Multivariate smoothing, model selection

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Recap

- How GAMs work
- How to include detection info
- Simple spatial-only models
- How to check those models

Univariate models are fun, but…

Ecology is not univariate

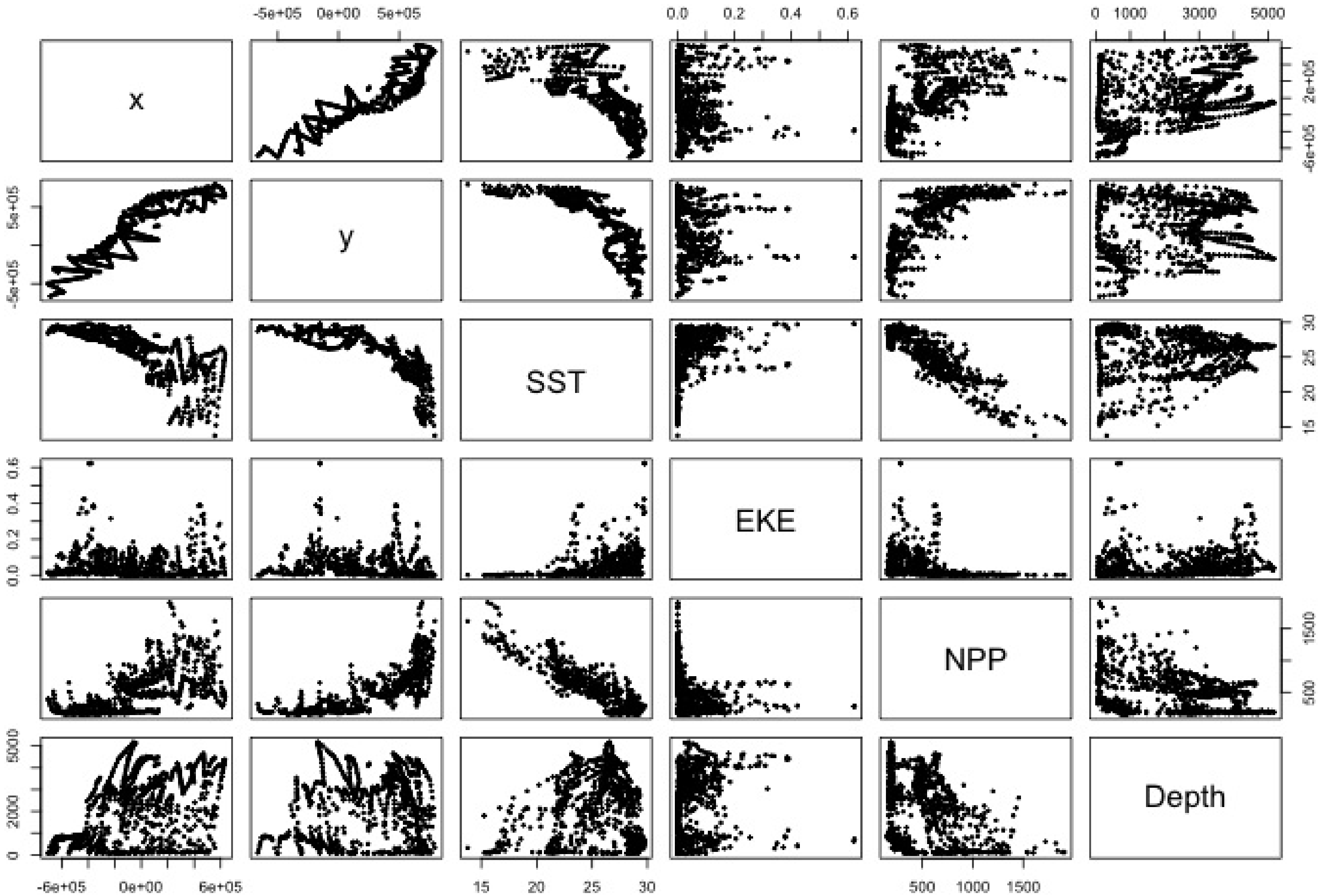
- Many variables affect distribution
- Want to model the **right** ones
- Select between possible models
  - Smooth term selection
  - Response distribution
- Large literature on model selection

Tobler’s first law of geography

“Everything is related to everything else, but near things are more related than distant things”

Tobler (1970)

Implications of Tobler's law



What can we do about this?

- Careful inclusion of smooths
- Fit models using robust criteria (REML)
- Test for concurvity
- Test for sensitivity

Models with multiple smooths

Adding smooths

- Already know that + is our friend
- Add everything then remove smooth terms?

```
dsm_all_tw <- dsm(count~s(x, y, bs="ts") +
                        s(Depth, bs="ts") +
                        s(DistToCAS, bs="ts") +
                        s(SST, bs="ts") +
                        s(EKE, bs="ts") +
                        s(NPP, bs="ts"),
                      ddf.obj=df_hr,
                      segment.data=segs, observation.data=obs,
                      family=tw(), method="REML")
```

Now we have a huge model, what do we do?

Smooth term selection

- Classically two main approaches:
  - Stepwise - path dependence
  - All possible subsets - computationally expensive

Removing terms by shrinkage

- Remove smooths using a penalty (shrink the EDF)
- Basis `"ts"` - thin plate splines with shrinkage
- "Automatic"

p-values

- p-values can be used
- They are **approximate**
- Reported in `summary`
- Generally useful though

Let’s employ a mixture of these techniques

How do we select smooth terms?

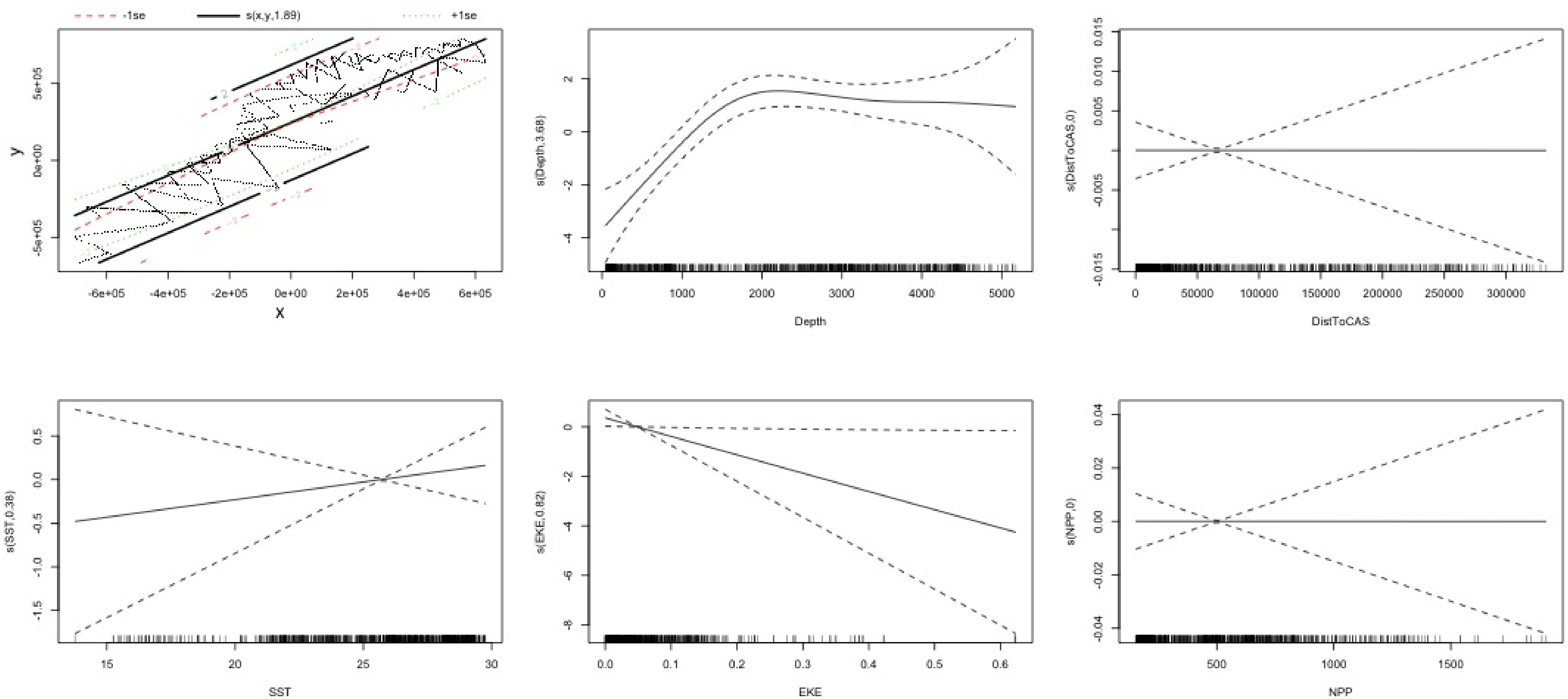
1. Look at EDF
   - Terms with EDF<1 may not be useful
   - These can usually be removed
2. Remove non-significant terms by p-value
   - Decide on a significance level and use that as a rule

Example of selection

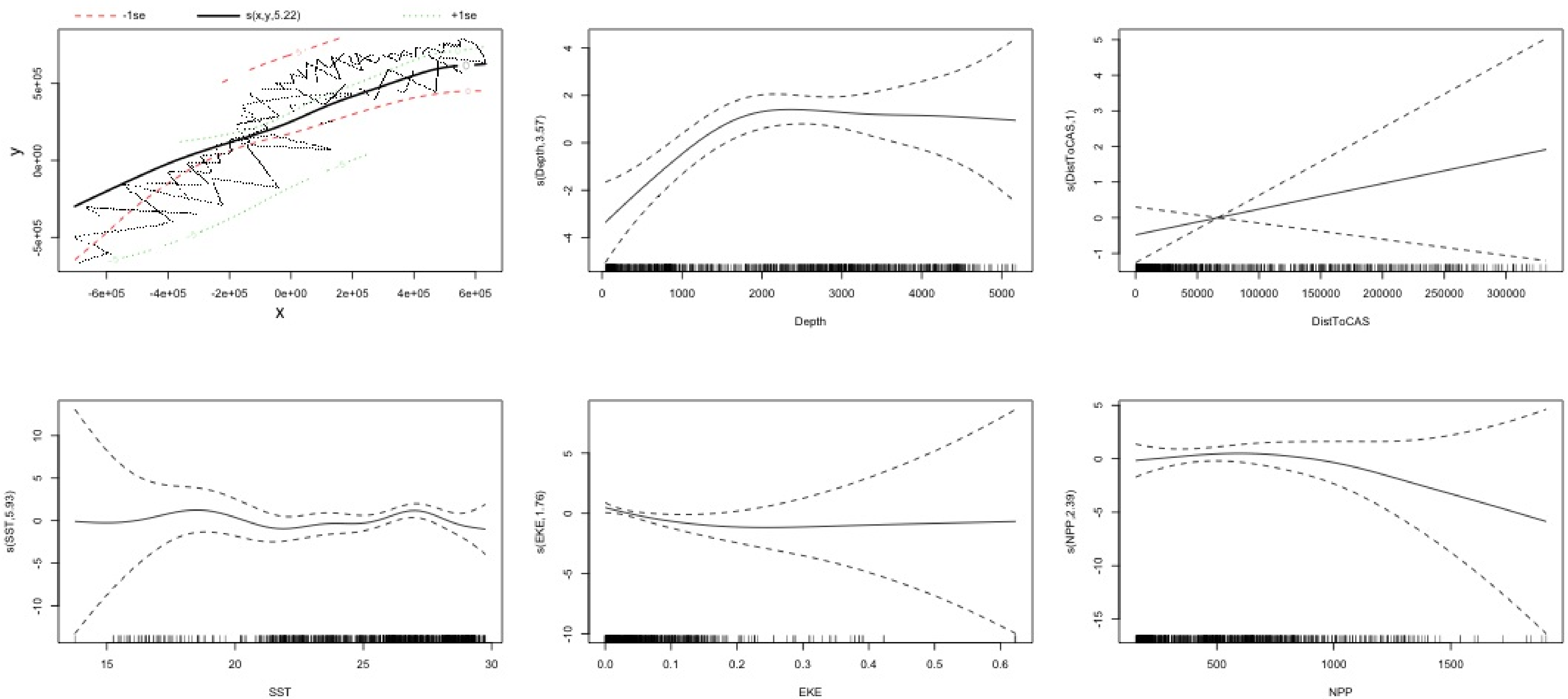
Selecting smooth terms

```
Family: Tweedie(p=1.277)
Link function: log

Formula:
count ~ s(x, y, bs = "ts") + s(Depth, bs = "ts") + s(DistToCAS,
    bs = "ts") + s(SST, bs = "ts") + s(EKE, bs = "ts") + s(NPP,
    bs = "ts") + offset(off.set)

Parametric coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -20.260      0.234  -86.59   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Approximate significance of smooth terms:
                   edf Ref.df     F  p-value
s(x,y)       1.888e+00     29 0.705 3.56e-06 ***
s(Depth)     3.679e+00      9 4.811 2.15e-10 ***
s(DistToCAS) 3.936e-05      9 0.000   0.6798
s(SST)       3.831e-01      9 0.063   0.2160
s(EKE)       8.196e-01      9 0.499   0.0178 *
s(NPP)       1.587e-04      9 0.000   0.8361
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

R-sq.(adj) =   0.11   Deviance explained =   35%
-REML = 385.04  Scale est. = 4.5486    n = 949
```

Shrinkage in action



Same model with no shrinkage



Let's remove some smooth terms & refit

```
dsm_all_tw_rm <- dsm(count~s(x, y, bs="ts") +
                        s(Depth, bs="ts") +
                        #s(DistToCAS, bs="ts") +
                        #s(SST, bs="ts") +
                        s(EKE, bs="ts"),#+
                        #s(NPP, bs="ts"),
                      ddf.obj=df_hr,
                      segment.data=segs, observation.data=obs,
                      family=tw(), method="REML")
```

What does that look like?

```
Family: Tweedie(p=1.279)
Link function: log

Formula:
count ~ s(x, y, bs = "ts") + s(Depth, bs = "ts") + s(EKE, bs =
"ts") +
    offset(off.set)

Parametric coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -20.258      0.234  -86.56   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Approximate significance of smooth terms:
            edf Ref.df     F  p-value
s(x,y)   1.8969     29 0.707 1.76e-05 ***
s(Depth) 3.6949      9 5.024 1.08e-10 ***
s(EKE)   0.8106      9 0.470   0.0216 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

R-sq.(adj) =  0.105   Deviance explained = 34.8%
-REML = 385.09  Scale est. = 4.5733    n = 949
```

Removing EKE...

```
Family: Tweedie(p=1.268)
Link function: log

Formula:
count ~ s(x, y, bs = "ts") + s(Depth, bs = "ts") + offset(off.set)

Parametric coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -20.3088     0.2425  -83.75   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Approximate significance of smooth terms:
           edf Ref.df     F  p-value
s(x,y)   6.443     29 1.322 4.75e-08 ***
s(Depth) 3.611      9 4.261 1.49e-10 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

R-sq.(adj) =  0.141   Deviance explained = 37.8%
-REML = 389.86  Scale est. = 4.3516    n = 949
```

General strategy

For each response distribution and non-nested model structure:

1. Build a model with the smooths you want
2. Make sure that smooths are flexible enough (`k=...`)
3. Remove smooths that have been shrunk
4. Remove non-significant smooths

Comparing models

Nested vs. non-nested models

- Compare `~s(x)+s(depth)` with `~s(x)`
  - nested models
- What about `s(x) + s(y)` vs. `s(x, y)`
  - don't want to have all these in the model
  - not nested models

Measures of ”fit”

- Two listed in `summary`
  - Deviance explained
  - Adjusted R^2
- Deviance is a generalisation of R^2
- Highest likelihood value (*saturated* model) minus estimated model value
- (These are usually not very high for DSMs)

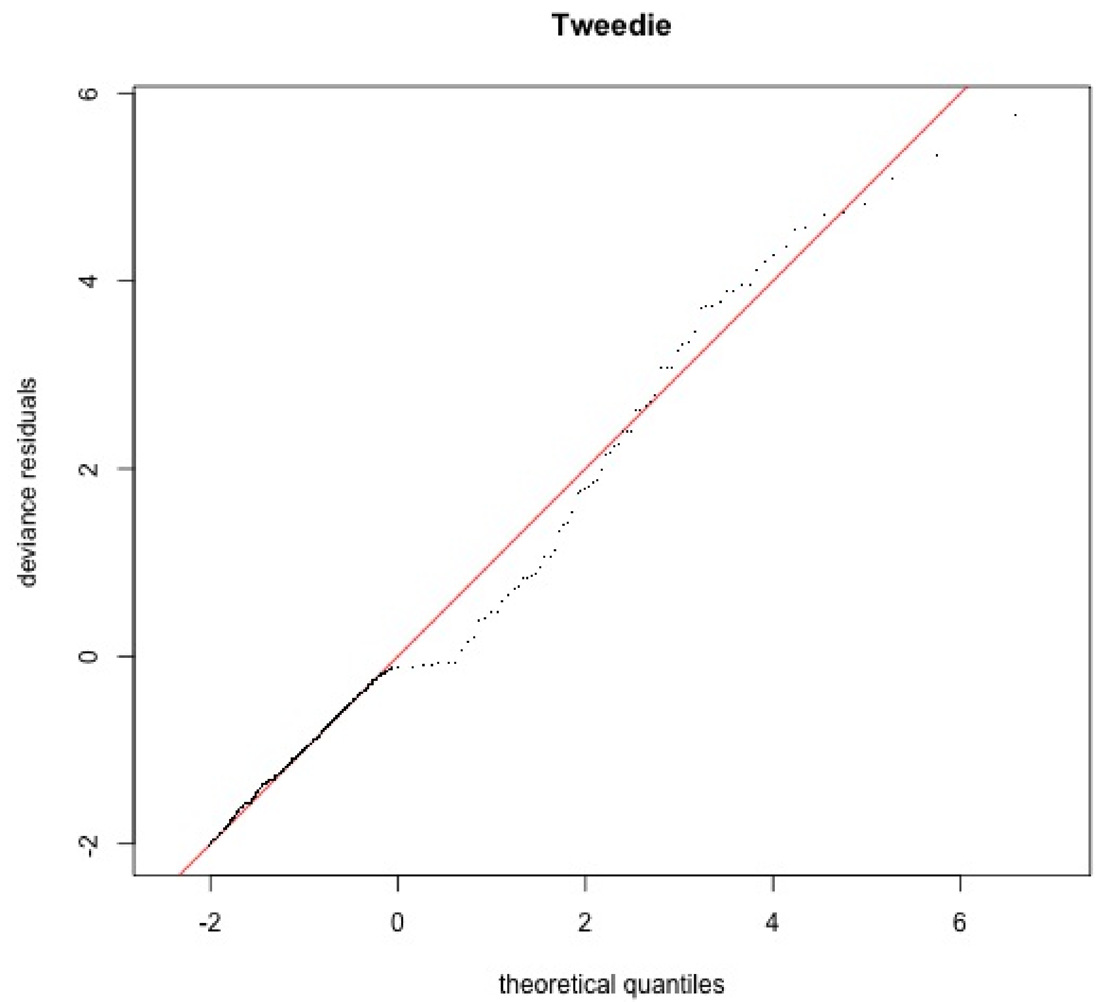
A quick note about REML scores

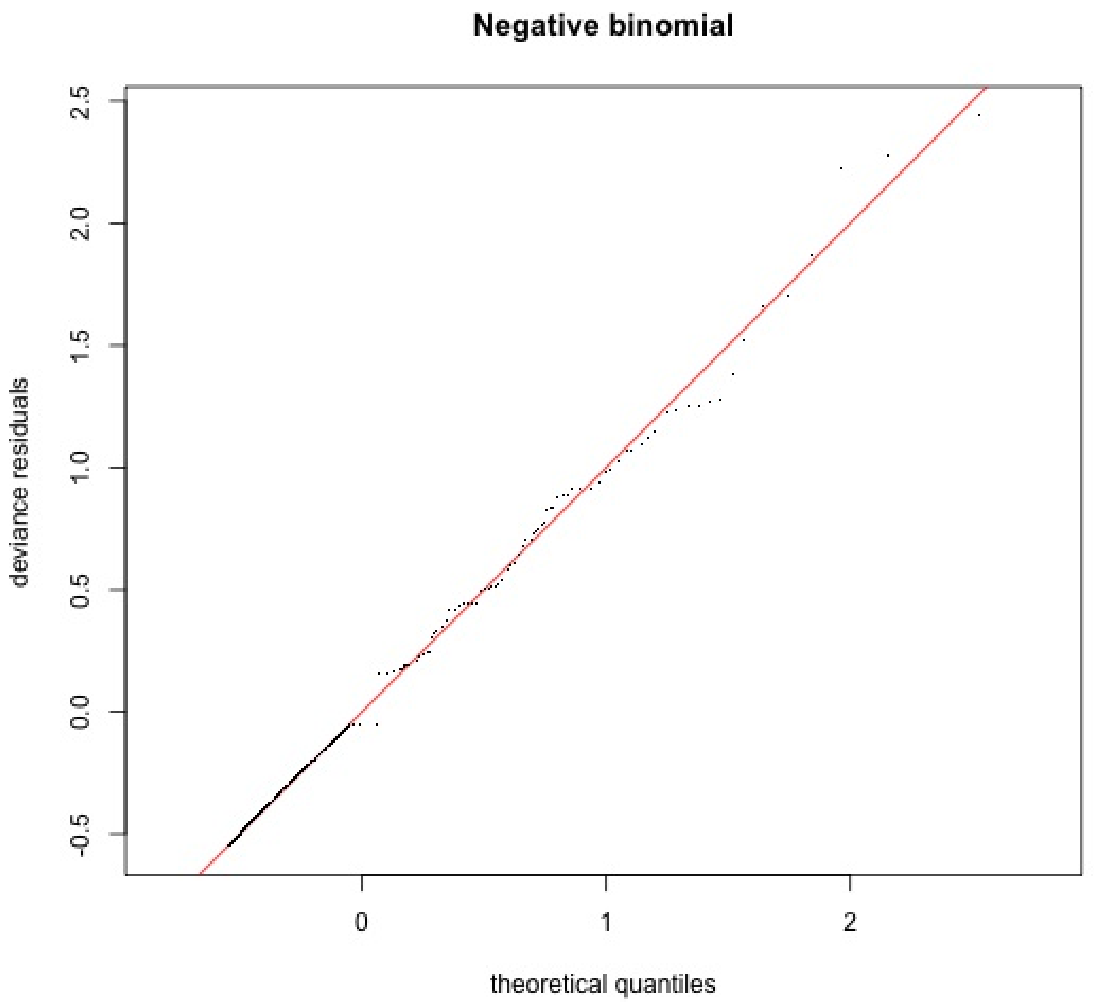
- Use REML to select the smoothness
- Can also use the score to do model selection
- **BUT** only compare models with the same fixed effects
  - (i.e. same “linear terms” in the model)
- $\Rightarrow$ **All terms** must be penalised (e.g. `bs="ts"`)
- Alternatively set `select=TRUE` in *gam()*

Selecting between response distributions

Goodness of fit tests

- Q-Q plots
- Closer to the line == better





Going back to concurvity

"How much can one smooth be approximated by one or more other smooths?"

Concurvity (model/smooth)

```
concurvity(dsm_all_tw)
```

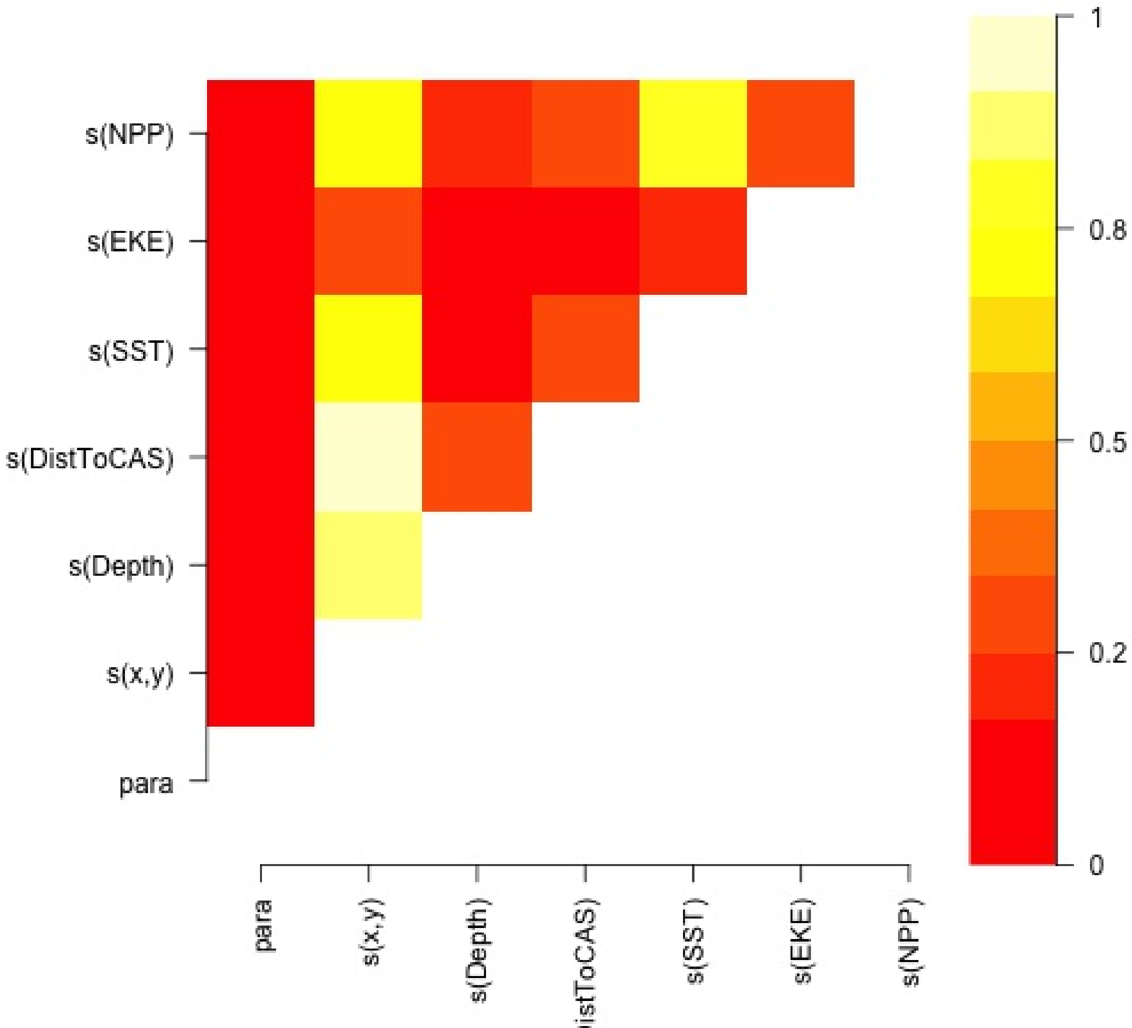
```
                    para    s(x,y)  s(Depth) s(DistToCAS)    s(SST)
s(EKE)
worst    2.539199e-23 0.9963493 0.9836597    0.9959057 0.9772853
0.7702479
observed 2.539199e-23 0.8571723 0.8125938    0.9882995 0.9525749
0.6745731
estimate 2.539199e-23 0.7580838 0.9272203    0.9642030 0.8978412
0.4906765
            s(NPP)
worst    0.9727752
observed 0.9483462
estimate 0.8694619
```

Concurvity between smooths

```
concurvity(dsm_all_tw, full=FALSE)$estimate
```

```
                    para       s(x,y)    s(Depth) s(DistToCAS)
para         1.000000e+00 4.700364e-26 4.640330e-28 6.317431e-27
s(x,y)       8.687343e-24 1.000000e+00 9.067347e-01 9.568609e-01
s(Depth)     1.960563e-25 2.247389e-01 1.000000e+00 2.699392e-01
s(DistToCAS) 2.964353e-24 4.335154e-01 2.568123e-01 1.000000e+00
s(SST)       3.614289e-25 5.102860e-01 3.707617e-01 5.107111e-01
s(EKE)       1.283557e-24 1.220299e-01 1.527425e-01 1.205373e-01
s(NPP)       2.034284e-25 4.407590e-01 2.067464e-01 2.701934e-01
                   s(SST)       s(EKE)       s(NPP)
para         5.042066e-28 3.615073e-27 6.078290e-28
s(x,y)       7.205518e-01 3.201531e-01 6.821674e-01
s(Depth)     1.232244e-01 6.422005e-02 1.990567e-01
s(DistToCAS) 2.554027e-01 1.319306e-01 2.590227e-01
s(SST)       1.000000e+00 1.735256e-01 7.616800e-01
s(EKE)       2.410615e-01 1.000000e+00 2.787592e-01
s(NPP)       7.833972e-01 1.033109e-01 1.000000e+00
```

Visualising concurvity between terms



- Previous matrix output visualised
- Diagonal/lower triangle left out for clarity
- High values (yellow) = BAD

Path dependence

Sensitivity

- General path dependency?
- What if there are highly concurve smooths?
- Is the model is sensitive to them?

What can we do?

- Fit variations excluding smooths
  - Concurve terms that are excluded early on
- Appendix of Winiarski et al (2014) has an example

Sensitivity example

- `s(Depth)` and `s(x, y)` are highly concurve (0.9067)
- Refit removing `Depth` first

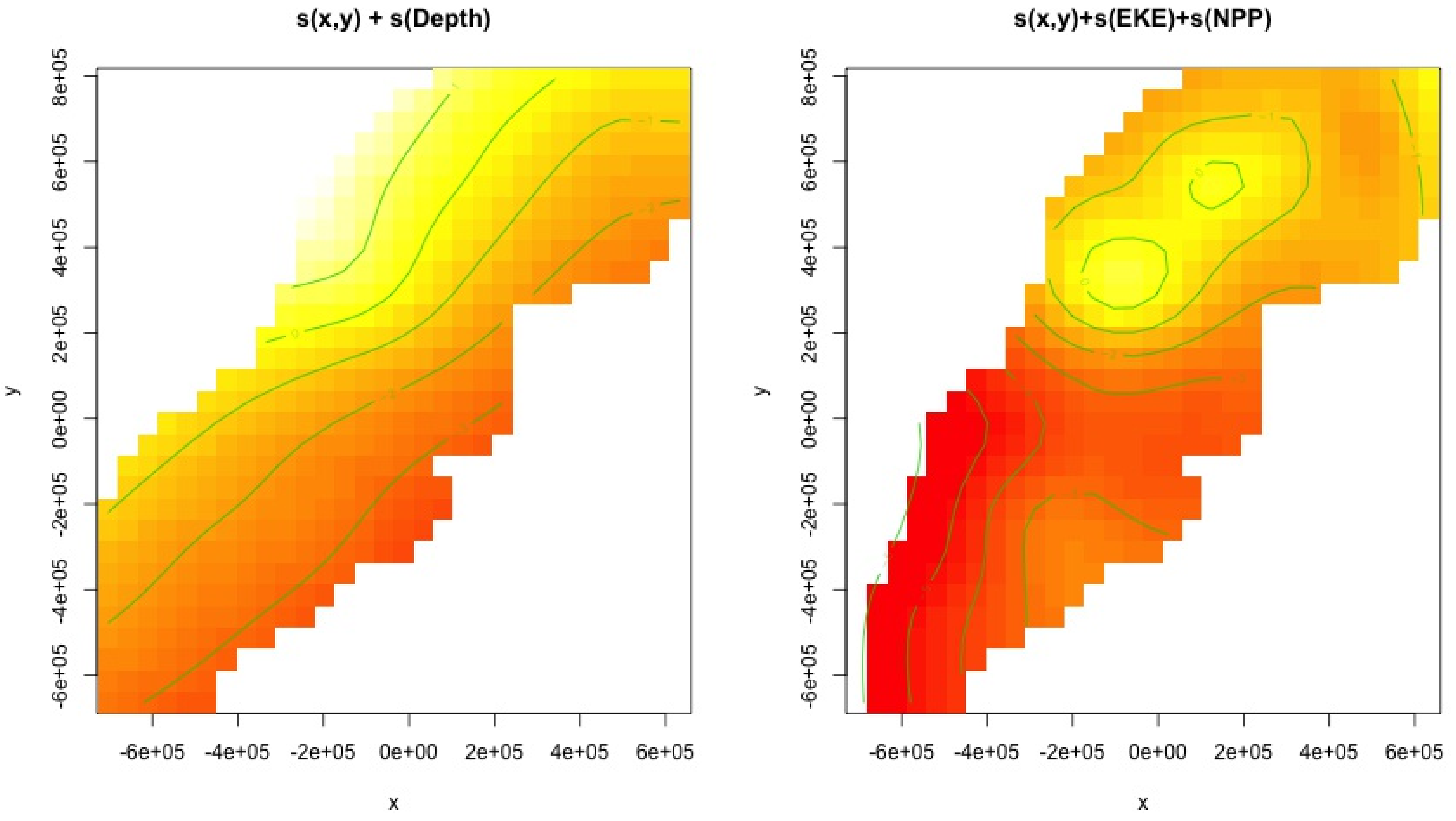
```
# with depth
```

```
                edf Ref.df        F      p-value
s(x,y)   6.442980     29 1.321650 4.754400e-08
s(Depth) 3.611038      9 4.261229 1.485902e-10
```

```
# without depth
```

```
                edf Ref.df         F      p-value
s(x,y) 13.7777929     29 2.5891485 1.161562e-12
s(EKE)  0.8448441      9 0.5669749 1.050441e-02
s(NPP)  0.7994168      9 0.3628134 3.231807e-02
```

Comparison of spatial effects



Sensitivity example

- Refit removing x and y…

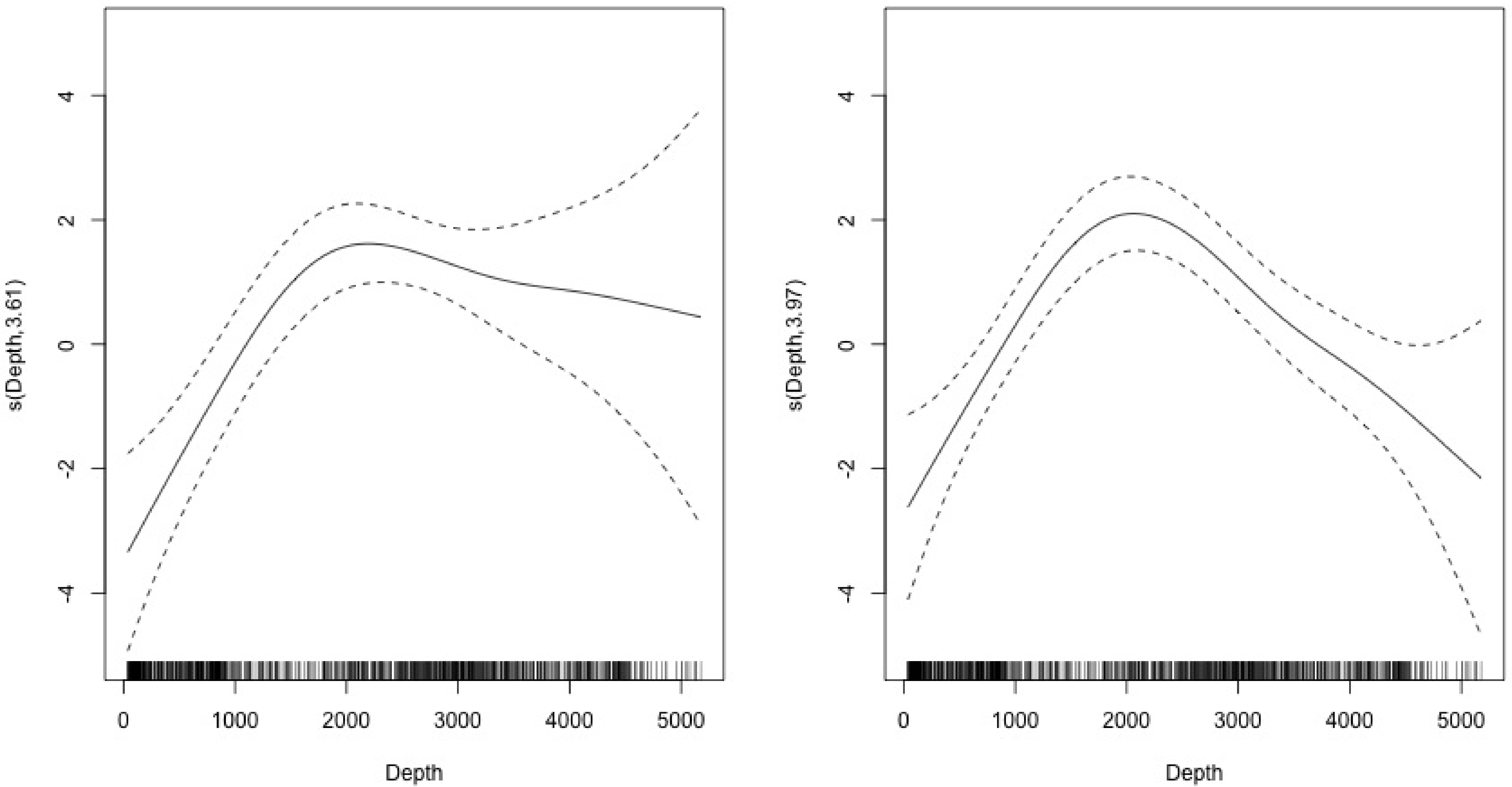
```
# without xy
```

```
                edf Ref.df        F      p-value
s(SST)   4.583260      9 3.244322 3.118815e-06
s(Depth) 3.973359      9 6.799043 4.125701e-14
```

```
# with xy
```

```
                edf Ref.df        F      p-value
s(x,y)   6.442980     29 1.321650 4.754400e-08
s(Depth) 3.611038      9 4.261229 1.485902e-10
```

Comparison of depth smooths



Comparing those three models...

Name	Rsq	Deviance
`s(x,y) + s(Depth)`	0.1411	37.82
`s(x,y)+s(EKE)+s(NPP)`	0.1159	34.40
`s(SST)+s(Depth)`	0.1213	35.76

- “Full” model still explains most deviance
- No depth model requires spatial smooth to “mop up” extra variation
- We'll come back to this when we do prediction

Recap

Recap

- Adding smooths
- Removing smooths
  - p-values
  - shrinkage
- Comparing models
- Comparing response distributions
- Sensitivity