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PHYS 265

Mine Crafting: A Study of Free Fall

I. Introduction

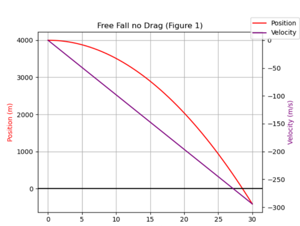
Understanding the behavior of falling bodies has been one of the most common problems throughout history; from warfare, aircraft, Newton’s apple, and much more. The study of falling objects and the effects of different forces such as Gravity, Coriolis, and parameters such as drag have a major impact on the dynamics of bodies in free fall. This report analyzes the motion of a body during free fall with increasing complexity. I provide detailed explanation while communicating clearly so a layperson can understand, while also giving accurate details, and explanations of the underlying mechanics. First I start with just basic gravity and explore an object falling down a mineshaft, then I account for drag, then use an accurate model of variable gravity that much more accurately describes the free fall of a body. Then I treat the mineshaft as going through the entirety of the Earth, which highlights harmonic motion. Next I dive in a nonhomogeneous Earth and the impact of non-uniform density on the free fall of an object. Finally I will take us to the moon and explore some fun and interesting differences between an object falling on the moon and on the Earth.

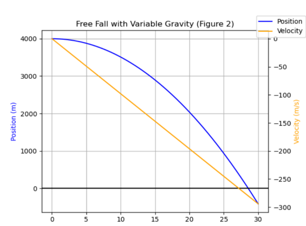
II. Fall Time with Drag and Variable Gravity

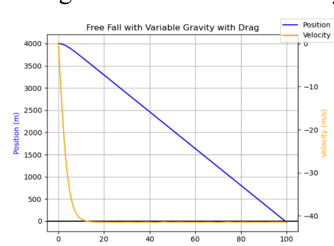
I first start off with the Earth and evaluate the fall time of a 1 kilogram object down a mineshaft that is 4 km in depth. I evaluate the fall time under the 3 following conditions

1.Free fall with standard gravity and no drag
2.Free fall with variable gravity and no drag
3.Free fall with variable gravity and drag

For Free fall with standard gravity and no drag I used the following differential equations. $\frac{dy}{dt} = v$ and $\frac{dv}{dt} = -g$ with v being the velocity and g being the gravitation constant of 9.81 meters per second squared. The fall time calculated for this scenario was 28.6 seconds. For Free fall with variable gravity and no drag. I replaced g for radial function g(r) = $g(\frac{r}{R})$. Using this equation yielded 28.6 seconds as well. This can be explained since within a 4km mineshaft the effect of a variable gravity will not have a significant impact. For Variable gravity with drag I used my previous variable gravity equation and added a drag force. The fall time for this was 99.4 seconds. This shows that even over a relatively short 4 km drop, air drag increases the fall time by a factor of nearly



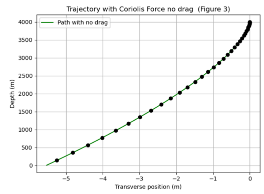




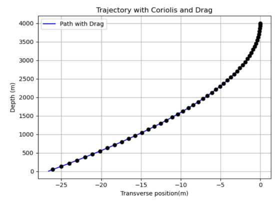
four.

III. Feasibility of Depth Measurement

Next step is to analyze the effects of the Coriolis effect on the free fall of the object. I wanted to see if the Coriolis Force would cause the object to bump into the walls of the mineshaft, causing the free fall to be interrupted. When the Coriolis force was analyzed without drag, I calculated its total displacement in the transverse direction was 5.5 meters. Assuming the mineshaft has a diameter of 5 meters and we drop the object from the center, it would bump into the wall before it reached the bottom of the shaft.



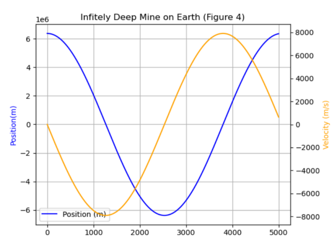
Next, I accounted for drag and the results were even more pronounced.With drag included, the transverse displacement increased dramatically to 26.6 meters..



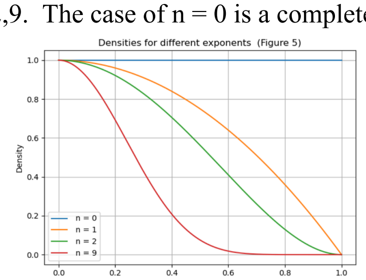
With a result of 5.5 meters without drag and a result of 26.6 meters with drag it is clear that this is not feasible for accurate depth measurement. The Coriolis effect would prevent the object from actually reaching the bottom of the mine in both the case with drag and without.

IV. Homogenous and Non-Homogenous Models

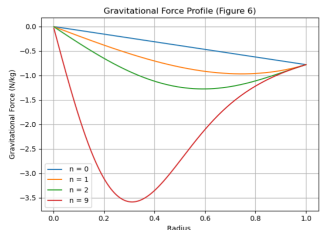
Next, I examine how a planet's internal density distribution affects the fall time through a central tunnel. But before going into the non-homogeneous example I will first start with an "infinitely" deep mine through the Earth pole to pole. For this example I used my previous variable gravity model. With this model the object exhibits harmonic motion and reaches the center of the earth at a time of 1266.5 seconds with a velocity of 7910.8 meters per second in the downward direction.



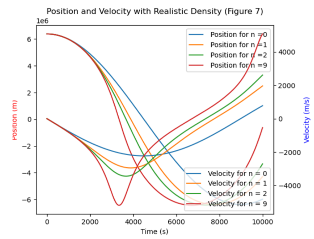
Next I explore the distributed density model using the equation $\rho(r) = \rho n(1 - \frac{r^2}{R})^n$, pn being the normalizing constant, R being the radius of the earth, n being evaluated with values: 0,1,2,9. The case of n = 0 is a completely uniform earth. First I explored the different values of n effect on the density.



Then using nested functions within for loops I was able to calculate the force for each of these n values using the integral definition of Mass. $M = \int_V \rho(r) dV = \int_0^{2\pi}\int_0^{\pi}\int_0^{R_\oplus} \rho(r) r^2 \sin\phi \, dr \, d\phi \, d\theta = 4\pi \int_0^{R_\oplus} \rho(r) r^2 \, dr$



Finally to put it all together I want to again plot the velocity and position of an object in free fall through an "mine" all the way through the earth.



With the non-homogenous earth accounted for my fall times calculated to reach the center of the earth drastically changed for n = 0 I calculated a time of 4491.9 seconds and a Velocity of 2230.4 meters per second. For n = 9 a time of 3345.5 seconds and a velocity of 5193.4 meters per seconds. From this data I can see that as the density increased the fall time shortened and the velocity increased which makes sense due to the increased effect of higher density concentration. Finally I went to the moon where I "dug" another hole through the center of the beloved natural satellite. Using a similar model to the variable gravity in part section 2 I calculated a fall time of 1626.8 seconds with a velocity of 1678.0 meters per second. I found the Moon to be about 61% less dense than Earth. And derived a relationship between fall time and density, and found it to be inversely proportional to the square root of density. Doing a simple test calculation allowed me to calculate a fall time of 1621.1 seconds. This matched my numerical result of 1626.8 seconds very closely, confirming the theoretical relationship.

V. Discussion and Future Work

Throughout this project I explored a relatively simple dynamical problem at first, then explored the problem in increasing complexity, by the addition of more realistic forces and mechanics. This we first started with simple free fall down a mine shaft, only accounting for the constant gravitational force, and making my way to account for density distribution variable gravity and the coriolis force. While this exercise has been enlightening in many ways. There is still much work that needs to be done in the future regarding free fall. This is not just a theoretical problem, as stated in the introduction, this research has applications to the military, space travel, sports, and much more. While I am happy with the work so far, exploring even more complex and realistic models in the future will be a valuable and interesting direction. First there is a basic assumption that was made, that being the earth is completely spherical. With future work I would like to explore with more advanced equipment, calculated density distribution of a non-spherical earth. Another assumption made is that the gravity is linear within the Earth, which is not true, the earth is made of several layers which are not uniform density causing non-linear fluctuations in the gravitational force.