurring after two preceding tags $t1$ and $t2$.
    - $P(t3|t2,t1)=\frac{Count\ of\ the\ bigram\ (t1,t2,t3)}{Count\ of\ the\ tag\ (t1,t2)}$
  - **Trigram Tag Counts**
    - Sentence 1: DT → NN → VBD → IN → DT → NN
      - P(VBD|DT,NN) → DT → NN → VBD
      - P(IN|NN,VBD) → NN → VBD → IN
      - P(DT|VBD,IN) → VBD → IN → DT
      - P(NN|IN,DT) → IN → DT → NN
    - Sentence 2: DT → NN → VBD → RB
      - P(VBD|DT,NN) → DT → NN → VBD
      - P(RB|NN,VBD) → NN → VBD → RB
    - Sentence 3: DT → NN → VBD → RB
      - P(VBD|DT,NN) → DT → NN → VBD
      - P(RB|NN,VBD) → NN → VBD → RB
    - Trigram Transition Counts
      - P(VBD|DT, NN) - 3
      - P(IN|NN, VBD) - 1
      - P(RB|NN, VBD) - 2
      - P(DT|VBD, IN) - 1
      - P(NN|IN, DT) - 1

P(VBD|DT, NN)	**DT → NN → VBD**	**3**
P(IN|NN, VBD)	NN → VBD → IN	1
P(DT|VBD,IN)	VBD → IN → DT	1
P(NN|IN,DT)	IN → DT → NN	1
P(RB|NN,VBD)	NN → VBD → RB	2

- Trigram Probabilities
  - P(VBD| DT, NN) = C(DT, NN, VBD) / C(DT, NN) = 3 / 4
  - P(RB| NN, VBD) = C(NN, VBD, RB) / C(NN, VBD) = 2 / 3
  - P(IN|NN, VBD)=C((NN, VBD, IN) / C(NN, VBD)=1/3
  - P(DT|VBD, IN)=1
  - P(NN|IN, DT)=1

Test Sentence: "The dog ran quickly."

- Step 1: initialize the tag sequence for each word in the sentence
  - T =(DT,NN,VB,RB)
- Step 2: Compute Emission Probabilities for the Test Sentence
  - P("The"|DT)= 3/4
  - P("dog"|NN)=1/4
  - P("ran"|VBD)=1/3
  - P("quickly"|RB)=1/2
- Step 3: Compute Transition Probabilities for the Test Sentence
  - P(DT) = 1
  - P(NN|DT) = 1
  - P(VBD|NN) = 3/4
  - P(RB|VBD) = 2/3
  - P(VBD| DT, NN) =3 / 4
  - P(RB| NN, VBD) =2 / 3
- Step 4: Calculate Joint Probability

  $P(T|W)= P(DT) \times P(the|DT) \times P(NN|DT) \times P(dog|NN) \times P(VBD|NN) \times P(ran|VBD) \times P(RB|VBD) \times P(quickly|RB) \times P(VBD|DT,NN) \times P(RB|NN,VBD)$

  $P(T|W)= 1\times (3/4)\times 1\times (1/4)\times (3/4)\times (1/3)\times (2/3)\times (1/2)\times 1= 1/48 \approx 0.50$
- Given the probabilities computed from the training corpus, the most probable tag sequence for the test sentence "The dog ran quickly" is: **The/DT dog/NN ran/VB quickly/RB**.
- The probability of this tag sequence occurring with these words is approximately 0.50