Tanner Krebs, Gerardo Lopez, William Thornton
Math 314
Due: 11/0217

COSC/MATH - 314
Assignment 7

Problem 1:

(a.) Find an integer x between 0 and 69 that satisfies simultaneously x = 2 (mod 7) and x = 3 (mod 10). (Note: by the Chinese Remainder Theorem, we know that there is such an integer.)

(b.) Find another integer y (different from the x from (a)) such that y = 2 (mod 7) and y = 3 (mod 10).

Problem 2: Use the repeated squaring method to calculate each of the following (show the main steps for each calculation): $3^7(mod 12)$, $16^{10}(mod 230)$, $5^{14}(mod 26)$, $4^{22}(mod 11)$, $3^{65}(mod 71)$.

Problem 3: Use Fermat?s Little Theorem (and other methods if you need) to calculate the following (all moduli are prime numbers): $99^{101}(mod 101)$, $94^{66}(mod 67)$, $4968732^{7540}(mod 7541)$, $65^{144}(mod 73)$, $65^{143}(mod 73)$.

Problem 4: Show that $2^{56} + 3^{56}$ is divisible by 17.

Problem 5: Find three different integers a, b, and c such that $2^a = 2^b = 2^c = 1$ (mod 19).

Problem 6:

(a.) Show that 2 is a primitive root (mod 11)

(b.) Find all natural numbers x that satisfy the equation $2^{3x} = 2$ (mod 11). (Note: There are infinitely many natural numbers that satisfy the given equation. "Find all" means to present a description of all of them.)