~~as highly complementary to their own, but it is distinct in several ways. For the most part, we focus on the implications of illiquidity for durable expenditures rather than for non-durable spending because durable spending is substantially more important for understanding business cycle behavior. Since our motivation is understanding how micro consumption dynamics influence aggregate business cycles, our model also features a variety of aggregate shocks, and we explore the implications of general equilibrium.~~

~~Finally, our paper is closely related to Caballero, Engel, and Haltiwanger (1995, 1997) and Bachmann, Caballero, and Engel (2013), which argued for time-varying responsiveness arising from lumpy firm behavior. Besides the obvious difference that we study households rather than firms, there are several distinctions between our analyses. In Caballero, Engel, and Haltiwanger (1995, 1997), they imputed capital and employment gaps and explored their aggregate implications. Their gap imputation relies on an assumption that firms' reset targets follow a random walk, while our procedure requires no such assumption. Bachmann, Caballero, and Engel (2013) built a quantitative GE model of firm investment and targeted various aggregate time-series facts to address concerns that these early papers were not robust to general equilibrium and lacked quantitative realism. However, they did not test their model implications in micro data.[12]~~

~~To summarize, our analysis overlaps in part with many papers, but we believe we are the first paper to jointly explore the micro and macro implications of household durable adjustment in an estimated, structural GE model. We believe the synthesis of microdata, structural modeling, and general equilibrium is important for providing an accurate assessment of the impact of policy changes.~~

2. MODEL AND ESTIMATION

2.1. *Model Description*

Our baseline model for estimation is a standard incomplete markets model with the addition of household durable consumption subject to fixed costs of adjustment. Households maximize expected discounted utility of a consumption aggregate, and they are subject to idiosyncratic earnings shocks as well as borrowing constraints. In this section, we describe the partial equilibrium version of the model with no aggregate shocks, and in the following sections, we discuss the addition of aggregate shocks: first in partial and then in general equilibrium.

[12]The literature on firm lumpiness must also contend with issues that are not present in our household environment. In particular, it can make a large quantitative difference whether these models are calibrated to match firm versus establishment moments, and it is not clear what level of aggregation corresponds to an economic decision maker. In contrast, for household level durable adjustment, the correct level of aggregation does not have any such ambiguity.

Households solve:

$$\max_{c_t^i, d_t^i, a_t^i} E \sum \beta^t \left(\frac{\left[(c_t^i)^v (d_t^i)^{1-v} \right]^{1-\gamma} - 1}{1-\gamma} \right),$$

s.t.

$$c_t^i = wh\eta_t^i(1-\tau) + (1+r)a_{t-1}^i + d_{t-1}^i(1-\delta_d) - d_t^i - a_t^i - A(d_t^i, d_{t-1}^i),$$

$$a_t^i \geq -(1-\theta)d_t^i;\ d_t^i \geq 0,$$

$$\log \eta_t^i = \rho_\eta \log \eta_{t-1}^i + \varepsilon_t^i \quad \text{with} \quad \varepsilon_t^i \sim N(0, \sigma_\eta),$$

where c_t^i, d_t^i, and a_t^i are household i's non-durable consumption, durable stock, and liquid assets, respectively. The parameter β is the quarterly discount factor, v is the relative weight on non-durable consumption in period utility, and $1/\gamma$ is the intertemporal elasticity of substitution.[13] η_t^i represents shocks to idiosyncratic labor earnings, h is a household's fixed[14] hours of work, while w and r are the aggregate wage and interest rate, δ_d is the depreciation rate of durables, and τ is a proportional payroll tax. Finally, $A(d_t^i, d_{t-1}^i)$ is the fixed adjustment cost that households face when adjusting their durable stock. We assume that A takes the form

$$A(d, d_{-1}) = \begin{cases} 0 & \text{if } d = \left[1 - \delta_d(1-\chi)\right] d_{-1}, \\ F^d(1-\delta_d)d_{-1} + F^t wh\eta_t^i & \text{else.} \end{cases}$$

Following Bachmann, Caballero, and Engel (2013), $0 \leq \chi \leq 1$ is a "required maintenance" parameter. Positive values of χ represent the fact that some maintenance is required to continue enjoying the flows from durable consumption, for example, fixing a flat tire on a car or fixing a broken furnace in a house.[15] When a household adjusts its durable stock, it must pay fixed adjustment costs that take two forms. First, they lose a fixed fraction of the value of their durable stock. These costs correspond to brokers' fees, titling costs, etc. Second, households face some time cost of adjusting their durable holdings. These costs correspond to, for example, the time involved in searching for a new house or in researching which car to purchase. We allow for this

[13] Piazzesi and Schneider (2007) provided some evidence in favor of the Cobb–Douglas period utility function. Note the Cobb–Douglas utility function also means we can normalize the service flows from durables to be equal to the stock without loss of generality.

[14] Endogenizing hours complicates the model and does not affect our main conclusions.

[15] In previous versions of this paper, we considered an adjustment cost function that allowed households to endogenously choose the amount of maintenance between 0 and 1 without paying the fixed adjustment cost. This led to similar results but substantially increases the computational burden of the model, which makes estimation infeasible.

general specification because these two adjustment costs may interact differently with the business cycle. The opportunity cost of time is procyclical, so that time costs will tend to generate countercyclical durable adjustment. Conversely, fixed costs that are proportional to the stock of durables have the most bite when income is low and tend to generate procyclical durable adjustment. Estimating a specification with both costs allows the data to inform their relative importance.

Given these assumptions, the infinite horizon problem can be recast recursively as

$$V(a_{-1}, d_{-1}, \eta) = \max\bigl[V^{\text{adjust}}(a_{-1}, d_{-1}, \eta), V^{\text{noadjust}}(a_{-1}, d_{-1}, \eta)\bigr],$$

with

$$\begin{aligned}
&V^{\text{adjust}}(a_{-1}, d_{-1}, \eta) = \max_{c,d,a} \frac{\bigl[c^{v} d^{1-v}\bigr]^{1-\gamma}}{1-\gamma} + \beta E_{\varepsilon} V\bigl(a, d, \eta'\bigr), \\
&\text{s.t.} \\
&c = wh\eta(1-\tau) + (1+r)a_{-1} + d_{-1}(1-\delta_d) \\
&\qquad - d - a - F^{d}(1-\delta_d)d_{-1} - F^{t}wh\eta, \\
&a > -(1-\theta)d, \\
&\log\eta' = \rho_\eta \log\eta + \varepsilon \quad \text{with} \quad \varepsilon \sim N(0, \sigma_\eta), \\
&V^{\text{noadjust}}(a_{-1}, d_{-1}, \eta) \\
&\quad = \max_{c,a} \frac{\bigl[c^{v} d^{1-v}\bigr]^{1-\gamma}}{1-\gamma} + \beta E_{\varepsilon} V\bigl(a, d_{-1}\bigl(1-\delta_d(1-\chi)\bigr), \eta'\bigr), \\
&\text{s.t.} \\
&c = wh\eta(1-\tau) + (1+r)a_{-1} - \delta_d \chi d_{-1} - a, \\
&a > -(1-\theta)d, \\
&\log\eta' = \rho_\eta \log\eta + \varepsilon \quad \text{with} \quad \varepsilon \sim N(0, \sigma_\eta).
\end{aligned}$$

We now turn to a discussion of how we estimate the parameters of the model. The computational solution of the model is discussed in Appendix D.

~~2.2. *Estimation*~~

~~To decrease computational burden, our estimation procedure proceeds in two steps: we first calibrate some subset of parameters for which we have reliable external evidence. We then estimate the remaining parameters using an indirect inference procedure, which we describe shortly.~~

~~liquidity is amplified in her model by rather volatile taste shocks for goods that can only be paid for with cash (e.g. many home and auto repairs). It is naturally hard to identify these fundamentally unobserved shocks and their size in the data. A more serious empirical problem is that the use of credit cards has become much more widespread in the last 20 years; in the model this should imply a fall in the size of the puzzle group not seen in the data. Adding a (costly) cash-out option on the credit card to the model, as is now common, could also further reduce the implied size of the puzzle group. In total, this demand for cash might certainly be a contributing factor, but it seems unlikely that it is the central explanation of the credit card debt puzzle. Finally, note that in a model with both a Hicksian motive for holding liquid assets and a precautionary borrowing motive, the two would reinforce each other.~~

3 Model

3.1 Bellman Equation

We consider potentially infinitely lived households characterized by a vector, $\mathbf{S}_t$, of the following state variables: end-of-period gross debt (D_{t-1}), end-of-period gross assets (A_{t-1}), market income (Y_t), permanent income (P_t), an unemployment indicator, $u_t \in \{0,1\}$, and an indicator for whether the household is currently excluded from new borrowing, $x_t \in \{0,1\}$. In each period the households choose *consumption*, C_t, and *debt*, D_t, to maximize expected discounted utility.

Postponing the specification of the exogenous and stochastic income process to section 3.3, the household optimization problem is given in recursive form by

$$
\begin{aligned}
V(\mathbf{S}_t) &= \max_{D_t, C_t} \frac{C_t^{1-\rho}}{1-\rho} + \beta \cdot \mathbb{E}_t\left[V(\mathbf{S}_{t+1})\right] && (3.1) \\
&\text{s.t.} \\
A_t &= (1+r_a) \cdot A_{t-1} + Y_t - C_t && (3.2) \\
&\quad -\underbrace{r_d \cdot D_{t-1}}_{\text{interest}} - \underbrace{\lambda \cdot D_{t-1}}_{\text{installment}} + \underbrace{(D_t - (1-\lambda) \cdot D_{t-1})}_{\text{new debt}} \\
N_t &= A_t - D_t && (3.3) \\
D_t &\leq \max\left\{\underbrace{(1-\lambda) \cdot D_{t-1}}_{\text{old contract}}, \mathbf{1}_{x_t=0} \cdot \underbrace{(\eta \cdot N_t + \varphi \cdot P_t)}_{\text{new contract}}\right\} && (3.4) \\
A_t, D_t, C_t &\geq 0 && (3.5)
\end{aligned}
$$

where ρ is the risk aversion coefficient, β is the discount factor, r_a is the (real) interest rate on *assets*, r_d is the (real) interest rate on *debt* and $\lambda \in [0,1]$ is the *minimum payment due rate*. Equation (3.2) is the budget constraint, (3.3) defines end-of-period (financial) net worth, and (3.4) is the borrowing constraint. The model is closed by assuming that the households are required to "die without debt" (i.e $N_T \geq 0$ in some infinitely distant terminal period $T \to \infty$). We only cover the case $r_d > r_a$. We denote the optimal debt and consumption functions by $D^\star(\mathbf{S}_t)$ and $C^\star(\mathbf{S}_t)$.

We assume that x_t transitions according to a first order Markov process. The (unconditional) risk of losing access to the credit market is given by $\pi_{x,*}^{lose}$, and the chance of re-gaining access is given by $\pi_{x,*}^{gain}$. Conditional on unemployment we assume that the risk of losing access to the credit market is given by $\pi_{x,u}^{lose} = \chi_{lose} \cdot \pi_{x,w}^{lose}$, where $\pi_{x,w}^{lose}$ is the risk of losing access conditional on employment (in our calibration we choose χ_{lose} and let $\pi_{x,w}^{lose}$ adjust to match the chosen unconditional transition probabilities).

3.2 The Borrowing Constraint

Our specification of the debt contract is obviously simplistic, but it serves our purpose, and only add one extra state variable to the standard model. If $\eta > 0$ asset-rich households are allowed to take on more debt even though there is no formal collaterization. We allow *gearing* in this way to be as general as possible, and we use end-of-period timing and update the effect of income on the borrowing constraint period-by-period following the standard approach in buffer-stock models.[8]

The crucial departure from the canonical buffer-stock model is that we assume that the debt contract is *partially irrevocable from the lender side.* This provides the first term ("old contract") in the maximum operator in borrowing constraint (3.4), implying that the households can always continue to borrow up to the *remaining principal* of their current debt contract (i.e. $(1-\lambda) \cdot D_{t-1}$). The second term ("new contract") is a more standard borrowing constraint and only needs to be satisfied if the households want to take on *new debt* ($D_t > (1-\lambda) \cdot D_{t-1}$). Hereby current debt can potentially relax the households borrowing constraint in future

[8] Note than a borrowing constraint such as $D_t \leq A_t + \alpha \cdot P_t$ would be problematic because it would allow the households to take on infinitely much debt for a given level of consumption. A similar problem would also arise with $D_t \leq A_{t-1} + \alpha \cdot P_t$ in the time limit if $r_d = r_a$.

periods and it thus provides extra liquidity. This implies that it might be optimal for the households to make choices such that both $D_t > 0$ and $A_t > 0$; i.e to simultaneously be a borrower and a saver.

If there was only one-period debt (i.e. $\lambda = 1$) it would never be optimal for the households to simultaneously have both positive assets and positive debt because the option value of borrowing today would disappear. Consequently it would not be necessary to keep track of assets and debts separately and the model could be written purely in terms of net worth.[9] This would also imply that (3.4) could be rewritten as

$$N_t \geq -\frac{\mathbf{1}_{x_t=0} \cdot \varphi}{1+\eta} \cdot P_t \tag{3.6}$$

showing that our model nests the canonical buffer-stock consumption model a la Carroll (1992, 1997, 2012) as a limiting case for $\lambda \to 1$.

3.3 Income

The income process is given by

$$\begin{aligned} Y_{t+1} &= \tilde{\xi}\left(u_{t+1}, \xi_{t+1}\right) \cdot P_{t+1} \\ P_{t+1} &= \Gamma \cdot \psi_{t+1} \cdot P_t \\ \tilde{\xi}\left(u_{t+1}\right) &\equiv \begin{cases} \mu & \text{if } u_{t+1} = 1 \\ \frac{\xi_{t+1} - u_* \cdot \mu}{1-u_*} & \text{if } u_{t+1} = 0 \end{cases} \\ u_{t+1} &= \begin{cases} 1 & \text{with probability } u_* \\ 0 & \text{else} \end{cases} \end{aligned}$$

where ξ_t and ψ_t are respectively *transitory* and *permanent* mean-one log-normal income shocks[10] (with finite lower and upper supports), and u_* is the unemployment rate.[11] Because we have fully permanent shocks, we introduce a small constant mortality rate in the simulation exercise to keep the distribution of income finite.

[9] If $N_t \geq 0$ then $A_t = N_t$ and $D_t = 0$, and if $N_t < 0$ then $D_t = -N_t$ and $A_t = 0$.

[10] Note that the *unconditional* expectation of Y_{t+1} thus is $\Gamma \cdot P_t$.

[11] Throughout the paper we will continue to interpret u_t as unemployment, but it could also proxy for a range of other large shocks to both income and consumption. This would relax the model's tight link between unemployment and a higher risk of a negative shock to the availability of new borrowing.

~~lieved. It is common practice to calibrate economic models using externally estimated parameters and the effect of children on the marginal utility of consumption (or equivalence scales) are often calibrated from external microeconometric sources. For example, Cagetti (2003) uses the estimates from Attanasio, Banks, Meghir and Weber (1999) when analyzing wealth accumulation and precautionary savings over the life cycle and Scholz, Seshadri and Khitatrakun (2006) use equivalence scales from Citro and Michael (1995) when analyzing retirement savings of American households.~~

~~The results are also related to a growing strand of literature estimating models of intertemporal behavior related to household demographics. While I estimate a model in which the age of all (three) children can affect the marginal utility of consumption, all existing studies in this strand of literature estimate models in which children can affect households in much more restricted ways.[7] Motivated by the results in Browning and Ejrnæs (2009), I allow the age of all children to be potentially important.~~

~~The rest of the paper proceeds as follows. In the next section, I augment a standard imperfect markets life cycle model with the potential presence of children. Section 3 presents the Danish administrative registers and Section 4 discusses how some model parameters are calibrated to the Danish environment. Section 5 presents the estimation results and model fit while Section 6 investigates the robustness of the results. Finally, I conclude.~~

2 A Model of Consumption in the Presence of Children

The theoretical framework used throughout this study is purposely very similar to the underlying models in, e.g., Attanasio, Banks, Meghir and Weber (1999) and Browning and Ejrnæs (2009). The model is based on the buffer-stock model pioneered by Deaton (1991) and Carroll (1992) and first structurally estimated in Gourinchas and Parker (2002). A novelty of this study is that I augment the standard buffer-stock model with the potential presence of children and allow the marginal utility of consumption to be affected by the number and age of all children.

Households work until an exogenously given retirement age, T_r, and die with certainty at age T after consuming all available resources. In all preceding periods, households chose the level of consumption, C_t, that solves the optimization problem

$$\max_{C_t} \mathbb{E}_t \left[\sum_{\tau=t}^{T_r-1} \beta^{\tau-t} v(\mathbf{z}_t;\theta) u(C_\tau) + \beta^{T_r-t} V_{T_r}(M_{T_r}, \mathbf{z}_{T_r}) \right], \tag{1}$$

[7] For example, Love (2010) assumes that children arrive with two years intervals and Sommer (2014) assumes that there is two types of children: Children living at home and children, who have left the household. The model in Hong and Ríos-Rull (2012) is independent of the age of dependents while in the recent working papers by Blundell, Dias, Meghir and Shaw (forthcoming) and Adda, Dustmann and Stevens (forthcoming) only the age of the youngest child matters.

where utility is CRRA, $u(C_t) = C_t^{1-\rho}/(1-\rho)$, and $v(\mathbf{z}_t;\theta)$ is a taste shifter in which θ is the loadings on the number and age of children, contained in $\mathbf{z}_t$. As most of the existing literature, I follow Attanasio, Banks, Meghir and Weber (1999) and let children affect the *marginal value* of consumption.[8]

Households solve (1) subject to the intertemporal budget constraint,

$$M_{t+1} = R(M_t - C_t) + Y_{t+1},$$

where R is the gross real interest rate, M_t is resources available for consumption in beginning of period t and Y_t is beginning-of-period income. Retirees are not allowed to be net-borrowers, $A_t = M_t - C_t \geq 0, \forall t \geq T_r$, while working households can borrow up to a fraction of their permanent income $A_t \geq -\kappa P_t \, \forall t$, $\kappa \geq 0$.

Income Process. Income follows a stochastic process when working,

$$\begin{aligned} Y_t &= P_t \varepsilon_t, \ \forall t < T_r, \\ P_t &= G_t P_{t-1} \eta_t, \ \forall t < T_r, \end{aligned}$$

where P_t denotes permanent income, G_t is real gross permanent income growth, η_t is a mean one permanent income shock, $\log \eta_t \sim \mathcal{N}(-\sigma_\eta^2/2, \sigma_\eta^2)$, and ε_t is a mean one transitory income shock taking the value μ with probability $\wp$ and otherwise log normal,[9]

$$\begin{aligned} \varepsilon_t &= \begin{cases} \mu & \text{with probability } \wp \\ (\tilde{\varepsilon}_t - \mu\wp)/(1-\wp) & \text{with probability } 1-\wp \end{cases} \\ \log \tilde{\varepsilon}_t &\sim \mathcal{N}(-\sigma_\varepsilon^2/2, \sigma_\varepsilon^2). \end{aligned}$$

When retired, the income process is a deterministic constant fraction $\varkappa \leq 1$ of permanent income at retirement, $Y_t = \varkappa P_{T_r}$, $\forall t \geq T_r$. This way of modeling the replacement rate in retirement is fairly standard and a similar approach is also applied in, for example, Kaplan and Violante (2010) and Blundell, Low and Preston (2013). While the latter assumes a replacement rate of zero, the former study implements a benefit sys-

[8]Alternatively, the household composition could be included as a scaling of resources or consumption (equivalence scaling), as done in, e.g., Fernández-Villaverde and Krueger (2007). See Bick and Choi (2013) for an analysis of different approaches to and implied behavior from inclusion of household demographics in life cycle models.

[9]This specification of the transitory income shock follows that of Carroll (1997) and ensures that $\mathbb{E}_t[\varepsilon_{t+1}] = 1$, regardless of the values of μ, $\wp$ and σ_ε^2. Note, that by setting $\wp = 0$ the specification is close to that in Carroll (1997) where I multiply $\tilde{\varepsilon}$ with the factor $(1-\wp)^{-1}$ to ensure a mean of one while Carroll (1997, p. 6) is not explicit about how he ensures that $\mathbb{E}_t[\varepsilon_{t+1}] = 1$.

tem aimed at approximating the US pension scheme.[10]

Fertility. Households are fertile from age 15 to 43 and children arrive with a known probability distribution depending on the age of the wife, educational attainment, and the number of children already present in the household.[11] Children leave home at age 21 and do not influence household consumption in subsequent periods. Households can have at most three children for which the age is contained in $\mathbf{z}_t$,

$$\mathbf{z}_t = (\text{age of child } 1_t, \text{ age of child } 2_t, \text{ age of child } 3_t) \in \{\text{NC}, [0, 20]\}^3,$$

where NC refers to "no child" and the oldest child is denoted child one, the second oldest child as child two and the third oldest child as child three.

A novelty of this study is that I keep track of the age of *all* (three) children inside the household. To the best of my knowledge, this has not previously been done in dynamic models of intertemporal consumption and savings behavior. To circumvent the computational cost of keeping track of the age of all children, strict assumptions on the timing of children are typically imposed.[12] Knowing the age of each child is, however, necessary to allow for an arbitrary child, age and scale effect of children on the marginal utility of consumption. To reduce the otherwise very large state space, I use the imposed ordering of children in $\mathbf{z}_t$: Because children leave the household at age 21, $\mathbf{z}_t$ evolves according to

$$\begin{aligned}
\mathbf{z}_{1,t+1} &= \begin{cases} \tilde{\mathbf{z}}_{2,t+1} & \text{if } \tilde{\mathbf{z}}_{1,t+1} \geq 21 \\ \tilde{\mathbf{z}}_{1,t+1} & \text{else} \end{cases} \\
\mathbf{z}_{2,t+1} &= \begin{cases} \tilde{\mathbf{z}}_{3,t+1} & \text{if } \tilde{\mathbf{z}}_{1,t+1} \geq 21 \\ \tilde{\mathbf{z}}_{2,t+1} & \text{else} \end{cases} \\
\mathbf{z}_{3,t+1} &= \begin{cases} NC & \text{if } \tilde{\mathbf{z}}_{1,t+1} \geq 21 \\ \tilde{\mathbf{z}}_{3,t+1} & \text{else} \end{cases}
\end{aligned} \tag{2}$$

[10] Blundell, Low and Preston (2013) also implement a version with $\varkappa = .5$ in their robustness checks. Kaplan and Violante (2010, p. 65) write that "*Benefits are then scaled proportionately so that a worker earning average labor income each year is entitled to a replacement rate of 45 percent...*".

[11] Love (2010); Hong and Ríos-Rull (2012) and Blundell, Dias, Meghir and Shaw (forthcoming) also assume that children arrive probabilistically. The same is true in Adda, Dustmann and Stevens (forthcoming) although, in their model, households essentially has limited control over contraceptive effectiveness.

[12] To reduce the computational complexity, Scholz and Seshadri (2009) assume that households choose the *number* of children to have at age 27, such that all children arrive simultaneously. Love (2010), on the other hand, assumes that children arrive with two years intervals, and Sommer (2014) assumes that there is two types of children: Children living at home and children, who have left the household. Alternatively, Blundell, Dias, Meghir and Shaw (forthcoming) assumes that only the youngest child matters.

where the age of the children is re-arranged if the oldest child leaves the household. The age of each child (denoted with subscript j) evolves according to

$$\tilde{\mathbf{z}}_{j,t+1} = \begin{cases} 0 & \text{if } b_{t+1} = 1, \mathbf{z}_{j,t} = NC \text{ and } \mathbf{z}_{<j,t} \neq NC \text{ and } \mathbf{z}_{>j,t} = NC \\ \mathbf{z}_{j,t} + 1 & \text{if } \mathbf{z}_{j,t} \neq NC \\ NC & \text{else} \end{cases} \tag{3}$$

where the first case refers to a childbirth ($b_{t+1} = 1$) that occurs with an age and household size dependent probability, $\pi_{t+1}(\mathbf{z}_t)$, and the last two conditions in the first case ensures the relevant ordering of the children.[13] For example, if a household has one child and has another in the next period, we have that $b_{t+1} = 1$ and $\mathbf{z}_{1,t} \neq NC, \mathbf{z}_{2,t} = \mathbf{z}_{3,t} = NC$ and, thus $\tilde{\mathbf{z}}_{1,t+1} = \mathbf{z}_{1,t} + 1$, $\tilde{\mathbf{z}}_{2,t+1} = 0$ and $\tilde{\mathbf{z}}_{3,t} = NC$.

While I assume a stochastic process for childbirth, an alternative deterministic fertility process assumed in, e.g., Gourinchas and Parker (2002); Cagetti (2003) and Browning and Ejrnæs (2009), is sometimes adopted. In their framework, households know with perfect foresight at the beginning of adulthood when and how many children they will have.[14]

Retirement. Following Gourinchas and Parker (2002), among others, V_{T_r} is a retirement value function, summarizing in a parsimonious way *all* post-retirement savings motives. The retirement value function is assumed to be given as

$$V_{T_r}(M_{T_r}, \mathbf{z}_{T_r}) = \gamma \max_{C_j \in [0, M_j] \forall j = T_r, \ldots, T} \left\{ v(\mathbf{z}_{T_r}; \theta) u(C_{T_r}) + \sum_{s=T_r+1}^{T} \beta^{s-T_r} v(\mathbf{z}_s; \theta) u(C_s) \right\}$$

where γ is a parsimonious way of adjusting for all post retirement motives such as survival and income uncertainty, retirement benefits and bequest motives. In the following, I refer to γ as a "retirement motive" although it summarizes *all* potential ignored post-retirement motives and should not be interpreted as a parameter measuring the importance of the retirement motive.

The specification in, e.g., Kaplan and Violante (2010) and Blundell, Low and Preston (2013) is similar to assuming $\gamma = 1$. Naturally, and as I show in the robustness section, γ is closely related to the replacement rate κ and thus provides an additional degree of freedom in the model to correct for any potential miss-calibration of the replacement rate in retirement.

[13] I use "$< j$" and "$> j$" subscripts to denote all indices below and above j, respectively.

[14] In the robustness section, I estimate this alternative model and find similar results as in the baseline stochastic fertility model.

~~of the control group B in Q2 incorporates the reaction to the news, and thus the addition of the lagged rebate in the regression does not fully resolve the problem.~~

~~In spite of these difficulties in mapping directly β_2 to an MPC, we maintain that the rebate coefficient is an informative statistic: only if the true MPC out of the check is sizable and the MPC out of the news is small, can the rebate coefficient be as large as is empirically estimated. The advantage of the structural model is that it enables one to identify all the separate components of equation (2). As a result, it allows one to quantify the current and lagged MPCs out of an income shock, out of an anticipated income change, and out of the news of a future change in income—all magnitudes that are essential for policy analysis.~~

3. A LIFE-CYCLE MODEL WITH LIQUID AND ILLIQUID ASSETS

Our framework integrates the Baumol–Tobin inventory-management model of money demand into an incomplete-markets life-cycle economy. We first describe the full model; next, we use a series of examples to highlight the economic mechanisms at work.

3.1. *Model Description*

Demographics. The stationary economy is populated by a continuum of households, indexed by i. Age is indexed by $j = 1, 2, \dots, J$. Households retire at age J^w and retirement lasts for J^r periods.

Preferences. Households have an Epstein–Zin–Weil objective function defined recursively by

$$V_{ij} = \left[(1-\beta)\left(c_{ij}^{\phi} s_{ij}^{1-\phi}\right)^{1-\sigma} + \beta\left\{\mathbb{E}_j\left[V_{i,j+1}^{1-\gamma}\right]\right\}^{(1-\sigma)/(1-\gamma)}\right]^{1/(1-\sigma)}, \tag{4}$$

where $c_{ij} \geq 0$ is consumption of nondurables and $s_{ij} \geq 0$ is the service flow from housing for household i at age j. The parameter β is the discount factor, ϕ measures the weight of nondurables relative to housing services in period-utility, γ regulates risk aversion, and $1/\sigma$ is the elasticity of intertemporal substitution.[15]

Idiosyncratic Earnings. In any period during the working years, household labor earnings (in logs) are given by

$$\log y_{ij} = \chi_j + \alpha_i + z_{ij}, \tag{5}$$

[15] Piazzesi, Schneider, and Tuzel (2007) offered both (i) microevidence from CEX on the variation of housing expenditure share across different household types, and (ii) time-series evidence on the relationship between the aggregate expenditure share and the relative price of housing services. Both dimensions of the data suggest an elasticity of substitution between nondurable and housing consumption very close to 1, which is the Cobb–Douglas case that we adopt in our preference specification.

where χ_j is a deterministic age profile common across all households, α_i is a household-specific fixed effect, and z_{ij} is a stochastic idiosyncratic component that obeys the conditional c.d.f. $\Gamma^z(z_{j+1}, z_j)$.

Assets. Households can hold a liquid asset m_{ij} and an illiquid asset a_{ij}. The illiquid asset pays a gross financial return $1/q^a$, whereas positive balances of the liquid asset pay $1/q^m$. When the household wants to make deposits into, or withdrawals from, the illiquid account, it must pay a transaction cost κ.[16] The trade-off between these two savings instruments is that the illiquid asset earns a higher return, in the form of capital gain and consumption flow, but its adjustments are subject to the transaction cost. Households start their working lives with an exogenously given quantity of each asset.

Illiquid assets are restricted to be always nonnegative, $a_{ij} \geq 0$. Because of the prevalence of housing among commonly held illiquid assets (see Section 5), we let the stock of illiquid assets a_{ij} yield a utility flow with proportionality parameter $\zeta > 0$. Households are also free to purchase or rent out housing services $h_{ij} \geq -\zeta a_{ij}$ on the market.[17] As a result, $s_{ij} = \zeta a_{ij} + h_{ij}$.

We allow borrowing in the liquid asset to reflect the availability of unsecured credit up to an ad hoc limit, $\underline{m}_{j+1}(y_{ij})$, expressed as a function of current labor earnings. The interest rate on borrowing is denoted by $1/\bar{q}^m$ and we define the function $q^m(m_{i,j+1})$ to encompass both the case $m_{i,j+1} \geq 0$ and the case $m_{i,j+1} < 0$.

Financial returns to the liquid and illiquid assets, as well as the borrowing rate, are exogenous. Two reasons dictate the choice of abstracting from the equilibrium determination of returns. First, the total outlays from the 2001 rebate amounted to less than 0.1% of aggregate net worth, surely not enough to move asset prices significantly. Second, 83% of aggregate wealth is held by the top quintile of the distribution (Díaz-Giménez, Glover, and Ríos-Rull (2011, Table 6)), and the portfolio allocation of such households is unlikely to be affected by the receipt of a \$500 check from the government.[18]

Government. Government expenditures G are not valued by households. Retirees receive social security benefits $p(\chi_{J^w}, \alpha_i, z_{iJ^w})$, where the arguments proxy for average gross lifetime earnings. The government levies proportional taxes on consumption expenditures (τ^c) and on asset income (τ^a, τ^m), a payroll tax $\tau^{ss}(y_{ij})$ with an earnings cap, and a progressive tax on labor income $\tau^y(y_{ij})$. There is no deduction for interest paid on unsecured borrowing. We denote

[16]It is straightforward to allow for a utility cost or a time cost proportional to labor income rather than a monetary cost of adjustment. We have experimented with both types of costs and obtained similar results in both cases. See Kaplan and Violante (2011).

[17]This assumption adds realism to the model. Technically, it is useful because, with our Cobb–Douglas period-utility specification, housing services are an essential consumption good and, without a rental market, even the poorest households would be forced to pay the transaction cost in order to deposit into the illiquid account to start enjoying a minimum amount of housing services.

[18]In simulations, the aggregate stock of illiquid wealth increases by only 0.14% during the first year of the transition, an amount hardly large enough to have an impact on the rate of return.

the combined income tax liability function as $\mathcal{T}(y_{ij}, a_{ij}, m_{ij})$. For retirees, the same tax function applies with y_{ij} taking the value $p(\cdot)$. Finally, we let the government issue one-period debt B at price q^g.

Household Problem. We use a recursive formulation of the problem. Let $\mathbf{s}_j = (m_j, a_j, \alpha, z_j)$ be the vector of individual states at age j. The value function of a household at age j is $V_j(\mathbf{s}_j) = \max\{V_j^0(\mathbf{s}_j), V_j^1(\mathbf{s}_j)\}$, where $V_j^0(\mathbf{s}_j)$ and $V_j^1(\mathbf{s}_j)$ are the value functions conditional on not adjusting and adjusting (i.e., depositing into or withdrawing from) the illiquid account, respectively. This decision takes place at the beginning of the period, after receiving the current endowment shock, but before consuming.[19]

Consider a household of age j. If $V_j^0(\mathbf{s}_j) \geq V_j^1(\mathbf{s}_j)$, the household chooses not to adjust its illiquid assets and solves the dynamic problem

$$V_j^0(\mathbf{s}_j) = \max_{c_j, h_j, m_{j+1}} \left[(1-\beta)\left(c_j^{\phi} s_j^{1-\phi}\right)^{1-\sigma} + \beta\left\{\mathbb{E}_j\left[V_{j+1}^{1-\gamma}\right]\right\}^{(1-\sigma)/(1-\gamma)}\right]^{1/(1-\sigma)} \tag{6}$$

subject to:

$$\begin{aligned}
&\left(1+\tau^c\right)(c_j + h_j) + q^m(m_{j+1})m_{j+1} = y_j + m_j - \mathcal{T}(y_j, a_j, m_j),\\
&s_j = h_j + \zeta a_j,\\
&q^a a_{j+1} = a_j,\\
&c_j \geq 0, \quad h_j \geq -\zeta a_j, \quad m_{j+1} \geq -\underline{m}_{j+1}(y_j),\\
&y_j = \begin{cases} \exp(\chi_j + \alpha + z_j), & \text{if } j \leq J^w,\\ p(\chi_{J^w}, \alpha, z_{J^w}), & \text{otherwise,}\end{cases}
\end{aligned}$$

where z_j evolves according to the conditional c.d.f. Γ_j^z.

If $V_j^0(\mathbf{s}_j) < V_j^1(\mathbf{s}_j)$, the household adjusts its holding of illiquid assets and solves

$$\begin{aligned}
V_j^1(\mathbf{s}_j) = \max_{c_j, h_j, m_{j+1}, a_{j+1}} &\left[(1-\beta)\left(c_j^{\phi} s_j^{1-\phi}\right)^{1-\sigma}\right.\\
&\left. + \beta\left\{\mathbb{E}_j\left[V_{j+1}^{1-\gamma}\right]\right\}^{(1-\sigma)/(1-\gamma)}\right]^{1/(1-\sigma)}
\end{aligned} \tag{7}$$

subject to:

$$\begin{aligned}
&\left(1+\tau^c\right)(c_j + h_j) + q^m(m_{j+1})m_{j+1} + q^a a_{j+1}\\
&\quad = y_j + m_j + a_j - \kappa - \mathcal{T}(y_j, a_j, m_j),
\end{aligned}$$

[19]Because of this timing, after the earnings shock the household can always choose to pay the transaction cost, access the illiquid account, and use all its resources to finance consumption. Hence, our model *does not* feature a cash-in-advance (CIA) constraint. See Jovanovic (1982) for an exhaustive discussion of the difference between models with transaction costs and models with CIA constraints.

$$s_j = h_j + \zeta a_j,$$

$$c_j \geq 0, \quad h_j \geq -\zeta a_j, \quad m_{j+1} \geq -\underline{m}_{j+1}(y_j), \quad a_{j+1} \geq 0,$$

$$y_j = \begin{cases} \exp(\chi_j + \alpha + z_j), & \text{if } j \leq J^w, \\ p(\chi_{J^w}, \alpha, z_{J^w}), & \text{otherwise.} \end{cases}$$

Appendix E in Supplemental Material (Kaplan and Violante (2014)) describes the computational algorithm used to solve problems (6) and (7).

Balanced Budget. The government always respects its intertemporal budget constraint

$$G + \sum_{j=J^w+1}^{J} \int p(y_{J^w})\, d\mu_j + \left(\frac{1}{q^g} - 1\right) B = \tau^c \sum_{j=1}^{J} \int c_j\, d\mu_j + \sum_{j=1}^{J} \int \mathcal{T}(y_j, a_j, m_j)\, d\mu_j, \tag{8}$$

where μ_j is the distribution of households of age j over the individual state vector $\mathbf{s}_j$.

~~4. HAND-TO-MOUTH HOUSEHOLDS IN MODEL AND DATA~~

~~In this section, we first illustrate, by means of numerical examples, how hand-to-mouth behavior arises endogenously in our model, even when agents hold positive illiquid wealth. Next, we measure hand-to-mouth households in the Survey of Consumer Finances.~~

~~4.1. *Behavior in the Model: The "Wealthy Hand-to-Mouth"*~~

~~For ease of exposition, we focus on a stylized version of the model with time-separable preferences ($\gamma = \sigma$), without service flow from illiquid assets ($\phi = 1$, $\zeta = 0$), with logarithmic period-utility, deterministic labor income ($z_j \equiv 0$), and no taxes ($\mathcal{T}(\cdot) = \tau^c \equiv 0$). Moreover, we assume that $\bar{q}^m < q^a < q^m$. The second inequality states that the illiquid asset has a higher return and the first one ensures that households do not borrow to deposit into the illiquid account.~~

~~*Two Euler Equations*. Consumption and portfolio decisions are characterized by a short-run Euler equation (EE-SR) that corresponds to borrowing or saving in the liquid asset, and a long-run Euler equation that corresponds to (dis)saving in the illiquid asset (EE-LR). In periods where the working house-~~

~~the covariance matrix (of log earnings), abstracting from higher order moments.[17] This approach faces two difficulties. First, the empirical evidence documented in this paper shows large deviations from lognormality, in which case the third and fourth moments contain valuable information. Second, the covariance matrix estimation makes it difficult to select among alternative models of earnings risk, because it is difficult to judge the relative importance—from an economic standpoint—of the covariances that a given model matches well and those that it does not.~~

~~With these considerations in mind, we follow a different approach that relies on matching the kinds of moments presented above.[18] We believe that economists can more easily judge whether or not each one of these moments is relevant for the economic questions they have in hand. We conducted a battery of diagnostic tests on each set of moments to get a better idea about what elementary components (e.g., persistent shocks, normal mixtures, heterogeneous profiles, etc.) have the potential to generate those features. We conducted similar diagnostic analyses on the other cross sectional moments (standard deviation, skewness, and kurtosis) as well as on impulse response moments. The rich persistence pattern necessitated mixing multiple AR(1) processes, whereas the variation in the second to fourth moments over the life cycle and with earnings levels seemed impossible to match without introducing explicit dependence of mixing probabilities on age and earnings. Therefore, we make these features part of our benchmark.~~

6.1 A Flexible Stochastic Process

The most general econometric process we estimate has the following features: (i) a heterogeneous income profiles (hereafter, HIP) component of linear form; (ii) a mixture of two AR(1) processes,[19] denoted by z_1 and z_2, where each component receives a new innovation in a given year with probability $p_j \in [0, 1]$ for $j = 1, 2$; (iii) a nonemployment

[17]To be clear, GMM or minimum distance estimation that is used to match such moments does not require the assumption of lognormality for consistency. But abstracting away from moments higher than covariances is a reflection of the belief that higher-order moments do not contain independent information, which relies on the lognormality assumption.

[18]This approach is in the spirit of Browning *et al.* (2010), Altonji *et al.* (2013), and Guvenen and Smith (2014).

[19]Geweke and Keane (2007) study how regression models can be smoothly mixed, and our modeling approach shares some similarities with their framework.

shock ν; and (iv) an i.i.d. shock ε. Here is the full specification:

$$Y_t^i = (1 - \nu_t^i) \exp\left(g(t) + \alpha^i + \beta^i t + z_{1,t}^i + z_{2,t}^i + \varepsilon_t^i\right) \tag{2}$$

$$z_{1,t}^i = \rho_1 z_{1,t-1}^i + \eta_{1t}^i \tag{3}$$

$$z_{2,t}^i = \rho_2 z_{2,t-1}^i + \eta_{2t}^i, \tag{4}$$

where $t = (age - 24)/10$ denotes normalized age, and for $j = 1, 2$:

$$\text{Innovations to AR(1)'s:} \quad \eta_{jt} \sim \begin{cases} -p_{jt}^i \mu_j & \text{with pr. } 1 - p_{jt}^i \\ \mathcal{N}((1 - p_{jt}^i)\mu_j, \sigma_j^i) & \text{with pr. } p_{jt}^i \end{cases} \tag{5}$$

$$\text{Indiv.-specific innov. var.:} \quad \log\left(\sigma_j^i\right) \sim \mathcal{N}(\log \overline{\sigma}_j - \frac{\widetilde{\sigma}_j^2}{2}, \widetilde{\sigma}_j) \tag{6}$$

$$\text{Initial value of AR(1) process:} \quad z_{j0}^i \sim \mathcal{N}(0, \sigma_j^i \sigma_{j0}). \tag{7}$$

$$\text{Nonemployment shock:} \quad \nu_t^i \sim \begin{cases} 0 & \text{with pr. } 1 - p_{\nu t}^i \\ \min\{1, \text{Expon.}(\lambda)\} & \text{with pr. } p_{\nu t}^i \end{cases} \tag{8}$$

The term $g(t)$ in (2) is a quadratic polynomial in age and captures the life-cycle component of earnings that is common to all individuals. The random vector (α^i, β^i) is drawn from a multivariate normal distribution with zero mean and a covariance matrix to be estimated.[20] Each AR(1) component, z_1 and z_2, receives a shock drawn from a mixture of a Gaussian distribution and a mass point chosen so that the innovations have zero mean. Furthermore, we allow the variance of each innovation to be individual-specific, with a lognormal distribution with mean $\overline{\sigma}_j$ and a standard deviation proportional to $\widetilde{\sigma}_j$.[21] We also allow for heterogeneity in the initial conditions of the persistent processes, $z_{1,0}^i$ and $z_{2,0}^i$, given in equation (8).[22] Since the specifications of z_1 and z_2 are the same so far, we need an identifying assumption to distinguish between the two, so, without

[20] One possible source of heterogeneity in the growth rate, β^i, could be human capital accumulation in the presence of ability heterogeneity. See, e.g., Huggett *et al.* (2011) and Guvenen *et al.* (2014a).

[21] Chamberlain and Hirano (1999) and Browning *et al.* (2010) find strong evidence of such heterogeneity. Our preliminary analysis also confirmed this prediction, which is why we allow for this feature. Furthermore, such heterogeneity in innovation variances can also give rise to excess kurtosis in the cross-sectional distribution of income changes, as we found in Section 3, even when each individual receives Gaussian innovations. Thus, modeling this feature is important for understanding the determinants of kurtosis we documented earlier.

[22] This specification allows the standard deviation of the initial condition to be proportional to the individual-specific standard deviation σ_j^i.

loss of generality, we impose $\rho_1 < \rho_2$. Finally, with probability $p^i_{\nu t}$, an individual receives a nonemployment shock ($\nu_t > 0$) whose duration has an exponential distribution with mean δ, truncated at 1, the interpretation being of full year nonemployment (i.e., zero annual income).

The age and income dependence of moments is captured by allowing the mixture probabilities to depend on age and the persistent component of earnings ($z_1 + z_2$):[23]

$$
\begin{aligned}
p^i_{jt} &= \frac{\exp\left(\xi^i_{j,t-1}\right)}{1+\exp\left(\xi^i_{j,t-1}\right)}, \qquad (9)\\
\xi^i_{jt} &= a_j + b_j \times t + c_j \times \left(z^i_{1,t} + z^i_{2,t}\right) + d_j \times \left(z^i_{1,t} + z^i_{2,t}\right) \times t,
\end{aligned}
$$

for $j = 1, 2$. The equation for $p_{\nu t}$ is the same as (9) but $\xi^i_{j,t-1}$ is replaced with ξ^i_{jt}. This completes the description of the stochastic process.

~~Estimation Procedure~~

~~We estimate the parameters of the stochastic process described by equations (2) to (9) using the method of simulated moments (MSM). The empirical targets are: (i) the standard deviation, skewness, and kurtosis of one year and five year earnings growth; and (ii) moments describing the impulse response functions; and (iii) moments from the life cycle profile of average earnings. In addition, the within cohort variance of log earnings levels is a key dimension of the data that has been extensively studied in previous research. For both completeness, and consistency with earlier work, we include these variances (summarized in Figure A.18) as a fourth set of moments.~~

~~As discussed earlier, the descriptive analysis relied on log earnings changes because of its familiarity and required us to drop individuals with low earnings—below $\overline{Y}_{\min}$—in year t or $t+k$. In the estimation, we target the same set of moments but this time computed using arc percent changes, which allows us to include individuals who enter and exit market work. This way, the estimated income process will capture not only income shocks in the intensive margin but also the nature and persistence of shocks that move workers in and out of the labor market. The counterparts of all the figures in Sections 3 and 4 using arc percent change are reported in Appendix B.2. They exhibit the same qualitative patterns as the log difference versions presented above, but their~~

[23]We have also considered an alternative specification where the innovation variances are functions of earnings and age. After extensive experimentation with this formulation, we have found it to perform very poorly.

~~individuals who are full-year nonemployed in year t conditional on recent earnings in the data and in the model. The fit is quite good, with a small discrepancy at the lowest end.~~[27] ~~This large heterogeneity in nonemployment rates is behind the estimated model's ability to generate the substantially different patterns documented for high- and low-income individuals in the previous sections.~~

~~Finally, Figure 12b shows another moment of the data that was not targeted in the estimation but is important for the analysis of wealth concentration in the next section. It shows the log density of earnings (levels—not changes) in the data, along with the counterparts from the benchmark estimation and the Gaussian process. As seen here, the benchmark process generates a Pareto tail that matches the data almost perfectly whereas the Gaussian process (as expected) generates a lognormal tail that underestimates the number of individuals with very high earnings. We will return to this point in the next section.~~

7 Life-Cycle Consumption-Savings Model

We now investigate some key implications of the rich earnings process estimated in the previous section for life-cycle consumption-savings behavior. Consider a continuum of individuals that participate in the labor market for the first T_W years of their life, retire at that age, and die at age $T > T_W$. Individuals have standard CRRA preferences over consumption and hence supply labor inelastically.

Individual earnings follow the stochastic process described by equations (2) to (9). Although this process has many parameters, all dynamics are captured through only two state variables ($z_{1,t}^i$ and $z_{2,t}^i$), which makes it relatively straightforward to embed it into a dynamic programming problem. Let $\Upsilon^i \equiv (\alpha^i, \beta^i, \sigma_1^i, \sigma_2^i)$ denote the parameters capturing fixed (ex ante) heterogeneity. We construct a four-dimensional discrete grid over Υ^i where each grid point corresponds to a worker type k. Therefore, some aspects of an individual's problem depends on his (discrete) ex ante type k, whereas others—including the dynamics of income—are drawn from continuous distributions that are also fully individual specific.

Individuals can borrow or save using a risk-free asset with gross return R, where

[27] We have also generated similar graphs from the Survey of Income and Program Participation (SIPP) data set for various unemployment outcomes (duration, incidence, etc.) as a function of past two-year average income, which shows the same strong decline in unemployment outcomes with past earnings.

borrowing is subject to an age-dependent worker-type-specific limit, denoted by $\underline{A}_t^k$. Then the budget constraint is given by

$$\begin{aligned} c_t^i + a_{t+1}^i &= a_t^i R + Y_t^{\mathrm{disp},i}, \qquad \forall t, & (10)\\ Y_t^{\mathrm{disp},i} &= \max\left\{\underline{Y}, \widetilde{Y}_t^i\right\}^{1-\tau}, \qquad t = 1, \ldots, T_W, & (11)\\ Y_t^{\mathrm{disp},i} &= \left(\widetilde{Y}_R^k\right)^{1-\tau}, \qquad t = T_W + 1, \ldots, T, & (12)\\ a_{t+1}^i &\geq \underline{A}_t^k, \qquad \forall t, & (13) \end{aligned}$$

where c_t^i and a_t^i denote consumption and asset holdings, respectively, and $Y_t^{\mathrm{disp},i}$ is disposable income, which differs from gross income, $\widetilde{Y}_t^i$, in two ways. First, the government provides social insurance by guaranteeing a minimum level of income, $\underline{Y}$, to all individuals—more on this in a moment. Second, the government imposes a progressive tax on after-transfer income. Following Benabou (2000) and Heathcote *et al.* (2014), we take this tax function to have a power form, with exponent $1-\tau$. This specification has been found to fit the U.S. data quite well. Finally, retirees receive pension income, $\widetilde{Y}_R^k$, specified to mimic the U.S. Social Security Administration's OASDI system.[28] Appendix D contains further details. The dynamic programming problem of an individual is

$$\begin{aligned} V_t^i\left(a_t^i, z_{1,t}^i, z_{2,t}^i; \Upsilon^k\right) &= \frac{\left(c_t^i\right)^{1-\gamma}}{1-\gamma} + \beta \mathbb{E}_t\left[V_{t+1}^i\left(a_{t+1}^i, z_{1,t+1}^i, z_{2,t+1}^i; \Upsilon^k\right)\right] \\ \text{s.t.} & \\ & \text{equations (10)-(13), and } V_{T+1}^i \equiv 0, \end{aligned}$$

where β is the subjective discount factor, and γ governs risk aversion. This problem can be solved using standard numerical techniques; see computational Appendix D.

~~7.1 Calibration~~

~~Households enter the labor market at age 25, retire at 60 ($T_W = 36$), and die at 85 ($T = 60$). The coefficient of relative risk aversion γ is set to 2 as a conservative choice. The discount factor, β, is calibrated to replicate the wealth to income ratio of 4 as reported in the Flow of Funds Z1 tables.[29] R is set to 3%, and $\underline{A}_t^k$ is set equal to~~

[28] The dependence of pension on worker type k rather than individual income is to avoid introducing another state variable.

[29] The total wealth-to-income ratio is defined to be total asset holdings in the population relative to the sum of total before-tax labor income and capital income.

~~in both homeownership rates and nonhousing consumption in the two economies in which the elasticity of substitution between housing and nonhousing consumption is low. When the elasticity of substitution between housing and nonhousing consumption is high, homeownership rates do not move much, but nonhousing consumption declines substantially. Finally, a lower expected house price growth aggravates the negative effect of current house price declines on the homeownership rates, house or rent value, and nonhousing consumption. To conclude, our article reconciles to some extent the differences in theoretical as well as empirical estimations of the intratemporal elasticity of substitution between housing and nonhousing consumption. It nevertheless remains a challenge as to what is the relevant parameter to use for conducting policy experiments given their different implications.~~

~~To the best of our knowledge, our article represents one of the first structural estimations of housing preference parameters that are consistent with both time series and cross-sectional evidence on households' housing consumption and savings decisions. Estimating a rich life-cycle model allows us to address potential biases directly by replicating them in the simulation. The recent paper by Bajari et al. (2013) is the closest in spirit to our article. There are, however, significant differences. The first difference is methodology. Bajari et al. adopt a two-step approach. In the first step, reduced-form decision rules are estimated together with the law of motion for state variables. The structural parameters are then estimated in the second step using simulation based on the reduced-form decision rule in the first step. In contrast, we solve the decision rules endogenously instead of imposing reduced forms. In addition, we explicitly model and estimate households' tenure decisions. The second and more important difference between the two papers is different target moments, which the models are estimated to match. There is much less variation in aggregate house prices during Bajari et al.'s sample period (1983–93) than during ours (1984–2005). Furthermore, we make use of cross-sectional heterogeneity by studying areas with high, medium, and low house prices separately. These additional heterogeneities turn out to be crucial in explaining the differences in our estimates. When we reestimate our model using similar life-cycle aggregate moments as in Bajari et al. (2013), not surprisingly, we obtain a much larger elasticity of substitution between housing and nonhousing consumption.~~

~~The rest of the article proceeds as follows: In Section 2, we present a life-cycle model of housing choices with an adjustment cost. In Section 3, we lay out our estimation strategy and describe the data sources. Section 4 discusses our main findings and implications. Section 5 presents alternative estimations. We further explore model implications in Section 6. We conclude and point to future extensions in Section 7.~~

2. THE MODEL ECONOMY

Our modeling strategy extends that of Yao and Zhang (2005) and Li and Yao (2007) by admitting a flexible specification of elasticity of substitution between housing and other consumption.

2.1. *Demographics and Preferences.* A household lives for a maximum of T periods. Let λ_j denote the probability that the household is alive at time j conditional on being alive at time $j-1$, $j = 0, \ldots, T$. We set $\lambda_0 = 1$, $\lambda_T = 0$, and $0 < \lambda_j < 1$ for all $0 < j < T$.

The household derives utility from consuming housing services H_t and nonhousing goods C_t, as well as from bequeathing wealth Q_t. The utility function from consumption demonstrates a CES between the two goods, modified to incorporate a demographic effect:

$$U(C_t, H_t; N_t) = N_t \frac{\left[(1-\omega)(\frac{C_t}{N_t})^{1-\frac{1}{\zeta}} + \omega(\frac{H_t}{N_t})^{1-\frac{1}{\zeta}}\right]^{\frac{1-\gamma}{1-\frac{1}{\zeta}}}}{1-\gamma} \tag{1}$$

$$= N_t^{\gamma} \frac{\left[(1-\omega)C_t^{1-\frac{1}{\zeta}} + \omega H_t^{1-\frac{1}{\zeta}}\right]^{\frac{1-\gamma}{1-\frac{1}{\zeta}}}}{1-\gamma},$$

where N_t denotes the exogenously given effective family size to capture the economies of scale in household consumption. The parameter ω controls the expenditure share on housing services, and ζ governs the degree of intratemporal substitutability between housing and nondurable consumption goods. The bequest function $B(Q_t)$ will be discussed later.

The household retires exogenously at $t = J$ $(0 < J < T)$. Before retirement, the household receives labor income Y_t,

$$Y_t = P_t^Y \varepsilon_t, \tag{2}$$

where ε_t is the transitory shock to Y_t and P_t^Y is the permanent labor income at time t that follows:

$$P_t^Y = \exp\{f(t)\} P_{t-1}^Y \nu_t^s \nu_t^i. \tag{3}$$

The deterministic component $f(t)$ is a function of age. The term ν_t^s represents the aggregate shock to permanent labor income that is shared by all agents in the economy, whereas ν_t^i is the idiosyncratic shock to permanent labor income. We assume that $\{\ln \varepsilon_t, \ln \nu_t^s, \ln \nu_t^i\}$ are independently and identically normally distributed with mean $\{-0.5\sigma_\varepsilon^2, -0.5\sigma_{\nu^s}^2 + \mu_\nu^s, -0.5\sigma_{\nu^i}^2\}$ and variance $\{\sigma_\varepsilon^2, \sigma_{\nu^s}^2, \sigma_{\nu^i}^2\}$, respectively.[4] After retirement, a household's permanent labor income does not change any more, and the household receives a constant fraction θ of its permanent labor income before retirement,

$$P_t^Y = P_J^Y, \quad \text{for } t = J, \ldots, T, \tag{4}$$

$$Y_t = \theta P_t^Y, \quad \text{for } t = J, \ldots, T. \tag{5}$$

2.2. *Housing and Financial Markets.* A household can either own a house or rent housing services by paying a fraction α $(0 < \alpha < 1)$ of the market value of the rental house. A renter's housing tenure is denoted as $D_t^o = 0$, and a homeowner's housing tenure is $D_t^o = 1$.

We assume that house price P_t^H is exogenously given by the following stochastic process:

$$P_t^H = P_{t-1}^H \xi_t^s \xi_t^i, \tag{6}$$

where ξ_t^s represents the aggregate shock to the house price shared by all households and ξ_t^i is the idiosyncratic shock unique to the household. ξ_t^s and ξ_t^i follow an i.i.d. normal process with mean $\{-0.5\sigma_{H^s}^2 + \mu_H^s, -0.5\sigma_{H^i}^2\}$ and variance $\{\sigma_{H^s}^2, \sigma_{H^i}^2\}$. In addition, we assume that the aggregate house price shock ξ_t^s is correlated with the aggregate income shock ν_t^s with a correlation coefficient of ρ.[5]

A household can finance home purchases with a mortgage. The mortgage balance denoted by M_t needs to satisfy the following collateral constraint at all times:

$$0 \leq M_t \leq (1 - \delta) P_t^H H_t, \tag{7}$$

[4] The labor income process largely follows that of Carroll and Samwick (1997), which is also adopted in Cocco et al. (2005) and Gomes and Michaelides (2005), with the exception that we allow part of the permanent income to be driven by an economy-wide aggregate shock.

[5] Flavin and Yamashita (2002), Campbell and Cocco (2007), and Yao and Zhang (2005) also make similar assumptions about house price dynamics. The innovation here is that we split the permanent price shock into an aggregate component and an idiosyncratic component and allow the aggregate component to be correlated with the aggregate shock that affects individual permanent income. Campbell and Cocco (2007) assume the correlation to be 1.

where $0 \leq \delta \leq 1$, and $P_t^H H_t$ denotes the value of the house at time t.[6]

We assume that households can save in liquid assets, which earn a constant risk-free rate r, but they cannot hold noncollateralized debt. We further assume that the mortgage borrowing rate r_m is higher than the risk-free rate r. It follows immediately then that households will always want to repay their mortgages and will not hold mortgages and liquid assets simultaneously. We let S_t denote the net financial wealth. Finally, in order to keep the house quality constant, a homeowner is required to spend a fraction ψ $(0 \leq \psi \leq 1)$ of the house value on repair and maintenance.

At the beginning of each period, each household receives a moving shock, D_t^m, that takes a value of 1 if the household has to move for reasons that are exogenous to our model, and 0 otherwise. The moving shock does not affect a renter's decision since he does not incur any moving costs. When a homeowner receives a moving shock $(D_t^m = 1)$, he is forced to sell the house.[7] Selling a house incurs a transaction cost that is a fraction ϕ of the market value of the existing house. The selling decision, D_t^s, is 1 if the homeowner sells and 0 otherwise. A homeowner who does not have to move for exogenous reasons can choose to sell the house voluntarily. Following a home sale—for either exogenous or endogenous reasons—a homeowner faces the same decisions as a renter coming into period t and is free to buy or rent for the following period.

2.3. *Wealth Accumulation and Budget Constraints.* We denote the household's spendable resources upon home sale by Q_t.[8] It follows that

$$Q_t = \max\left\{S_{t-1}\left[1 + r(S_{t-1})\right] + Y_t + P_{t-1}^H D_{t-1}^o H_{t-1}\xi_t^s \xi_t^i (1-\phi), \eta P_t^Y\right\}. \tag{8}$$

As discussed earlier, if S_{t-1} is nonnegative, the household has paid off its mortgage and, hence, $r(S_{t-1}) = r$; if S_{t-1} is negative, it represents the mortgage the household holds on the house. As a result, $r(S_{t-1}) = r_m$. The last term ηP_t^Y denotes government transfers. Following Hubbard et al. (1994, 1995) and De Nardi et al. (2010), we assume that government transfers provide a wealth floor that is proportional to the household's permanent labor income. The intertemporal budget constraint, therefore, can be written as

$$Q_t = \begin{cases} C_t + S_t + \alpha P_t^H H_t, & \text{rent at } t-1 \text{ and } t(D_{t-1}^o = D_t^o = 0); \\ C_t + S_t + \alpha P_t^H H_t, & \text{own at } t-1, \text{ sell and rent at } t(D_{t-1}^o = D_t^s = 1, D_t^o = 0); \\ C_t + S_t + (1+\psi) P_t^H H_t, & \text{rent at } t-1, \text{ own at } t(D_{t-1}^o = 0, D_t^o = 1); \\ C_t + S_t + (1+\psi) P_t^H H_t, & \text{own at } t-1, \text{sell and own at } t(D_{t-1}^o = D_t^s = D_t^o = 1); \\ C_t + S_t + (1+\psi-\phi) P_t^H H_t, & \text{own the same H at } t-1 \text{ and } t(D_{t-1}^o = D_t^o = 1, D_t^s = 0). \end{cases} \tag{9}$$

The third term on the right-hand side of the budget constraint represents housing expenditure when households have various different house tenure status. Note that for the last group of households who stayed in their houses, we need to subtract from their expenditure the housing

[6] By applying collateral constraints to both newly initiated mortgages and ongoing loans, we effectively rule out default. Default on mortgages was, until recently, relatively rare in reality. According to the Mortgage Bankers Association, the annual 90+ days default rate for a prime fixed-rate mortgage loan was around 2% prior to 2007. When house prices experience a sequence of adverse shocks (as during the recent crisis) that amounts to a loss of 20% or more of house equity, this assumption would imply that households will be forced to sell more frequently in the model than in reality.

[7] We assume that house prices in the old and new locations are the same. This is obviously a simplifying assumption as in reality households can move to states that have very different house price movement. In Figure A3 in the online appendix, using data from the Current Population Survey, we show that the annual rate at which households move across states, however, has been small, averaging between 2% and 3% from 1984 to 2005.

[8] Under this definition, conditional on selling the house, a homeowner's problem is identical to that of a renter with same age t, permanent income P_t^Y, housing price $P_t H$, and liquidated wealth Q_t.

selling cost, which was subtracted from wealth in hand on the left-hand side defined in Equation (8).

2.4. *Bequest Function.* We assume that upon death, a household distributes its spendable resources Q_t equally among L beneficiaries to finance their nonhousing good and rented housing services consumption for one period. Parameter L thus determines the strength of bequest motives. Under the assumption of CES utility, the beneficiary's expenditure on nonhousing good and housing service consumption is the following function of house price:

$$\frac{C_t}{C_t + \alpha P_t^H H_t} = \frac{(1-\omega)^\zeta}{(1-\omega)^\zeta + \omega^\zeta (\alpha P_t^H)^{1-\zeta}}. \tag{10}$$

Therefore, the bequest function is defined by

$$\begin{aligned} B(Q_t) &= L \frac{\left[(1-\omega)\left(\frac{Q_t}{L}\frac{(1-\omega)^\zeta}{(1-\omega)^\zeta+\omega^\zeta(\alpha P_t^H)^{1-\zeta}}\right)^{1-\frac{1}{\zeta}} + \omega\left(\frac{Q_t}{L}\frac{\omega^\zeta(\alpha P_t^H)^{-\zeta}}{(1-\omega)^\zeta+\omega^\zeta(\alpha P_t^H)^{1-\zeta}}\right)^{1-\frac{1}{\zeta}}\right]^{\frac{1-\gamma}{1-\frac{1}{\zeta}}}}{1-\gamma} \\ &= L^\gamma Q_t^{1-\gamma} \frac{\left[(1-\omega)\left(\frac{(1-\omega)^\zeta}{(1-\omega)^\zeta+\omega^\zeta(\alpha P_t^H)^{1-\zeta}}\right)^{1-\frac{1}{\zeta}} + \omega\left(\frac{\omega^\zeta(\alpha P_t^H)^{-\zeta}}{(1-\omega)^\zeta+\omega^\zeta(\alpha P_t^H)^{1-\zeta}}\right)^{1-\frac{1}{\zeta}}\right]^{\frac{1-\gamma}{1-\frac{1}{\zeta}}}}{1-\gamma}. \end{aligned} \tag{11}$$

2.5. *The Optimization Problem.* We denote $X_t = \{D_{t-1}^o, Q_t, P_t^Y, P_t^H, H_{t-1}\}$ the vector of state variables. The household at state X_t solves the following optimization problem:

$$V_t(X_t) = \max_{A_t}\{\lambda_t\,[U(C_t, H_t; N_t) + \beta E_t[V_{t+1}(X_{t+1})]] + (1-\lambda_t)B(Q_t)\} \tag{12}$$

subject to the mortgage collateral borrowing constraint (Equation (7)), wealth processes (Equation (8)), and the intertemporal budget constraints (Equation (9)), where β is the time discount factor and $A_t = \{C_t, H_t, S_t, D_t^o, D_t^s\}$ is the vector of choice variables.

An analytical solution for the optimization problem does not exist. We thus derive numerical solutions through backward induction of value function. The online Appendix A provides details of our numerical method.

~~2.6. *Characterization of Individual Housing and Consumption Behavior.* Qualitatively, at a given household age, the effects of wealth-to-income ratio and house value-to-income ratio on the household's optimal decision rules are similar to those reported in Yao and Zhang (2005). A renter's house tenure decision is largely determined by the renter's wealth-to-income ratio as depicted in Figure A4 in the online appendix. In this figure, we plot a renter's house tenure decision as a function of the renter's wealth-to-permanent-labor-income ratio for a given median house price. Two observations emerge. First, the more wealth a renter has relative to income, the more likely the renter will buy, as more wealth on hand enables the renter to afford the down payment for a house of desired value. The wealth-to-income ratio that triggers homeownership increases late in life, reflecting the short expected duration of an older household.[9] Once a household becomes a homeowner, the household will stay in the house as long as its house-value-to-income ratio is not too far from the optimal level it would have chosen as a renter in order to avoid incurring transaction costs~~.

[9] Note that the wealth-to-income cutoff ratio for homeownership is not tilted upward for the very young as in Yao and Zhang (2005). This is because we target homeownership rates for married stable households between the ages 25 and 34, whereas Yao and Zhang (2005) target much lower ownership rates for the very young, and a much higher cutoff ratio is needed to generate the low rates.