EXERCISE I

Q.1 In how many ways can clean & clouded (overcast) days occur in a week assuming that an entire day is either clean or clouded.

Q.2 Four visitors A, B, C & D arrive at a town which has 5 hotels. In how many ways can they disperse themselves among 5 hotels, if 4 hotels are used to accommodate them.

Q.3 If the letters of the word "VARUN" are written in all possible ways and then are arranged as in a dictionary, then the rank of the word VARUN is :
(A) 98 (B) 99 (C) 100 (D) 101

Q.4 How many natural numbers are there from 1 to 1000 which have none of their digits repeated.

Q.5 A man has 3 jackets, 10 shirts, and 5 pairs of slacks. If an outfit consists of a jacket, a shirt, and a pair of slacks, how many different outfits can the man make?

Q.6 There are 6 roads between A & B and 4 roads between B & C.
(i) In how many ways can one drive from A to C by way of B?
(ii) In how many ways can one drive from A to C and back to A, passing through B on both trips ?
(iii) In how many ways can one drive the circular trip described in (ii) without using the same road more than once.

Q.7 (i) How many car number plates can be made if each plate contains 2 different letters of English alphabet, followed by 3 different digits.
(ii) Solve the problem, if the first digit cannot be 0. (Do not simplify)

Q.8 (i) Find the number of four letter word that can be formed from the letters of the word HISTORY. (each letter to be used at most once)
(ii) How many of them contain only consonants?
(iii) How many of them begin & end in a consonant?
(iv) How many of them begin with a vowel?
(v) How many contain the letters Y?
(vi) How many begin with T & end in a vowel?
(vii) How many begin with T & also contain S?
(viii) How many contain both vowels?

Q.9 If repetitions are not permitted
(i) How many 3 digit numbers can be formed from the six digits 2, 3, 5, 6, 7 & 9 ?
(ii) How many of these are less than 400 ?
(iii) How many are even ?
(iv) How many are odd ?
(v) How many are multiples of 5 ?

Q.10 In how many ways can 5 letters be mailed if there are 3 different mailboxes available if each letter can be mailed in any mailbox.

Q.11 Every telephone number consists of 7 digits. How many telephone numbers are there which do not include any other digits but 2 , 3 , 5 & 7 ?

Q.12 (a) In how many ways can four passengers be accommodate in three railway carriages, if each carriage can accommodate any number of passengers.

(b) In how many ways four persons can be accommodated in 3 different chairs if each person can occupy only one chair.

Q.13 How many of the arrangements of the letter of the word "LOGARITHM" begin with a vowel and end with a consonant?

Q.14 Number of natural numbers between 100 and 1000 such that at least one of their digits is 7, is
(A) 225 (B) 243 (C) 252 (D) none

Q.15 How many four digit numbers are there which are divisible by 2 .

Q.16 In a telephone system four different letter P, R, S, T and the four digits 3, 5, 7, 8 are used. Find the maximum number of "telephone numbers" the system can have if each consists of a letter followed by a four-digit number in which the digit may be repeated.

Q.17 Find the number of 5 lettered palindromes which can be formed using the letters from the English alphabets.

Q.18 Number of ways in which 7 different colours in a rainbow can be arranged if green is always in the middle.

Q.19 Two cards are drawn one at a time & without replacement from a pack of 52 cards. Determine the number of ways in which the two cards can be drawn in a definite order.

Q.20 Numbers of words which can be formed using all the letters of the word "AKSHI", if each word begins with vowel or terminates in vowel .

Q.21 A letter lock consists of three rings each marked with 10 different letters. Find the number of ways in which it is possible to make an unsuccessful attempts to open the lock.

Q.22 How many 10 digit numbers can be made with odd digits so that no two consecutive digits are same.

Q.23 It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?

Q.24 If no two books are alike, in how many ways can 2 red, 3 green, and 4 blue books be arranged on a shelf so that all the books of the same colour are together?

ANSWERS

Q.1 128 Q.2 120 Q.3 C Q.4 738 Q.5 150 Q.6 (i) 24; (ii) 576; (iii) 360
Q.7 (i) 26 . 25 . 10 . 9 . 8 = 468000 ; (ii) 26 . 25 . 9 . 9 . 8 = 421200 Q.8 (i) 840 (ii) 120 (iii) 400 (iv) 240 (v) 480 (vi) 40 (vii) 60 (viii) 240 Q.9 (i) 120 (ii) 40 (iii) 40 (iv) 80 (v) 20
Q.10 243 ways Q.11 4^7 Q.12 (a) 3^4 ; (b) 24 Q.13 90720 Q.14 C Q.15 4500
Q.16 1024 Q.17 26^3 Q.18 720 Q.19 2652 Q.20 84 Q.21 999 Q.22 $5\cdot4^9$
Q.23 2880 Q.24 1728

EXERCISE II

Q.1 How many of the 900 three digit numbers have at least one even digit?
(A) 775 (B) 875 (C) 450 (D) 750

Q.2 The number of natual numbers from 1000 to 9999 (both inclusive) that do not have all 4 different digits is
(A) 4048 (B) 4464 (C) 4518 (D) 4536

Q.3 The number of different seven digit numbers that can be written using only three digits 1, 2 & 3 under the condition that the digit 2 occurs exactly twice in each number is :
(A) 672 (B) 640 (C) 512 (D) none

Q.4 Out of seven consonants and four vowels, the number of words of six letters, formed by taking four consonants and two vowels is (Assume that each ordered group of letter is a word):
(A) 210 (B) 462 (C) 151200 (D) 332640

Q.5 All possible three digits even numbers which can be formed with the condition that if 5 is one of the digit, then 7 is the next digit is :
(A) 5 (B) 325 (C) 345 (D) 365

Q.6 The number of positive integral 'x' satisfying the equation $1! + 2! + 3! + ... + (x!) = (N)^2$ for some natural N, is :
(A) 0 (B) one (C) two (D) infinite

Q.7 The number of six digit numbers that can be formed from the digits 1, 2, 3, 4, 5, 6 & 7 so that digits do not repeat and the terminal digits are even is :
(A) 144 (B) 72 (C) 288 (D) 720

Q.8 A new flag is to be designed with six vertical strips using some or all of the colours yellow, green, blue and red. Then, the number of ways this can be done such that no two adjacent strips have the same colour is
(A) 12×81 (B) 16×192 (C) 20×125 (D) 24×216

Q.9 In how many ways can 5 colours be selected out of 8 different colours including red, blue, and green
(a) if blue and green are always to be included,
(b) if red is always excluded,
(c) if red and blue are always included but green excluded?

Q.10 A 5 digit number divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 & 5 without repetition. The total number of ways this can be done is :

(A) 3125 (B) 600 (C) 240 (D) 216

Q.11 Number of 9 digits numbers divisible by nine using the digits from 0 to 9 if each digit is used atmost once is K . 8 ! , then K has the value equal to ______ .

Q.12 Number of natural numbers less than 1000 and divisible by 5 can be formed with the ten digits, each digit not occuring more than once in each number is ______ .

ANSWERS

Q.1	A	Q.2	B	Q.3	A	Q.4	C	Q.5	D	Q.6	C	Q.7	D
Q.8	A	Q.9	(a) 20, (b) 21, (c) 10	Q.10	D	Q.11	17	Q.12	154				

EXERCISE III

Q.1 Find the number of ways in which letters of the word VALEDICTORY be arranged so that the vowels may never be separated.

Q.2 How many numbers between 400 and 1000 (both exclusive) can be made with the digits 2,3,4,5,6,0 if
(a) repetition of digits not allowed. (b) repetition of digits is allowed.

Q.3 Number of odd integers between 1000 and 8000 which have none of their digits repeated, is
(A) 1014 (B) 810 (C) 690 (D) 1736

Q.4 If ${}^{20}P_r = 13\times {}^{20}P_{r-1}$, then the value of r is ___________.

Q.5 The number of ways in which 5 different books can be distributed among 10 people if each person can get at most one book is :
(A) 252 (B) 10^5 (C) 5^{10} (D) ${}^{10}C_5.5!$

Q.6 The product of all odd positive integers less than 10000, is
(A) $\frac{(10000)!}{(5000!)^2}$ (B) $\frac{(10000)!}{2^{5000}}$ (C) $\frac{(9999)!}{2^{5000}}$ (D) $\frac{(10000)!}{2^{5000}\cdot(5000)!}$

Q.7 The 9 horizontal and 9 vertical lines on an 8 × 8 chessboard form 'r' rectangles and 's' squares. The ratio $\frac{s}{r}$ in its lowest terms is
(A) $\frac{1}{6}$ (B) $\frac{17}{108}$ (C) $\frac{4}{27}$ (D) none

Q.8 There are 720 permutations of the digits 1, 2, 3, 4, 5, 6. Suppose these permutations are arranged from smallest to largest numerical values, beginning from 1 2 3 4 5 6 and ending with 6 5 4 3 2 1.
(a) What number falls on the 124th position?
(b) What is the position of the number 321546?

Q.9 A student has to answer 10 out of 13 questions in an examination . The number of ways in which he can answer if he must answer atleast 3 of the first five questions is :
(A) 276 (B) 267 (C) 80 (D) 1200

Q.10 The number of three digit numbers having only two consecutive digits identical is
(A) 153 (B) 162 (C) 180 (D) 161

Q.11 Number of 3 digit numbers in which the digit at hundreath's place is greater than the other two digit is
(A) 285 (B) 281 (C) 240 (D) 204

Q.12 Number of permutations of 1, 2, 3, 4, 5, 6, 7, 8 and 9 taken all at a time, such that the digit
1 appearing somewhere to the left of 2
3 appearing somewhere to the left of 4 and
5 appearing somewhere to the left of 6, is
(*e.g.* 815723946 would be one such permutation)
(A) 9 · 7! (B) 8! (C) 5! · 4! (D) 8! · 4!

ANSWERS

Q.1	967680	Q.2 (a) 60 ; (b) 107	Q.3	D	Q.4	8	Q.5	D	Q.6	D		
Q.7	B	Q.8 (a) 213564, (b) 267th	Q.9	A	Q.10	B	Q.11	A	Q.12	A		

EXERCISE IV

Q.1 The interior angles of a regular polygon measure 150º each . The number of diagonals of the polygon is
(A) 35 (B) 44 (C) 54 (D) 78

Q.2 Number of different natural numbers which are smaller than two hundred million & using only the digits 1 or 2 is :
(A) $(3) . 2^8 - 2$ (B) $(3) . 2^8 - 1$ (C) $2 (2^9 - 1)$ (D) none

Q.3 5 Indian & 5 American couples meet at a party & shake hands . If no wife shakes hands with her own husband & no Indian wife shakes hands with a male, then the number of hand shakes that takes place in the party is :
(A) 95 (B) 110 (C) 135 (D) 150

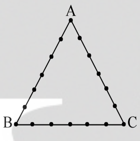
Q.4 The number of n digit numbers which consists of the digits 1 & 2 only if each digit is to be used atleast once, is equal to 510 then n is equal to:
(A) 7 (B) 8 (C) 9 (D) 10

Q.5 Number of six digit numbers which have 3 digits even & 3 digits odd, if each digit is to be used atmost once is ______ .

Q.6 The tamer of wild animals has to bring one by one 5 lions & 4 tigers to the circus arena. The number of ways this can be done if no two tigers immediately follow each other is ______.

Q.7 18 points are indicated on the perimeter of a triangle ABC (see figure).

How many triangles are there with vertices at these points?



(A) 331 (B) 408 (C) 710 (D) 711

Q.8 A committee of 5 is to be chosen from a group of 9 people. Number of ways in which it can be formed if two particular persons either serve together or not at all and two other particular persons refuse to serve with each other, is
(A) 41 (B) 36 (C) 47 (D) 76

Q.9 A question paper on mathematics consists of twelve questions divided into three parts A, B and C, each containing four questions . In how many ways can an examinee answer five questions, selecting atleast one from each part .
(A) 624 (B) 208 (C) 2304 (D) none

Q.10 If m denotes the number of 5 digit numbers if each successive digits are in their descending order of magnitude and n is the corresponding figure, when the digits are in their ascending order of magnitude then $(m - n)$ has the value
(A) ${}^{10}C_4$ (B) ${}^{9}C_5$ (C) ${}^{10}C_3$ (D) ${}^{9}C_3$

ANSWERS

Q.1	C	Q.2	A	Q.3	C	Q.4	C
Q.5	64800	Q.6	43200	Q.7	D	Q.8	A
Q.9	A	Q.10	B				

EXERCISE V

Q.1 There are m points on a straight line AB & n points on the line AC none of them being the point A. Triangles are formed with these points as vertices, when
(i) A is excluded (ii) A is included. The ratio of number of triangles in the two cases is:
(A) $\frac{m+n-2}{m+n}$ (B) $\frac{m+n-2}{m+n-1}$ (C) $\frac{m+n-2}{m+n+2}$ (D) $\frac{m(n-1)}{(m+1)(n+1)}$

Q.2 Number of ways in which 7 green bottles and 8 blue bottles can be arranged in a row if exactly 1 pair of green bottles is side by side, is (Assume all bottles to be alike except for the colour).
(A) 84 (B) 360 (C) 504 (D) 514

Q.3 In a certain algebraical exercise book there are 4 examples on arithmetical progressions, 5 examples on permutation – combination and 6 examples on binomial theorem . Number of ways a teacher can select for his pupils atleast one but not more than 2 examples from each of these sets, is ______ .

Q.4 The kindergarten teacher has 25 kids in her class . She takes 5 of them at a time, to zoological garden as often as she can, without taking the same 5 kids more than once. Find the number of visits, the teacher makes to the garden and also the number of of visits every kid makes.

Q.5 There are n persons and m monkeys (m > n). Number of ways in which each person become the owner of one monkey is
(A) n^m (B) m^n (C) ${}^mP_n$ (D) mn

Q.6 Seven different coins are to be divided amongst three persons . If no two of the persons receive the same number of coins but each receives atleast one coin & none is left over, then the number of ways in which the division may be made is :
(A) 420 (B) 630 (C) 710 (D) none

Q.7 Let there be 9 fixed points on the circumference of a circle . Each of these points is joined to every one of the remaining 8 points by a straight line and the points are so positioned on the circumference that atmost 2 straight lines meet in any interior point of the circle . The number of such interior intersection points is :
(A) 126 (B) 351 (C) 756 (D) none of these

Q.8 The number of 5 digit numbers such that the sum of their digits is even is :
(A) 50000 (B) 45000 (C) 60000 (D) none

Q.9 A forecast is to be made of the results of five cricket matches, each of which can be win, a draw or a loss for Indian team. Find
(i) the number of different possible forecasts
(ii) the number of forecasts containing 0, 1, 2, 3, 4 and 5 errors respectively

Q.10 The number of ways in which 8 distinguishable apples can be distributed among 3 boys such that every boy should get atleast 1 apple & atmost 4 apples is $K \cdot {}^7P_3$ where K has the value equal to
(A) 14 (B) 66 (C) 44 (D) 22

Q.11 A women has 11 close friends. Find the number of ways in which she can invite 5 of them to dinner, if two particular of them are not on speaking terms & will not attend together.

Q.12 A rack has 5 different pairs of shoes. The number of ways in which 4 shoes can be chosen from it so that there will be no complete pair is
(A) 1920 (B) 200 (C) 110 (D) 80

Q.13 An old man while dialing a 7 digit telephone number remembers that the first four digits consists of one 1's, one 2's and two 3's. He also remembers that the fifth digit is either a 4 or 5 while has no memorising of the sixth digit, he remembers that the seventh digit is 9 minus the sixth digit. Maximum number of distinct trials he has to try to make sure that he dials the correct telephone number, is
(A) 360 (B) 240 (C) 216 (D) none

Q.14 Number of different ways in which 8 different books can be distributed among 3 students, if each student receives atleast 2 books is ______.

Q.15 There are 10 seats in a double decker bus, 6 in the lower deck and 4 on the upper deck. Ten passengers board the bus, of them 3 refuse to go to the upper deck and 2 insist on going up. The number of ways in which the passengers can be accommodated is _____. (Assume all seats to be duly numbered)

Q.16 In how many different ways a grandfather along with two of his grandsons and four grand daughters can be seated in a line for a photograph so that he is always in the middle and the two grandsons are never adjacent to each other.

ANSWERS

Q.1	A	Q.2	C	Q.3	3150	Q.4	${}^{25}C_5$; ${}^{24}C_4$	Q.5	C	Q.6	B
Q.7	A	Q.8	B	Q.9	243 ; 1, 10, 40, 80, 80, 32			Q.10	D	Q.11	378
Q.12	D	Q.13	B	Q.14	2940	Q.15	${}^{4}C_2 . 2! \, {}^{6}C_3 . 3! \, 5!$ or 172800			Q.16	528

EXERCISE VI

Q.1 Find the number of permutations of the "AUROBIND" in which vowels appear in an alphabetical order.

Q.2 The greatest possible number of points of intersection of 9 different straight lines & 9 different circles in a plane is:
(A) 117 (B) 153 (C) 270 (D) none

Q.3 If as many more words as possible be formed out of the letters of the word "DOGMATIC" then the number of words in which the relative order of vowels and consonants remain unchanged is ______ .

Q.4 Number of ways in which 7 people can occupy six seats, 3 seats on each side in a first class railway compartment if two specified persons are to be always included and occupy adjacent seats on the same side, is $(5\,!) \cdot k$ then k has the value equal to :
(A) 2 (B) 4 (C) 8 (D) none

Q.5 Number of ways in which 9 different toys be distributed among 4 children belonging to different age groups in such a way that distribution among the 3 elder children is even and the youngest one is to receive one toy more, is :
(A) $\frac{(5!)^2}{8}$ (B) $\frac{9!}{2}$ (C) $\frac{9!}{3!(2!)^3}$ (D) none

Q.6 In an election three districts are to be canvassed by 2, 3 & 5 men respectively . If 10 men volunteer, the number of ways they can be alloted to the different districts is :
(A) $\frac{10\,!}{2\,!\,3\,!\,5\,!}$ (B) $\frac{10\,!}{2\,!\,5\,!}$ (C) $\frac{10\,!}{(2\,!)^2\;5\,!}$ (D) $\frac{10\,!}{(2\,!)^2\;3\,!\,5\,!}$

Q.7 Let P_n denotes the number of ways in which three people can be selected out of 'n' people sitting in a row, if no two of them are consecutive. If, $P_{n+1} - P_n = 15$ then the value of 'n' is :
(A) 7 (B) 8 (C) 9 (D) 10

Q.8 Number of ways in which 8 people can be arranged in a line if A and B must be next each other and C must be somewhere behind D, is equal to
(A) 10080 (B) 5040 (C) 5050 (D) 10100

Q.9 A has 3 maps and B has 9 maps. All the 12 maps being distinct. Determine the number of ways in which they can exchange their maps if each keeps his initial number of maps.

Q.10 Number of three digit number with atleast one 3 and at least one 2 is
(A) 58 (B) 56 (C) 54 (D) 52

Paragraph for question nos. 11 to 13

Consider the word W = MISSISSIPPI

Q.11 If N denotes the number of different selections of 5 letters from the word W = MISSISSIPPI then N belongs to the set

(A) {15, 16, 17, 18, 19} (B) {20, 21, 22, 23, 24}
(C) {25, 26, 27, 28, 29} (D) {30, 31, 32, 33, 34}

Q.12 Number of ways in which the letters of the word W can be arranged if atleast one vowel is separated from rest of the vowels

(A) $\frac{8!\cdot 161}{4!\cdot 4!\cdot 2!}$ (B) $\frac{8!\cdot 161}{4\cdot 4!\cdot 2!}$ (C) $\frac{8!\cdot 161}{4!\cdot 2!}$ (D) $\frac{8!}{4!\cdot 2!}\cdot\frac{165}{4!}$

Q.13 If the number of arrangements of the letters of the word W if all the S's and P's are separated is $(K)\left(\frac{10!}{4!\cdot 4!}\right)$ then K equals

(A) $\frac{6}{5}$ (B) 1 (C) $\frac{4}{3}$ (D) $\frac{3}{2}$

Paragraph for Question Nos. 14 to 16

16 players $P_1, P_2, P_3, ...P_{16}$ take part in a tennis tournament. Lower suffix player is better than any higher suffix player. These players are to be divided into 4 groups each comprising of 4 players and the best from each group is selected for semifinals.

Q.14 Number of ways in which 16 players can be divided into four equal groups, is

(A) $\frac{35}{27}\prod_{r=1}^{8}(2r-1)$ (B) $\frac{35}{24}\prod_{r=1}^{8}(2r-1)$ (C) $\frac{35}{52}\prod_{r=1}^{8}(2r-1)$ (D) $\frac{35}{6}\prod_{r=1}^{8}(2r-1)$

Q.15 Number of ways in which they can be divided into 4 equal groups if the players P_1, P_2, P_3 and P_4 are in different groups, is :

(A) $\frac{(11)!}{36}$ (B) $\frac{(11)!}{72}$ (C) $\frac{(11)!}{108}$ (D) $\frac{(11)!}{216}$

Q.16 Number of ways in which these 16 players can be divided into four equal groups, such that when the best player is selected from each group, P_6 is one among them, is $(k)\frac{12!}{(4!)^3}$. The value of k is :

(A) 36 (B) 24 (C) 18 (D) 20

ANSWERS

Q.1	${}^8C_4 \cdot 4!$	Q.2	C	Q.3	719	Q.4	C	Q.5	C
Q.6	A	Q.7	B	Q.8	B				
Q.9	${}^{12}C_3 - 1 = 219$	or	${}^3C_1\cdot{}^9C_1 + {}^3C_2\cdot{}^9C_2 + {}^3C_3\cdot{}^9C_3$			Q.10	D	Q.11	C
Q.12	B	Q.13	B	Q.14	A	Q.15	C	Q.16	D

EXERCISE VII

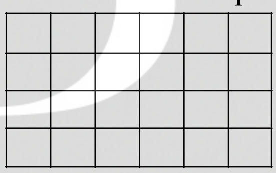
Q.1 There are 10 red balls of different shades & 9 green balls of identical shades. Then the number of arranging them in a row so that no two green balls are together is
(A) $(10\,!)\,.\,{}^{11}P_9$ (B) $(10\,!)\,.\,{}^{11}C_9$ (C) $10\,!$ (D) $10\,!\ 9\,!$

Q.2 Number of ways in which n distinct objects can be kept into two identical boxes so that no box remains empty, is ______ .

Q.3 A shelf contains 20 different books of which 4 are in single volume and the others form sets of 8, 5 and 3 volumes respectively. Number of ways in which the books may be arranged on the shelf, if the volumes of each set are together and in their due order is
(A) $\dfrac{20!}{8!\,5!\,3!}$ (B) 7! (C) 8! (D) 7 . 8!

Q.4 If all the letters of the word "QUEUE" are arranged in all possible manner as they are in a dictionary, then the rank of the word QUEUE is :
(A) 15^{th} (B) 16^{th} (C) 17^{th} (D) 18^{th}

Q.5 Number of rectangles in the grid shown which are not squares is



(A) 160 (B) 162 (C) 170 (D) 185

Q.6 All the five digit numbers in which each successive digit exceeds its predecessor are arranged in the increasing order of their magnitude. The 97^{th} number in the list does not contain the digit
(A) 4 (B) 5 (C) 7 (D) 8

Q.7 The number of combination of 16 things, 8 of which are alike and the rest different, taken 8 at a time is _____.

Q.8 The number of different ways in which five 'dashes' and eight 'dots' can be arranged, using only seven of these 13 'dashes' & 'dots' is :
(A) 1287 (B) 119 (C) 120 (D) 1235520

Q.9 There are n identical red balls & m identical green balls . The number of different linear arrangements consisting of "n red balls but not necessarily all the green balls" is ${}^{x}C_{y}$ then
(A) $x = m + n\,,\ y = m$ (B) $x = m + n + 1\,,\ y = m$
(C) $x = m + n + 1\,,\ y = m + 1$ (D) $x = m + n\,,\ y = n$

Q.10 Consider a determinant of order 3 all whose entries are either 0 or 1. Five of these entries are 1 and four of them are '0'. Also $a_{ij} = a_{ji}\ \forall\ 1 \le i, j \le 3$. Find the number of such determinants.

Q.11 How many different arrangements are possible with the term $a^2b^4c^5$ written at full length.

Q.12 Find the number of 4 digit numbers starting with 1 and having exactly two identical digits.

Q.13 Number of ways in which 5 A's and 6 B's can be arranged in a row which reads the same backwards and forwards, is

ANSWERS

Q.1	B	Q.2	$2^{n-1} - 1$	Q.3	C	Q.4	C	Q.5	A	Q.6	B
Q.7	256	Q.8	C	Q.9	B	Q.10	12	Q.11	6930		
Q.12	432	Q.13	10								

EXERCISE VIII

Q.1 Number of different words that can be formed using all the letters of the word "DEEPMALA" if two vowels are together and the other two are also together but separated from the first two is :
(A) 960 (B) 1200 (C) 2160 (D) 1440

Q.2 The number of ways in which 10 boys can take positions about a round table if two particular boys must not be seated side by side is :
(A) 10 (9) ! (B) 9 (8) ! (C) 7 (8) ! (D) none

Q.3 In a unique hockey series between India & Pakistan, they decide to play on till a team wins 5 matches . The number of ways in which the series can be won by India, if no match ends in a draw is :
(A) 126 (B) 252 (C) 225 (D) none

Q.4 Number of cyphers at the end of ${}^{2002}C_{1001}$ is
(A) 0 (B) 1 (C) 2 (D) 200

Q.5 Three vertices of a convex *n* sided polygon are selected. If the number of triangles that can be constructed such that none of the sides of the triangle is also the side of the polygon is 30, then the polygon is a
(A) Heptagon (B) Octagon (C) Nonagon (D) Decagon

Q.6 A gentleman invites a party of $m + n$ $(m \neq n)$ friends to a dinner & places m at one table T_1 and n at another table T_2 , the table being round . If not all people shall have the same neighbour in any two arrangement, then the number of ways in which he can arrange the guests, is
(A) $\frac{(m+n)!}{4\ mn}$ (B) $\frac{1}{2}\ \frac{(m+n)!}{mn}$ (C) $2\ \frac{(m+n)!}{mn}$ (D) none

Q.7 There are 12 guests at a dinner party . Supposing that the master and mistress of the house have fixed seats opposite one another, and that there are two specified guests who must always, be placed next to one another ; the number of ways in which the company can be placed, is:
(A) 20 . 10 ! (B) 22 . 10 ! (C) 44 . 10 ! (D) none

Q.8 Let P_n is the number of ways of selecting 3 people out of 'n' sitting in a row , if no two of them are consecutive and Q_n is the corresponding figure when they are in a circle . If $P_n - Q_n = 6$, then 'n' =
(A) 8 (B) 9 (C) 10 (D) 12

Q.9 Define a 'good word' as a sequence of letters that consists only of the letters A, B and C and in which A never immidiately followed by B, B is never immediately followed by C, and C is never immediately followed by A. If the number of n-letter good words are 384, find the value of n.

Q.10 Six married couple are sitting in a room. Find the number of ways in which 4 people can be selected so that
(a) they do not form a couple (b) they form exactly one couple
(c) they form at least one couple (d) they form atmost one couple

Q.11 Fifty college teachers are surveyed as to their possession of colour TV, VCR and tape recorder. Of them, 22 own colour TV, 15 own VCR and 14 own tape recorders. Nine of these college teachers own exactly two items out of colour TV, VCR and tape recorders ; and, one college teacher owns all three. how many of the 50 college teachers own none of three, colour TV, VCR or tape recorder?
(A) 4 (B) 9 (C) 10 (D) 11

Q.12 There are counters available in x different colours. The counters are all alike except for the colour. The total number of arrangements consisting of y counters, assuming sufficient number of counters of each colour, if no arrangement consists of all counters of the same colour is :
(A) $x^y - x$ (B) $x^y - y$ (C) $y^x - x$ (D) $y^x - y$

Q.13 There are $(p + q)$ different books on different topics in Mathematics. $(p \neq q)$
If L = The number of ways in which these books are distributed between two students X and Y such that X get p books and Y gets q books.

M = The number of ways in which these books are distributed between two students X and Y such that one of them gets p books and another gets q books.

N = The number of ways in which these books are divided into two groups of p books and q books then,
(A) L = M = N (B) L = 2M = 2N (C) 2L = M = 2N (D) L = M = 2N

The question given below contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct. Choose the correct alternative.

Q.14 Statement 1: The sum $^{40}C_0 \cdot {}^{60}C_{10} + {}^{40}C_1 \cdot {}^{60}C_9 + \ldots\ldots\ldots + {}^{40}C_{10} \cdot {}^{60}C_0$ equals $^{100}C_{10}$.
because
Statement 2: Number of ways of selecting 10 students out of 40 boys and 60 girls is $^{100}C_{10}$.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

MATCH THE COLUMN:

Q.15

	Column-I		Column-II
(A)	In a plane a set of 8 parallel lines intersect a set of n parallel lines, that goes in another direction, forming a total of 1260 parallelograms. The value of n is equal to	(P)	6
		(Q)	9
(B)	If $\frac{^{n-1}P_3}{^{n}P_4} = \frac{1}{9}$ then n is equal to	(R)	10
(C)	Number of ways in which 5 persons A, B, C, D and E can be seated on round table if A and D do not sit next to each other	(S)	12
(D)	Number of cyphers at the end of the number $^{50}P_{25}$		

ANSWERS

Q.1	D	Q.2	C	Q.3	A	Q.4	B	Q.5	C	Q.6	A	Q.7	A
Q.8	C	Q.9	8	Q.10	240, 240, 255, 480			Q.11	C	Q.12	A	Q.13	C
Q.14	A	Q.15	(A) R; (B) Q; (C) S; (D) P										

EXERCISE IX

Q.1 Product of all the even divisors of $N = 1000$, is
(A) $32 \cdot 10^2$ (B) $64 \cdot 2^{14}$ (C) $64 \cdot 10^{18}$ (D) $128 \cdot 10^6$

Q.2 Let m denote the number of ways in which 4 different books are distributed among 10 persons, each receiving none or one only & let n denote the number of ways of distribution if the books are all alike,then
(A) $m = 4n$ (B) $n = 4m$ (C) $m = 24n$ (D) none

Q.3 If the number of ways in which we can arrange n ladies & n gentlemen at a round table so that 2 ladies or 2 gentlemen may not sit next to one another is 86400, then the value of n equals
(A) 8 (B) 7 (C) 6 (D) 9

Q.4 There are six periods in each working day of a school. Number of ways in which 5 subjects can be arranged if each subject is allotted at least one period and no period remains vacant is
(A) 210 (B) 1800 (C) 360 (D) 120

Q.5 The number of all possible selections of one or more questions from 10 given questions, each equestion having an alternative is :
(A) 3^{10} (B) $2^{10} - 1$ (C) $3^{10} - 1$ (D) 2^{10}

Q.6 A team of 8 students goes on an excursion, in two cars, of which one can seat 5 and the other only 4. If internal arrangement inside the car does not matter then the number of ways in which they can travel, is
(A) 91 (B) 182 (C) 126 (D) 3920

Q.7 The number of divisors of the number 21600 is _____ and the sum of these divisors is ______.

Q.8 10 BOYS& 2 GIRLS are sitting in a row. The number of ways in which exactly 3 BOYS sit between 2 GIRLS is ______ .

Q.9 The number of ways of choosing a committee of 2 women & 3 men from 5 women & 6 men, if Mr. A refuses to serve on the committee if Mr. B is a member & Mr. B can only serve, if Miss C is the member of the committee, is :
(A) 60 (B) 84 (C) 124 (D) none

Q.10 Six persons A, B, C, D, E and F are to be seated at a circular table . The number of ways this can be done if A must have either B or C on his right and B must have either C or D on his right is :
(A) 36 (B) 12 (C) 24 (D) 18

Q.11 There are 2 identical white balls, 3 identical red balls and 4 green balls of different shades. The number of ways in which they can be arranged in a row so that atleast one ball is separated from the balls of the same colour, is :
(A) $6\,(7\,! - 4\,!)$ (B) $7\,(6\,! - 4\,!)$ (C) $8\,! - 5\,!$ (D) none

Q.12 Sameer has to make a telephone call to his friend Harish, Unfortunately he does not remember the 7 digit phone number. But he remembers that the first three digits are 635 or 674, the number is odd and there is exactly one 9 in the number. The maximum number of trials that Sameer has to make to be successful is
(A) 10,000 (B) 3402 (C) 3200 (D) 5000

Q.13 Six people are going to sit in a row on a bench. A and B are adjacent, C does not want to sit adjacent to D. E and F can sit anywhere. Number of ways in which these six people can be seated, is
(A) 200 (B) 144 (C) 120 (D) 56

Q.14 Given 11 points, of which 5 lie on one circle, other than these 5, no 4 lie on one circle . Then the maximum number of circles that can be drawn so that each contains atleast three of the given points is :
(A) 216 (B) 156 (C) 172 (D) none

Q.15 One hundred management students who read at least one of the three business magazines are surveyed to study the readership pattern. It is found that 80 read Business India, 50 read Business world, and 30 read Business Today. Five students read all the three magazines. How many read exactly two magazines?
(A) 50 (B) 10 (C) 95 (D) 25

Q.16 Find the number of 10 digit numbers using the digits 0, 1, 2, ... 9 without repetition. How many of these are divisible by 4.

Q.17 A four digit number is called a doublet if any of its digit is the same as only one neighbour. For example, 1221 is a doublet but 1222 is not. Number of such doublets are
(A) 2259 (B) 2268 (C) 2277 (D) 2349

ANSWERS

Q.1	C	Q.2	C	Q.3	C	Q.4	B	Q.5	C	Q.6	C		
Q.7	72, 78120			Q.8	$^{10}C_3 . 3 ! 2 ! . 8 !$			Q.9	C	Q.10	D	Q.11	A
Q.12	B	Q.13	B	Q.14	B	Q.15	A	Q.16	$(20)\cdot 8!$	Q.17	B		

EXERCISE X

Q.1 There are 100 different books in a shelf. Number of ways in which 3 books can be selected so that no two of which are neighbours is

(A) ${}^{100}C_3 - 98$ (B) ${}^{97}C_3$ (C) ${}^{96}C_3$ (D) ${}^{98}C_3$

Q.2 Two classrooms A and B having capacity of 25 and (n–25) seats respectively. A_n denotes the number of possible seating arrangements of room 'A', when 'n' students are to be seated in these rooms, starting from room 'A' which is to be filled up full to its capacity. If $A_n - A_{n-1} = 25!\,({}^{49}C_{25})$ then 'n' equals

(A) 50 (B) 48 (C) 49 (D) 51

Q.3 The sum of all numbers greater than 1000 formed by using digits 1, 3, 5, 7 no digit being repeated in any number is :

(A) 72215 (B) 83911 (C) 106656 (D) 114712

Q.4 Number of positive integral solutions satisfying the equation $(x_1 + x_2 + x_3)(y_1 + y_2) = 77$, is

(A) 150 (B) 270 (C) 420 (D) 1024

Q.5 Distinct 3 digit numbers are formed using only the digits 1, 2, 3 and 4 with each digit used at most once in each number thus formed. The sum of all possible numbers so formed is

(A) 6660 (B) 3330 (C) 2220 (D) none

Q.6 The streets of a city are arranged like the lines of a chess board . There are m streets running North to South & 'n' streets running East to West . The number of ways in which a man can travel from NW to SE corner going the shortest possible distance is :

(A) $\sqrt{m^2 + n^2}$ (B) $\sqrt{(m-1)^2 \cdot (n-1)^2}$ (C) $\dfrac{(m+n)!}{m!\,.\,n!}$ (D) $\dfrac{(m+n-2)!}{(m-1)!\,.\,(n-1)!}$

Q.7 An ice cream parlour has ice creams in eight different varieties . Number of ways of choosing 3 ice creams taking atleast two ice creams of the same variety, is :

(A) 56 (B) 64 (C) 100 (D) none

(Assume that ice creams of the same variety are identical & available in unlimited supply)

Q.8 There are 12 books on Algebra and Calculus in our library, the books of the same subject being different. If the number of selections each of which consists of 3 books on each topic is greatest then the number of books of Algebra and Calculus in the library are respectively:

(A) 3 and 9 (B) 4 and 8 (C) 5 and 7 (D) 6 and 6

Q.9 The sum of all the numbers formed from the digits 1, 3, 5, 7, 9 which are smaller than 10,000 if repetion of digits is not allowed, is (where S = (1+3+5+7+9))

(A) (28011)S (B) (28041)S (C) (28121)S (D) (29152)S

MULTIPLE CORRECT

Q.10 The combinatorial coefficient C(n, r) is equal to

(A) number of possible subsets of r members from a set of n distinct members.

(B) number of possible binary messages of length n with exactly r 1's.

(C) number of non decreasing 2-D paths from the lattice point (0, 0) to (r, n).

(D) number of ways of selecting r things out of n different things when a particular thing is always included plus the number of ways of selecting 'r' things out of n, when a particular thing is always excluded.

Q.11 Identify the correct statement(s).

(A) Number of naughts standing at the end of $\underline{|125}$ is 30 .

(B) A telegraph has 10 arms and each arm is capable of 9 distinct positions excluding the position of rest. The number of signals that can be transmitted is $10^{10} - 1$.

(C) Number of numbers greater than 4 lacs which can be formed by using only the digits 0, 2, 2, 4, 4 and 5 is 90.

(D) In a table tennis tournament, every player plays with every other player. If the number of games played is 5050 then the number of players in the tournament is 100.

Q.12 There are 10 questions, each question is either True or False. Number of different sequences of answers (assuming all answer are not correct) is also equal to

(A) Number of ways in which a normal coin tossed 10 times would fall in a definite order if both Heads and Tails are present.

(B) Number of ways in which a multiple choice question containing 10 alternatives with one or more than one correct alternatives can be answered.

(C) Number of ways in which it is possible to draw at least one coin with 10 coins of different denominations taken some or all at a time.

(D) Number of different selections of 10 indistinguishable things taken some or all at a time.

Q.13 The continued product, 2 . 6 . 10 . 14 ...... to n factors is equal to

(A) ${}^{2n}C_n$ (B) ${}^{2n}P_n$

(C) $(n + 1)(n + 2)(n + 3)\ ......\ (n + n)$ (D) none

Q.14 The Number of ways in which five different books to be distributed among 3 persons so that each person gets at least one book, is equal to the number of ways in which

(A) 5 persons are allotted 3 different residential flats so that and each person is alloted at most one flat and no two persons are alloted the same flat.

(B) number of parallelograms (some of which may be overlapping) formed by one set of 6 parallel lines and other set of 5 parallel lines that goes in other direction.

(C) 5 different toys are to be distributed among 3 children, so that each child gets at least one toy.

(D) 3 mathematics professors are assigned five different lecturers to be delivered , so that each professor gets at least one lecturer.

Q.15 The combinatorial coefficient ${}^{n-1}C_p$ denotes

(A) the number of ways in which *n* things of which p are alike and rest different can be arranged in a circle.

(B) the number of ways in which *p* different things can be selected out of *n* different thing if a particular thing is always excluded.

(C) number of ways in which *n* alike balls can be distributed in *p* different boxes so that no box remains empty and each box can hold any number of balls.

(D) the number of ways in which $(n-2)$ white balls and p black balls can be arranged in a line if black balls are separated, balls are all alike except for the colour.

Q.16 Maximum number of permutations of 2n letters in which there are only a's & b's,taken all at a time is :

(A) ${}^{2n}C_n$ (B) $\frac{2}{1}\cdot\frac{6}{2}\cdot\frac{10}{3}\ ......\ \frac{4n-6}{n-1}\cdot\frac{4n-2}{n}$

(C) $\frac{n+1}{1}\cdot\frac{n+2}{2}\cdot\frac{n+3}{3}\cdot\frac{n+4}{4}\ ......\ \frac{2n-1}{n-1}\cdot\frac{2n}{n}$ (D) $\frac{2^n\cdot[1\cdot3\cdot5\ ......\ (2n-3)(2n-1)]}{n!}$

Q.17 Number of ways in which 3 numbers in A.P. can be selected from 1, 2, 3, ... n is :

(A) $\left(\frac{n-1}{2}\right)^2$ if n is even (B) $\frac{n(n-2)}{4}$ if n is odd

(C) $\frac{(n-1)^2}{4}$ if n is odd (D) $\frac{n(n-2)}{4}$ if n is even

Q.18 If P(n, n) denotes the number of permutations of n different things taken all at a time then P(n, n) is also identical to (where $0 \le r \le n$)

(A) $r! \cdot P(n, n-r)$ (B) $(n-r) \cdot P(n, r)$ (C) $n \cdot P(n-1, n-1)$ (D) $P(n, n-1)$

Q.19 Which of the following statements are correct?

(A) Number of words that can be formed with 6 only of the letters of the word "CENTRIFUGAL" if each word must contain all the vowels is $3 \cdot 7!$

(B) There are 15 balls of which some are white and the rest black. If the number of ways in which the balls can be arranged in a row, is maximum then the number of white balls must be equal to 7 or 8. Assume balls of the same colour to be alike.

(C) There are 12 things, 4 alike of one kind, 5 alike and of another kind and the rest are all different. The total number of combinations of atleast one thing is equal to 240.

(D) Number of selections that can be made of 6 letters from the word "COMMITTEE" is 35.

MATCH THE COLUMN:

Q.20

	Column-I		Column-II
(A)	Number of increasing permutations of m symbols are there from the n set numbers $\{a_1, a_2, \ldots, a_n\}$ where the order among the numbers is given by $a_1 < a_2 < a_3 < \ldots a_{n-1} < a_n$ is	(P)	n^m
(B)	There are m men and n monkeys. Number of ways in which every monkey has a master, if a man can have any number of monkeys	(Q)	${}^mC_n$
(C)	Number of ways in which n red balls and $(m-1)$ green balls can be arranged in a line, so that no two red balls are together, is (balls of the same colour are alike)	(R)	${}^nC_m$
(D)	Number of ways in which 'm' different toys can be distributed in 'n' children if every child may receive any number of toys, is	(S)	m^n

Q.21

	Column-I		Column-II
(A)	Four different movies are running in a town. Ten students go to watch these four movies. The number of ways in which every movie is watched by atleast one student, is (Assume each way differs only by number of students watching a movie)	(P)	11
		(Q)	36
(B)	Consider 8 vertices of a regular octagon and its centre. If T denotes the number of triangles and S denotes the number of straight lines that can be formed with these 9 points then the value of (T – S) equals		
(C)	In an examination, 5 children were found to have their mobiles in their pocket. The Invigilator fired them and took their mobiles in his possession. Towards the end of the test, Invigilator randomly returned their mobiles. The number of ways in which at most two children did not get their own mobiles is	(R)	52
		(S)	60
(D)	The product of the digits of 3214 is 24. The number of 4 digit natural numbers such that the product of their digits is 12, is		
(E)	The number of ways in which a mixed double tennis game can be arranged from amongst 5 married couple if no husband & wife plays in the same game, is	(T)	84

SUBJECTIVE

Q.22 A commitee of 10 members is to be formed with members chosen from the faculties of Arts, Economics, Education, Engineering, Medicine and Science. Number of possible ways in which the faculties representation be distributed on this committee, is ________.
(Assume every department contains more than 10 members).

Q.23 How many ways are there to seat *n* married couples ($n \geq 3$) around a table such that men and women alternate and each women is not adjacent to her husband.

Q.24 10 identical ball are distributed in 5 different boxes kept in a row and labled A, B, C, D and E. Find the number of ways in which the ball can be distributed in the boxes if no two adjacent boxes remain empty.

Q.25 The number of non negative integral solution of the inequation $x + y + z + w \leq 7$ is ____ .

Q.26 On the normal chess board as shown, I_1 & I_2 are two insects which starts moving towards each other. Each insect moving with the same constant speed. Insect I_1 can move only to the right or upward along the lines while the insect I_2 can move only to the left or downward along the lines of the chess board. Prove that the total number of ways the two insects can meet at same point during their trip is equal to

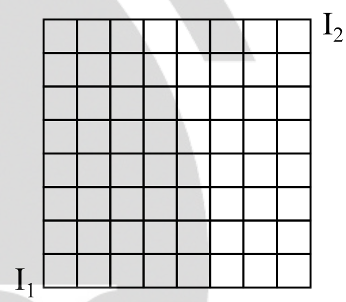
$$\binom{9}{8}\binom{10}{7}\binom{11}{6}\binom{12}{5}\binom{13}{4}\binom{14}{3}\binom{15}{2}\binom{16}{1}$$

OR

$$2^8\binom{1}{1}\binom{3}{2}\binom{5}{3}\binom{7}{4}\binom{9}{5}\binom{11}{6}\binom{13}{7}\binom{15}{8}$$

OR

$$\binom{2}{1}\binom{6}{2}\binom{10}{3}\binom{14}{4}\binom{18}{5}\binom{22}{6}\binom{26}{7}\binom{30}{8}$$



Q.27 How many numbers greater than 1000 can be formed from the digits 112340 taken 4 at a time.

Q.28 Tom has 15 ping-pong balls each uniquely numbered from 1 to 15. He also has a red box, a blue box, and a green box.
(a) How many ways can Tom place the 15 distinct balls into the three boxes so that no box is empty?
(b) Suppose now that Tom has placed 5 ping-pong balls in each box. How many ways can he choose 5 balls from the three boxes so that he chooses at least one from each box?

Q.29 Find the number of ways in which 12 identical coins can be distributed in 6 different purses, if not more than 3 & not less than 1 coin goes in each purse.

Q.30 In how many ways it is possible to select six letters, including at least one vowel from the letters of the word "F L A B E L L I F O R M ".

ANSWERS

Q.1 D Q.2 A Q.3 C Q.4 C Q.5 A Q.6 D Q.7 B
Q.8 D Q.9 B Q.10 A,B,D Q.11 B,C Q.12 B, C Q.13 B, C Q.14 B,C,D
Q.15 B, D Q.16 A, B, C, D Q.17 C, D Q.18 A, C, D Q.19 A, B, D
Q.20 (A) R; (B) S; (C) Q; (D) P Q.21 (A) T; (B) R; (C) P; (D) Q; (E) S
Q.22 3003 Q.23 $n!(n-1)! - 2(n-1)!$ Q.24 771 ways Q.25 330 Q.26 12870
Q.27 159 Q.28 (a) $3^{15} - 3 \cdot 2^{15} + 3$; (b) 2250 Q.29 141 Q.30 296