

RECURSION - CLASS 1

Da /

Recursion :-

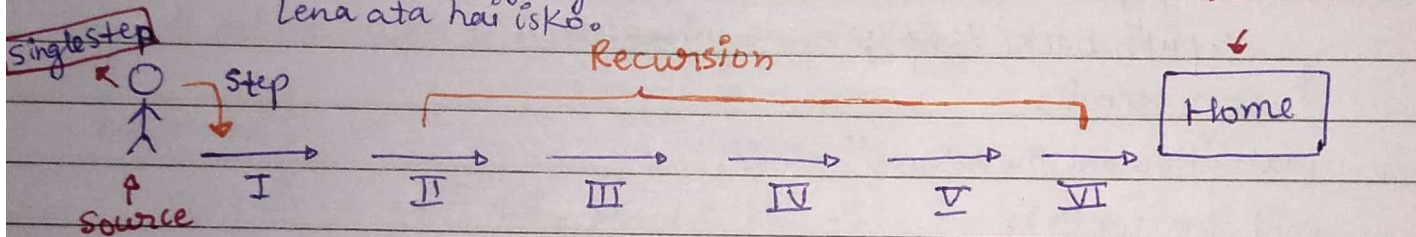
↳ Bookish Term → when a function calls itself directly/indirectly

↳ In-depth → Solution of Bigger Problem depends → Solution of chhoti problem of same type.

∴ We can apply recursion in this case.

↳ Bhaiya ki Line → 1 case turn solve Karo, baki recursion sambhal lega

Problem statement - Isko ghr jaha hai but ek time pr single step lena ata hai isko.



Bigger Problem

$$\text{Step}(6) = 1 + \text{step}(5)$$

∴ we can apply recursion in this case.

Single step

Recursion

Problem Statement -

$$\text{solve}(n) \xrightarrow{\text{find}} 2^n$$

Bigger → 2^n
Problem

$$\text{solve}(n) = 2^n$$

$$\text{solve}(n) = 2 \times 2^{n-1}$$

$$\begin{aligned} \text{∴ solve}(n-1) \\ = 2^{n-1} \end{aligned}$$

$$\text{Solve}(n) = 2 \times \text{solve}(n-1)$$

Big Problem

Choti Problem

∴ same type of problems so we can apply recursion here.

Spiral

Date / /

Ps → solve(n) → (n → 1) counting print

i.e. solve(n-1) → (n-1 → 1) counting print krega

Solve(n) → n, n-1, n-2, ..., 1

Solve(n) = n, (n-1, n-2, ..., 1)
Recursion

Big Problem → Solve(n) = n, solve(n-1) → small problem of same type

∴ we can apply Recursion in this case

→ Factorial

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$

Solve(5) = 5!

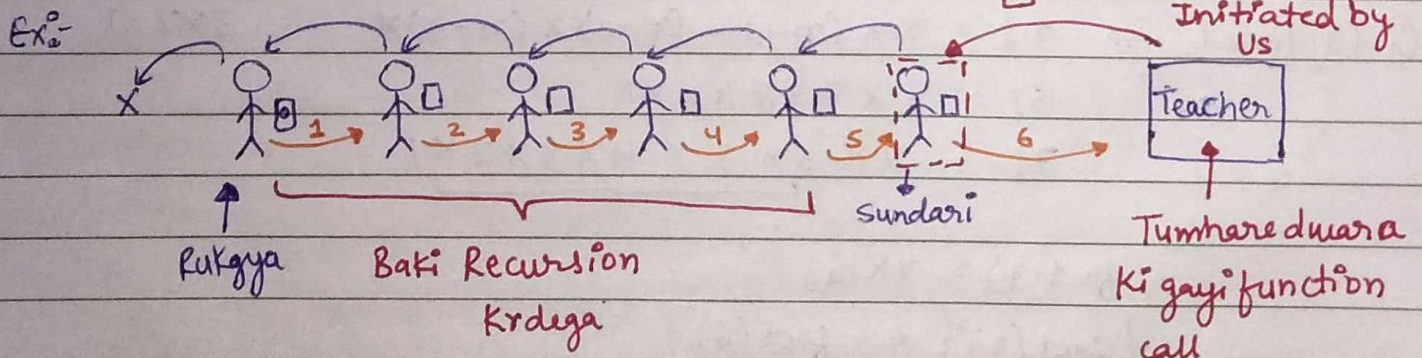
Solve(5) = 5 × 4 × 3 × 2 × 1
4!

Solve(4) = 4! = 4 × 3 × 2 × 1

Solve(5) = 5 × 4!

Big Problem → Solve(5) = 5 × solve(4) → Ye Recursion Krdega
Ye hum Krnge
choti problem

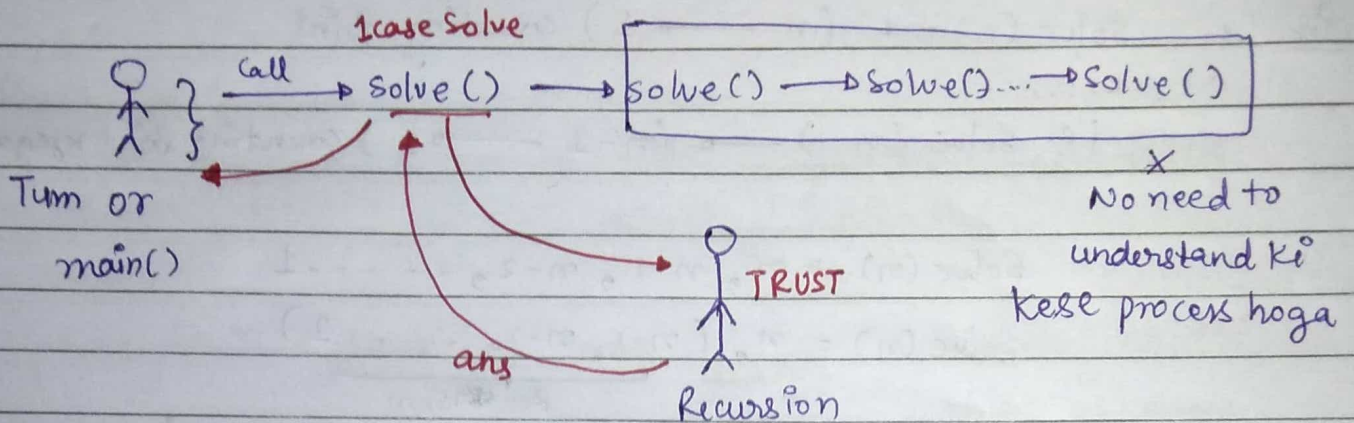
Rule → if piche → Present
↳ pass the paper
→ if → piche → absent
↳ ruk jao



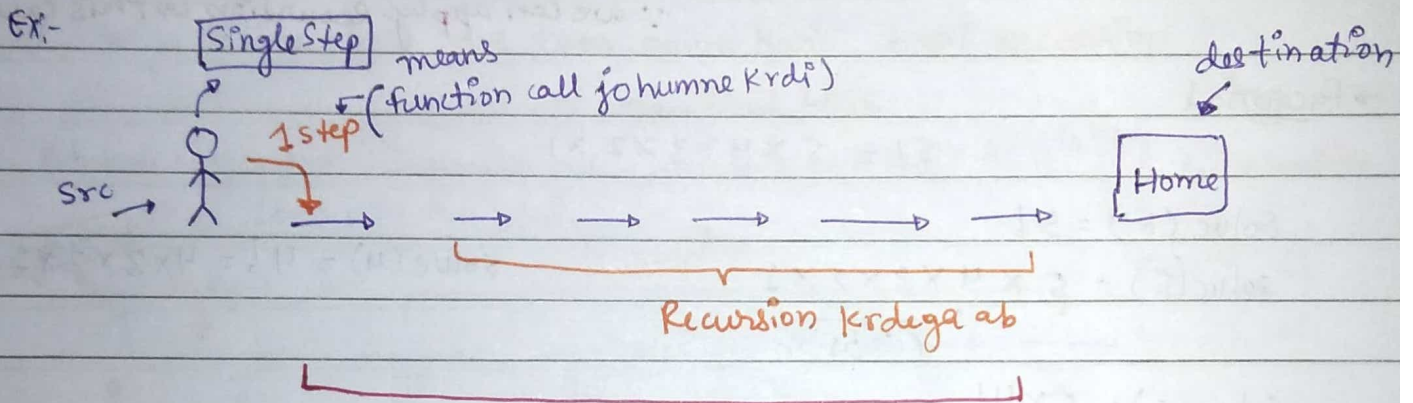
Rule → Aage wale ko
is blank paper pe +1 likh
Kaledo

Teacher ne sundari
ko paper diya tha to
usse hi wapis chahiye !!

Piche k process se isko ko
lena dena nhi !! Spiral



∴ Recursion pe trust rakho ki ye ans laake dega bs.



$$\begin{aligned} \text{Overall Ans} &= 1 + \text{Recursion Ans} \\ \text{Big} &= 1 + \text{choti} \end{aligned}$$

Q Factorial → $n! = n \times (n-1) \times (n-2) \times (n-3) \times \dots \times 3 \times 2 \times 1$

$5! = 5 \times 4 \times 3 \times 2 \times 1$

$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$\text{fact}(7) = 7 \times 6!$

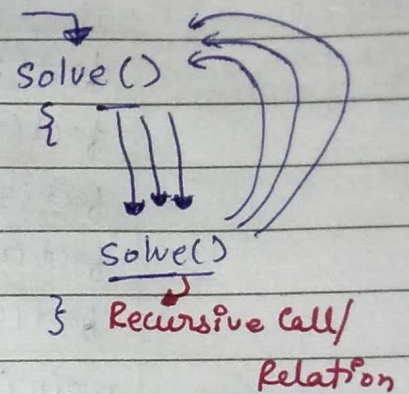
$\text{fact}(7) = 7 \times \text{fact}(6)$

$\text{fact}(n) = n \times \text{fact}(n-1)$

Date / ... /

Parts of Recursive code :-

- Base Case → mandatory
 - ↳ To stop the recursion
- Recursive Call → mandatory
or
Recursive Relation
- Processing → optional



code ⇒ `int factorial(int n) {`

`// base case`

`if (n == 1) return 1;`

`if (n == 0) return 1;`

`// Processing`

`// Recursive Relation`

`int recursionKaAns = factorial(n-1);`

`// Processing`

`int finalAns = n * recursionKaAns;`

`return finalAns;`

`}`

output - 120

`int main() {`

`cout << factorial(5);`

`return 0;`

`}`

start
`fact(5)` $n=5$

`if (n == 0 || n == 1) → x`

`return 1;`

`int recAns = fact(n-1);`

`int finalAns = n * recAns;`

`return finalAns;`

5 × 24

120

(I)

`fact(4)`

$n=4$

`if (n == 0 || n == 1) → x`

`return 1;`

`int recAns = fact(n-1);`

`int finalAns = n * recAns;`

`return finalAns;`

4 × 6

24

(B)

`fact(3)`

$n=3$

`if (n == 0 || n == 1) → x`

`return 1;`

`int recAns = fact(n-1);`

`int finalAns = n * recAns;`

`return finalAns;`

3 × 2

6

(C)

`fact(2)`

$n=2$

`if (n == 0 || n == 1) → x`

`return 1;`

`int recAns = fact(n-1);`

`int finalAns = n * recAns;`

`return finalAns;`

2 × 1

2

(D)

`fact(1)`

`if (n == 0 || n == 1) → ✓`

`return 1;`

`int recAns = fact(n-1);`

`int finalAns = n * recAns;`

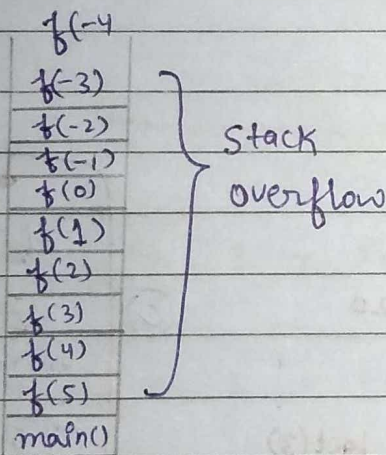
`return finalAns;`

Spiral

			Popped from stack and returned 1
fact(1)	fact(1)		
fact(2)	fact(2)		Popped & returned 2
fact(3)	fact(3)		Popped & returned 6
fact(4)	fact(4)		Popped & returned 24
fact(5)	fact(5)		Popped & returned 120
main()	main()		

∴ function call stack

→ If we don't apply Base Case, then stack will filled with calls and it arise stack overflow condition and memory gets occupied by the program.



Q i/p $\rightarrow n \rightarrow$ print counting from "n" to "1"

$f(n) = n$ to 1

$f(n-1) = n-1$ to 1

void printCounting (int n) {

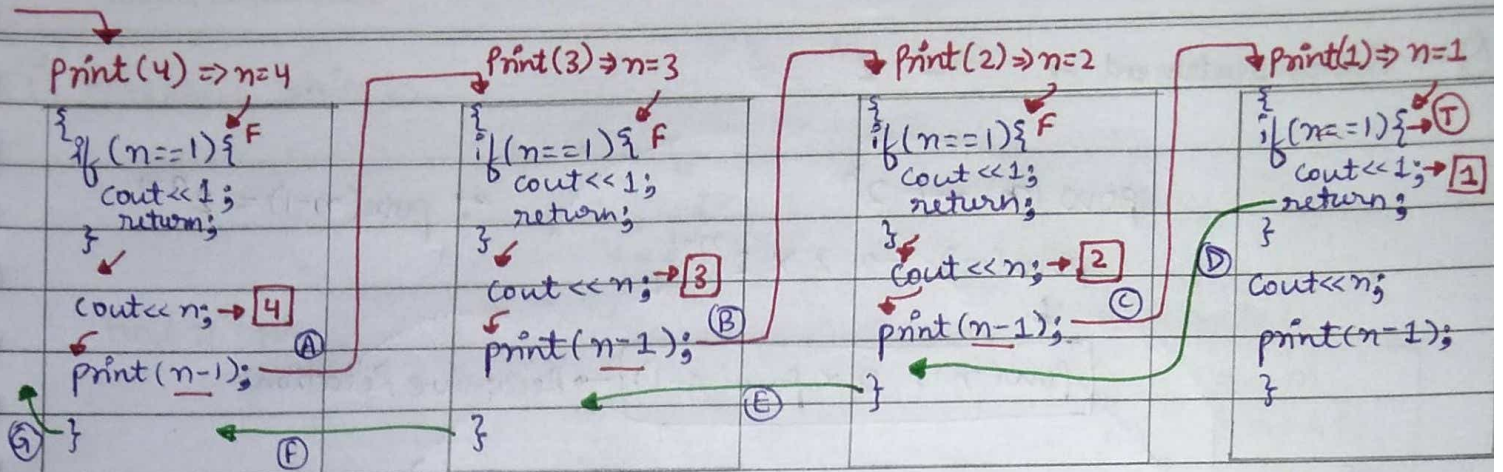
Base Case \rightarrow if (n==1) { //B.C \rightarrow if (n==0) return;

count << 1;
return;

Processing \rightarrow { count << n << " ";

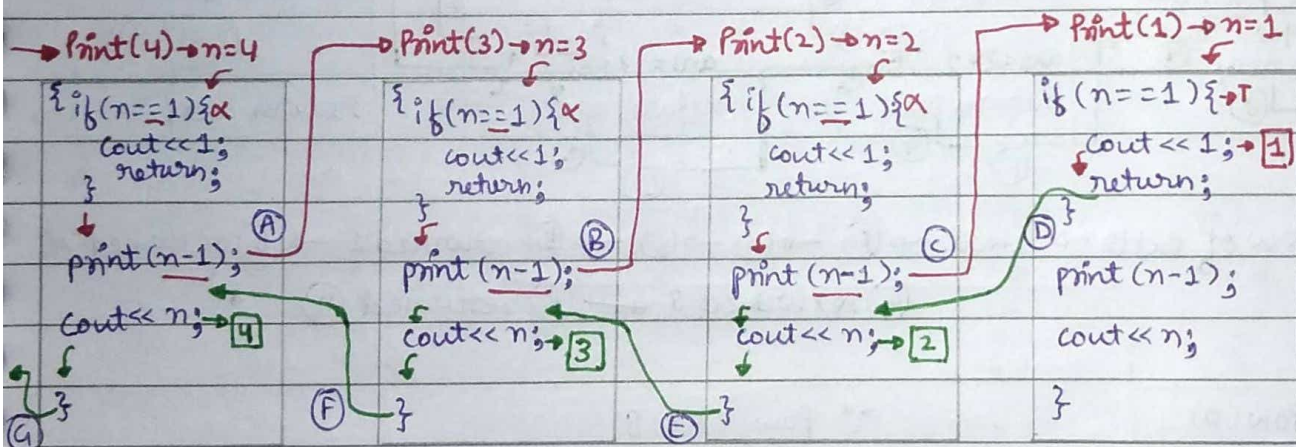
Recursive Call \rightarrow printCounting(n-1);

}



function	Print(1)	\therefore It will print \rightarrow 4 3 2 1
call	Print(2)	and popped out accordingly.
stack	Print(3)	
	Print(4)	\therefore Here, the function is ending with Recursive Relation. So, it's called Tail Recursion.
	main()	

\rightarrow when we change the order of Processing and Recursive Relation.



\therefore It will print \rightarrow 1 2 3 4

function	Print(1)	$\&$ popped out
call	Print(2)	
stack	Print(3)	\therefore Here, the Recursive Relation is before processing
	Print(4)	it's called Head Recursion.
	main()	

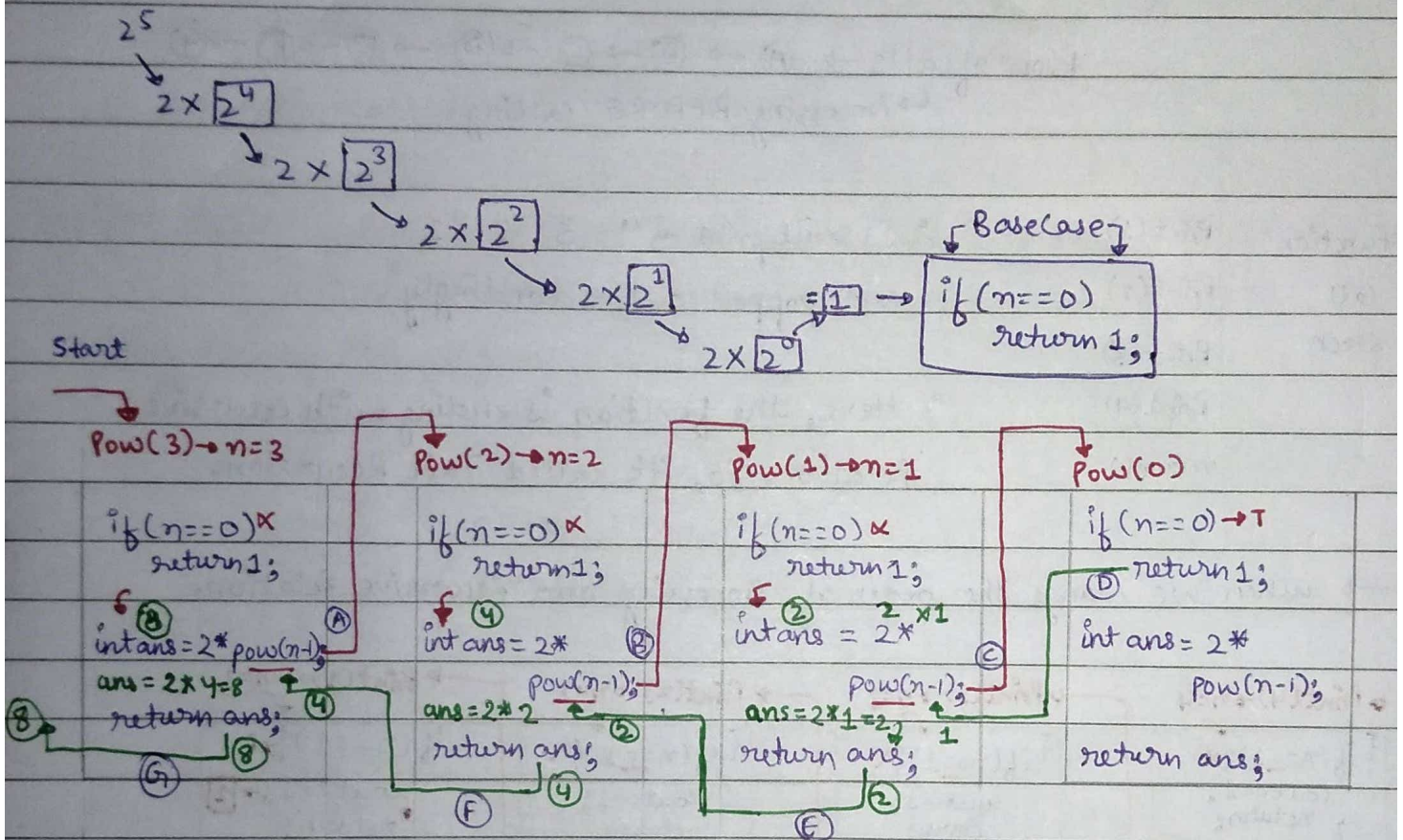
Q → Problem Statement → find 2^n

$$\text{pow}(n) \rightarrow 2^n$$

$$\because \text{pow}(n-1) = 2^{n-1}$$

$$\text{pow}(n) \rightarrow 2 \times 2^{n-1}$$

$$\boxed{\text{pow}(n) = 2 \times \text{pow}(n-1)} \rightarrow \text{Recursive Relation}$$



Flow of calls → start → (A) → (B) → (C) → (D) returned 1 → (E) returned 2
(F) returned 4 → (G) returned 8

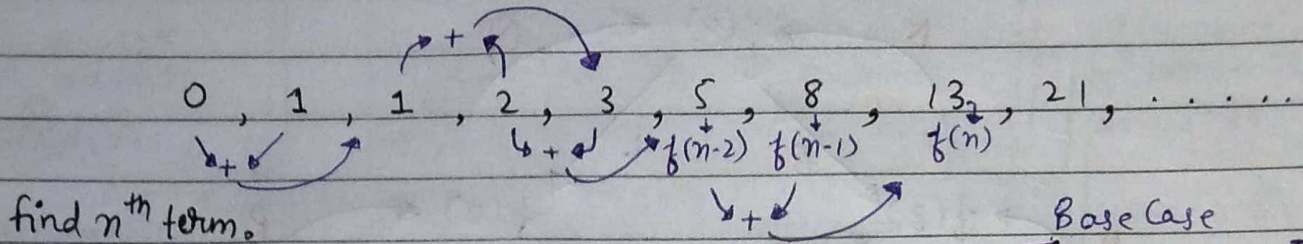
Pow(0)
Pow(1)
Pow(2)
Pow(3)
main()

$$\because \text{Pow}(3) = 8$$

↳ Each call will pop out after returning particular ans.

function call stack

Q Fibonacci Series:-



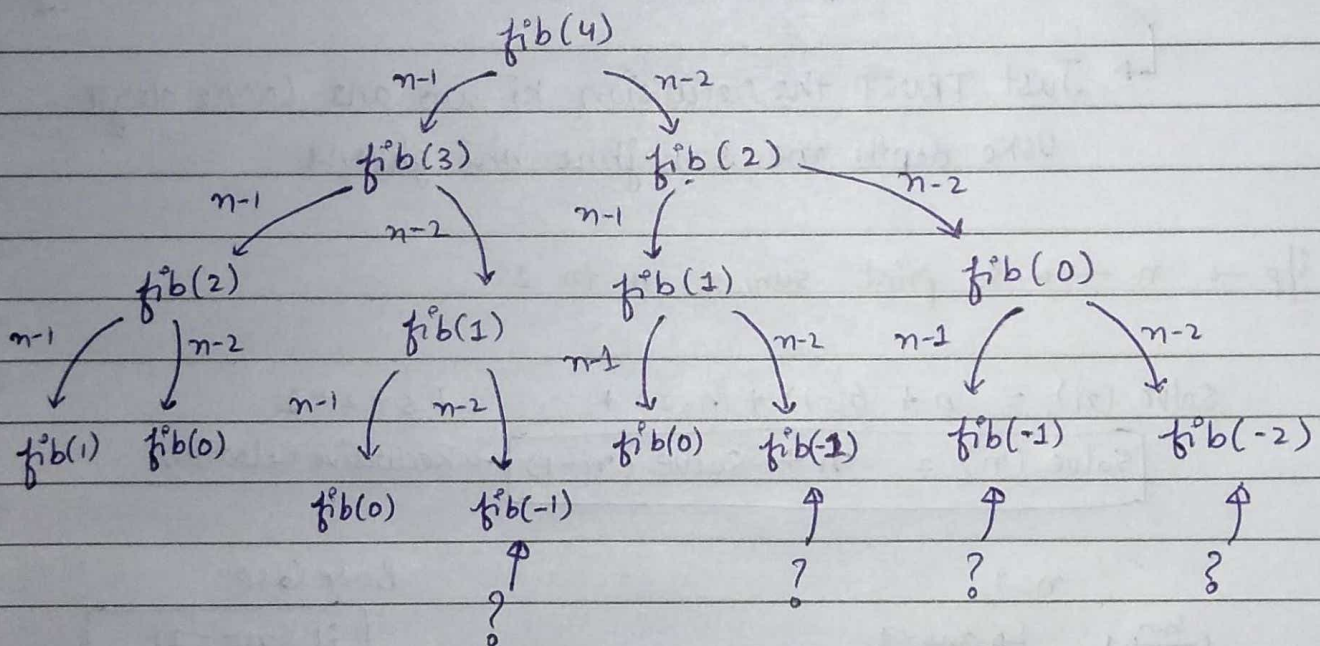
Code:-

```

void fib(int n) {
    // Base Case
    if (n == 0) return 0;
    if (n == 1) return 1;
    // R.R
    int ans = fib(n-1) + fib(n-2);
    return ans;
}

```

Recursion Tree



$\text{fib}(0)$ ke niche ans kaise nikalu nhi pta
or

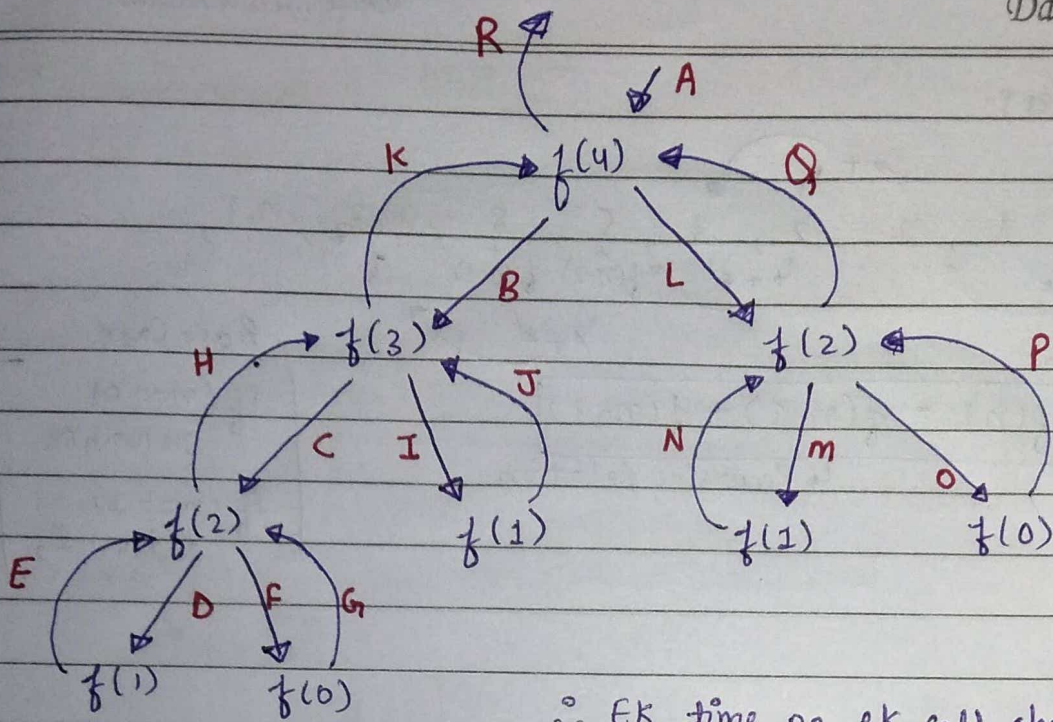
$\text{fib}(1)$ ke niche kaise nikalu nhi pta so yahi hai Base Case, inki values pta honi chahiye.

```

if (n == 0)
    return 0;
if (n == 1)
    return 1;

```

Base Case



∴ Ek time pe ek call chalegi

→ [To understand Recursion, you need to first understand Recursion]

↳ Just TRUST the recursion ki wo ans Laake dega
Usko depth me samajhne nhi ghusna

Q i/p → n → so print sum of n to 1

$$\text{Solve}(n) = n + (n-1) + (n-2) + \dots + 3 + 2 + 1$$

$$\boxed{\text{Solve}(n) = n + \text{Solve}(n-1)} \rightarrow \text{Recursive Relation}$$

$$n=1$$

$$1 \xrightarrow{\text{sum}} 1 \quad \text{Ans} = 1$$

Base Case

$$\boxed{\text{if } (n == 1)}$$

return 1;

Code :-

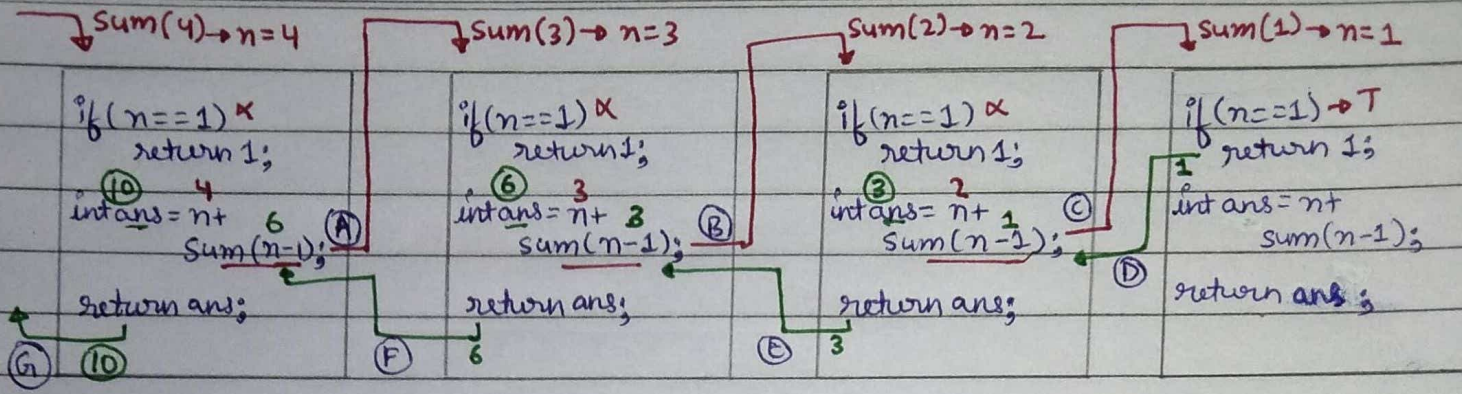
```
int sum(int n) {
```

```
    // Base Case
```

```
    if (n == 1)
        return 1;
```

```
    int ans = n + sum(n-1);
```

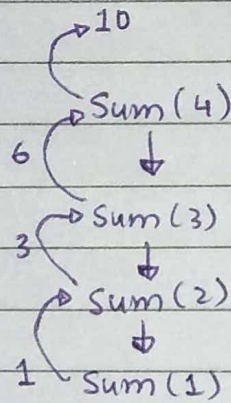
```
    return ans;
```

$$\text{Sum}(4) = 10 \quad (1 + 2 + 3 + 4)$$

• All entries will be pop out after returning particular value.

	Sum(1)
function	Sum(2)
call	Sum(3)
stack	Sum(4)
	main()



Recursion
Tree