$\lambda y.y$

Assignment 5

① ⓐ $(\lambda z.z)\ (\lambda y.yy)\ (\lambda x.xa)$

$(\lambda y.yy)\ (\lambda x.xa)$

$(\lambda x.xa)(\lambda x.xa)$

$(\lambda x.xa)\,a$

aa

ⓑ $(\lambda z.z)(\lambda z.zz)(\lambda z.zy)$

$(\lambda z.zz)\ (\lambda z.zy)$

$(\lambda z.zy)(\lambda z.zy)$

$(\lambda z.zy)\,y$

yy

ⓒ $(\lambda x.\lambda y.xyy)(\lambda a.a)b$

$(\lambda y.(\lambda a.a)yy)\,b$

[redacted]

$(\lambda a.a)bb$

bb

(d) $(\lambda x.\lambda y.xyy)(\lambda y.y)y$

~~$(\lambda x.\lambda y.xyy)$~~

$(\lambda x.\lambda a.xaa)(\lambda b.b)y$ Alpha equivalence

$(\lambda a.(\lambda b.b)aa)y$

$(\lambda b.b)yy$

yy

(e) $(\lambda x.xx)(\lambda y.yx)z$

$((\lambda y.yx)(\lambda y.yx))z$

$((\lambda y.yx)x)z$

xxz

(f) $(\lambda x.(\lambda y.(xy))y)z$

$(\lambda x.(\lambda a.(xa))y)z$ Alpha equivalence

$(\lambda a.(za))y$

zy

(g) $((\lambda x.xx)(\lambda y.y))(\lambda y.y)$

$(\lambda y.y)(\lambda y.y)(\lambda y.y)$

$(\lambda y.y)(\lambda y.y)$

$(\lambda y.y)$

(h) $(((\lambda x.\lambda y.(xy))(\lambda y.y))w)$

$(((\lambda x.\lambda a.(xa))(\lambda y.y))w)$ Alpha equivalence

$(\lambda a.(\lambda y.y)a)w$

~~$(\lambda a.a)(w)$~~ $(\lambda y.y)w$

w

(i) $(\lambda x.y)((\lambda y.yyy)(\lambda x.xxx))$

$(\lambda x.y)((\lambda a.aaa)(\lambda b.bbb))$ Alpha equivalence

$(\lambda x.y)((\lambda b.bbb)(\lambda b.bbb)(\lambda b.bbb))$

$(\lambda x.y((\lambda b.bbbbbbbbb$

$(\lambda x.y)((\lambda x.xxx)(\lambda x.xxx)(\lambda x.xxx))$

we can repeat the beta reduction process for x as many times as we wish.

0 -> λs.(λz.z) 1 -> λsz.s(z) 2 -> λsz.s(s(z))
λsz.z

Date

No.

Successor function -> λwyx.y(wyx)

Addition Example 2+3 (λsz.s(sz))(λwyx.y(wyx))
(λuv.u(u(uv))) 2+3

Multiplication function -> (λxyz.x(yz))
2·2 -> (λxyz.x(yz))22

T ≡ λxy.x -> True

F ≡ λxy.y -> False

AND function of two arguments
Λ ≡ λxy.xy(λuv.v) ≡ λxy.xyF

OR function of two arguments
V ≡

(2) (a) or false true = true

or = λx.λy.((x true)y)
true = λx.λy.x
false = λx.λy.y

or false true false true
(λx.λy.((x true)y))(λx.λy.y)(λx.λy.x)
λy.((false true)y) true
= (false true) true
= ((λx.λy.y) true) true
= (λy.y) true
= true

(b) [redacted] +22=4

1 = λf.λy.fy
2 = λf.λy.f(fy)
3 = λf.λy.f(f(fy))
4 = λf.λy.f(f(f(fy)))

N1+N2 = λx.λy.(N1 x)((N2 x)y)

= λx.λy.(2 x)((2 x)y)
= λx.λy.((λf.λy.f(fy))x)((2x)y)
= λx.λy.(λy.x(xy))((2x)y)
= λx.λy.(λy.x(xy))((λy.x(xy))y)
= λx.λy.(λy.x(xy))(λy.x(xy))
= λx.λy.x(x(xy)) Alpha equi
= λf.λy.f(f(fy))
= 4

(c) succ 2 = 3

$2 = \lambda f.\lambda y.f(fy)$

$3 = \lambda f.\lambda y.f(f(fy))$

$succ = \lambda z.\lambda f.\lambda y.f(zfy)$

succ 2

$(\lambda z.\lambda f.\lambda y.f(zfy))\,2$

$\lambda f.\lambda y.f(2fy)$

$\lambda f.\lambda y.f((\lambda f.\lambda y.f(fy))fy)$

$\lambda f.\lambda y.f((\lambda y.f(fy))y)$

$\lambda f.\lambda y.f(f(fy)) = 3$ //