ТЕТРАДЬ

для ДЗ по физике

учени________ класса ______

РК6-36Б школы ______

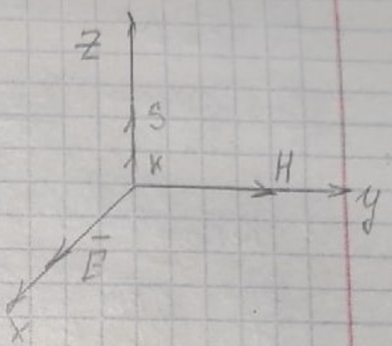
Сергеевой

Дианы

18- вариант

12 листов

Задача 4



Дано:

$\bar S(z,t) = \bar S_m \cos^2(\omega t - kz)$

$k = 0.41\ м^{-1}$

$S_m = 26 \frac{Дж}{с \cdot м^2}$

Найти:

$\bar H(z,t)$ - ?

$\bar E(z,t)$ - ?

ω - ?

$\langle \bar S \rangle$ - ?

$\langle S \rangle$ - ?

$\langle |\bar j_{cm}| \rangle$ - ? $\bar j_{cm}$?

$K_{eд}$ - ?

Решение:

• В общем виде:

$$\bar E(z,t) = \bar E_m e^{i(\omega t - \bar k \bar z + \varphi_0)}$$

$$\bar H(z,t) = \bar H_m e^{i(\omega t - \bar k \bar z + \varphi_0)}$$

$$\bar k \bar z = k_x x + k_y y + k_z z = k_z z = kz$$

$$\Rightarrow \bar E(z,t) = \bar E_m e^{i(\omega t - kz + \varphi_0)}$$

$$\bar H(z,t) = \bar H_m e^{i(\omega t - kz + \varphi_0)}$$

Из ур-я Максвелла в диф. форме: $rot\,\bar E = -\frac{\partial \bar B}{\partial t} = -\mu_0 \frac{\partial \bar H}{\partial t}$

$E_y = |\bar E| = E$ $\quad H_z = |\bar H| = H$

$E_x = E_z = 0$ $\quad H_y = H_x = 0$

$$rot\,\bar E = \begin{vmatrix} \bar i & \bar j & \bar k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} \ominus$$ ~ ротор векторного поля $\bar E$

$$\ominus \begin{vmatrix} \bar i & \bar j & \bar k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E & 0 & 0 \end{vmatrix} = \frac{\partial E(z,t)}{\partial z}\bar j - \frac{\partial E(z,t)}{\partial y}\bar k =$$

$$\frac{\partial}{\partial x}\left(E_m e^{i(\omega t - kz + \varphi_0)}\right)\bar{j} = -kE_m e^{i(\omega t - kz + \varphi_0)}\bar{j}$$

$$\frac{\partial \bar{H}}{\partial t} = \frac{\partial H_x}{\partial t}\bar{i} + \frac{\partial H_y}{\partial t}\bar{j} + \frac{\partial H_z}{\partial t}\bar{k} = \frac{\partial H}{\partial t}\bar{j}$$

$$\Rightarrow -kE_m e^{i(\omega t - kz + \varphi_0)}\bar{j} = -\mu_0 \frac{\partial H}{\partial t}\bar{j}$$

$$\frac{\partial H}{\partial t} = \frac{kE_m}{\mu_0} e^{i(\omega t - kz + \varphi_0)}$$

$$H = \int \frac{kE_m}{\mu_0} e^{i(\omega t - kz + \varphi_0)} dt = \frac{kE_m}{\mu_0 \omega} e^{i(\omega t - kz + \varphi_0)}$$

$$H_m = \frac{kE_m}{\mu_0 \omega}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi\nu}{\lambda\nu} = \frac{\omega}{c}, \quad c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$ – скорость света в вакууме

$$\Rightarrow \omega = kc$$

$$\Rightarrow H_m = \frac{E_m}{\mu_0 c}$$

т.к. $\bar{S}(z,t) = \bar{S}_m \cos^2(\omega t - kz)$ и $\bar{S} = [\bar{E}; \bar{H}]$

$$\bar{S} = [\bar{E}\bar{H}] = E_m \cos(\omega t - kz + \varphi_0) \cdot H_m \cos(\omega t - kz + \varphi_0) =$$

$$= \frac{E_m^2}{\mu_0 c} \cos^2(kct - kz + \varphi_0)$$

$$S_m \cos^2(kct - kz) = \frac{E_m^2}{\mu_0 c} \cos^2(kct - kz + \varphi_0)$$

! Должно быть равно при любой фазе колебаний

$\Rightarrow \varphi_0 = 0$

$$\Rightarrow S_m = \frac{E_m^2}{\mu_0 c}$$

$E_m = \sqrt{\mu_0 c S_m}$

$E = \sqrt{\mu_0 c S_m} \cos(kct - kz)$

$\bar{E}(z,t) = \sqrt{\mu_0 c S_m} \cos(kct - kz)\bar{i}$

• $H(x,t) = H_m \cos(kct - kz) = \frac{E_m}{\mu_0 c} \cos(kct - kz) = \frac{1}{\mu_0 c}\sqrt{\mu_0 c S_m} \cos(kct - kz) = \sqrt{\frac{S_m}{\mu_0 c}} \cos(kct - kz)$

$\bar{H}(z,t) = \sqrt{\frac{S_m}{\mu_0 c}} \cos(kct - kz)\bar{j}$

• $w = w_E + w_H = \frac{\varepsilon_0 E^2(z,t)}{2} + \frac{\mu_0 H^2(z,t)}{2}$

w_E - объёмн. плотность энергии электрич. поля

w_H - объёмн. плотн. энергии магнитн. поля

$w(z,t) = \frac{\varepsilon_0}{2}\mu_0 c S_m \cos^2(kct - kz) + \frac{\mu_0 S_m}{2\mu_0 c}\cos^2(kct - kz) = \frac{S_m}{c}\cos^2(kct - kz)$

• $\langle \bar{S} \rangle = \frac{1}{T}\int_0^T \bar{S}(t)dt$

$\langle \bar{S} \rangle = \frac{1}{T} \bar{S}_m \cos^2(kct - kz)dt = \frac{\bar{S}_m}{T}\int_0^T \cos^2(kct - kz)dt =$

т. к. $\langle \cos^2(kct-kz)\rangle = \frac{1}{T}\int_0^T \cos^2(kct-kz)\,dt = \frac{1}{2T}\int_0^T (1+$
$+\cos(2(kct-kz))dt = 1/2$

$(T = \frac{2\pi}{\omega} = \frac{2\pi}{kc})$

$\langle \bar{S}\rangle = \frac{1}{T}\int_0^T \bar{S}_m \cos^2(kct-kz)\,dt = \bar{S}_m/2$

$\langle \bar{S}\rangle = \frac{S_m}{2}\bar{i}$

• Среднее за период колебаний значение плотности потока энергии:

$\langle S\rangle = \frac{1}{T}\int_0^T S(t)\,dt =$

Модуль вектора Пойнтинга

$\langle S\rangle = \frac{1}{T}\int_0^T S_m \cos^2(kct-kz)\,dt = \frac{S_m}{2}$

• $\bar{j}_{см} = \frac{\partial \bar{D}}{\partial t}$ $\qquad$ $\bar{D}$ - вектор электрического смещения

$\bar{D} = \varepsilon\varepsilon_0\bar{E}$, $\varepsilon = 1$

$\bar{D} = \varepsilon_0\sqrt{\mu_0 c S_m}\cos(kct-kz)\bar{i}$

$\bar{j}_{см} = \frac{\partial \bar{D}}{\partial t} = -\varepsilon_0 kc\sqrt{\mu_0 c S_m}\sin(kct-kz)\bar{i} =$
$= -k\sqrt{\varepsilon_0 c S_m}\sin(kct-kz)\bar{i}$

• $\langle |\bar{j}_{см}|\rangle = \frac{1}{T}\int_0^T \left|k\sqrt{\varepsilon_0 c S_m}\sin(kct-kz)\right|dt =$
$= \frac{kc}{2\pi}k\sqrt{\varepsilon_0 c S_m}\int_0^{\frac{2\pi}{kc}} |\sin(kct-kz)|\,dt =$

$$= \frac{kc}{2\pi} k \sqrt{\varepsilon_0 c S_m} \left(\int_0^{\pi/kc} \sin(kct - kz)dt \int_{\pi/kc}^{2\pi/kc} (-\sin(kct - kz))dt \right) =$$

$$= \frac{kc}{2\pi} k \sqrt{\varepsilon_0 c S_m} \int_0^{\pi/kc} \sin(kct - kz)dt =$$

$$= \frac{k\sqrt{\varepsilon_0 c S_m}}{\pi} (-\cos(\pi - kz) + \cos(-kz)) = \frac{k\sqrt{\varepsilon_0 c S_m}}{\pi} =$$

$$= \frac{2k}{\pi} \sqrt{\varepsilon_0 c S_m}$$

$$\langle |\overline{j_{cm}}| \rangle = \frac{2k}{\pi} \sqrt{\varepsilon_0 c S_m}$$

- $\overline{K_{eo}} = \frac{\overline{S}}{c^2}$

$$K_{eo}(z,t) = \frac{w(z,t)}{c}$$

$$K_{eo}(z,t) = \frac{S_m}{c} \cos^2(kct - kz)$$

- Волновое ур-е для магнитн. и электрич. компонент

Общий вид:

$$\frac{\partial^2 \overline{E}}{\partial t^2} = v^2 \left(\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 \overline{E}}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} \right)$$

$$\frac{\partial^2 \overline{H}}{\partial t^2} = v^2 \left(\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 \overline{H}}{\partial y^2} + \frac{\partial^2 \overline{H}}{\partial z^2} \right)$$

$$v = \frac{1}{\varepsilon_0 \mu_0 \varepsilon \mu}$$

$$\frac{\partial^2 E}{\partial t^2} = c^2\frac{\partial^2 \bar{E}}{\partial y^2}$$

$$\frac{\partial^2 H}{\partial t^2} = c^2\frac{\partial^2 \bar{H}}{\partial y^2}$$

$$E(x,t) = \sqrt{\mu_0 c S_m}\cos(ks-kz)i$$

$$\bar{H}(x,t) = \sqrt{\frac{S_m}{\mu_0 c}}\cos(kct-kz)\bar{j}$$

$$\frac{\partial^2 E}{\partial t^2} = -k^2c^2\sqrt{\mu_0 c S_m}\cos(kct-kz)i$$

$$\frac{\partial^2 E}{\partial y^2} = -k^2\sqrt{\mu_0 c S_m}\cos(kct-kz)\bar{i}$$

$$\frac{\partial^2 H}{\partial t^2} = -k^2c^2\sqrt{\frac{S_m}{\mu_0 c}}\cos(kct-kz)\bar{j}$$

$$\frac{\partial^2 H}{\partial y^2} = -k^2\sqrt{\frac{S_m}{\mu_0 c}}\cos(kct-kz)\bar{j}$$

При $t=0$: