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I. Introduction

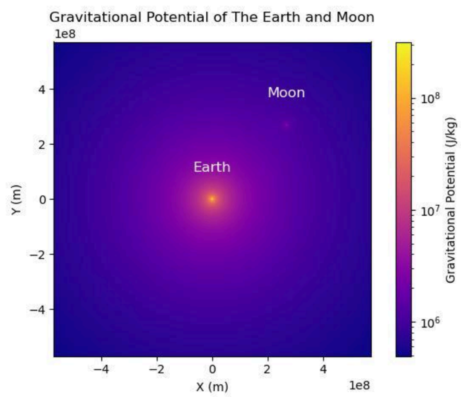
The Apollo Mission plans to send a man to the moon. This will take ingenuity, resolve, and the collaboration of physicists, computer scientists, engineers, and politicians. Within this report, I will detail the underlying physics, present plots and figures relevant to the Apollo missions, and break down the mathematics of a rocket launch. I will explain physics and mathematics in a way that will be understandable to the layman. Space flight involves some of the most complicated physics and mathematics currently in use. But to even start analyzing the process of space flight, we must first understand some of the basics. This includes the gravitational potential of the Earth, the gravitational potential of the Earth-Moon system, as well as the gravitational forces a spacecraft would experience within the Earth-Moon system. Along with this, I will provide data on how far a Saturn V rocket will travel with a given amount of fuel, which is crucial to leaving the gravitational pull of the Earth.

II. The gravitational potential of the Earth-Moon system

We define the gravitational potential at a distance r from a mass M as:

$$\Phi(r) = -\frac{GM}{r}$$

Where G is the gravitational constant, M is the mass of the given body, and r is the distance from that mass. Since we want the gravitational potential of the Earth-Moon system, we can calculate this function for the Earth and the Moon separately, then sum the two functions to obtain the total gravitational potential energy. I was then able to plot and visualize the Earth-Moon system. From this plot, we can clearly see that the gravitational potential energy around the Earth is much larger than that of the Moon. This makes sense because the Earth is significantly more massive than the Moon.

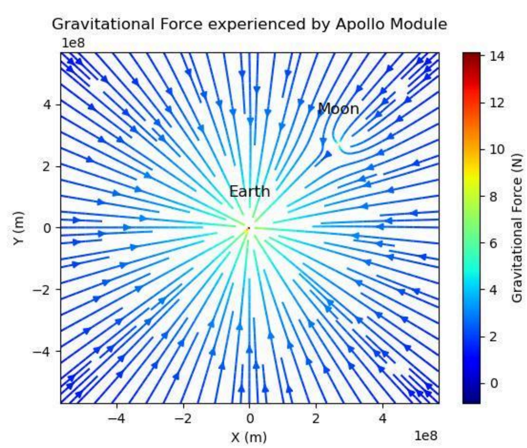


III. The Gravitational Force of the Earth-Moon system

After giving an overview of the Gravitational Potential of the Earth-Moon system, it is important to also see the forces that this system would have on a command module from a Saturn V rocket. The Gravitational force between two bodies is given by the following equation.

$$\mathbf{F}_{12} = -G\frac{m_1 m_2}{r^2}\hat{\mathbf{r}}_{12}$$

Where F is the Force between the 2 objects, m1 is the mass of the earth or moon, and m2 is the mass of the Apollo module, and r is the distance between the objects. We can see this system in the following plot.



From this plot we can see that the gravitational force experienced by the module is much higher closer to the earth and the moon, and falls off the farther the module gets from the two bodies.

IV. Projected Performance of the Saturn V Rocket Stage 1

Understanding the gravitational potential and forces between the Earth and Moon system is only the first step. We must also understand if we can actually launch the rocket from the Earth's surface to space. For the first stage of the rocket launch we would like to know that altitude achieved with a given mass of fuel. This has been calculated to be 64546.79 meters with an initial "wet mass" of 2.4×10^{6} kilograms with a total burn time of 129 seconds.

V. Discussion and Future Work

While conducting this project I had to make several assumptions, the first of these was treating the Earth and Moon as a point particle for simplicity of calculations. This introduced a problem of a singularity, which was addressed within my code by treating the potential and force to be 0 at that singularity. During future work we plan to model the Earth and Moon more realistically, as 3-dimensional bodies, this will take much more advanced calculations, but will give a much more accurate picture. A recent report from NASA was received that gave different results for the first state of the Saturn V rocket. Their data suggests a burn time of 160 seconds and an altitude of 70 km, opposed to my burn time of 129 and altitude of approximately 64km. The disparity of these calculations can be understood since when I did my calculations, I did not account for drag or air resistance. I had only accounted for the force of gravity. With more funding we will be able to conduct future work and experiments that will more accurately be able to model these complex systems, with a hope of reaching the moon safely.