Ouroboros

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High Level Overview

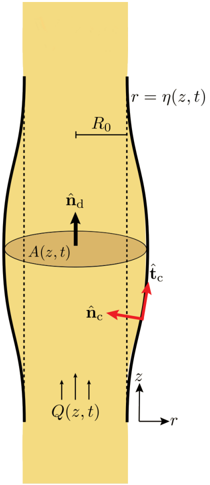
Can we make a soliton gas?

- One-Dimensional Fluid System
- Solitons (and other discrete structures) possible
- Need to use some statistics...

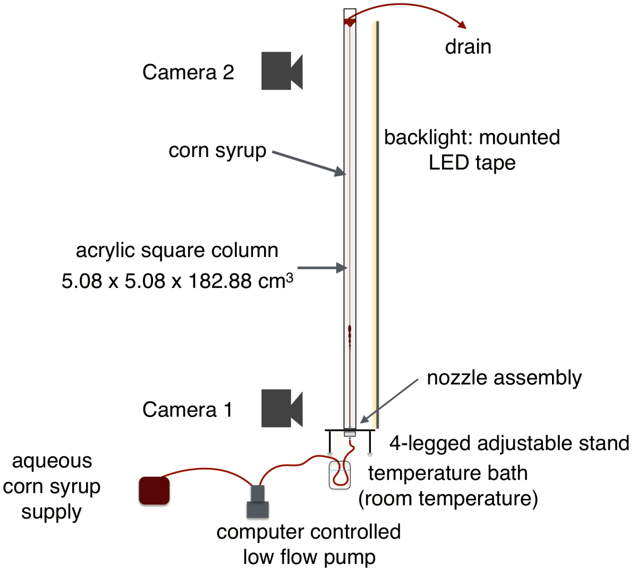
What's our Environment?

Viscous Fluid Conduits

- Two viscous fluids, with inner forming axisymmetric conduit.
- Exterior Fluid: $\rho^{(e)}$ density and $\mu^{(e)}$ viscosity
- Interior Fluid: $\rho^{(i)}$ density and $\mu^{(i)}$ viscosity
- $\rho^{(i)} < \rho^{(e)} \Rightarrow$ buoyant flow
- $\mu^{(i)} << \mu^{(e)} \Rightarrow$ minimal drag
- Re $<< 1 \Rightarrow$ low Reynold's number



Experimental Setup



Notes on Solitons

Our system is governed by the Conduit Equation,

$$A_t + \left(A^2\right)_z - \left(A^2\left(A^{-1}A_t\right)_z\right)_z = 0$$

- Solitons are *solitary travelling waves*.
- Solitons are a special solution to the conduit equation of the form

$$A(z,t) = f(\zeta) = f(z - ct)$$

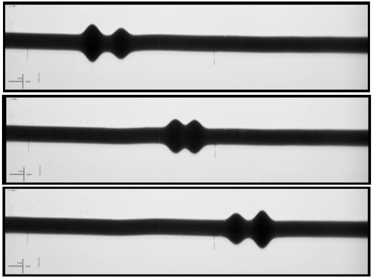
- Solitons have Non-linear characteristics, most notably their speed is determined by their non-dimensionalized amplitude (a).

$$c = \frac{a^2 - 2a^2 \ln a - 1}{2a - a^2 - 1}$$

Conduit Soliton

Soliton - Soliton Interactions

- Two solitons can interact if a bigger one chases a smaller one.
- The solitons' speed and amplitude are preserved save for a phase-shift



Phase Shift

What's a Soliton Gas?

- A *Soliton* is a solitary travelling wave, while a *Gas* can be thought of as a collection of particles interacting.
- In our system, our gas is one-dimensional, as we're limited to the interior of the conduit.
- A soliton gas has inherent random behavior dictated by two random variables:
  1. Frequency of solitons, Z
  2. Soliton amplitude, A
- In a "true" soliton gas,

$$Z_g \sim Poisson(\lambda)$$

Where λ corresponds to the "density" of the gas.

How do we find a Soliton Gas?

- Remember, true gas has $Z_g \sim Poisson(\lambda)$!
- Due to the phase-shift from soliton-soliton interactions, any input distribution Z will converge to Z_g.
- The Amplitude distribution A is preserved over time.
- Given some initial spacing distribution Z_0, we need to measure how close it is to our ideal distribution Z_g.

Numerical Simulations

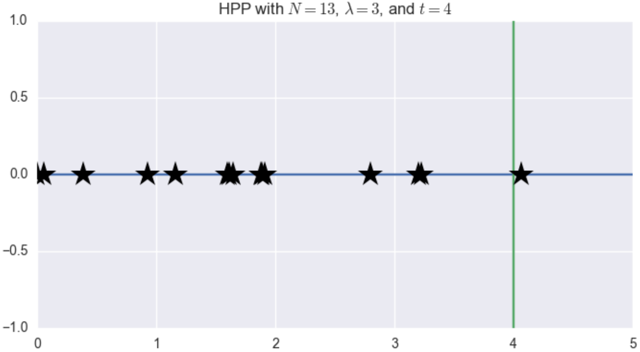
- Experiments are hard and time consuming. Much easier to just run simulations.
- Can simulate using adaptive step RK4 with periodic boundary conditions.
- In this system, with periodic boundary conditions, recurrence is guaranteed.

Why Periodic Boundary Conditions?

- Recurrence is guaranteed.
- Therefore we'll run two simulations simultaneously, one on $[0, L]$ and the other on $[0, 2L]$.
- At each timestep we'll check "poissonness" of each. If they differ by more than 20%, we restart with new initial conditions.
- Since we have recurrence, eventually the simulation on $[0, L]$ will tend back to initial conditions. We want to stop before then.
- Similarly, due to phase shifts, poissonness should increase and then decrease. We want to stop before this.

What are we calling "Poissonness"?

- We've established that a soliton gas should have Poisson-distributed solitons.
- This means that we can look at the problem as a Poisson-Point Process, i.e. at any given point in space we should see points appear over time.



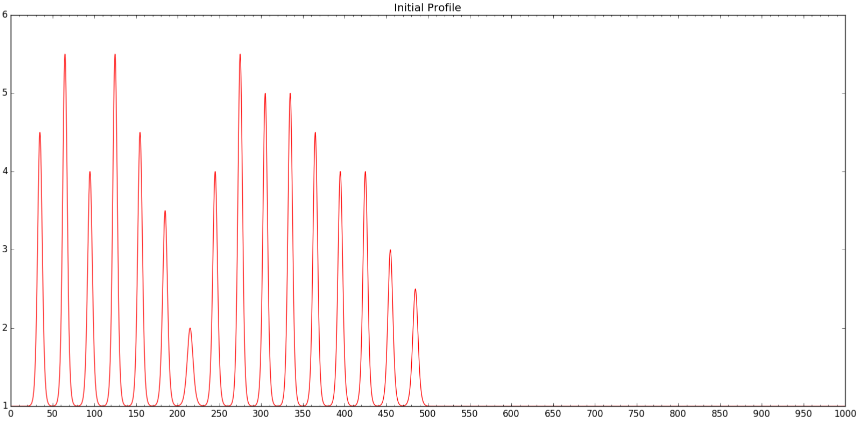
Meaning....

- Since this is a Poisson-Point Process the gap between points is exponentially distributed.
- Therefore the "Poissonness" is how close our gaps of our solitons are to the exponential distribution.
- We use the residual sum squared on the QQ-plot as a metric of "distance" from one distribution to the other. This is our Poissonness metric.

Initial Conditions

- We have two random variables to simulate.
- Very first case is easy
  - Z is one soliton per "unit"
  - A is $Unif(\{2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5, 7\})$
- After a restart on $2L$ and $4L$ things are trickier... We can't feed initial conditions where solitons overlap (makes solver grumpy) so instead we need to simulate initial conditions with same Poissonness as last timestep (before we quit)

Plot of Initial Conditions

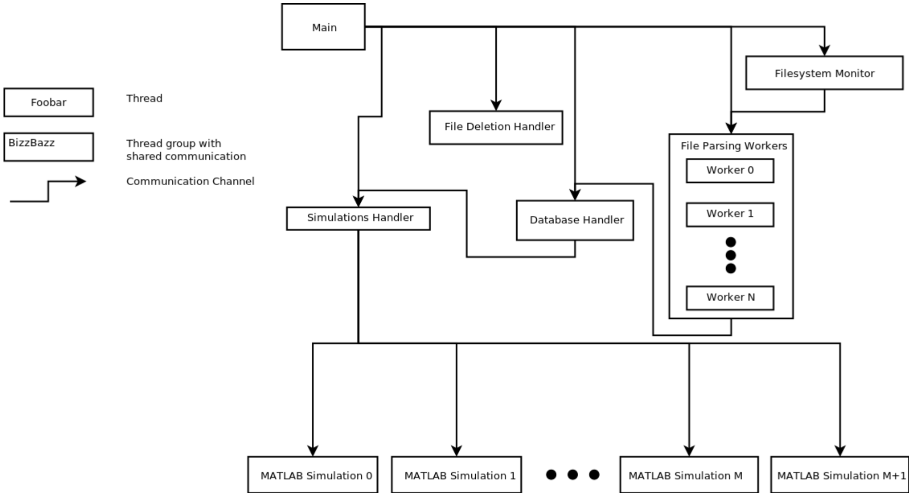


Leveraging Parallelism

The big flaw so far is that we're only looking at a single run of the simulation. We could easily get bad results from only a single run.

Let's instead consider running a hundred different simulations simultaneously, or even a thousand. We have to adjust our simulation to be able to handle running in a massively parallel environment such as the CU supercomputer, Summit.

Leveraging Parallelism



Preliminary Results

References and Acknowledgements

References

- D. S. Agafontsev and V. E. Zakharov, Nonlinearity 28, 2791 (2015).

Acknowledgements

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