

Q1 It is clear from the color theory lecture that an object reflects the light color that it is made up of. As a result when light falls on tiles of different colors which are made up of combinations of some colors, some tiles reflect the incident light because it is made up of ^{same} constituent color and appears to glow. Whereas some tiles absorb the incident light as that light color is not present in the constituent colors, that the tile is made up of. Hence those tiles appear muted and devoid of their original color.

- Effect under Yellow light
Since none of the tiles is made up of yellow as its constituent color except for yellow, the effect is almost negligible and it is not hard to solve the puzzle.

- Effect under red light
Now the white, Yellow, Orange and Red tiles have red color as one ~~other~~ of their component colors. As a result of which when red light is incident on these tiles, they will reflect the red

light and all these tiles appears to be red or a shade of red. This make it really difficult to solve the cube.

- Result. If we compare the difficulty of the cube under Yellow and Red light, based on the above explanation, we can conclude that it is much more difficult to solve the cube under red light.

Q 2

Solⁿ

$$x = \frac{x}{x+y+z}$$

$$y = \frac{y}{x+y+z}$$

$$[(1-x-y)/y]y = \left[1 - \frac{x}{x+y+z} - \frac{y}{x+y+z} \right] y$$

$$\left[\frac{y}{x+y+z} \right]$$

$$= \left[\frac{x+y+z-x-y}{x+y+z} \right] x$$

$$x$$

$$x+y+z$$

$$= x+y+z-x-y$$

$$= z$$

$$\therefore [(1-x-y)/y]y = z$$

- Comment with reasons whether this algorithm will work efficiently.

Solⁿ There are two types of color gamuts, Additive and subtractive. Subtractive style is also known as CMYK (Cyan, Magenta, Yellow, Key). This algorithm will work well because human eye is better at perceiving the relationship between the colors than the absolute colors. While printing the image has a white border because of which there is a color cast as the eyes will adjust according to the surrounding white paper and thus the perception will not be affected even though nearest color on the printer color gamut is selected.

- The cartoon image has constant color tones and we know that human eye is good at adapting of color relationship, since the color tone is constant our eyes will be able to perceive the image better. Since there is a significant difference in the color tone for a changing color tone real image this algorithm will not perform better.

- The translation failure occurs because some colors in the source may not be in the destination color space. These out of gamut colors can change the look of the image. Color Management has methods of interpreting out of ~~bound~~ gamut colors. This method is called out of gamut colors and mapping them into destination gamut color space. This method is called rendering intents. Few rendering intents are

- Perceptual Rendering Intent.
This intent is most suited for photographs that contains out of gamut colors. It maintains the relationship between colors to produce the best results.

- Saturation Rendering Intent.
It produces more concentrated solid colors in business graphics like graphs and charts. Color may be less accurate than those produced by rendering intents.

Q3 $P(X) = x^k, P(Y) = 1 - x^k$

We know that since there are only 2 symbols $P(X) + P(Y) = 1$

$$P(Y) = 1 - P(X)$$

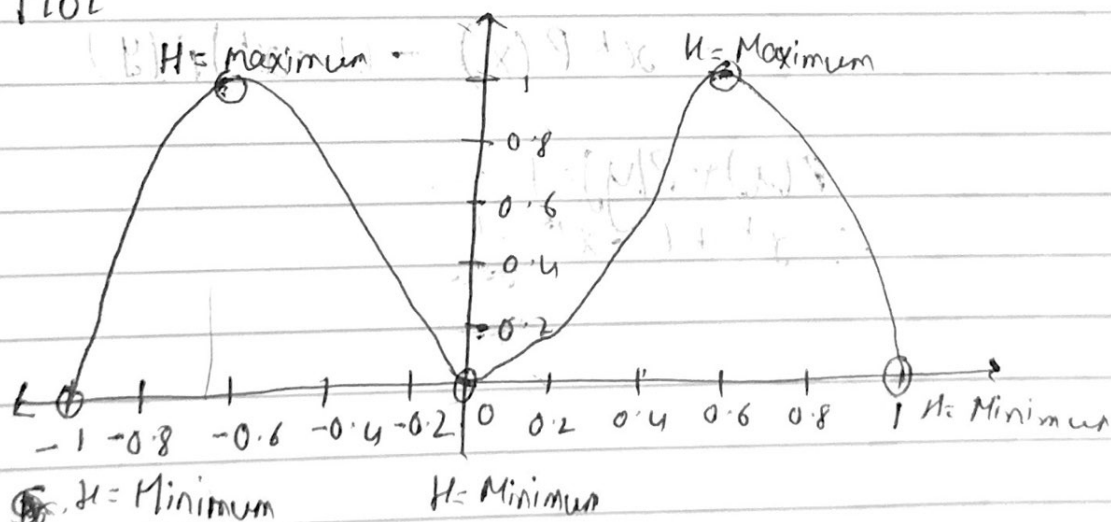
$$H = -\sum P_i \log(P_i)$$

$$\begin{aligned} H &= -P(X) \log(P(X)) - P(Y) \log(P(Y)) \\ &= -x^k \log x^k - (1 - x^k) \log(1 - x^k) \\ &= -[x^k \log x^k - (1 - x^k) \log(1 - x^k)] \end{aligned}$$

for $x = k = 2$

$$H = -[x^2 \log x^2 - (1 - x^2) \log(1 - x^2)]$$

• Plot



- From the plot we can see that H is minimum at $x = -1, 0, 1$.
We know that H is minimum when probability of an one symbol is 1

$$\therefore P(x) = 1 \quad \text{or} \quad P(y) = 1$$

$$x^k = 1 \quad \text{or} \quad 1 - x^k = 1$$

$$\therefore x^2 = 1 \quad \text{or} \quad x^2 = 0$$

$$\therefore x = \pm 1 \quad \text{or} \quad x = 0$$

$$\therefore H \text{ is minimum at } x = -1, 0, 1$$

- $H = -[x^k \log x^k + (1-x^k)(\log(1-x^k))]$
 $H = 0$ is the minimum value

$$\therefore x^k \log x^k + (1-x^k) \log(1-x^k) = 0$$

- We know that entropy is maximum when probability is equally distributed
 $\therefore P(x) = P(y)$

$$x^2 = 1 - x^2$$

$$2x^2 = 1$$

$$\therefore x^2 = \frac{1}{2}$$

$$\therefore x = \pm 0.707$$

$$H \text{ is maximum at } x = -0.707, +0.707$$

- H is maximum when $P(X) = P(Y)$

$$\therefore P(X) = P(Y)$$

$$\therefore x^K = 1 - x^K$$

$$2x^K = 1$$

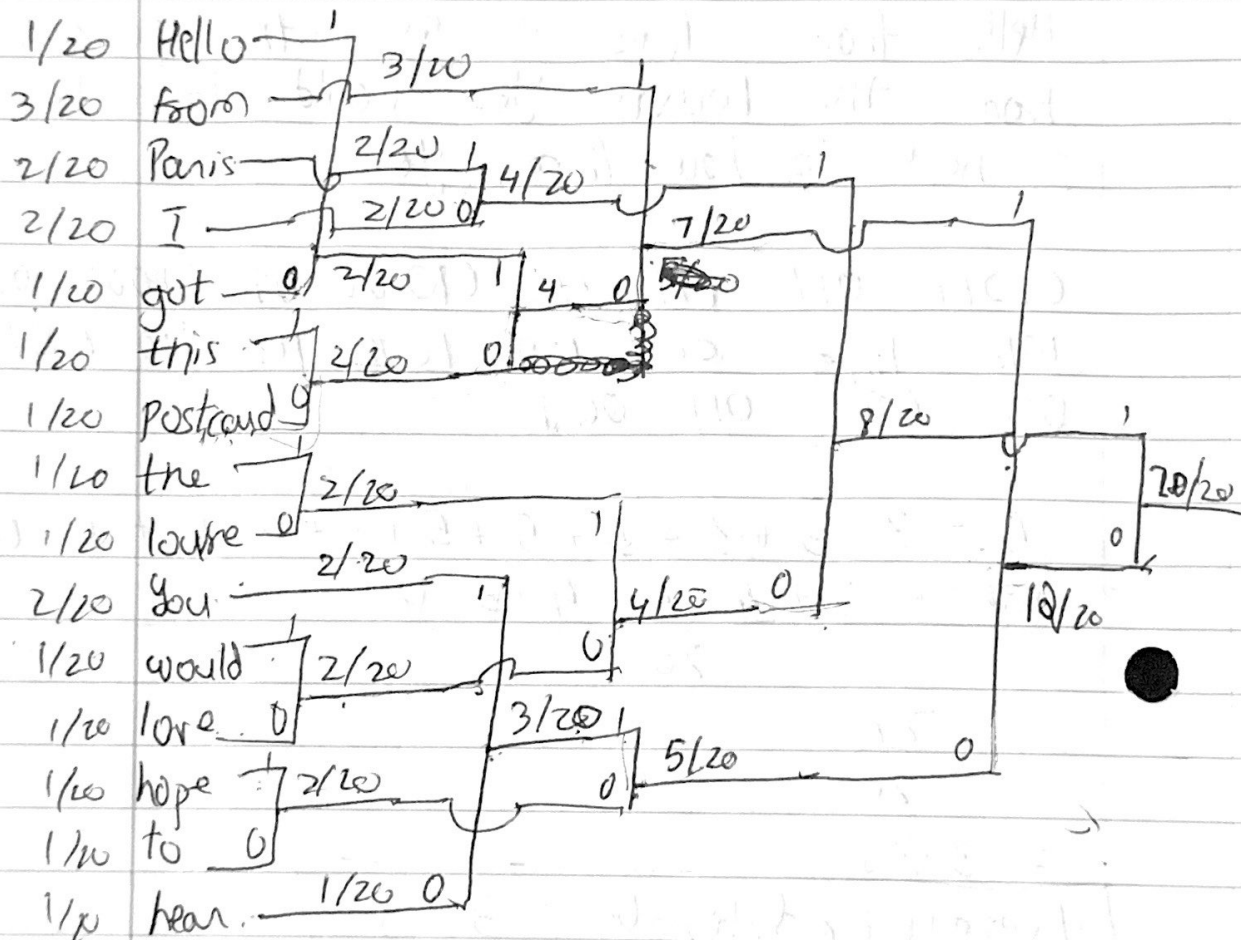
$$\therefore x^K = \frac{1}{2}$$

$$\therefore x = \sqrt[K]{\frac{1}{2}}$$

84

Hello from Paris I got this postcard
from the Louvre You would love Paris
I hope to hear from you.

	count	Probability
Hello	1	1/20
from	3	3/20
Paris	2	2/20
I	2	2/20
got	1	1/20
this	1	1/20
postcard	1	1/20
the	1	1/20
Louvre	1	1/20
You	2	2/20
Would	1	1/20
love	1	1/20
Hope	1	1/20
to	1	1/20
hear	1	1/20



Hello	01011 (5)	you	0011 (4)
from	011 (3)	would	1001 (4)
Paris	111 (3)	love	01000 (4)
I	110 (3)	hope	0001 (4)
got	01010 (5)	to	0000 (4)
this	01001 (5)	hear	0010 (4)
postcard	01000 (5)		
the	1011 (4)		
Louvre	1010 (4)		

Hello from Paris I got this postcard
from the Louvre. You would love Paris
I hope to hear from you.

01011 011 111 110 0101001001 01000 011
1011 1010 0011 1001 1000 111 110 0001
0000 0010 011 001

5 + 3 + 3 + 3 + 5 + 5 + 5 + 3 + 4 + 4 + 4 + 4 + 4
~~5~~ + 3 + 3 + 4 + 4 + 4 + 3 + 4

20

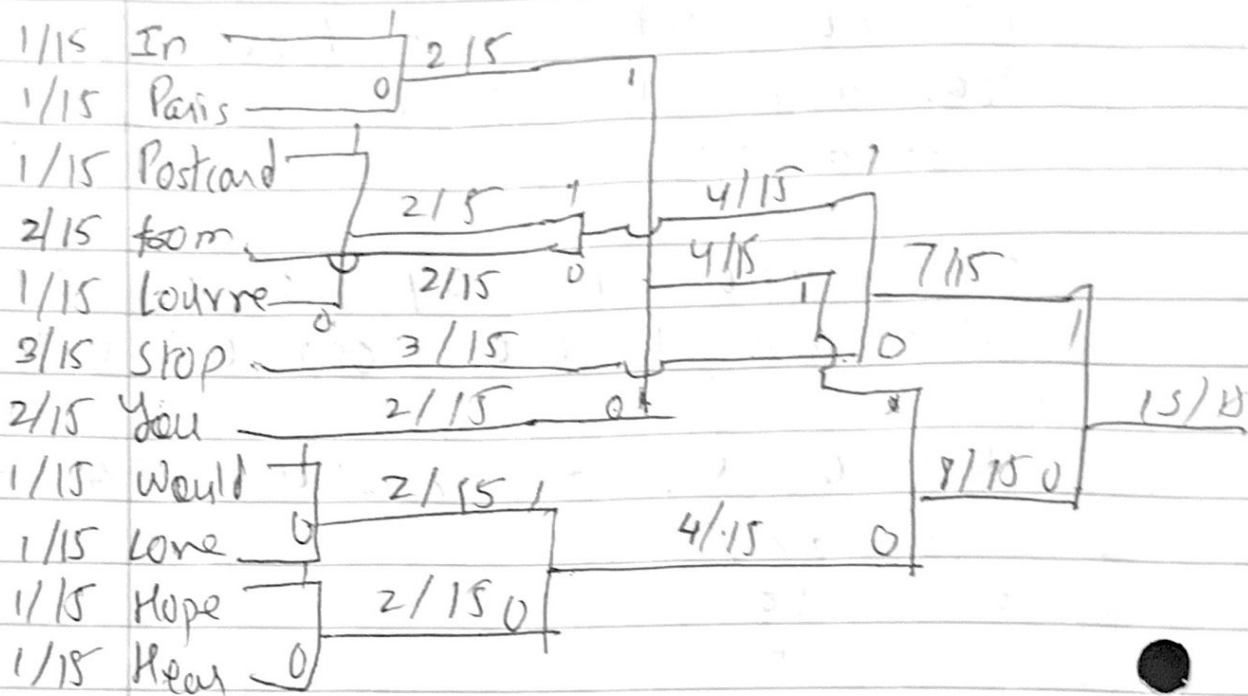
$$= \frac{77}{20}$$

$$= 3.85$$

Average code length = 3.85

• Shortened Message

In	1	1/15	Love	1	1/15
Paris	1	1/15	hope	1	1/15
Postcard	1	1/15	hear	1	1/15
from	2	2/15			
Louvre	1	1/15			
Stop	3	3/15			
You	2	2/15			
would	1	1/15			



In 0111

Paris 0110

Postcard 1111

from 0111

Louvre 1110

Stop 10

You 010

Would 0011

Love 0010

Hope 0001

Hear 0000

In Paris Postcard from Louvre Stop
 You would love stop hope hear from you
 soon stop

0111 0110 1111 011 1110 10 010
 0011 0010 10 0001 0000 011 010 10

$$\begin{aligned}
 &4 + 4 + 4 + 3 + 4 + 2 + 3 + 4 + 4 + 2 + 4 + 4 + 4 + 3 \\
 &\quad + 3 + 3 + 2 \quad / 15 \\
 &= \frac{50}{15} \\
 &= 3.33
 \end{aligned}$$

Average code length = 3.33

$$H = \sum p_i \log_2(1/p_i)$$

For the first message the entropy is

$$\begin{aligned}
 H &= \sum p_i \log_2(1/p_i) \\
 H &= 3.784
 \end{aligned}$$

For the second message the entropy is

$$\begin{aligned}
 H &= \sum p_i \log_2(1/p_i) \\
 \therefore H &= 3.323
 \end{aligned}$$

Since the entropy of the first message is higher it is quantitatively better.

We are using the same algorithm for both message, thus both are qualitatively equal.