Technical University of Denmark

*Written examination, date:* 9. December 2014  *Enclosure: 12 pages*

*Course name:* Multivariate Statistics *Course number:* 02409

*Aids allowed:* All

*Exam duration:* 4 hours

*Weighting:* The questions are given equal weight

This exam is answered by:

____________________ ____________________ __________

(name) (signature) (study no.)

There is a total of 30 questions for the 6 problems. The answers to the 30 questions must be written into the table below.

| Problem | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
|---|---|---|---|---|---|---|---|---|---|---|
| Question | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 2.1 | 2.2 | 2.3 | 2.4 |
| Answer | | | | | | | | | | |

| Problem | 2 | 3 | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 |
|---|---|---|---|---|---|---|---|---|---|---|
| Question | 2.5 | 3.1 | 3.2 | 3.3 | 3.4 | 3.5 | 4.1 | 4.2 | 4.3 | 4.4 |
| Answer | | | | | | | | | | |

| Problem | 5 | 5 | 5 | 5 | 5 | 6 | 6 | 6 | 6 | 6 |
|---|---|---|---|---|---|---|---|---|---|---|
| Question | 5.1 | 5.2 | 5.3 | 5.4 | 5.5 | 6.1 | 6.2 | 6.3 | 6.4 | 6.5 |
| Answer | | | | | | | | | | |

The possible answers for each question are numbered from 1 to 6. If you enter a wrong number, you may correct it by crossing the wrong number in the table and writing the correct answer immediately below. If there is any doubt about the meaning of a correction then the question will be considered not answered.

**Only the front page must be returned.** The front page must be returned even if you do not answer any of the questions or if you leave the exam prematurely. Drafts and/or comments are not considered, only the numbers entered above are registered.

A correct answer gives 5 points, a wrong answer gives – 1 point. Unanswered questions or a 6 (corresponding to “don’t know”) give 0 points. The total number of points needed for a satisfactorily answered exam is determined at the final evaluation of the exam. Especially note that the grade 10 may be given even if only one answer is wrong or unanswered.

Remember to write your name, signature, and study number on the front page.

*Please note, that there is one and only one correct answer to each question. Furthermore, some of the possible alternative answers may not make sense. When the text refers to SAS-output the values may be rounded to fewer decimal places than in the output itself. The enclosures do not necessarily contain all the output generated by the given SAS programs. Please check that all pages of the exam paper and the enclosures are present.*

# Problem 1

We consider the model

$$\begin{bmatrix} X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \\ X_3 & Y_3 & Z_3 \\ X_4 & Y_4 & Z_4 \\ X_5 & Y_5 & Z_5 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 2 \\ 1 & -1 & -1 \\ 1 & 0 & -2 \\ 1 & 1 & -1 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} \alpha_x & \alpha_y & \alpha_z \\ \beta_x & \beta_y & \beta_z \\ \gamma_x & \gamma_y & \gamma_z \end{bmatrix} + \begin{bmatrix} \varepsilon_1 & \delta_1 & \varphi_1 \\ \varepsilon_2 & \delta_2 & \varphi_2 \\ \varepsilon_3 & \delta_3 & \varphi_3 \\ \varepsilon_4 & \delta_4 & \varphi_4 \\ \varepsilon_5 & \delta_5 & \varphi_5 \end{bmatrix}$$

Where the error terms $[\varepsilon_i \quad \delta_i \quad \varphi_i]'$, $i = 1,2,3,4,5,$ are independent and normally distributed $N_3(\mathbf{0}, \mathbf{\Sigma})$, and where $\mathbf{\Sigma}$ is the unknown dispersion matrix

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_z^2 \end{bmatrix}$$

We assume that we obtained the following observations

$$\begin{bmatrix} 2 & 4 & 6 \\ 1 & 4 & 5 \\ 3 & 2 & 2 \\ 4 & 3 & 5 \\ 0 & 2 & 2 \end{bmatrix}$$

With the usual notation we have

$$\boldsymbol{x}'\boldsymbol{x} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 14 \end{bmatrix}$$

*The problem continues on the next page*

## Question 1.1.

The maximum likelihood estimator for $\alpha_x$ becomes

1 ☐ 1

2 ☐ 2

3 ☐ 3

4 ☐ 4

5 ☐ 5

6 ☐ Don't know

## Question 1.2.

The covariance between the maximum likelihood estimators for $\alpha_x$ and $\beta_x$ becomes

1 ☐ $\sigma_{xy}$

2 ☐ $0$

3 ☐ $\frac{1}{10}$

4 ☐ $\sigma_{xz}$

5 ☐ $\frac{1}{5}$

6 ☐ Don't know

## Question 1.3.

The covariance between the maximum likelihood estimators for $\alpha_x$ and $\alpha_y$ becomes

1 ☐ $\frac{1}{10}$

2 ☐ $0$

3 ☐ $\frac{1}{5}\sigma_{xy}$

4 ☐ $\frac{1}{10}\sigma_{xy}$

5 ☐ $\frac{1}{5}\sigma_{xz}$

6 ☐ Don't know

*The problem continues on the next page*

## Question 1.4.

We now want to test the hypothesis

$$H_0 : \begin{bmatrix} \beta_y & \beta_z \\ \gamma_y & \gamma_z \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

against all alternatives. This hypothesis may also be written

$$H_0\text{: } \boldsymbol{A} \begin{bmatrix} \alpha_x & \alpha_y & \alpha_z \\ \beta_x & \beta_y & \beta_z \\ \gamma_x & \gamma_y & \gamma_z \end{bmatrix} \boldsymbol{B}' = \boldsymbol{C} \text{ against } H_1\text{: } \boldsymbol{A} \begin{bmatrix} \alpha_x & \alpha_y & \alpha_z \\ \beta_x & \beta_y & \beta_z \\ \gamma_x & \gamma_y & \gamma_z \end{bmatrix} \boldsymbol{B}' \neq \boldsymbol{C}$$

Here the matrix $\boldsymbol{A}$ is:

1 ☐ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

2 ☐ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

3 ☐ $\begin{bmatrix} 0 & 1 & 1 \end{bmatrix}$

4 ☐ $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

5 ☐ $\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$

6 ☐ Don't know.

## Question 1.5.

The matrix $\boldsymbol{B}$ is:

1 ☐ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2 ☐ $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3 ☐ $\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$

4 ☐ $\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$

5 ☐ $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

6 ☐ Don't know.

*The problem continues on the next page*

### Question 1.6.

If the hypothesis $H_0$ is true then the distribution of the usual test statistic is:

1 ☐ U(2, 2, 1)

2 ☐ U(1, 1, 2)

3 ☐ U(1, 2, 1)

4 ☐ U(2, 2, 2)

5 ☐ U(2, 2, 5)

6 ☐ Don't know.

# Problem 2

We consider a normally distributed random variable

$$\begin{bmatrix} X \\ Y \end{bmatrix}$$

that represents a measurement of a two-dimensional property on subjects that belong to one of two populations $\pi_1$ or $\pi_2$. The dispersion matrix is equal to

$$\mathbf{\Sigma} = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}$$

The mean value depends on which population the subject belongs to:

$$E\left(\begin{bmatrix} X \\ Y \end{bmatrix}\right) = \begin{cases} \boldsymbol{\mu}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \ if\ \pi_1\ is\ true \\ \\ \boldsymbol{\mu}_2 = \begin{bmatrix} 4 \\ 4 \end{bmatrix} \ if\ \pi_2\ is\ true \end{cases}$$

We finally assume that the prior probabilities of belonging to either population is 0.5. We immediately obtain

$$\mathbf{\Sigma}^{-1} = \begin{bmatrix} \frac{4}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{4}{3} \end{bmatrix}$$

$$\boldsymbol{\mu}_1'\mathbf{\Sigma}^{-1}\boldsymbol{\mu}_1 = \frac{4}{3} \quad , \quad \boldsymbol{\mu}_2'\mathbf{\Sigma}^{-1}\boldsymbol{\mu}_2 = \frac{64}{3} \quad , \quad (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2)'\mathbf{\Sigma}^{-1}(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2) = 12$$

*The problem continues on the next page*

## Question 2.1.

The linear discriminant function Z for distinguishing between the two populations is:

1 ☐ $-2X + Y$

2 ☐ $-2X - 2Y + 10$

3 ☐ $X + Y - 12$

4 ☐ $X - 12$

5 ☐ $-2X - 2Y$

6 ☐ Don’t know

## Question 2.2.

If $\pi_2$ is true then the mean of Z is:

1 ☐ $-6$

2 ☐ $-3$

3 ☐ $0$

4 ☐ $3$

5 ☐ $6$

6 ☐ Don’t know.

## Question 2.3.

The variance of Z is:

1 ☐ $6$

2 ☐ $\frac{64}{3}$

3 ☐ $36$

4 ☐ $12$

5 ☐ $\frac{4}{3} \times \frac{64}{3}$

6 ☐ Don’t know.

*The problem continues on the next page*

### Question 2.4.

Let $\Phi(x)$ be the cumulative distribution function for a $N(0,1)$-distributed random variable. If $\pi_2$ is true then the probability of misclassification is :

1 ☐ 0.05

2 ☐ $\Phi(12)$

3 ☐ $\Phi(\sqrt{3})$

4 ☐ $1 - \Phi(\sqrt{3})$

5 ☐ 0.95

6 ☐ Don't know.

### Question 2.5.

If the prior probability for $\pi_2$ is 0.9 (and 0.1 for $\pi_1$ ) the the above probability of misclassification becomes:

1 ☐ 0.023

2 ☐ $1 - \Phi(2.37)$

3 ☐ $\Phi(2.37)$

4 ☐ $\Phi(10.2)$

5 ☐ $0.0000$

6 ☐ Don't know.

# Problem 3

Enclosure A belongs to this problem. The data are the first 20 records of gene expression data taken from www.biostat.jhsph.edu/~ririzarr/Teaching/649/. It is not necessary to be familiar with gene expression data in order to solve the problems. The variables are called X and Y and we wish to see how well ln(Y) may be predicted based on ln(X).

We do a regression analysis with ln(Y) = lny as dependent variable and ln(X) = lnx as independent variable.

*The problem continues on the next page*

## Question 3.1.

The 95% confidence interval for the coefficient to lnx is:

1 ☐ $[0.96429 - t(18)_{0.975} \times 0.03336,\ 0.96429 + t(18)_{0.975} \times 0.03336]$

2 ☐ $\left[0.96429 - t(18)_{0.975} \times \sqrt{0.01014},\ 0.96429 + t(18)_{0.975} \times \sqrt{0.01014}\right]$

3 ☐ $[0.94929 - t(18)_{0.975} \times 0.01014, 0.94929 + t(18)_{0.975} \times 0.01014]$

4 ☐ $\left[0.96429 - t(18)_{0.975} \times \sqrt{0.03336},\ 0.96429 + t(18)_{0.975} \times \sqrt{0.03336}\right]$

5 ☐ $[0.94929 - 1.96 \times 0.1007,\ 0.94929 + 1.96 \times 0.1007]$

6 ☐ Don't know.

## Question 3.2.

The length of the 95% prediction interval for the last observation is:

1 ☐ $2 \times \sqrt{0.0230^2 + 0.01014}$

2 ☐ $2 \times t(18)_{0.975} \times \sqrt{0.0230^2 + 1}$

3 ☐ $2 \times t(18)_{0.975} \times \sqrt{0.0230 + 0.01014}$

4 ☐ $2 \times t(18)_{0.975} \times \sqrt{0.0230^2 + 0.01014}$

5 ☐ $2 \times t(18)_{0.975} \times \sqrt{0.01014}$

6 ☐ Don't know.

## Question 3.3.

We now remove each of the observations one at a time. The observation that - when removed- will cause the numerically least change in its predicted value is no:

1 ☐ 2

2 ☐ 6

3 ☐ 10

4 ☐ 15

5 ☐ 20

6 ☐ Don't know.

*The problem continues on the next page*

### Question 3.4.
The number of observations that have as well an extreme RStudent residual as an extreme (i.e. large) leverage is:

1 ☐ 0

2 ☐ 1

3 ☐ 2

4 ☐ 3

5 ☐ 4

6 ☐ Don’t know.

### Question 3.5.
For which observation do we have the worst prediction of lny?

1 ☐ 2

2 ☐ 6

3 ☐ 9

4 ☐ 14

5 ☐ 18

6 ☐ Don’t know.

# Problem 4

Enclosure B belongs to this problem. The data are measurements of the average number of miles per gallon gasoline a number of cars could drive under controlled circumstances. The original data source is R.M. Heavenrich, J.D. Murrell, and K.H. Hellman, Light Duty Automotive Technology and Fuel Economy Trends Through 1991, U.S. Environmental Protection Agency, 1991 (EPA/AA/CTAB/91-02). The number of cases is 82 and the variables measured were:

1. vol: Cubic feet of cabin space
2. hp: Engine horsepower
3. mpg: Average miles per gallon
4. sp: Top speed (mph)
5. wt: Vehicle weight (100 lb)

*The problem continues on the next page*

The fuel consumption (mpg) is the independent variable, and in order to improve linearity we have taken the logarithm of mpg. Furthermore we have added the logarithm and the square of the independent variables to the data set. Firstly we consider the model that contains ‘vol  lnhp  sp  wt’.

## Question 4.1.

The diagnostic plots show that one might consider adding two quadratic terms to the model. The variables whose squares should be added are:

1 ☐ (vol, lnhp)

2 ☐ (vol, sp)

3 ☐ (vol, wt)

4 ☐ (lnhp, sp)

5 ☐ (lnhp, wt)

6 ☐ Don’t know.

## Question 4.2.

How many residuals (RStudent) fall outside the usual [-2, 2] interval:

1 ☐ 1

2 ☐ 5

3 ☐ 9

4 ☐ 13

5 ☐ 17

6 ☐ Don’t know.

*The problem continues on the next page*

### Question 4.3.

We now consider the model that contains 'wt lnhp sqsp sqhp' and we want to make a single test in order to see whether we may assume that the coefficients to sqsp and sqhp can be assumed to be simultaneously equal to 0. The usual test statistic is:

1 ☐ $\frac{(0.74641-0.44873)/1}{0.44873/79}$

2 ☐ $\frac{(0.56239-0.44873)/2}{0.44873/79}$

3 ☐ $\frac{(0.56239-0.41289)/2}{0.44873/77}$

4 ☐ $\frac{(0.56239-0.74641)/2}{0.44873/77}$

5 ☐ $\frac{(0.56239-0.44873)/2}{0.44873/77}$

6 ☐ Don't know.

### Question 4.4.

The distribution under the null hypothesis of the statistic above is:

1 ☐ t(79)

2 ☐ F(1, 79)

3 ☐ F(2, 79)

4 ☐ U(2, 2, 79)

5 ☐ F(2, 77)

6 ☐ Don't know.

# Problem 5

Enclosure C with SAS program and SAS output belongs to this problem. The measurements are estimated correlations between 176 observed values of different properties of offspring of alfalfa (lucerne). The data is described in Example 5.4, p. 238 in the course book Ersbøll & Conradsen (2012). The variables measured are also given in the table below.

*The problem continues on the next page*

| Variable no. & name | Unit of measure | Explanation |
|---|---|---|
| x1: Type of growth | Grade 1 - 9 | 1 = growth is lying down, 9 = growth is upright |
| x2: Regrowth after winter | Grade 1 - 9 | 1 = worst, 9 = best |
| x3: Ability to creep | Grade 1 - 9 | 1 = no runners, 9 = most runners |
| x4: Activity | Grade 1 - 9 | 1 = weakest, 9 = strongest |
| x5: Time of blooming | Grade 1 - 9 | 1 = latest blooming, 9 = earliest blooming |
| x6: Plant height | cm | |
| x7: Seed weight | g per plant | |
| x8: Plant weight | g per plant after drying | |
| x9: Percent seed | % | Calculated per plant by means of (7) and (8) |

## Question 5.1.

How many principal components must we include if we want to explain at least 80% of the total variation ?

1 ☐ 2

2 ☐ 3

3 ☐ 4

4 ☐ 5

5 ☐ 6

6 ☐ Don't know

## Question 5.2.

The degrees of freedom for the usual test statistic for testing the hypothesis that the smallest 3 eigenvalues are equal against all alternatives is:

1 ☐ 5

2 ☐ 10

3 ☐ 15

4 ☐ 20

5 ☐ 25

6 ☐ Don't know.

*The problem continues on the next page*

## Question 5.3.

We now consider a factor analysis with three factors of the data considered above. Consider the following statements on a plant having a large value of an arbitrary factor

A. A small plant with an upright growth.
B. A plant that immediately looks healthy: Early blooming, good height and weight, lot of seeds - absolute and relative.
C. A plant with very little seed.
D. A plant with good 'dynamic' properties: A heavy plant that grows fine after winter, that has many runners and shows good activity.
E. A plant that irrespective of whether it is upright or lying down looks good and has good 'dynamic' properties.

For (VARIMAX rotated factor 1, VARIMAX rotated factors 2, VARIMAX rotated factors 3) the following characterization is adequate:

1 ☐ (E, B, D)

2 ☐ (D, C, A)

3 ☐ (A, B, C)

4 ☐ (A, D, C)

5 ☐ (B, D, A)

6 ☐ Don't know.

## Question 5.4.

What fraction of the total variance will be explained by the first VARIMAX rotated factor?

1 ☐ 0.17187+0.13527+0.79486

2 ☐ 0.3475

3 ☐ $0.679644^2$

4 ☐ $2.7197/9$

5 ☐ $2.5733320^2 + 1.3926576^2$

6 ☐ Don't know.

*The problem continues on the next page*

### Question 5.5.

What fraction of the variation of the first variable x1 is explained by the third VARIMAX rotated factor?

1 ☐ 0.79486

2 ☐ 0.679644

3 ☐ $0.679644^2$

4 ☐ $0.79486^2$

5 ☐ 1.3926576/9

6 ☐ Don’t know.

# Problem 6

Enclosure D belongs to this problem. The data are due to Littell, Freund and Spector (1991). They are measurements on the effect of three different weightlifting programs. There were taken measurements at 7 different time points, the same for the three treatments. They were

- RI: The number of repetitions of weightlifting was increased as subjects became stronger.
- WI: The amount of weight was increased as subjects became stronger.
- CONT: Control group with no training.

Since there may be large individual differences between the strength of the subjects participating in the study we have normalized the last two measurements by dividing them with the first measurement yielding the new variables RS6 and RS7. We shall only consider these relative measures of the strength of the subjects.

### Question 6.1.

We now want to compare the three treatments with a multivariate analysis of variance of the relative strengths at times 6 and 7. Then the null hypothesis distribution of the usual test statistic Wilk’s Lambda is:

1 ☐ U(2, 1, 55)

2 ☐ U(2, 2, 54)

3 ☐ U(2, 3, 56)

4 ☐ F(2, 54)

5 ☐ F(3, 56)

6 ☐ Don’t know.

*The problem continues on the next page*

## Question 6.2.

Under the usual assumptions the estimate for the variance of RS6 is equal to:

1 ☐ $\sqrt{0.0059531967/2}$

2 ☐ 0.0059531967/2

3 ☐ 0.0349065

4 ☐ 0.0349065/54

5 ☐ $\sqrt{0.0349065/54}$

6 ☐ Don't know.

## Question 6.3.

We now only consider the active treatment groups WI and RI, and we want to test whether the means of $[RS6 \quad RS7]'$ are the same for the two treatment groups. The usual test statistic becomes:

1 ☐ $\frac{16+21-2-1}{2(16+21-2)}\frac{16\times 21}{16+21}0.16700$

2 ☐ 0.0068386375

3 ☐ $\frac{16+21-2-1}{2(16+21-2)}\frac{16\cdot 21}{16+21}0.14851$

4 ☐ $\frac{57-2}{56\times 2}0.14851^2$

5 ☐ $\frac{16+21-2-1}{2(16+21-2)}\frac{16\times 21}{16+21}0.81667748^2$

6 ☐ Don't know.

## Question 6.4.

The distribution under the null hypothesis of the test statistic above is:

1 ☐ t(54)

2 ☐ F(1, 54)

3 ☐ F(2, 34)

4 ☐ U(2, 2, 34)

5 ☐ F(2, 54)

6 ☐ Don't know.

*The problem continues on the next page*

**Question 6.5.**

We are now interested in using the values of $[RS6 \quad RS7]'$ in distinguishing between the two training programs with a decision function of the form $[RS6 \quad RS7]' \begin{bmatrix} a \\ b \end{bmatrix} > c$ . Rounded to integers the values of a, b and c should be chosen as

1 ☐ $27, -32, -6$

2 ☐ $21, 28, 33$

3 ☐ $24, -4, 21$

4 ☐ $20, 18, 8$

5 ☐ $16, 12, 8$

6 ☐ Don’t know.

# Enclosure A – SAS program

```
/*The gene dataset consists of the first 20 observations from the sasuser.gene
dataset taken from (http://www.biostat.jhsph.edu/~ririzarr/Teaching/649)*/;

data gene;
set sasuser.gene;
if _n_<21;
run;
data gene;
set gene;
lny = log(y);
lnx = log(x);
run;

Title 'The observations in the gene expression dataset';
proc print data=gene; run;

ods graphics on;

proc reg data=gene plots(label)=all;
model lny=lnx/r influence;
run;

ods graphics off;
```



# Enclosure A – SAS Output

'

**The observations in the gene expression dataset**

| Obs | y | x | lny | lnx |
|---|---|---|---|---|
| 1 | 1012.92 | 403.10 | 6.92060 | 5.99917 |
| 2 | 1777.01 | 888.12 | 7.48269 | 6.78911 |
| 3 | 1838.03 | 1015.10 | 7.51645 | 6.92274 |
| 4 | 1878.87 | 1039.59 | 7.53843 | 6.94658 |
| 5 | 2481.68 | 1352.98 | 7.81669 | 7.21007 |
| 6 | 718.38 | 375.99 | 6.57700 | 5.92957 |
| 7 | 631.74 | 334.46 | 6.44848 | 5.81251 |
| 8 | 801.87 | 418.60 | 6.68694 | 6.03692 |
| 9 | 735.21 | 287.91 | 6.60016 | 5.66266 |
| 10 | 1154.28 | 547.40 | 7.05124 | 6.30517 |
| 11 | 876.57 | 400.69 | 6.77602 | 5.99319 |
| 12 | 1764.01 | 794.54 | 7.47535 | 6.67776 |
| 13 | 2068.58 | 915.77 | 7.63462 | 6.81976 |
| 14 | 9284.50 | 4295.62 | 9.13610 | 8.36535 |
| 15 | 2157.77 | 1100.30 | 7.67683 | 7.00334 |
| 16 | 4633.86 | 2245.81 | 8.44114 | 7.71682 |
| 17 | 1647.83 | 878.70 | 7.40722 | 6.77845 |
| 18 | 2891.35 | 1549.99 | 7.96948 | 7.34600 |
| 19 | 858.48 | 439.49 | 6.75517 | 6.08562 |
| 20 | 1838.49 | 895.51 | 7.51670 | 6.79739 |

# Enclosure A – SAS Output

**The REG Procedure**
**Model: MODEL1**
**Dependent Variable: lny**

| | |
|---|---|
| Number of Observations Read | 20 |
| Number of Observations Used | 20 |

| Analysis of Variance | | | | | |
|---|---|---|---|---|---|
| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
| Model | 1 | 8.47517 | 8.47517 | 835.78 | <.0001 |
| Error | 18 | 0.18253 | 0.01014 | | |
| Corrected Total | 19 | 8.65770 | | | |

| | | | |
|---|---|---|---|
| Root MSE | 0.10070 | R-Square | 0.9789 |
| Dependent Mean | 7.37136 | Adj R-Sq | 0.9777 |
| Coeff Var | 1.36610 | | |

| Parameter Estimates | | | | | |
|---|---|---|---|---|---|
| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
| Intercept | 1 | 0.94929 | 0.22328 | 4.25 | 0.0005 |
| lnx | 1 | 0.96429 | 0.03336 | 28.91 | <.0001 |



# Enclosure A – SAS Output

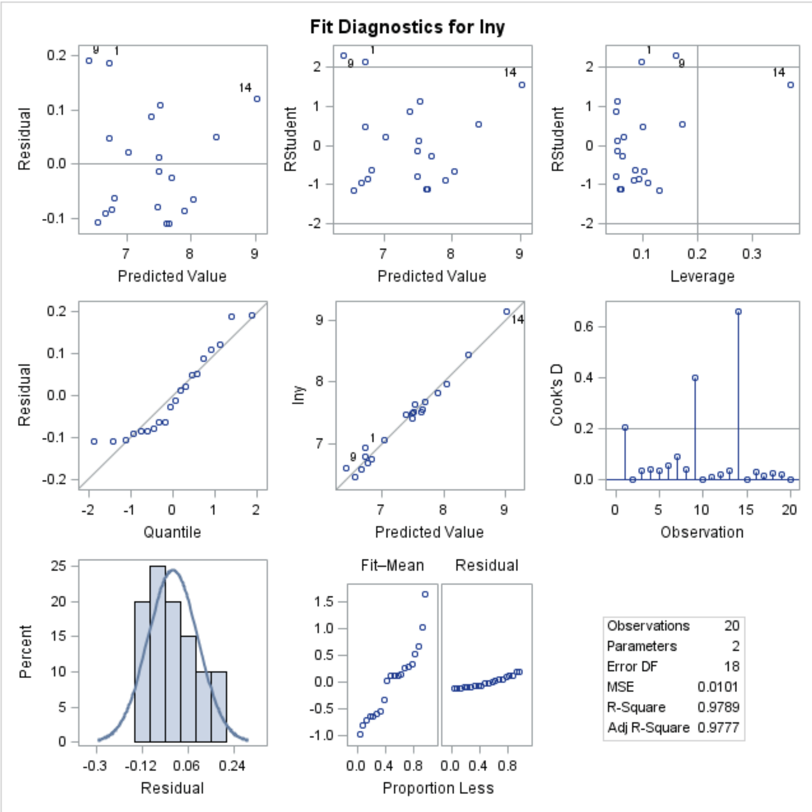
**The REG Procedure**
**Model: MODEL1**
**Dependent Variable: lny**

| Output Statistics | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Obs | Dep. Variable | Pred. Value | Std Err. Mean Predict | Residual | Std Err. Residual | Student Resi-dual | -2-1 0 1 2 | Cook's D | Hat Diag H | DFFITS |
| 1 | 6.9206 | 6.7342 | 0.0315 | 0.1864 | 0.0956 | 1.949 | \|*** | 0.206 | 0.0979 | 0.7023 |
| 2 | 7.4827 | 7.4960 | 0.0229 | -0.0133 | 0.0981 | -0.135 | \| | 0.001 | 0.0518 | -0.0308 |
| 3 | 7.5165 | 7.6248 | 0.0242 | -0.1084 | 0.0978 | -1.108 | **\| | 0.038 | 0.0576 | -0.2758 |
| 4 | 7.5384 | 7.6478 | 0.0245 | -0.1094 | 0.0977 | -1.120 | **\| | 0.039 | 0.0590 | -0.2825 |
| 5 | 7.8167 | 7.9019 | 0.0290 | -0.0852 | 0.0964 | -0.883 | *\| | 0.035 | 0.0832 | -0.2645 |
| 6 | 6.5770 | 6.6671 | 0.0332 | -0.0901 | 0.0951 | -0.948 | *\| | 0.055 | 0.1085 | -0.3297 |
| 7 | 6.4485 | 6.5542 | 0.0361 | -0.1057 | 0.0940 | -1.125 | **\| | 0.094 | 0.1288 | -0.4360 |
| 8 | 6.6869 | 6.7706 | 0.0306 | -0.0837 | 0.0959 | -0.872 | *\| | 0.039 | 0.0926 | -0.2767 |
| 9 | 6.6002 | 6.4097 | 0.0402 | 0.1904 | 0.0923 | 2.062 | \|**** | 0.402 | 0.1591 | 0.9975 |
| 10 | 7.0512 | 7.0293 | 0.0254 | 0.0219 | 0.0974 | 0.225 | \| | 0.002 | 0.0638 | 0.0572 |
| 11 | 6.7760 | 6.7285 | 0.0316 | 0.0476 | 0.0956 | 0.498 | \| | 0.014 | 0.0988 | 0.1612 |
| 12 | 7.4753 | 7.3886 | 0.0225 | 0.0868 | 0.0981 | 0.884 | \|* | 0.021 | 0.0500 | 0.2016 |
| 13 | 7.6346 | 7.5255 | 0.0231 | 0.1091 | 0.0980 | 1.113 | \|** | 0.035 | 0.0528 | 0.2647 |
| 14 | 9.1361 | 9.0159 | 0.0612 | 0.1202 | 0.0800 | 1.503 | \|*** | 0.661 | 0.3691 | 1.1945 |
| 15 | 7.6768 | 7.7025 | 0.0253 | -0.0257 | 0.0975 | -0.264 | \| | 0.002 | 0.0629 | -0.0665 |
| 16 | 8.4411 | 8.3905 | 0.0418 | 0.0506 | 0.0916 | 0.553 | \|* | 0.032 | 0.1726 | 0.2473 |
| 17 | 7.4072 | 7.4857 | 0.0229 | -0.0785 | 0.0981 | -0.800 | *\| | 0.017 | 0.0515 | -0.1845 |
| 18 | 7.9695 | 8.0330 | 0.0321 | -0.0635 | 0.0954 | -0.665 | *\| | 0.025 | 0.1016 | -0.2201 |
| 19 | 6.7552 | 6.8176 | 0.0296 | -0.0624 | 0.0963 | -0.648 | *\| | 0.020 | 0.0862 | -0.1958 |
| 20 | 7.5167 | 7.5039 | 0.0230 | 0.0128 | 0.0980 | 0.130 | \| | 0.000 | 0.0521 | 0.0297 |

| | |
|---|---|
| Sum of Residuals | 0 |
| Sum of Squared Residuals | 0.18253 |
| Predicted Residual SS (PRESS) | 0.24540 |

# Enclosure A – SAS Output

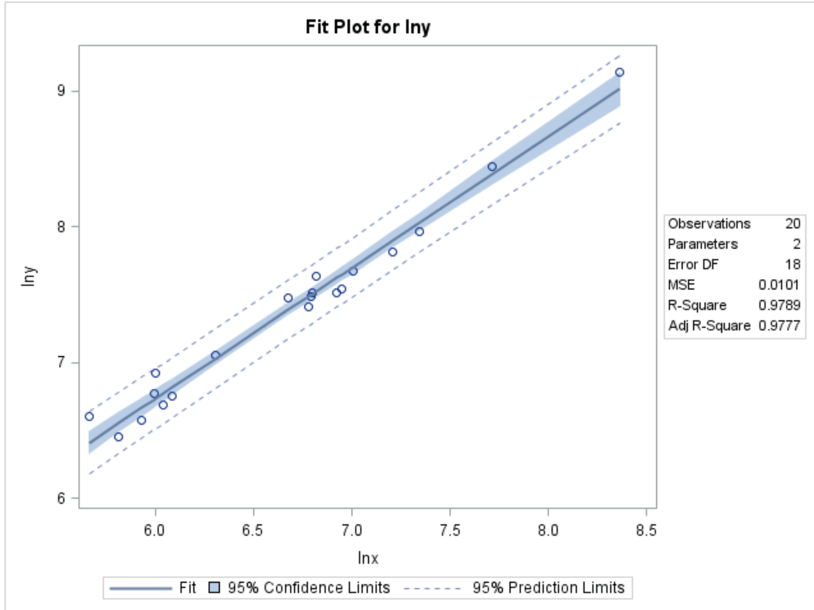
,






# Enclosure A – SAS Output

,



# Enclosure B – SAS program

```
/*Passenger Car Mileage data from R.M. Heavenrich, J.D. Murrell, and K.H.
Hellman: Light Duty Automotive Technology and Fuel Economy Trends Through
1991, U.S. Environmental Protection Agency, 1991 (EPA/AA/CTAB/91-02).
Data available at (http://lib.stat.cmu.edu/DASL/Datafiles/carmpgdat.html)*/;

data car;
set sasuser.car;
lnmpg=log(mpg); lnvol=log(vol); lnhp=log(hp); lnsp=log(sp);
   lnwt=log(wt);
sqvol=vol*vol; sqhp=hp*hp; sqsp=sp*sp; sqwt=wt*wt;
run;

Title 'The first 20 observations in the car mileage dataset with squares and
logarithms added';
proc print data=car (obs=20);
run;

ods graphics on;

Title 'Model lnmpg = vol lnhp sp wt';
proc reg plots(label)=all;
model lnmpg = vol lnhp sp wt;
run;

Title 'Stepwise regression on all variables';
proc reg plots(label)=all;
model lnmpg = vol lnvol sqvol hp lnhp sqhp sp lnsp sqsp wt lnwt sqwt /
   r    selection=stepwise;
run;

ods graphics off;
```



# Enclosure B – SAS Output

**The first 20 observations in the car mileage dataset without squares and logarithms added**

| Obs | make | mpg | vol | hp | sp | wt |
|---|---|---|---|---|---|---|
| 1 | GM/GeoMe | 65.4 | 89 | 49 | 96 | 17.5 |
| 2 | GM/GeoMe | 56.0 | 92 | 55 | 97 | 20.0 |
| 3 | GM/GeoMe | 55.9 | 92 | 55 | 97 | 20.0 |
| 4 | SuzukiSw | 49.0 | 92 | 70 | 105 | 20.0 |
| 5 | Daihatsu | 46.5 | 92 | 53 | 96 | 20.0 |
| 6 | GM/GeoSp | 46.2 | 89 | 70 | 105 | 20.0 |
| 7 | GM/GeoSp | 45.4 | 92 | 55 | 97 | 20.0 |
| 8 | HondaCiv | 59.2 | 50 | 62 | 98 | 22.5 |
| 9 | HondaCiv | 53.3 | 50 | 62 | 98 | 22.5 |
| 10 | Daihatsu | 43.4 | 94 | 80 | 107 | 22.5 |
| 11 | SubaruJu | 41.1 | 89 | 73 | 103 | 22.5 |
| 12 | HondaCiv | 40.9 | 50 | 92 | 113 | 22.5 |
| 13 | HondaCiv | 40.9 | 99 | 92 | 113 | 22.5 |
| 14 | SubaruJu | 40.4 | 89 | 73 | 103 | 22.5 |
| 15 | SubaruJu | 39.6 | 89 | 66 | 100 | 22.5 |
| 16 | SubaruJu | 39.3 | 89 | 73 | 103 | 22.5 |
| 17 | ToyotaTe | 38.9 | 91 | 78 | 106 | 22.5 |
| 18 | HondaCiv | 38.8 | 50 | 92 | 113 | 22.5 |
| 19 | ToyotaTe | 38.2 | 91 | 78 | 106 | 22.5 |
| 20 | FordEsco | 42.2 | 103 | 90 | 109 | 25.0 |

# Enclosure B – SAS Output

### Model lnmpg= vol lnhp sp wt

**The REG Procedure**
**Model: MODEL1**
**Dependent Variable: lnmpg**

**Analysis of Variance**

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|---|---|---|---|---|---|
| Model | 4 | 6.95846 | 1.73961 | 245.58 | <.0001 |
| Error | 77 | 0.54544 | 0.00708 | | |
| Corrected Total | 81 | 7.50390 | | | |

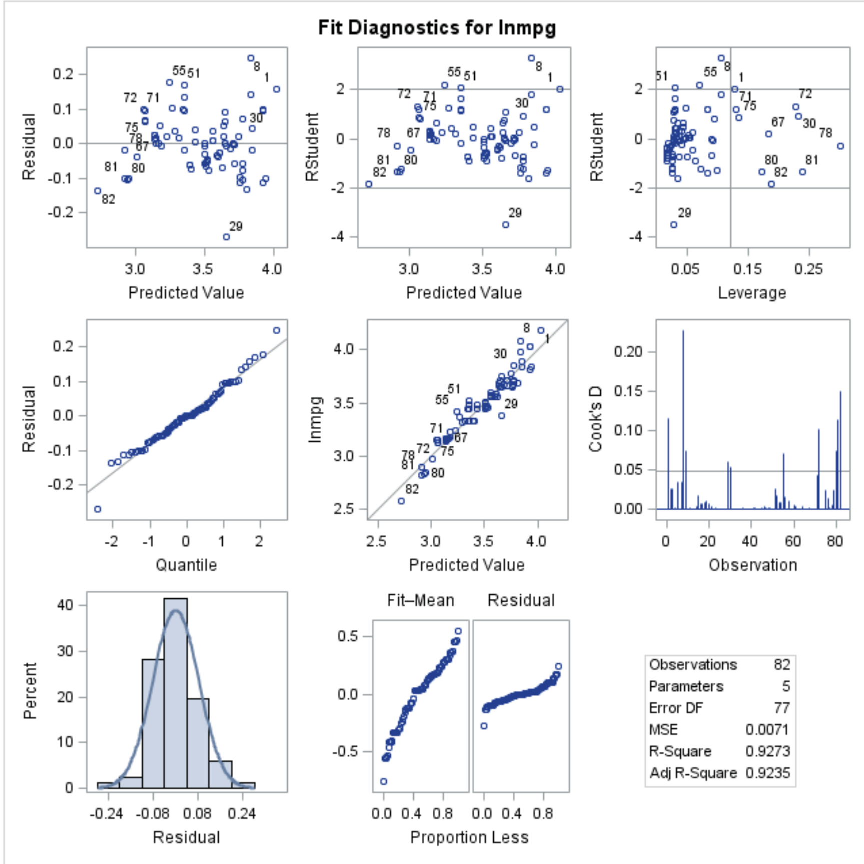
| | | | |
|---|---|---|---|
| Root MSE | 0.08416 | R-Square | 0.9273 |
| Dependent Mean | 3.47571 | Adj R-Sq | 0.9235 |
| Coeff Var | 2.42151 | | |

**Parameter Estimates**

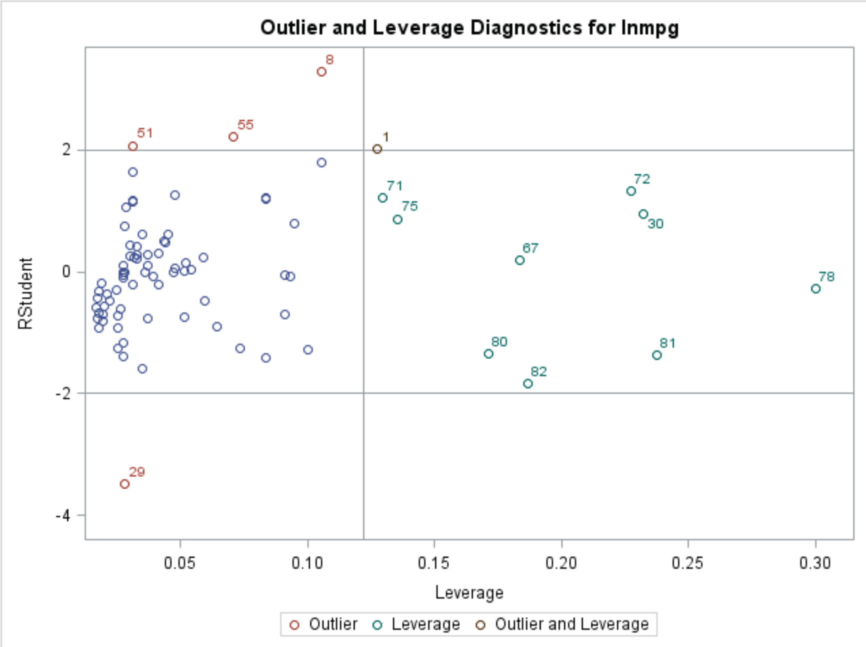
| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > \|t\| |
|---|---|---|---|---|---|
| Intercept | 1 | 5.75631 | 0.29230 | 19.69 | <.0001 |
| vol | 1 | -0.00012456 | 0.00052666 | -0.24 | 0.8137 |
| lnhp | 1 | -0.45030 | 0.14998 | -3.00 | 0.0036 |
| sp | 1 | 0.00388 | 0.00292 | 1.33 | 0.1879 |
| wt | 1 | -0.01943 | 0.00396 | -4.91 | <.0001 |



# Enclosure B – SAS Output

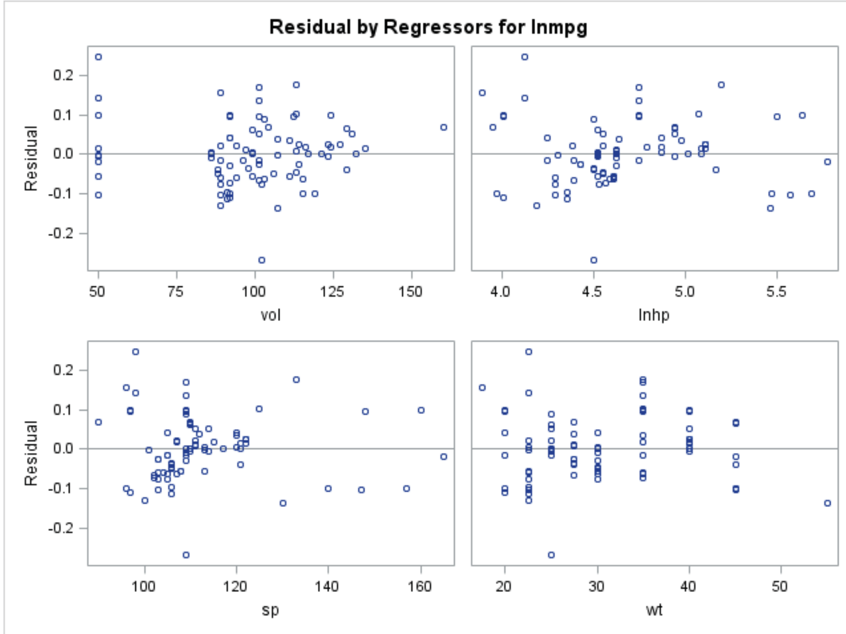


# Enclosure B – SAS Output






# Enclosure B – SAS Output



### Stepwise regression on all variables

**The REG Procedure**
**Model: MODEL1**
**Dependent Variable: lnmpg**

**Stepwise Selection: Step 1**

**Variable wt Entered: R-Square = 0.9005 and C(p) = 57.1574**

| Analysis of Variance | | | | | |
|---|---|---|---|---|---|
| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
| Model | 1 | 6.75750 | 6.75750 | 724.27 | <.0001 |
| Error | 80 | 0.74641 | 0.00933 | | |
| Corrected Total | 81 | 7.50390 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 4.57248 | 0.04213 | 109.92158 | 11781.4 | <.0001 |
| wt | -0.03548 | 0.00132 | 6.75750 | 724.27 | <.0001 |

**Bounds on condition number: 1, 1**

**Stepwise Selection: Step 2**

**Variable lnhp Entered: R-Square = 0.9251 and C(p) = 25.8365**

| Analysis of Variance | | | | | |
|---|---|---|---|---|---|
| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
| Model | 2 | 6.94151 | 3.47076 | 487.54 | <.0001 |
| Error | 79 | 0.56239 | 0.00712 | | |
| Corrected Total | 81 | 7.50390 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 5.38578 | 0.16415 | 7.66396 | 1076.56 | <.0001 |
| lnhp | -0.24895 | 0.04897 | 0.18401 | 25.85 | <.0001 |
| wt | -0.02417 | 0.00250 | 0.66353 | 93.21 | <.0001 |

**Bounds on condition number: 4.7282, 18.913**



**Stepwise Selection: Step 9**

**Variable lnhp Removed: R-Square = 0.9450 and C(p) = 4.7654**

| Analysis of Variance | | | | | |
|---|---|---|---|---|---|
| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
| Model | 5 | 7.09101 | 1.41820 | 261.04 | <.0001 |
| Error | 76 | 0.41289 | 0.00543 | | |
| Corrected Total | 81 | 7.50390 | | | |

| Variable | Parameter Estimate | Standard Error | Type II SS | F Value | Pr > F |
|---|---|---|---|---|---|
| Intercept | 16.83379 | 2.14108 | 0.33583 | 61.82 | <.0001 |
| hp | 0.01770 | 0.00511 | 0.06520 | 12.00 | 0.0009 |
| sqhp | -0.00004828 | 0.00001000 | 0.12668 | 23.32 | <.0001 |
| sp | -0.15797 | 0.02794 | 0.17370 | 31.97 | <.0001 |
| sqsp | 0.00060887 | 0.00010033 | 0.20007 | 36.83 | <.0001 |
| lnwt | -1.37447 | 0.22678 | 0.19957 | 36.73 | <.0001 |

**All variables left in the model are significant at the 0.1500 level.**

**No other variable met the 0.1500 significance level for entry into the model.**

| Summary of Stepwise Selection | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Step | Variable Entered | Variable Removed | Number Vars In | Partial R-Square | Model R-Square | C(p) | F Value | Pr > F |
| 1 | wt | | 1 | 0.9005 | 0.9005 | 57.1574 | 724.27 | <.0001 |
| 2 | lnhp | | 2 | 0.0245 | 0.9251 | 25.8365 | 25.85 | <.0001 |
| 3 | sqsp | | 3 | 0.0022 | 0.9273 | 24.8135 | 2.39 | 0.1265 |
| 4 | sqhp | | 4 | 0.0129 | 0.9402 | 9.2295 | 16.67 | 0.0001 |
| 5 | | wt | 3 | 0.0016 | 0.9386 | 9.3999 | 2.06 | 0.1555 |
| 6 | hp | | 4 | 0.0026 | 0.9412 | 7.8355 | 3.44 | 0.0675 |
| 7 | sp | | 5 | 0.0019 | 0.9432 | 7.1937 | 2.60 | 0.1109 |
| 8 | lnwt | | 6 | 0.0023 | 0.9455 | 6.0503 | 3.18 | 0.0784 |
| 9 | | lnhp | 5 | 0.0005 | 0.9450 | 4.7654 | 0.72 | 0.3974 |

# Enclosure C – SAS Program

```
/*The data comes from Example 5.4 p. 238 in Ersbøll & Conradsen (2012)*/;

data crop(type=corr);
infile cards;
input _type_ $ _name_ $ x1-x9;
cards;
N    . 176    176    176    176    176    176    176    176    176
CORR x1  1.000 -0.033  0.116  0.018  0.131 -0.207  0.035 -0.087  0.041
CORR x2 -0.033  1.000  0.711  0.515  0.125  0.199 -0.025  0.348 -0.066
CORR x3  0.116  0.711  1.000  0.440  0.022  0.039 -0.133  0.218 -0.157
CORR x4  0.018  0.515  0.440  1.000  0.201  0.517  0.071  0.689 -0.081
CORR x5  0.131  0.125  0.022  0.201  1.000  0.496  0.487  0.168  0.486
CORR x6 -0.207  0.199  0.039  0.517  0.496  1.000  0.453  0.559  0.367
CORR x7  0.035 -0.025 -0.133  0.071  0.487  0.453  1.000  0.360  0.947
CORR x8 -0.087  0.348  0.218  0.689  0.168  0.559  0.360  1.000  0.128
CORR x9  0.041 -0.066 -0.157 -0.081  0.486  0.367  0.947  0.128  1.000
;
run;

Title 'The crop data';
proc print data=crop;
run;

ods graphics on;
proc factor
  data=crop
  rotate=varimax
  plots=(scree initloadings(vector) loadings(vector));
run;
ods graphics off;
```



# Enclosure C – SAS Output

### The crop data

| Obs | _type_ | _name_ | x1 | x2 | x3 | x4 | x5 | x6 | x7 | x8 | x9 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | N | | 176 | 176 | 176 | 176 | 176 | 176 | 176 | 176 | 176 |
| 2 | CORR | x1 | 1.000 | -0.033 | 0.116 | 0.018 | 0.131 | -0.207 | 0.035 | -0.087 | 0.041 |
| 3 | CORR | x2 | -0.033 | 1.000 | 0.711 | 0.515 | 0.125 | 0.199 | -0.025 | 0.348 | -0.066 |
| 4 | CORR | x3 | 0.116 | 0.711 | 1.000 | 0.440 | 0.022 | 0.039 | -0.133 | 0.218 | -0.157 |
| 5 | CORR | x4 | 0.018 | 0.515 | 0.440 | 1.000 | 0.201 | 0.517 | 0.071 | 0.689 | -0.081 |
| 6 | CORR | x5 | 0.131 | 0.125 | 0.022 | 0.201 | 1.000 | 0.496 | 0.487 | 0.168 | 0.486 |
| 7 | CORR | x6 | -0.207 | 0.199 | 0.039 | 0.517 | 0.496 | 1.000 | 0.453 | 0.559 | 0.367 |
| 8 | CORR | x7 | 0.035 | -0.025 | -0.133 | 0.071 | 0.487 | 0.453 | 1.000 | 0.360 | 0.947 |
| 9 | CORR | x8 | -0.087 | 0.348 | 0.218 | 0.689 | 0.168 | 0.559 | 0.360 | 1.000 | 0.128 |
| 10 | CORR | x9 | 0.041 | -0.066 | -0.157 | -0.081 | 0.486 | 0.367 | 0.947 | 0.128 | 1.000 |

### The FACTOR Procedure

| | |
|---|---|
| Input Data Type | Correlations |
| N Set/Assumed in Data Set | 176 |
| N for Significance Tests | 176 |

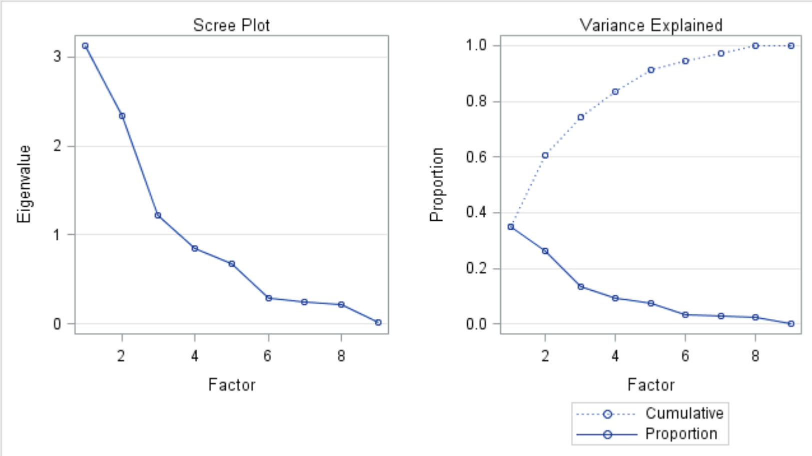
### The FACTOR Procedure
### Initial Factor Method: Principal Components

**Prior Communality Estimates: ONE**

| Eigenvalues of the Correlation Matrix: Total = 9 Average = 1 | | | | |
|---|---|---|---|---|
| | Eigenvalue | Difference | Proportion | Cumulative |
| 1 | 3.12713262 | 0.78562655 | 0.3475 | 0.3475 |
| 2 | 2.34150607 | 1.12443569 | 0.2602 | 0.6076 |
| 3 | 1.21707038 | 0.37454816 | 0.1352 | 0.7429 |
| 4 | 0.84252222 | 0.15878292 | 0.0936 | 0.8365 |
| 5 | 0.68373929 | 0.38994087 | 0.0760 | 0.9124 |
| 6 | 0.29379842 | 0.04096571 | 0.0326 | 0.9451 |
| 7 | 0.25283270 | 0.03277307 | 0.0281 | 0.9732 |
| 8 | 0.22005963 | 0.19872097 | 0.0245 | 0.9976 |
| 9 | 0.02133867 | | 0.0024 | 1.0000 |

**3 factors will be retained by the MINEIGEN criterion.**

# Enclosure C – SAS Output




| Factor Pattern | | | |
|---|---|---|---|
| | Factor1 | Factor2 | Factor3 |
| x1 | -0.03215 | -0.02210 | 0.82348 |
| x2 | 0.47109 | 0.66538 | 0.21979 |
| x3 | 0.30601 | 0.71369 | 0.39707 |
| x4 | 0.66989 | 0.55564 | -0.11180 |
| x5 | 0.62336 | -0.32613 | 0.26750 |
| x6 | 0.79859 | -0.07914 | -0.33676 |
| x7 | 0.68698 | -0.62850 | 0.11367 |
| x8 | 0.73621 | 0.26529 | -0.28829 |
| x9 | 0.56702 | -0.70865 | 0.19869 |

| Variance Explained by Each Factor | | |
|---|---|---|
| Factor1 | Factor2 | Factor3 |
| 3.1271326 | 2.3415061 | 1.2170704 |

| Final Communality Estimates: Total = 6.685709 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| x1 | x2 | x3 | x4 | x5 | x6 | x7 | x8 | x9 |
| 0.679644 | 0.712957 | 0.760665 | 0.769980 | 0.566497 | 0.757409 | 0.879880 | 0.695504 | 0.863172 |



# Enclosure C – SAS Output

**The FACTOR Procedure**
**Rotation Method: Varimax**

| Orthogonal Transformation Matrix | | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| 1 | 0.74239 | 0.59769 | -0.30269 |
| 2 | -0.63256 | 0.77418 | -0.02277 |
| 3 | 0.22073 | 0.20837 | 0.95282 |

| Rotated Factor Pattern | | | |
|---|---|---|---|
| | Factor1 | Factor2 | Factor3 |
| x1 | 0.17187 | 0.13527 | 0.79486 |
| x2 | -0.02264 | 0.84248 | 0.05167 |
| x3 | -0.13662 | 0.81816 | 0.26946 |
| x4 | 0.12117 | 0.80725 | -0.32194 |
| x5 | 0.72812 | 0.17583 | 0.07362 |
| x6 | 0.56859 | 0.34586 | -0.56079 |
| x7 | 0.93267 | -0.05229 | -0.08533 |
| x8 | 0.31511 | 0.58534 | -0.50358 |
| x9 | 0.91307 | -0.16832 | 0.03382 |

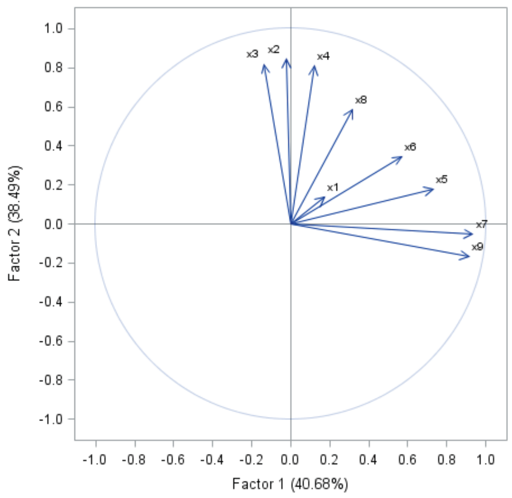
| Variance Explained by Each Factor | | |
|---|---|---|
| Factor1 | Factor2 | Factor3 |
| 2.7197196 | 2.5733320 | 1.3926576 |

| Final Communality Estimates: Total = 6.685709 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| x1 | x2 | x3 | x4 | x5 | x6 | x7 | x8 | x9 |
| 0.679644 | 0.712957 | 0.760665 | 0.769980 | 0.566497 | 0.757409 | 0.879880 | 0.695504 | 0.863172 |

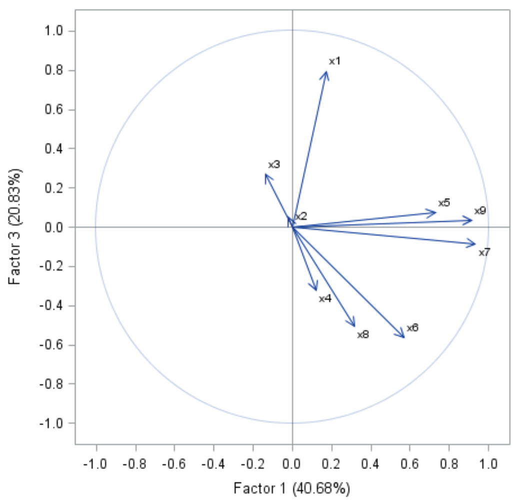
**The FACTOR Procedure**
**Rotation Method: Varimax**

# Enclosure C – SAS Output

**Rotated Factor Pattern**



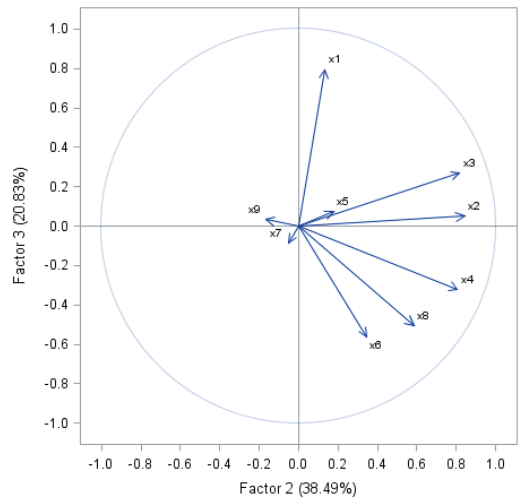

**Rotated Factor Pattern**






# Enclosure C – SAS Output

**Rotated Factor Pattern**



# Enclosure D – SAS Program

```
Title 'Strength data from Littell, Freund & Spector (1991)';
data weights;
input subject treatment $ s1 s2 s3 s4 s5 s6 s7;
cards;
1  cont  85 85 86 85 87 86 87
2  cont  80 79 79 78 78 79 78


20 cont  78 79 80 81 80 79 80
1  ri    79 79 79 80 80 78 80


16 ri    84 85 85 85 85 83 82
1  wi    84 85 84 83 83 83 84


21 wi    80 81 80 81 81 82 83
;
/*We add two new relative measures rs6 and rs7*/;
data weights;
set weights;
rs6=s6/s1; rs7=s7/s1;  run;

/*We discard the control group*/;
data weightsriwi;
set weights;
if treatment NE 'cont';  run;

Title 'Proc glm on the relative measures for all three treatments';
proc glm data=weights;
class treatment;
model rs6 rs7=treatment;
manova h=_all_/printe printh;
run;

Title 'Proc discrim on the relative measures for all three treatments';
proc discrim data=weights pool=yes  pcov;
class treatment;
var rs6 rs7;
run;

Title 'Proc discrim on the relative measures for the ri and the wi treatment';
proc discrim data=weightsriwi pool=yes  pcov;
class treatment;
var rs6 rs7;
run;
```

Only parts of the input data are shown. In total there are 20 cases from cont, 16 from ri and 21 from wi.



# Enclosure D – SAS Output

**Proc glm on the relative measures for all three treatments**

**The GLM Procedure**
**Multivariate Analysis of Variance**

| E = Error SSCP Matrix | | |
|---|---|---|
| | rs6 | rs7 |
| rs6 | 0.0349065144 | 0.0320103992 |
| rs7 | 0.0320103992 | 0.0389930628 |

| H = Type III SSCP Matrix for treatment | | |
|---|---|---|
| | rs6 | rs7 |
| rs6 | 0.0059531967 | 0.0068386375 |
| rs7 | 0.0068386375 | 0.0080267802 |

| Characteristic Roots and Vectors of: E Inverse * H, where H = Type III SSCP Matrix for treatment E = Error SSCP Matrix | | | |
|---|---|---|---|
| Characteristic Root | Percent | Characteristic Vector V'EV=1 | |
| | | rs6 | rs7 |
| 0.20681714 | 93.39 | 0.7631015 | 4.4249594 |
| 0.01463059 | 6.61 | -10.7384604 | 9.1744414 |

**MANOVA Test Criteria and F Approximations for the Hypothesis of No Overall treatment Effect**
**H = Type III SSCP Matrix for treatment**
**E = Error SSCP Matrix**

**S=2 M=-0.5 N=25.5**

| Statistic | Value | F Value | Num DF | Den DF | Pr > F |
|---|---|---|---|---|---|
| Wilks' Lambda | 0.81667748 | 2.82 | 4 | 106 | 0.0285 |
| Pillai's Trace | 0.18579367 | 2.77 | 4 | 108 | 0.0311 |
| Hotelling-Lawley Trace | 0.22144772 | 2.92 | 4 | 62.575 | 0.0281 |
| Roy's Greatest Root | 0.20681714 | 5.58 | 2 | 54 | 0.0062 |

**NOTE: F Statistic for Roy's Greatest Root is an upper bound.**
**NOTE: F Statistic for Wilks' Lambda is exact.**

# Enclosure D – SAS Output

## Proc discrim on the relative measures for all three treatments

### The DISCRIM Procedure

| | | | |
|---|---|---|---|
| Total Sample Size | 57 | DF Total | 56 |
| Variables | 2 | DF Within Classes | 54 |
| Classes | 3 | DF Between Classes | 2 |

| | |
|---|---|
| Number of Observations Read | 57 |
| Number of Observations Used | 57 |

| Class Level Information | | | | | |
|---|---|---|---|---|---|
| treatment | Variable Name | Frequency | Weight | Proportion | Prior Probability |
| cont | cont | 20 | 20.0000 | 0.350877 | 0.333333 |
| ri | ri | 16 | 16.0000 | 0.280702 | 0.333333 |
| wi | wi | 21 | 21.0000 | 0.368421 | 0.333333 |

| Pooled Within-Class Covariance Matrix, DF = 54 | | |
|---|---|---|
| Variable | rs6 | rs7 |
| rs6 | 0.0006464169 | 0.0005927852 |
| rs7 | 0.0005927852 | 0.0007220938 |

| Generalized Squared Distance to treatment | | | |
|---|---|---|---|
| From treatment | cont | ri | wi |
| cont | 0 | 0.62023 | 1.03154 |
| ri | 0.62023 | 0 | 0.14851 |
| wi | 1.03154 | 0.14851 | 0 |

| Linear Discriminant Function for treatment | | | |
|---|---|---|---|
| Variable | cont | ri | wi |
| Constant | -789.97582 | -821.58820 | -828.29052 |
| rs6 | 1119 | 1141 | 1120 |
| rs7 | 463.43389 | 473.18529 | 500.89391 |



# Enclosure D – SAS Output

## Proc discrim on the relative measures for the ri and the wi treatment

### The DISCRIM Procedure

| | | | |
|---|---|---|---|
| Total Sample Size | 37 | DF Total | 36 |
| Variables | 2 | DF Within Classes | 35 |
| Classes | 2 | DF Between Classes | 1 |

| | |
|---|---|
| Number of Observations Read | 37 |
| Number of Observations Used | 37 |

| Class Level Information | | | | | |
|---|---|---|---|---|---|
| treatment | Variable Name | Frequency | Weight | Proportion | Prior Probability |
| ri | ri | 16 | 16.0000 | 0.432432 | 0.500000 |
| wi | wi | 21 | 21.0000 | 0.567568 | 0.500000 |

| Pooled Within-Class Covariance Matrix, DF = 35 | | |
|---|---|---|
| Variable | rs6 | rs7 |
| rs6 | 0.0007003206 | 0.0006643357 |
| rs7 | 0.0006643357 | 0.0007811916 |

| Generalized Squared Distance to treatment | | |
|---|---|---|
| From treatment | ri | wi |
| ri | 0 | 0.16700 |
| wi | 0.16700 | 0 |

| Linear Discriminant Function for treatment | | |
|---|---|---|
| Variable | ri | wi |
| Constant | -748.96607 | -754.69084 |
| rs6 | 1125 | 1098 |
| rs7 | 346.41638 | 378.60181 |