Discrete Hydrodynamics

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1   High-Level Goal

Identify the role discretization plays in nonlinear hydrodynamic-like behavior

2   Sub-Project 1: Classification of the Riemann problem for the Toda lattice

- Physics model: Chain of particles with mass m, interacting with nearest neighbors. $x_n(t)$ is the horizontal position of mass n at time t.

Nondimensionalized equation

$$\partial_t^2 \tilde{x}_n = \exp\left(\tilde{x}_{n-1} - \tilde{x}_n\right) - \exp\left(\tilde{x}_n - \tilde{x}_{n+1}\right), \quad n \in \mathbb{Z} \tag{1}$$

At equilibrium,

$$x_n(0) = cn, \quad \partial_t x_n(0) = 0 \tag{2}$$

2.1   Milestones and Assignments

- [Everyone] Literature Review, with a focus on Toda lattices and

    - hydrodynamics, solitons
    - Whitham theory (Whitham equations are/should be in literature)
    - physical applications
    - shocks, rarefaction waves
    - original papers by Toda

- [Jessica] Numerical Code with appropriate boundary conditions

- [Michelle & Nevil] Determine/Identify the Riemann Problem for this system

- [Max R & Adam] Determine the (linear) dispersion relation for the system

- [Mingyu] Soliton solutions

- [Mingyu] Long wavelength assumption (hydrodynamic limit)

- [Max L] Nonlinear periodic solutions

- [Pat & Mingyu] Whitham equations

- [Ezio] Simple waves

- [Pat] Transformations of Eq. 1, e.g.

$$\partial_t a_k = b_{k+1} - b_k$$
$$\partial_t b_k = b_k \left(a_k - a_{k-1} \right)$$

(3)

- [Michelle] Symmetries, conservation laws