EXERCISE - 1

1. How many four digit numbers can be made by using the digits 1, 2, 3, 7, 8, 9 when
   (i) repetition of a digit is allowed?
   (ii) repetition of a digit is not allowed?
2. Find the total number of 9-digit numbers of different digits.
3. Find the total number of 4 digit number that are greater than 3000, that can be formed by using the digits 1, 2, 3, 4, 5, 6 (no digit is being repeated in any number).
4. How many numbers greater than 1000 or equal to, but not greater than 4000 can be formed with the digits 0, 1, 2, 3, 4, repetition of digits being allowed?
5. How many numbers between 400 and 1000 (both exclusive) which can be made with the digits 2,3,4,5,6,0 if
   (i) repetition of digits not allowed?
   (ii) repetition of digits is allowed?
6. A variable name in a certain computer language must be either an alphabet or a alphabet followed by a decimal digit. Find the total number of different variable names that can exist in that language.
7. Tanya typed a six-digit number, but the two 1's she typed did not show. What appeared was 2006. Find the number of different 6-digit numbers she would have typed.
8. A letter lock consists of three rings each marked with fifteen different letters. It is found that a man could open the lock only after he makes half the number of possible unsuccessful attempts to open the lock . If each attempt takes 10 seconds. Then find the minimum time he must have spent.
9. Find the number of 6-digit numbers that can be formed using 1, 2, 3, 4, 5, 6, 7 so that digits do not repeat and terminal digits are even.
10. Find the total number of numbers that can be formed by using all the digits 1, 2, 3, 4, 3, 2, 1 so that the odd digits always occupy the odd places.
11. Find the number of 6-digit numbers which have 3 digits even and 3 digits odd, if each digit is to be used atmost once.
12. Find the number of 4-digits numbers that can be made with the digits 1, 2, 3, 4 and 5 in which at least two digits are identical.
13. Find the number of 5-digit telephone numbers having atleast one of their digits is repeated.
14. Find the number of 3-digit numbers having only two consecutive digits identical.

15. Find the number of different matrices that can be formed with elements 0, 1, 2 or 3, each matrix having 4 elements.

16. Find the number of 6-digit numbers in which sum of the digits is even.

17. Find the number of 5-digit numbers divisible by 3 which can be formed using 0, 1, 2, 3, 4, 5 if repetition of digits is not allowed.

18. Find the number of 4-digit numbers divisible by 3 that can be formed by four different even digits.

19. Find the number of 5-digit numbers divisible by 6 which can be formed using 0, 1, 2, 3, 4, 5 if repetition of digits is not allowed.

20. Find the number of 5-digit numbers divisible by 4 which can be formed using 0, 1, 2, 3, 4, 5, when the repetition of digits is allowed

21. Natural numbers less than 10^4 and divisible by 4 and consisting of only the digits 0, 1, 2, 3, 4 and 5 (no repetition) are formed . Find the number of ways of formation of such number.

22. Find the number of natural numbers less than 1000 and divisible by 5 which can be formed with the ten digits, each digit not occurring more than once in each number.

23. Two numbers are chosen from 1, 3, 5, 7,…, 147, 149 and 151 and multiplied together. Find the number of ways which will give us the product a multiple of 5.

24. A 7-digit number divisible by 9 is to be formed by using 7 digits out of digits 1, 2, 3, 4, 5, 6, 7, 8, 9. Find the number of ways in which this can be done.

25. Find the number of 9-digits numbers divisible by nine using the digits from 0 to 9 if each digit is used atmost once.

26. Among 9! permutations of the digits 1, 2, 3,…, 9. Consider those arrangements which have the property that if we take any five consecutive positions, the product of the digits in those positions is divisible by 7. Find the number of such arrangements.

27. Find the number of distinct results which can be obtained when *n* distinct coins are tossed together.

28. Three distinct dice are rolled. Find the number of possible outcomes in which at least one die shows 5.

29. A telegraph has '*m*' arms and each arm is capable of '*n*' distinct positions including the position of rest. Find the total number of signals that can be made.

30. Find the number of possible outcomes in a throw of *n* distinct dice in which at least one of the dice shows an odd number.

31. Find the number of times the digit 5 will be written when listing integers from 1 to 1000.

32. Find the number of times of the digits 3 will be written when listing the integer from 1 to 1000.

33. If 33! is divisible by 2^n, then find the maximum value of n.

34. Let $E = \left\lfloor \frac{1}{3} + \frac{1}{50} \right\rfloor + \left\lfloor \frac{1}{3} + \frac{2}{50} \right\rfloor + \left\lfloor \frac{1}{3} + \frac{3}{50} \right\rfloor + \cdots$ upto 50 terms, then find the exponent of 2 in $(E)!$.

35. 3-digit numbers in which the middle one is a perfect square are formed using the digits 1 to 9 . Find their sum.

36. Find the sum of all the 4-digit even numbers which can be formed by using the digits 0, 1, 2, 3, 4 and 5 if repetition of digits is allowed.

37. Find sum of 5-digit numbers that can be formed using 0, 0, 1, 2, 3, 4.

38. Find sum of 5-digit numbers that can be formed using 0, 0, 1, 1, 2, 3.

39. The integers from 1 to 1000 are written in order around a circle. Starting at 1, every fifteenth number is marked (that is 1, 16, 31, etc.) This process is continued until a number is reached which has already been marked, then find the all unmarked numbers.

40. Let S be $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Find the number of subsets A of S such that. $x \in A$ and $2x \in S \Rightarrow 2x \in A$.

EXERCISE - 2

1. **(a)** Find 'n' if (i) ${}^{2n}C_3 : {}^{n}C_2 = 12 : 1$ (ii) ${}^{25}C_{n+5} = {}^{25}C_{2n-1}$

(b) Prove that ${}^{n-1}C_3 + {}^{n-1}C_4 > {}^{n}C_3$ if $n > 7$.

2. Find the number of positive integers satisfying the inequality ${}^{n+1}C_{n-2} - {}^{n+1}C_{n-1} \le 100$.

3. There are 20 questions in a questions paper. If no two students solve the same combination of questions but solve equal number of questions then find the maximum number of students who appeared in the examination.

4. In how many ways can 5 colours be selected out of 8 different colours including red, blue, and green

(i) if blue and green are always to be included,

(ii) if red is always excluded,

(iii) if red and blue are always included but green excluded?

5. The kindergarten teacher has 25 kids in her class. She takes 5 of them at a time, to zoological garden as often as she can, without taking the same 5 kids more than once. Find the number of visits, the teacher makes to the garden and also the number of of visits every kid makes.

6. A teacher takes 3 children from her class to the zoo at a time as often as she can, but does not take the same three children to the zoo more than once. She finds that she goes to the zoo 84 more than a particular child goes to the zoo. Find the number of children in her class.

7. A team of four students is to be selected from a total of 12 students. Find the total number of ways in which team can be selected such that two particular students refuse to be together and other two particular students wish to be together only.

8. A women has 11 close friends. Find the number of ways in which she can invite 5 of them to dinner, if two particular of them are not on speaking terms and will not attend together.

9. Four couples (husband and wife) decide to form a committee of four members. Find the number of different committees that can be formed in which no couple finds a place.

10. Find the number of ways in which a mixed double tennis game can be arranged from amongst 9 married couple if no husband and wife plays in the same game.

11. Find the number of ways of choosing a committee of 2 women and 3 men from 5 women and 6 men, if Mr. A refuses to serve on the committee if Mr. B is a member and Mr. B can only serve, if Miss C is the member of the committee.

12. Find the number of ways in which we can choose 3 squares on a chess board such that one of the squares has its two sides common to other two squares.

13. Find the number of ways of selecting three squares on a chessboard so that all the three be on a diagonal line of the board or parallel to it.

14. 5 Indian and 5 American couples meet at a party and shake hands. If no wife shakes hands with her husband and no Indian wife shakes hands with a male, then find the number of hand shakes that takes place in the party.

15. A person predicts the outcome of 20 cricket matches of his home team. Each match can result either in a win, loss or tie for the home team. Find the total number of ways in which he can make the predictions so that exactly 10 predictions are correct.

16. A forecast is to be made of the results of five cricket matches, each of which can be a win, a draw or a loss for Indian team. Find

(i) the number of different possible forecasts.

(ii) the number of forecasts containing 0, 1, 2, 3, 4 and 5 errors respectively.

17. A forecast is to be made of the results of five cricket matches, each of which can be a win or a draw or a loss for Indian team.

Let p = Number of forecasts with exactly 1 error

q = Number of forecasts with exactly 3 errors and

r = Number of forecasts with all five errors

then prove that $2q = 5r$, $8p = q$, and $2(p + r) > q$.

18. In a club election the number of contestants is one more than the number of maximum candidates for which a voter can vote. If the total number of ways is which a voter can vote be 62, then find the number of candidates.

19. Every one of the 10 available lamps can be switched on to illuminate certain Hall. Find the total number of ways in which the hall can be illuminated.

20. In a unique hockey series between India and Pakistan, they decide to play on till a team wins 5 matches . Find the number of ways in which the series can be won by India, if no match ends in a draw.

21. There are n different books and p copies of each in a library. Find the number of ways in which one or more books can be selected.

22. A class has n students. We have to form a team of the students by including atleast two students and also by excluding atleast two students. Find the number of ways of forming the team.

23. If the $(n + 1)$ numbers $a_1, a_2, a_3, \ldots, a_{n+1}$, be all different and each of them is a prime number, then find the number of different factors (other than 1) of $a_1^m \cdot a_2 \cdot a_3 \cdots a_{n+1}$.

24. In a certain algebraical exercise book there are 4 examples on arithmetical progressions, 5 examples on permutation-combination and 6 examples on binomial theorem. Find the number of ways a teacher can select for his pupils atleast one but not more than 2 examples from each of these sets.

25. Find the number of straight lines that can be drawn through any two points out of 10 points, of which 7 are collinear.

26. n lines are drawn in a plane such that no two of them are parallel and no three of them are concurrent. Find the number of different points at which these lines will cut each other.

27. Eight straight lines are drawn in the plane such that no two lines are parallel and no three lines are concurrent. Find The number of parts into which these lines divides the plane.

28. In a polygon no three diagonals are concurrent. If the total number of points of intersection of diagonals interior to the polygon be 70 then find the number of diagonals of the polygon.

29. In a plane there are two families of lines $y = x + r$, $y = -x + r$, where $r \in \{0, 1, 2, 3, 4\}$. Find the number of squares of diagonals of the length 2 formed by the lines.

30. Find the number of triangles whose vertices are at the vertices of an octagon but none of whose side happen to come from the sides of the octagon.

31. Let there be 9 fixed points on the circumference of a circle . Each of these points is joined to every one of the remaining 8 points by a straight line and the points are so positioned on the circumference that atmost 2 straight lines meet in any interior point of the circle. Find the number of such interior intersection points.

32. A bag contains 2 Apples, 3 Oranges and 4 Bananas. Find the number of ways in which 3 fruits can be selected if atleast one banana is always in the combination (Assume fruit of same species to be alike).

33. Find the number of selections of four letters from the letters of the word ASSASSINATION.

34. Find the number of ways to select 2 numbers from $\{0, 1, 2, 3, 4\}$ such that the sum of the squares of the selected numbers is divisible by 5 (repetition of numbers is allowed).

35. Find the number of ways in which we can choose 2 distinct integers from 1 to 100 such that difference between them is at most 10.

36. If a set A has m elements and another set B has n elements then find the number of functions from A to B.

37. Let $A = \{x : x \text{ is a prime number and } x < 30\}$. Find the number of different rational numbers whose numerator and denominator belongs to A.

38. Find the number of all three elements subsets of the set $\{a_1, a_2, a_3, \ldots, a_n\}$ which contain a_3.

39. If the total number of m-element subsets of the set $A = \{a_1, a_2, a_3, \ldots, a_n\}$ is k times the number of m-elements subsets containing a_4, then find n.

40. A set contains $(2n + 1)$ elements. Find the number of subsets of the set which contains at most n elements.

41. Find the number of subsets of the set $A = \{a_1, a_2, \ldots, a_n\}$ which contain even number of elements.

42. 'A' is a set containing 'n' distinct elements. A subset P of 'A' is chosen. The set 'A' is reconstructed by replacing the elements of P. A subset 'Q' of 'A' is again chosen. Find the number of ways of choosing P and Q so that $P \cap Q$ contains exactly two elements.

43. Find the number of ways of choosing triplets (x, y, z) such that $z \geq \max\{x, y\}$ and $x, y, z \in \{1, 2, \ldots, n, n+1\}$.

44. Find the number of ways in which the number 94864 can be resolved as a product of two factors.

45. Find the sum of the divisors of $2^5 \cdot 3^4 \cdot 5^2$.

46. In the decimal system of numeration, find the number of 6-digits numbers in which the digit in any place is greater than the digit to the left to it.

47. Find the number of 3-digit numbers of the form xyz such that $x < y$ and $z \leq y$.

48. Find the total number of 6-digit numbers $x_1\, x_2\, x_3\, x_4\, x_5\, x_6$ having the property $x_1 < x_2 \leq x_3 < x_4 < x_5 \leq x_6$.

49. The streets of a city are arranged like the lines of a chess board. There are m streets running North to South and 'n' streets running East to West. Find the number of ways in which a man can travel from NW to SE corner going the shortest possible distance.

50. Let there be $n \geq 3$ circles in a plane. Find the value of n for which the number of radical centres, is equal to the number of radical axes. (Assume that all radical axes and radical centre exist and are different)

51. Rajdhani express going from Bombay to Delhi stops at 4 intermediate stations. 10 passengers enter the train during the journey (including Bombay and 4 intermediate stations) with ten distinct tickets of two classes. Find the number of different sets of tickets they may have.

52. Find the number of functions f from the set $A = \{0, 1, 2\}$ into the set $B = \{0, 1, 2, 3, 4, 5, 6, 7\}$ such that $f(i) \le f(j)$ for $i < j$ and, $i, j \in A$.

53. Show that the number of ways of selecting n-objects out of $3n$-objects, n of which are alike and rest different is $2^{2n-1} + \binom{2n-1}{n-1}$.

54. Use a combinatorial argument to prove that:
(i) ${}^{2n}C_2 = 2 \cdot {}^{n}C_2 + n^2$ (ii) $r \cdot {}^{n}C_r = n\, {}^{n-1}C_{r-1}$

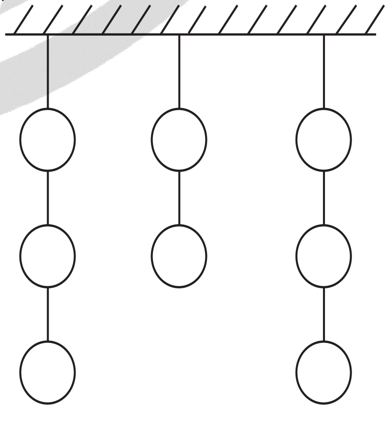
55. Prove (combinatorially) that
${}^{n}C_1 + 2\,{}^{n}C_2 + 3\,{}^{n}C_3 + \cdots + n\,{}^{n}C_n = n\,2^{n-1}$.

56. Prove (combinatorially) that
${}^{r}C_r + {}^{r+1}C_r + {}^{r+2}C_r + \cdots + {}^{n}C_r = {}^{n+1}C_{r+1}$, $r \le n$.

57. In a chess tournament, each participant was supposed to play exactly one game with each of the others. However, two participants withdraw after having played exactly 3 games each, but not with each other. The total number of games played in the tournament was 84. How many participants were there in all?

58. A positive integer n is called strictly ascending if its digits are in the increasing order. For example, 2368 and 147 are strictly ascending but 43679 is not. Find the number of strictly ascending numbers $< 10^9$.

59. The given figure shows 8 clay targets, arranged in 3 columns, to be shot by 8 bullets. Find the number of ways in which they can be shot, such that no target is shot before all the targets below it, if any, are first shot.



60. How many hexagons can be constructed by joining the vertices of a quindecagon (15 sides) if none of the sides of the hexagon is also the side of the 15-gon.

EXERCISE - 3

1. Find the value of r in following equations:

(i) ${}^{5}P_{r} = {}^{6}P_{r-1}$ **(ii)** ${}^{10}P_{r} = 720$ **(iii)** ${}^{20}P_{r} = 13 \times {}^{20}P_{r-1}$

2. In a railway compartment 6 seats are vacant on a berth. Find the number of ways in which 3 passengers sit on them.

3. Three men have 6 different trousers, 5 different shirts and 4 different caps. Find the number of different ways in which they can wear them.

4. Find the number of words of four letters containing equal number of vowels and consonants (repetition not allowed).

5. Find the number of words that can be formed using 6 consonants and 3 vowels out of 10 consonants and 4 vowels.

6. Find the number of ways in which the letters of the word ARRANGE can be made such that both R's do not come together.

7. Find the number of arrangements of the letters of the word BANANA is which the two 'N's do not appear adjacently.

8. We are required to form different words with the help of the letters of the word INTEGER. Let m_1 be the number of words in which I and N are never together and m_2 be the number of words which begin with I and end with R, then find m_1/m_2.

9. Find the number of arrangements that can be made with the letters of the word MATHEMATICS and also find the number of them, in which the vowels occur together.

10. Find the number of ways in which letters of the word VALEDICTORY be arranged so that the vowels may never be separated.

11. Find the number of different words which can be formed from the letters of the word LUCKNOW when

(i) all the letters are taken.

(ii) all the letters are taken and words begin with L.

(iii) all the letters are taken and the letters L and W respectively occupy the first and last places.

(iv) all the letters are taken and the vowels are always together.

12. Find the number of permutations of the word AUROBIND in which vowels appear in an alphabetical order.

13. If as many more words as possible be formed out of the letters of the word DOGMATIC then find the number of words in which the relative position of vowels and consonants remain unchanged.

14. Find the number of words which can be formed using all letters of the word 'Pataliputra' without changing the relative order of the vowels and consonants.

15. Find the total numbers of words that can be made by writing all letters of the word PARAMETER so that no vowel is between two consonants.

16. Find the total number of permutation of $n(n > 1)$ distinct things taken not more than r at a time and atleast 1, when each thing may be repeated any number of times.

17. Find the number of permutations of n distinct objects taken

(i) atleast r objects at a time

(ii) atmost r objects at a time

(Where repetition of the objects is allowed)

18. If the number of arrangements of $n - 1$ things from n distinct things is k times the number of arrangements of $n - 1$ things taken from n things in which two things are identical then find the value of k.

19. Find the number of different 7-digit numbers that can be written using only the three digits 1, 2 and 3 with the condition that the digit 2 occurs twice in each number.

20. Six identical coins are arranged in a row. Find the total number of ways in which the number of heads is equal to the number of tails.

21. There are n distinct white and n distinct black balls. Find the number of ways in which we can arrange these balls in a row so that neighboring balls are of different colours.

22. Find number of ways in which 6 girls and 6 boys can be arranged in a line if no two boys or no two girls are together.

23. Find the number of ways in which 3 boys and 3 girls (all are of different heights) can be arranged in a line so that boys as well as girls among themselves are in decreasing order of height (from left to right).

24. Find the number of ways in which 10 candidates $A_1, A_2, \ldots, A_{10}$ can be ranked so that A_1 is always above A_2.

25. Let A be a set of n (≥ 3) distinct elements. Find the number of triples (x, y, z) of the elements of A in which atleast two coordinates are equal.

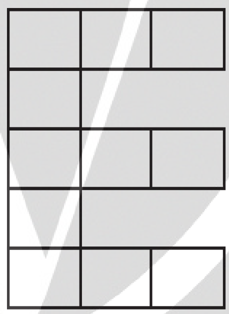
26. Find the number of ways of arranging m numbers out of 1, 2, 3, …, n so that maximum is $(n - 2)$ and minimum is 2 (repetitions of numbers is allowed) such that maximum and minimum both occur exactly once, $(n > 5, m > 3)$.

27. Eight chairs are numbered 1 to 8. Two women and three men wish to occupy one chair each. First the women choose the chairs from amongst the chairs marked 1 to 4, and then the men select the chairs from amongst the remaining. Find the number of possible arrangements.

28. There are 10 numbered seats in a double decker bus, 6 in the lower deck and 4 on the upper deck. Ten passengers board the bus, of them 3 refuse to go to the upper deck and 2 insist on going up. Find the number of ways in which the passengers can be accommodated.

29. In how many different ways a grandfather along with two of his grandsons and four grand daughters can be seated in a line for a photograph so that he is always in the middle and the two grandsons are never adjacent to each other.

30. Find the number of ways in which A A A B B B can be placed in the squares of the figure as shown so that no row remains empty.



31. The tamer of wild animals has to bring one by one 5 lions and 4 tigers to the circus arena. Find the number of ways this can be done if no two tigers immediately follow each other.

32. In a conference 10 speakers are present. If S_1 wants to speak before S_2 and S_2 wants to speak after S_3, then find the number of ways all the 10 speakers can give their speeches with the above restriction if the remaining seven speakers have no objection to speak at any number.

33. Find the total number of flags with three horizontal strips, in order, that can be formed using 2 identical red, 2 identical green and 2 identical white strips.

34. Messages are conveyed by arranging 4 white, 1 blue and 3 red flags on a pole. Flags of the same colour are alike. If a message is transmitted by the order in which the colours are arranged then the find the total number of messages that can be transmitted if exactly 6 flags are used.

35. Find number of arrangements of 4-letters taken from the word EXAMINATION.

36. Find number of ways in which an arrangement of 4-letters can be made from the letters of the word PROPORTION.

37. Find the number of permutations of the word ASSASSINATION taken 4 at a time.

38. The letters of the word TOUGH are written in all possible orders and these words are written out as in a dictionary, then find the rank of the word TOUGH.

39. The letters of the word SURITI are written in all possible orders and these words are written out as in a dictionary. What is the rank of the word SURITI?

40. There are 720 permutations of the digits 1, 2, 3, 4, 5, 6. Suppose these permutations are arranged from smallest to largest numerical values, beginning from 1 2 3 4 5 6 and ending with 6 5 4 3 2 1.

(a) What number falls on the 124th position?

(b) What is the position of the number 321546?

41. All the five digits number in which each successive digit exceeds its predecessor are arranged in the increasing order of their magnitude. Find the 97th number in the list.

42. All the 5 digit numbers, formed by permuting the digits 1, 2, 3, 4 and 5 are arranged in the increasing order. Find:

(i) the rank of 35421 (ii) the 100th number.

43. There are 11 seats in a row. Five people are to be seated. Find the number of seating arrangements, if

(i) the central seat is to be kept vacant;

(ii) for every pair of seats symmetric with respect to the central seat, one seat is vacant.

44. Find the number of ways in which six children of different heights can line up in a single row so that none of them is standing between the two children taller than him.

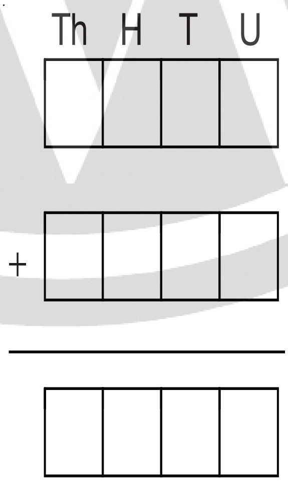
45. Define a 'good word' as a sequence of letters that consists only of the letters A, B and C and in which A never immediately followed by B, B is never immediately followed by C, and C is never immediately followed by A. If the number of n-letter good words are 384, then find the value of n.

46. There are 2 identical white balls, 3 identical red balls and 4 green balls of different shades. Find the number of ways in which they can be arranged in a row so that atleast one ball is separated from the balls of the same colour.

47. Eight identical rooks are to be placed on an 8×8 chess-board. Find the number of ways of doing this, so that no two rooks are in attacking positions.

48. How many arrangements of the 9 letters a, b, c, p, q, r, x, y, z are there such that y is between x and z? (Any two, or all three, of the letters x, y, z, may not be consecutive.)

49. In the figure, two 4-digit numbers are to be formed by filling the place with digits. Find the number of different ways in which these places can be filled by digits so that the sum of the numbers formed is also a 4-digit number and in no place the addition is with carrying.



50. Two n-digit integers (leading 0 allowed) are said to be equivalent if one is a permutation of the other. Thus 10075 and 01057 are equivalent. Find the number of 5-digit integers such that no two are equivalent.

EXERCISE - 4

1. A cabinet of ministers consists of 11 ministers, one minister being the chief minister. A meeting is to be held in a room having a round table and 11 chairs round it, one of them being meant for the chairman. Find the number of ways in which the ministers can take their chairs such that the chief minister occupying the chairman's place.

2. 20 persons were invited for a party. In how many ways can they and the host be seated at a circular table? In how many of these ways will two particular persons be seated on either side of the host?

3. In how many ways can 7 boys be seated at a round table so that two particular boys are

(i) next to each other (ii) separated.

4. A round table conference is to be held between 20 delegates of 2 countries. In how many ways can they be seated if two particular delegates

(i) always sit together (ii) never sit together.

5. There are 20 persons including two brothers. In how many ways can they be arranged on a round table if:

(i) There is exactly one person between the two brothers.

(ii) The two brothers are always separated.

(iii) What will be the corresponding answers if the two brothers were twins (alike in all respects)?

6. $2n$ chairs are arranged symmetrically around a table. There are $2n$ people, including A and B, who wish to occupy the chairs. Find the number of seating arrangements, if:

(i) A and B are next to each other;

(ii) A and B are diametrically opposite.

7. The 10 students of Batch B feel they have some conceptual doubt on circular permutation. Mr. Tiwari called them in discussion room and asked them to sit down around a circular table which is surrounded by 13 chairs. Mr. Tiwari told that his adjacent seat should not remain empty. Then find the number of ways, in which the students can sit around a round table if Mr. Tiwari also sit on a chair.

8. Find the number of ways in which 5 boys and 4 girls can be arranged on a circular table such that no two girls sit together and two particular boys are always together.

9. A person invites a party of 10 friends at dinner and place them

(i) 5 at one round table, 5 at the other round table.

(ii) 4 at one round table and 6 at other round table.

Find the ratio of number of circular permutation of case (i) to case (ii).

10. Six persons A, B, C, D, E and F are to be seated at a circular table. Find the number of ways this can be done if A must have either B or C on his right B must have either C or D on his right.

11. Find the number of ways in which 8 different flowers can be strung to form a garland so that 4 particular flowers are never separated.

12. Find the number of different garlands, that can be formed using 3 flowers of one kind and 3 flowers of other kind.

13. Find the number of seating arrangements of 6 persons at three identical round tables if every table must be occupied.

14. Let $1 \le n \le r$. The Stirling number of the first kind, $S(m, n)$, is defined as the number of arrangements of m distinct objects around n identical circular tables so that each table contains atleast one object. Show that:

(i) $S(m, 1) = (m - 1)!$;

(ii) $S(m, m - 1) = {}^mC_2, m \ge 2$.

15. Find the number of different ways of painting a cube by using a different colour for each face from six available colours.

(Any two colour schemes are called different if one cannot coincide with the other by a rotation of the cube.)

16. Find number of ways in which n things of which r alike and the rest distinct can be arranged in a circle distinguishing between clockwise and anti-clockwise arrangement.

EXERCISE - 5

1. Find the total number of ways of dividing 15 different things into groups of 8, 4 and 3 respectively.
2. Find the number of ways of distributing 50 identical things among 8 persons in such a way that three of them get 8 things each, two of them get 7 things each and remaining 3 get 4 things each.
3. Find the number of ways in which 14 men be partitioned into 6 committees where two of the committees contain 3 men each, and the others contain 2 men each.
4. If $3n$ different things can be equally distributed among 3 persons in k ways then find the number of ways to divide the $3n$ things in 3 equal groups.
5. Find the number of ways to give 16 different things to three persons A, B, C so that B gets 1 more than A and C gets 2 more than B.
6. Find the number of ways of distributing 10 different books among 4 students S_1, S_2, S_3 and S_4 such that S_1 and S_2 get 2 books each and S_3 and S_4 get 3 books each.
7. Find the number of different ways in which 8 different books can be distributed among 3 students, if each student receives at least 2 books.
8. Find the number of ways in which n different prizes can be distributed amongst m ($<n$) persons if each is entitled to receive at most $n-1$ prizes.
9. In a school there are two prizes for excellence in physics (Ist and IInd) two in Chemistry (Ist and IInd) and only 1 in Mathematics (Ist). In how many ways can these prizes be awarded to 20 students.
10. In an election three districts are to be canvassed by 2, 3 and 5 men respectively . If 10 men volunteer, then find the number of ways they can be allotted to the different districts.
11. A train time-table must be compiled for various days of the week so that two trains a day depart for three days, one train a day for two days and three trains a day for two days. Assuming all trains are identical how many different time-tables can be compiled?
12. In how many ways can 3 persons stay in 5 hotels? In how many of these each person stays in a different hotel.

13. 'n' different toys have to be distributed among 'n' children. Find the total number of ways in which these toys can be distributed so that exactly one child gets no toy.

14. Find the number of ways in which 7 different books can be given to 5 students if each can receive none, one or more books.

15. There are $(p + q)$ different books on different topics in Mathematics, where $p \neq q$. If L = The number of ways in which these books are distributed between two students X and Y such that X get p books and Y gets q books.
M = The number of ways in which these books are distributed between two students X and Y such that one of them gets p books and another gets q books.
N = The number of ways in which these books are divided into two groups of p books and q books.
Then prove that $2L = M = 2N$.

EXERCISE - 6

1. Find the number of ways to select 10 balls from an unlimited number of red, white, blue and green balls.

2. Find the number of ordered triples of positive integers which are solutions of the equation $x + y + z = 100$.

3. Find the number of integral solutions of $x_1 + x_2 + x_3 = 0$, with $x_i \geq -5$.

4. Find the number of integral solutions for the equation $x + y + z + t = 20$, where x, y, z, t are all ≥ -1.

5. Find the number of integral solutions of $a + b + c + d + e = 22$, subject to $a \geq -3$, $b \geq 1$, $c, d, e \geq 0$.

6. If a, b, c are three natural numbers in AP and $a + b + c = 21$ then find the possible number of values of the ordered triplet (a, b, c).

7. If a, b, c, d are odd natural numbers such that $a + b + c + d = 20$ then then find the number of values of the ordered quadruplet (a, b, c, d).

8. Find the number of non-negative integral solution of the equation, $x + y + 3z = 33$.

9. Find the number of integral solutions of the equation $3x + y + z = 27$, where x, y, $z > 0$.

10. If a, b, c are positive integers such that $a + b + c \leq 8$ then find the number of possible values of the ordered triplet (a, b, c).

11. Find the number of non-negative integral solution of the inequation
$x + y + z + w \leq 7$.

12. Find the number of non-negative even integral solutions of $x+y+z=100$.

13. Find the number of non-negative integral solutions of $x+y+z+w \le 23$.

14. Find the total number of positive integral solution of $15 < x_1 + x_2 + x_3 \le 20$.

15. Find the number of non-negative integer solutions of $(a+b+c)(p+q+r+s)=21$.

16. There are three piles of identical red, blue and green balls and each pile contains at least 10 balls. Find the number of ways of selecting 10 balls if twice as many red balls as green balls are to be selected.

17. Find the number of terms in a complete homogeneous expression of degree n in x, y and z.

18. In how many different ways can 3 persons A, B and C having 6 one rupee coins, 7 one rupee coins and 8 one rupee coins respectively donate 10 one rupee coins collectively.

(i) If each one giving at least one coin

(ii) If each one can give '0' or more coin.

Also answer the above questions for 15 rupees donation.

19. In an examination, the maximum marks for each of the three papers are 50 each. Maximum marks for the fourth paper is 100. Find the number of ways in which a candidate can score 60% marks on the whole.

20. Between two junction stations A and B, there are 12 intermediate stations. Find the number of ways in which a train can be made to stop at 4 of these stations so that no two of these halting stations are consecutive.

21. The minimum marks required for clearing a certain screening paper is 210 out of 300. The screening paper consists of '3' sections each of Physics, Chemistry and Mathematics Each section has 100 as maximum marks. Assuming there is no negative marking and marks obtained in each section are integers, find the number of ways in which a student can qualify the examination (Assuming no subjectwise cut-off limit).

22. Find the number of ways in which the sum of upper faces of four distinct dices can be six.

23. How many integers > 100 and $< 10^6$ have the digital sum $= 5$?

24. In how many ways can 14 be scored by tossing a fair die thrice?

25. Find the number of positive integral solutions of $abc = 30$.

26. Find The number of positive integral solutions of the equation $x_1 x_2 x_3 x_4 x_5 = 1050$.

27. Let y be an element of the set $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$ and x_1, x_2, x_3 be positive integers such that $x_1 x_2 x_3 = y$, then find the number of positive integral solutions of $x_1 x_2 x_3 = y$.

28. Let $x_i \in \mathbb{Z}$ such that $|x_1 x_2 \ldots x_{10}| = 1080000$. Find number of solutions.

29. Let $x_i \in \mathbb{Z}$ such that $x_1 x_2 \ldots x_{10} = 180000$. Find Number of solutions.

30. Let $x_i \in \mathbb{Z}$, such that $|x_1| + |x_2| + \cdots + |x_{10}| = 100$. Find number of solutions.

EXERCISE - 7

1. Find the numbers from 1 to 100 which are neither divisible by 2 nor by 3 nor by 7.

2. Find the number of numbers, from amongst 1, 2, 3, …, 500, which are divisible by none of 2, 3, 5.

3. Find the number of 3 element subsets of the set $\{1, 2, \ldots, 10\}$, in which the least element is 3 or the greatest element is 7.

4. Find the number of n digit numbers, which contain the digits 2 and 7, but not the digits 0, 1, 8, 9.

5. How many integers from 1 through 999 do not have any repeated digits?

6. Find the number of natural numbers less than or equal to 10^8 which are neither perfect squares, nor perfect cubes, nor perfect fifth powers.

7. In a certain state, license plates consist of from zero to three letters followed by from zero to four digits, with the provision, however, that a blank plate is not allowed.
(i) How many different license plates can the state produce?
(ii) Suppose 85 letter combinations are not allowed because of their potential for giving offense. How many different license plates can the state produce?

8. If the number of ways of selecting K coupons one by one out of an unlimited number of coupons bearing the letters A, T, M so that they cannot be used to spell the word MAT is 93, then find K.

9. How many positive integers divide 10^{40} or 20^{30}?

10. Find the number of permutations of letters a, b, c, d, e, f, g taken all together if neither 'beg' nor 'cad' pattern appear.

11. Find the number of permutations of the letters of the word HINDUSTAN such that neither the pattern 'HIN' nor 'DUS' nor 'TAN' appears.

12. Find the number of permutations of the 8 letters AABBCCDD, taken all at a time, such that no two adjacent letters are alike.

13. Find the number of non-negative integer solutions of $x_1 + x_2 + x_3 = 15$, subject to $x_1 \le 5$, $x_2 \le 6$, and $x_3 \le 7$.

14. According to the Gregorian calendar, a leap year is defined as a year n such that (i) n divides 4 but not 100; or (ii) n divides 400.
Find the number of leap years from the year 1000 to the year 3000, inclusive.

15. Find the number of onto functions from a set containing 6 elements to a set containing 3 elements.

16. How many 6-digit numbers contain exactly three different digits?

17. Let D_n be the nth derangement number. Prove that

(i) $D_n = (n-1)(D_{n-1} + D_{n-2}), n > 2;$

(ii) $\lim\limits_{n\to\infty} \dfrac{D_n}{n!} = \dfrac{1}{e}$

18. Show that n letters in n corresponding envelopes can be put such that none of the letters goes to the correct envelop is $n!\left(1-\dfrac{1}{1!}+\dfrac{1}{2!}-\dfrac{1}{3!}+\cdots+\dfrac{(-1)^n}{n!}\right)$ ways.

19. Five pairs of hand gloves of different colours are to be distributed to each of five people. Each person must get a left glove and a right glove. Find the number of distributions so that,exactly one person gets a proper pair.

20. Prove (combinatorially) that $\sum\limits_{r=1}^{n} r!r = (n+1)!-1.$

21. In maths paper there is a question on 'Match the column' in which column A contains 6 entries and each entry of column A corresponds to exactly one of the 6 entries given in column B written randomly. 2 marks are awarded for each correct matching and 1 mark is deducted from each incorrect matching. A student having no subjective knowledge decides to match all the 6 entries randomly. Find the number of ways in which he can answer, to get atleast 25% marks in this question.

22. Ten parabolas are drawn in a plane. Any two parabola intersect in four real, and distinct, points. No three parabola are concurrent. Find the total number of disjoint regions of the plane.

23. In how many ways can a 12 step staircase be climbed taking 1 step or 2 steps at a time?

24. A coin is tossed 10 times. Find the number of outcomes in which 2 heads are not successive.

25. Find the number of ways to pave a 1×7 rectangle by 1×1, 1×2, 1×3 tiles, if tiles of the same size are indistinguishable.

EXERCISE - 8

1. Find the number of ways in which n distinct objects can be put into two different boxes so that no box remains empty.

2. Find the number of ways in which n distinct objects can be kept into two identical boxes so that no box remains empty.

3. 10 identical balls are to be distributed in 5 different boxes kept in a row and labeled A, B, C, D and E. Find the number of ways in which the balls can be distributed in the boxes if no two adjacent boxes remain empty.

4. Find the number of distributions of 6 distinguishable objects in three distinguishable boxes such that each box contains an object.

5. Find the number of ways in which 12 identical coins can be distributed in 6 different purses, if not more than 3 and not less than 1 coin goes in each purse.

6. Find the number of ways in which 30 coins of one rupee each be given to six persons so that none of them receive less than 4 rupees.

7. Find the number of ways of wearing 8 distinguishable rings on 5 fingers of right hand.

8. 15 identical balls have to be put in 5 different boxes. Each box can contain any number of balls. Find total number of ways of putting the balls into box so that each box contains atleast 2 balls.

9. In how many ways can 3 blue, 4 red and 2 green balls be distributed in 4 distinct boxes? (Balls of the same colour are identical)

10. How many different ways can 15 Candy bars be distributed to Tanya, Manya, Shashwat and Adwik, if Tanya cannot have more than 5 candy bars and Manya must have at least two. Assume all Candy bars to be alike.

11. In how many ways, 16 identical coins can be distributed to 4 beggars when
(i) any beggar may get any number of coins?
(ii) every beggar gets atleast one coin?
(iii) every beggar gets atleast two coins?
(iv) every beggar gets atleast three coins?

12. Prove that the number of n digit quaternary sequences (whose digits are 0, 1, 2, and 3), in which each of the digits 2 and 3 appear atleast once, is $4^n - 2 \cdot 3^n + 2^n$.

13. Shivank has 15 ping-pong balls each uniquely numbered from 1 to 15. He also has a red box, a blue box, and a green box.
(i) How many ways can Shivank place the 15 distinct balls into the three boxes so that no box is empty?
(ii) Suppose now that Shivank has placed 5 ping-pong balls in each box. How many ways can he choose 5 balls from the three boxes so that he chooses at least one from each box?

14. In how many ways we can place 9 different balls in 3 different boxes such that in every box at least 2 balls are placed?

15. In how many ways can we put 12 different balls in three different boxes such that first box contains exactly 5 balls.

16. Five balls are to be placed in three boxes. Each can hold all the five balls. In how many different ways can we place the balls so that no box remains empty, if
(i) balls and boxes are all different?
(ii) balls are identical but boxes are different?
(iii) balls are different but boxes are identical?
(iv) balls as well as boxes are identical?

17. A man has 3 daughters. He wants to bequeath his fortune of 101 identical gold coins to them such that no daughter gets more share than the combined share of the other two. Find the number of ways of accomplishing this task.

18. There are six gates in an auditorium. suppose 20 delegates arrive. How many records could be there?

19. A man has to move 9 steps. He can move in 4 directions: left, light, forward, backward.

(i) In how many ways he can take 9 steps in 4 direction?

(ii) In how many ways he can move 9 steps if he has to take atleast one step in every direction.

(iii) In how many ways he can move 9 steps such that he finish his journey one step away (either left or right or forward or backward) from the starting position.

EXERCISE - 9

1. Prove that, among any 52 integers, two can always be found, such that the difference of their squares, is divisible by 100.

2. Show that, for any set of 10 points, chosen within a square, whose side is 3 units, there are two points, in the set, whose distance is at most $\sqrt{2}$.

3. There are 7 persons in a group, show that, some two of them, have the same number of acquaintances among them.

4. 51 points are scattered inside a square, with a side of one metre. Prove that some set of three of these points can be covered by a square, with side 20 cm.

5. Let $1 < a_1 < a_2 < a_3 < \ldots < a_{51} < 142$. Prove that, among the 50 consecutive differences $(a_i - a_{i-1})$ where i =1, 2, 3, …, 51, some value, must occur at least twelve times.

6. You are given 10 segments, such that, every segment is larger than 1 cm but shorter than 55 cm. Prove that, you can select three sides of a triangle, among these segments.

7. There are 9 cells in a 3×3 square. When these cells are filled by numbers 1, 2, 3 only, prove that, of the eight sums obtained, at least, two sums are equal.

8. Let there be given 9 lattice points in a 3-D Euclidean space. Show that, there is a lattice point, on the interior of one of the line segments joining two of these nine points.

9. Consider seven distinct positive integers, not exceeding 1706. Prove that, there are three of them, say a, b, c such that, $a < b + c < 4a$.

10. One million pine trees grow in a forest. It is known that, no pine tree, has more than 60000 pine needles in it. Show that, two pine trees in the forest must have the same number of pine needles.

11. In a circle of radius 16, there are placed 650 points; Prove that there exists a ring (annulus) of inner radius 2 and outer radius 3, which contains not less than 10 of the given points.

12. On a rectangular table of dimensions 120″ by 150″, we set 14001 marbles of size 1″ by 1″. Prove that, no matter how these are arranged, one can place a cylindrical glass with diameter of 5″ over atleast 8 marbles.

13. Let A be the set of 19 distinct integers, chosen from the AP 1, 4, 7, 10, …, 100. Prove that, there should be two distinct integers in A, such that, their sum is 104.

14. If a line is coloured in 11 colours, show that, there exist two points, whose distance apart, is an integer, which have the same colour.

15. Show that, given 12 integers, there exists two of them whose difference is divisible by 11.

16. Given eleven triangles, show that, some three of them belong to the same type (such as equilateral, isosceles, etc.)

17. A is a subset of the AP 2, 7, 12, …, 152. Prove that, there are two distinct elements of A whose sum is 159. What can you conclude if A has only 14 elements?

18. Given three points, in the interior of a right angled triangle, show that, two of them are at a distance not greater than the maximum of the lengths of the sides containing the right angle.

19. There are 90 cards numbered 10 to 99. A card is drawn and the sum of the digits of the number in the card is noted; show that if 35 cards are drawn, then, there are some three cards, whose sum of the digits are identical.

20. If in a class of 15 students, the total of the marks in a subject is 600, then show that, there is a group of 3 students, the total of whose marks is at least 120.

21. Let $ABCD$ be a square of side 20. Let T_i (i = 1, 2, …, 2000) be points in the interior of the square, such that, no three points from the set $S = \{A, B, C, D\} \subset T_i \; \forall \; i = 1, 2, 3, \ldots, 2000$ are collinear, Prove that, at least one triangle, with the vertices in S has area less than $\frac{1}{10}$.

22. 5 points are plotted inside a circle. Prove that, there exist two points, which form an acute angle with the centre of the circle.

23. Let A denote a subset of $\{1, 11, 21, 31, \ldots, 551\}$ having the property that, no two elements of A, add up to 552. Prove that A cannot have more than 28 elements.

24. Prove that, there exist two powers 3, which differ by a multiple of 2005.

25. All the points in the plane are coloured, using three colours. Prove that, there exists a triangle with vertices, having the same colour, such that, either it is isosceles or its angles are in geometric progression.

Answer Keys

EXERCISE - 1

1. (a) 1296, (b) 360
2. 9(9!)
3. 240
4. 376
5. (a) 60, (b) 107
6. 286
7. 15
8. Time required $= \frac{15\times15\times15-1}{2}\times\frac{10}{60\times60}=\frac{1687}{360}$ hrs.

   ≈ 4 hrs. 41 min. 10 Seconds $> 4\frac{1}{2}$ hrs.
9. 720
10. 18
11. 64800
12. 505
13. 69760
14. 162
15. 3×4^4
16. 45×10^4
17. 216
18. 36
19. 108
20. 1620
21. 103
22. 154
23. 1020
24. $4\cdot7!$
25. $17\cdot8!$
26. 8!
27. 2^n
28. 91.
29. n^m-1
30. 6^n-3^n
31. 300
32. 300
33. 31
34. 15
35. 134055
36. 1769580
37. 6399960
38. 2239986
39. Except $5k+1$, for $k=0, 1, 2, ..., 199$, all numbers will be unmarked.
40. 180

EXERCISE - 2

1. (i) $n=5$ (ii) $n=6,7$
2. 8
3. ${}^{20}C_{10}$
4. (a) 20 (b) 21 (c) 10
5. ${}^{25}C_5, {}^{24}C_4$
6. 10
7. 226
8. 378
9. 16
10. 1512
11. 124
12. 292
13. 135
15. ${}^{20}C_{10}\cdot2^{10}$
16. (i) 243 (ii)1, 10, 40, 80, 80, 32

17. $p = {}^5C_4 \cdot {}^2C_1 = 10,\ q = {}^5C_2\,({}^2C_1)^3 = 80$
$r = {}^5C_0({}^2C_1)^5 = 32$
$\Rightarrow 2q = 5r,\ 8p = q$, and $2(p + r) > q$
18. 6
19. 1023
20. 126
21. $(p + 1)^n - 1$
22. $2^n - 2n - 2$
23. $(m + 1)\,2^n - 1$
24. 3150
25. 25
26. ${}^nC_2$
27. 37
28. 20
29. 9
30. 16
31. 126
32. 6
33. 72
34. 5
35. 945
36. n^m
37. 91
38. ${}^{n-1}C_2$
39. mk
40. 2^{2n}
41. 2^{n-1}
42. ${}^nC_2 \cdot 3^{n-2}$
43. $\dfrac{(n+1)(n+2)(2n+3)}{6}$
44. 23
45. $63\times121\times31 = 3^2\cdot7^1\cdot11^2\cdot31$
46. 84
47. 276
48. ${}^{11}C_6$
49. $\dfrac{(m+n-2)!}{(m-1)!(n-1)!}$
50. 5
51. Total number of different tickets = 30 and number of selection = ${}^{30}C_{10}$
52. ${}^{10}C_3$
57. 15
58. $2^9 - 1$
59. 560
60. 140

EXERCISE - 3

1. (a) 4
(b) 3
(c) 8
2. ${}^6P_3$
3. ${}^6P_3 \times {}^5P_3 \times {}^4P_3$
4. 50400
5. ${}^{10}C_6 \times {}^4C_3 \times 9!$
6. 900
7. 40
8. 30
9. $\dfrac{11!}{(2!)^3}, \dfrac{8!}{(2!)^2}\times12$
10. 8!4!
11. (a) 7!, (b) 6!, (c) 5!, (d) 6!2!
12. ${}^8C_4 \cdot 4!$
13. 719
14. 3600
15. 1800
16. Number of ways = $n + n^2 + \cdots + n^r$
17. $\dfrac{n^r(n^{n-r+1}-1)}{n-1}$ and $\dfrac{n^{r+1}-1}{n-1}$
18. 2
19. ${}^7C_2\,2^5$
20. 20
21. $2(n!)^2$
22. $2 \cdot 6! \cdot 6!$
23. 20
24. $\dfrac{10!}{2}$
25. $3n^2 - 2n$
26. $m(m - 1)(n - 5)^{m-2}$
27. 1440
28. 172800
29. 528
30. 1620
31. 43200
32. ${}^{10}C_3\times2\times7!$
33. 24
34. 185
35. 2454
36. 758
37. 917
38. 89
39. 236

40. (a) 213564, (b) 267th

41. 24678

42. (i) 72nd, (ii) 51342

43. 3840

44. 32

45. 8

46. $6(7!-4!)$

47. $8!$

48. $\frac{9!}{3}$

49. 36×55^3

50. ${}^{14}C_5$

EXERCISE - 4

1. $10!$

2. $20!,\ 2\cdot18!$

3. (a) 240 (b) 480

4. (a) $2\cdot18!$ (b) $19!-2\cdot18!$

5. (a) $2\cdot18!$ (b) $19!-2\cdot18!$

(c) $18!\,(1/2)\{19!-2\cdot18!\}$

6. (i) $(2n-2)!\times2$ (ii) $(2n-2)!$

7. ${}^{10}C_2\times2!\times{}^{10}C_8\times8!$

8. 288

9. $\frac{24}{25}$

10. 18

12. 3

13. 225

15. 30

16. $\frac{(n-1)!}{r!}$

EXERCISE - 5

1. $\frac{15!}{8!4!3!}$

2. $\frac{(8!)}{(3!)^2(2!)}$

3. $\frac{14!}{(2!)^5\cdot(3!)^2\cdot4!}$

4. $\frac{k}{3!}$

5. $\frac{16!}{4!\,5!\,7!}$

6. 25200

7. 2940

8. m^n-m

9. $20^3,\ 19^2$

10. $\frac{10!}{2!\,3!\,5!}$

11. 210

12. 125, 60

13. $n!\,{}^nC_2$

14. 5^7

15. $L={}^{p+q}C_p\cdot{}^qC_q,\ M={}^{p+q}C_p\cdot{}^qC_q\times2!,\ N={}^{p+q}C_p\cdot{}^qC_q\cdot\Rightarrow$
$L=M/2=N$
$\Rightarrow 2L=M=2N$

EXERCISE - 6

1. 286

2. 4851

3. ${}^{17}C_2$

4. ${}^{27}C_3$

5. ${}^{28}C_4$

6. 13

7. $\binom{11}{3}$

8. 210

9. 100

10. 56

11. 330

12. ${}^{52}C_2$

13. ${}^{27}C_4$

14. 685

15. ${}^5C_2\cdot{}^{10}C_3+{}^9C_2\cdot{}^6C_3+{}^{23}C_2\cdot{}^4C_1+{}^{24}C_3\cdot{}^3C_1$

16. The possibilities are (0, 10, 0), (2, 7, 1), (4, 4, 2) and (6, 1, 3), where (r, b, g) denotes the number of red, blue and green balls.

17. $\frac{(n+2)(n+1)}{2}$

18. (i) 35 (ii) 47 (iii) $\binom{8}{2}$

19. 110551

20. ${}^9C_4$

21. ${}^{93}C_3$

22. 10

23. 246

24. 15

25. 27

26. 1875

27. 64

28. $2^{10}\binom{15}{6}\binom{12}{3}\binom{13}{4}$

29. $2^{9}\binom{15}{6}\binom{12}{3}\binom{13}{4}$

30. $\sum_{r=0}^{9} 2^{10-r}\binom{10}{r}\binom{99}{9-r}$

EXERCISE - 7

1. 28
2. 134
3. 33
4. $6^n - 5^n - 5^n + 4^n$
5. 738
6. 99989526
7. (a) $(1+26+26^2+26^3)\cdot(1+10+10^2+10^3+10^4)-1$
(b) $(1+26+26^2+26^3-85)\cdot(1+10+10^2+10^3+10^4)-1$
8. 5
9. 24×13^4
10. 2301
11. $7!-5!-5!+3!$
12. 169194
13. 864
14. 10
15. 485
16. 540
17. ${}^9C_2\cdot 360+{}^9C_3\cdot 540=58320$
20. 5400
22. 191
23. 101
24. 233
25. 144
26. 44

EXERCISE - 8

1. 2^n-2
2. $2^{n-1}-1$
3. 771
4. 540
5. 141
6. 462
7. $\dfrac{12!}{4!}$
8. ${}^9C_5$
9. 7000
10. 440
11. (i) ${}^{19}C_3$ (ii) ${}^{15}C_3$ (iii) ${}^{11}C_3$ (iv) ${}^{7}C_3$
13. (a) $3^{15}-3\cdot 2^{15}+3$, (b) 2250
14. 11508
15. ${}^{12}C_5\cdot 2^7$
16. (i) 150 (ii) 6 (iii) 25 (iv) 2
17. 1275
18. $\dfrac{25!}{5!}$
19. (a) 4^9 (b) 186480 (c) $4\times({}^9C_4)^2=63504$