

Written examination, date: 7. December 2010

Course name: Multivariate Statistics “Multivariat Statistik”.

Course no. : 02409

Exam duration: 4 hours

Aids allowed: All aids permitted

“Weighting”: The questions are given equal weight.

This exam is answered by:

(name)_____
(signature)_____
(study no.)

There is a total of 28 questions for the 9 problems. The answers to the 28 questions must be written into the table below.

Problem	1	1	1	1	1	1	1	1	2	2
Question	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	2.1	2.2
Answer										

Problem	2	2	2	3	3	3	3	3	4	4
Question	2.3	2.4	2.5	3.1	3.2	3.3	3.4	3.5	4.1	4.2
Answer										

Problem	4	4	5	5	6	7	8	9	XX	XX
Question	4.3	4.4	5.1	5.2	6.1	7.1	8.1	9.1	XX	XX
Answer									XX	XX

The possible answers for each question are numbered from 1 to 6. If you enter a wrong number, you may correct it by crossing the wrong number in the table and writing the correct answer immediately below. If there is any doubt about the meaning of a correction then the question will be considered not answered.

Only the front page must be returned. The front page must be returned even if you do not answer any of the questions or if you leave the exam prematurely. Drafts and/or comments are **not** considered, only the numbers entered above are registered.

A correct answer gives 5 points, a wrong answer gives -1 point. Unanswered questions or a 6 (corresponding to “don’t know”) gives 0 points. The total number of points, needed for a satisfactorily answered exam is determined at the final evaluation of the exam. Especially note that the grade 10 may be given even if only one answer is wrong or unanswered.

Remember to write your name, signature and study number on the front page.

Please note, that there is one and only one correct answer to each question. Furthermore, some of the possible alternative answers may not make sense. When the text refers to SAS-output the values may be rounded to fewer decimal places than in the output itself. Please check that all pages of the exam paper and the enclosure are present.

Problem 1.

Enclosure A with the SAS-program and the corresponding SAS-output belongs to this problem. #### indicates that information has been concealed (*Danish: skjult el. fjernet*).

The data are from a study on twenty engineering students and twenty pilots. Each variable corresponds to a score in a certain test.

The variables are:

Variable	Description
type	: engineering student (1), pilot (2)
x_1	: form relations
x_2	: dynamometer
x_3	: dotting
x_4	: sensory motor coordination
x_5	: perservation
x_6	: intelligence

Question 1.1.

The estimated value of $\text{Cov}(x_1, x_2)$ is calculated as:

1 ☐ $\frac{1}{39} \sum_{i=1}^{40} (x_{1,i} - 34.9)(x_{2,i} - 81.8)$

2 ☐ $\frac{1}{40} \sum_{i=1}^{40} (x_{1,i} - 34.9)(x_{2,i} - 81.8)$

3 ☐ $\frac{1}{39} \sum_{i=1}^{40} x_{1,i} \cdot x_{2,i} - 34.9 \cdot 81.8$

4 ☐ $\frac{1}{40} \sum_{i=1}^{40} x_{1,i} \cdot x_{2,i} - 34.9 \cdot 81.8$

5 ☐ $\frac{1}{39} \sum_{i=1}^{40} x_{1,i} \cdot x_{2,i}$

6 ☐ Don't know.

The problem continues on the next page

Question 1.2.

The space spanned by the two first eigenvectors of the variance-covariance matrix accounts for:

- 1 ☐ 25.5% of the variation of the original variables
- 2 ☐ 50.0% of the variation of the original variables
- 3 ☐ 75.5% of the variation of the original variables
- 4 ☐ 87.2% of the variation of the original variables
- 5 ☐ 94.8% of the variation of the original variables
- 6 ☐ Don't know.

Question 1.3.

The value of the first principal component for the first observation is:

- 1 ☐ $(22 - 34.9, 74 - 81.8, 223 - 214.7, 54 - 49.0, 254 - 265.3, 121 - 126.9)' \cdot (-0.04, 0.08, 0.78, -0.10, 0.58, 0.21)$
- 2 ☐ $(-0.04, 0.08, 0.78, -0.10, 0.58, 0.21) \cdot (22 - 34.9, 74 - 81.8, 223 - 214.7, 54 - 49.0, 254 - 265.3, 121 - 126.9)'$
- 3 ☐ $(22 - 34.9, 74 - 81.8, 223 - 214.7, 54 - 49.0, 254 - 265.3, 121 - 126.9)'$
- 4 ☐ $(22, 74, 223, 54, 254, 121)'$
- 5 ☐ $(-0.04, 0.08, 0.78, -0.10, 0.58, 0.21)$
- 6 ☐ Don't know.

Question 1.4.

The estimate of the correlation coefficient ρ_{13} is found as:

- 1 ☐ 1.0
- 2 ☐ $\frac{-81.8}{8.3 \cdot 37.0}$
- 3 ☐ -81.8
- 4 ☐ -1.0
- 5 ☐ $\frac{-81.8}{68.9 \cdot 1366.4}$
- 6 ☐ Don't know.

The problem continues on the next page

Question 1.5.

The usual test for the null-hypothesis: $\rho_{13|456} = 0$ has the following distribution (under the null-hypothesis):

- 1 ☐ $t(34)$
- 2 ☐ $t(35)$
- 3 ☐ $t(36)$
- 4 ☐ $t(37)$
- 5 ☐ $t(38)$
- 6 ☐ Don't know.

Question 1.6.

Consider the principal factor solution. The first element in that matrix is:

- 1 ☐ 1
- 2 ☐ 1.78
- 3 ☐ -0.10
- 4 ☐ -0.13
- 5 ☐ 0.30
- 6 ☐ Don't know.

Question 1.7.

Consider interpretation of the varimax-rotated factor solution. The variables mainly involved in the interpretation of each factor are:

- 1 ☐ varimax-factor 1: x_1, x_2 , varimax-factor 2: x_3, x_4 , varimax-factor 3: x_5, x_6
- 2 ☐ varimax-factor 1: x_2, x_3 , varimax-factor 2: x_1, x_4 , varimax-factor 3: x_5, x_6
- 3 ☐ varimax-factor 1: x_3, x_4 , varimax-factor 2: x_5, x_6 , varimax-factor 3: x_1, x_2
- 4 ☐ varimax-factor 1: x_1, x_3 , varimax-factor 2: x_4, x_6 , varimax-factor 3: x_2, x_5
- 5 ☐ varimax-factor 1: x_2, x_3 , varimax-factor 2: x_5, x_6 , varimax-factor 3: x_1, x_4
- 6 ☐ Don't know.

The problem continues on the next page

Question 1.8.

Now consider the principal factor solution. The fraction of variance of variable x_1 explained by using the three unrotated factors is:

- 1 ☐ 1.49
- 2 ☐ -0.36
- 3 ☐ 0.73
- 4 ☐ 0.81
- 5 ☐ 4.20
- 6 ☐ Don't know.

Problem 2.

Enclosure B with the SAS-program and the corresponding SAS-output belongs to this problem.

The data are from a study on twenty engineering students and twenty pilots. Each variable corresponds to a score in a certain test.

The variables are:

Variable	Description
type	: engineering student (1), pilot (2)
x_1	: form relations
x_2	: dynamometer
x_3	: dotting
x_4	: sensory motor coordination
x_5	: perservation

Question 2.1.

The analysis performed in the "proc discrim" step is:

- 1 ☐ a linear discriminant analysis with equal losses and equal priors
- 2 ☐ a linear discriminant analysis with unequal losses but with equal priors
- 3 ☐ a quadratic discriminant analysis with equal priors
- 4 ☐ a quadratic discriminant analysis with unequal priors
- 5 ☐ none of the above
- 6 ☐ Don't know.

The problem continues on the next page

Question 2.2.

Consider a linear discriminant analysis of the data. The first element of the pooled variance-covariance matrix is:

- 1 ☐ 68.9
- 2 ☐ $\frac{19 \cdot 68.2 + 19 \cdot 51.7}{38}$
- 3 ☐ $68.2 + 51.7$
- 4 ☐ $0.5(8.3 + 7.2)$
- 5 ☐ 8.3
- 6 ☐ Don't know.

Question 2.3. Consider a linear discriminant analysis of the data. $\|\hat{\mu}_1 - \hat{\mu}_2\|_{\hat{\Sigma}^{-1}}^2$ is estimated at:

- 1 ☐ 6.6
- 2 ☐ 27.7
- 3 ☐ $-112.3 + 133.7$
- 4 ☐ 4
- 5 ☐ 0.5
- 6 ☐ Don't know.

Question 2.4. Consider a linear discriminant analysis of the data. In that case Hotelling's T^2 in the two sample case could be used to test for difference in the means. The usual null-hypothesis distribution would then be:

- 1 ☐ $F(5,32)$
- 2 ☐ $F(5,34)$
- 3 ☐ $F(5,35)$
- 4 ☐ $F(5,36)$
- 5 ☐ $F(5,37)$
- 6 ☐ Don't know.

The problem continues on the next page

Question 2.5. The total number of misclassified observations seen is:

- 1 ☐ 1
- 2 ☐ 2
- 3 ☐ 3
- 4 ☐ 4
- 5 ☐ 5
- 6 ☐ Don't know.

Problem 3.

Enclosure B with the SAS-program and the corresponding SAS-output belongs to this problem.

The data are from a study on twenty engineering students and twenty pilots. Each variable corresponds to a score in a certain test.

The variables are:

Variable	Description
type	: engineering student (1), pilot (2)
x_1	: form relations
x_2	: dynamometer
x_3	: dotting
x_4	: sensory motor coordination
x_5	: perservation

Consider the variable "type" as the response or "y" variable. This is called a "dummy" variable. In the two-class case, linear discriminant analysis can then be substituted by linear regression analysis. An observation is then classified according to which value of the dummy variable is closest to the predicted value.

Question 3.1.

Using just variable x_1 in such an analysis would then result in:

- 1 ☐ 9 misclassifications
- 2 ☐ 13 misclassifications
- 3 ☐ 17 misclassifications
- 4 ☐ 19 misclassifications
- 5 ☐ 23 misclassifications
- 6 ☐ Don't know.

The problem continues on the next page

Question 3.2.

Variable selection for linear discriminant analysis in the two class case also works nicely by using the linear regression setup with a dummy y -variable. In this case the first variable to be included would be:

- 1 ☐ x_1
- 2 ☐ x_2
- 3 ☐ x_3
- 4 ☐ x_4
- 5 ☐ x_5
- 6 ☐ Don't know.

Question 3.3.

Consider the linear regression of type on x_1 . The degree of explanation is estimated at:

- 1 ☐ 0.15
- 2 ☐ 0.47
- 3 ☐ 6.8
- 4 ☐ 0.013
- 5 ☐ 0.11
- 6 ☐ Don't know.

The problem continues on the next page

Question 3.4.

Consider the linear regression of type on x_1 . Now consider the following values:

$$a_1 = 31.5, a_2 = 0.15, a_3 = 38, \mathbf{b}_1 = \begin{pmatrix} 2.33 \\ -0.024 \end{pmatrix}, \mathbf{b}_2 = \begin{pmatrix} 7.14 \\ -2.61 \end{pmatrix}, \mathbf{b}_3 = \begin{pmatrix} 0.10666 & -0.00290 \\ -0.00290 & 0.00008 \end{pmatrix}$$

In order to construct a 95% prediction interval for a new observation **I need** all of the following values:

- 1 ☐ a_1, a_2
- 2 ☐ $a_1, a_3, \mathbf{b}_1, \mathbf{b}_2$
- 3 ☐ $\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$
- 4 ☐ a_1, \mathbf{b}_3
- 5 ☐ $a_3, \mathbf{b}_1, \mathbf{b}_3$
- 6 ☐ Don't know.

Question 3.5.

Consider the linear regression of type on x_1 . The single observation which is most influential on the regression parameter for x_1 is observation number:

- 1 ☐ 1
- 2 ☐ 2
- 3 ☐ 9
- 4 ☐ 15
- 5 ☐ 30
- 6 ☐ Don't know.

Problem 4.

In a feeding experiment pigs were fed different diets. There were 6 pigs on each diet. Below is given a table of gain in weight y and initial weight x of the pigs.

Diet 1		Diet 2		Diet 3		Diet 4	
y	x	y	x	y	x	y	x
165	30	180	24	156	34	201	41
170	27	169	31	189	32	173	32
130	20	171	20	138	35	200	30
156	21	161	26	190	35	193	35
167	33	180	20	160	30	142	28
151	29	170	25	172	29	189	36

At least the following four models are feasible for the above mentioned data (the parameters may take different values for the different models):

A: $y_{ij} = \mu + \alpha_i + \beta_i \cdot x_{ij} + \varepsilon_{ij}, i = 1, \dots, 4, j = 1, \dots, 6$

B: $y_{ij} = \mu + \alpha_i + \beta \cdot x_{ij} + \varepsilon_{ij}, i = 1, \dots, 4, j = 1, \dots, 6$

C: $y_{ij} = \mu + \alpha_i + \varepsilon_{ij}, i = 1, \dots, 4, j = 1, \dots, 6$

D: $y_{ij} = \mu + \beta \cdot x_{ij} + \varepsilon_{ij}, i = 1, \dots, 4, j = 1, \dots, 6$

The four models have corresponding ANOVA tables:

Model A		
Source	Sum of squares	Degrees of freedom
Model	3905	7
Error	4196	16
Total	8101	23

Model B		
Source	Sum of squares	Degrees of freedom
Model	2846	4
Error	5255	19
Total	8101	23

The problem continues on the next page

Model C		
Source	Sum of squares	Degrees of freedom
Model	2163	3
Error	5938	20
Total	8101	23

Model D		
Source	Sum of squares	Degrees of freedom
Model	1236	1
Error	6865	22
Total	8101	23

Question 4.1.

The following sequence of successive testing is possible:

- 1 ☐ A, B, C
- 2 ☐ A, B, C, D
- 3 ☐ B, C, D
- 4 ☐ A, C, D
- 5 ☐ C, D
- 6 ☐ Don't know.

Question 4.2.

The residual variance for Model A is:

- 1 ☐ 557.86
- 2 ☐ 4196
- 3 ☐ 16
- 4 ☐ 262.25
- 5 ☐ 352.22
- 6 ☐ Don't know.

The problem continues on the next page

Question 4.3.

Assume $\alpha_i \neq \alpha_j, i \neq j$. Now consider a test of the hypothesis: $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4$. Under the null-hypothesis the usual test statistic is distributed as:

- 1 ☐ $F(1,18)$
- 2 ☐ $F(2,17)$
- 3 ☐ $F(3,16)$
- 4 ☐ $F(4,15)$
- 5 ☐ $F(5,14)$
- 6 ☐ Don't know.

Question 4.4.

For Model C consider a test of the hypothesis: "diet has no influence on weight-gain whatsoever". Under the null-hypothesis the usual test statistic is distributed as:

- 1 ☐ $F(2,21)$
- 2 ☐ $F(3,20)$
- 3 ☐ $F(4,19)$
- 4 ☐ $F(6,17)$
- 5 ☐ $F(5,18)$
- 6 ☐ Don't know.

Problem 5.

An experiment has been conducted to assess the effect of four nitrogen treatments in three blocks on several variables measured for Jonathan apples. Here we consider three of them: y_1 : total nitrogen, y_2 : potassium, y_3 : calcium. We will consider the multivariate response: $(y_1, y_2, y_3)'$. The results are given in the table below.

The problem continues on the next page

Treatment	Block	y_1	y_2	y_3
1	1	3580	8220	244
1	2	2870	7550	272
1	3	3110	8180	297
2	1	3040	10760	138
2	2	5810	11340	165
2	3	3010	9830	159
3	1	4250	11830	169
3	2	6950	12910	148
3	3	4680	11010	224
4	1	4700	8830	148
4	2	7230	10840	120
4	3	5760	9200	147

Question 5.1.

Under the null hypothesis, the usual test statistic for the hypothesis: "No treatment effect" is distributed as:

- 1 ☐ $U(\nu_1, \nu_2, 2)$
- 2 ☐ $U(\nu_1, \nu_2, 3)$
- 3 ☐ $U(\nu_1, \nu_2, 6)$
- 4 ☐ $U(\nu_1, \nu_2, 7)$
- 5 ☐ $U(\nu_1, \nu_2, 11)$
- 6 ☐ Don't know.

Question 5.2.

Consider the usual test statistic for the hypothesis: "No block effect". The transformation from the $U(\nu_1, \nu_2, \nu_3)$ -distribution to the F-distribution is

- 1 ☐ approximate because $\nu_1 \cdot \nu_2 = 6$
- 2 ☐ approximate because $\nu_3 = 6$
- 3 ☐ approximate because $\nu_1 = 3$
- 4 ☐ exact because $\nu_3 = 6$
- 5 ☐ exact because $\nu_2 = 2$
- 6 ☐ Don't know.

Problem 6.

Consider a general linear model $\mathbf{Y} = \mathbf{x}\boldsymbol{\theta} + \boldsymbol{\varepsilon}$, $\boldsymbol{\varepsilon} \in N(\mathbf{0}, \sigma^2\boldsymbol{\Sigma})$ with design matrix

$$\mathbf{x} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix} \text{ and corresponding parameter vector: } \boldsymbol{\theta} = \begin{pmatrix} \mu \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$$

We want to solve the normal equations using the generalised g2 inverse.

Question 6.1. The first step is to provide a reduced version \mathbf{x}^\dagger of the design matrix. For the generalised g2 inverse this is usually:

$$\mathbf{1} \square \mathbf{x}^\dagger = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{2} \square \mathbf{x}^\dagger = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\mathbf{3} \square \mathbf{x}^\dagger = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{pmatrix}$$

The problem continues on the next page

$$4 \square \mathbf{x}^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$5 \square \mathbf{x}^\dagger = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

6 ☐ Don't know.

Problem 7.

Corresponding to the values $t = 1, 2, 3$ we have the orthogonal polynomials: $\xi_0 = 1$ and $\xi_1 = t - 2$

Question 7.1.

Then ξ_2 equals:

1 ☐ t^2

2 ☐ $(t - 2)^2 - \frac{2}{3}$

3 ☐ $t^2 - 2$

4 ☐ $(t - 2)(t - 1)$

5 ☐ $t^2 - 3$

6 ☐ Don't know.

Problem 8.

Given: $D(\mathbf{Y}_i) = D\left[\begin{pmatrix} Y_{i1} \\ Y_{i2} \end{pmatrix}\right] = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$ exists.

Three observations are arranged as: $\mathbf{Y} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \\ Y_{31} & Y_{32} \end{pmatrix}$.

Question 8.1.

Then $D(\text{vc}(\mathbf{Y}))$ equals:

1 ☐ $\begin{pmatrix} a & a & a & b & b & b \\ a & a & a & b & b & b \\ a & a & a & b & b & b \\ b & b & b & c & c & c \\ b & b & b & c & c & c \\ b & b & b & c & c & c \end{pmatrix}$

2 ☐ $\begin{pmatrix} a & b & 0 & 0 & a & b \\ b & c & 0 & 0 & b & c \\ 0 & 0 & a & b & 0 & 0 \\ 0 & 0 & b & c & 0 & 0 \\ a & b & 0 & 0 & a & b \\ b & c & 0 & 0 & b & c \end{pmatrix}$

3 ☐ $\begin{pmatrix} a & 0 & 0 & b & 0 & 0 \\ 0 & a & 0 & 0 & b & 0 \\ 0 & 0 & a & 0 & 0 & b \\ b & 0 & 0 & c & 0 & 0 \\ 0 & b & 0 & 0 & c & 0 \\ 0 & 0 & b & 0 & 0 & c \end{pmatrix}$

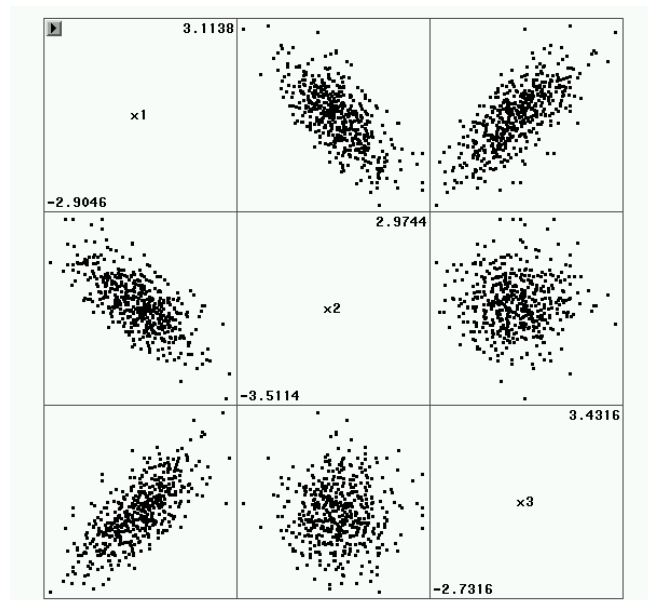
4 ☐ $\begin{pmatrix} a & b & 0 & 0 & 0 & 0 \\ b & c & 0 & 0 & 0 & 0 \\ 0 & 0 & a & b & 0 & 0 \\ 0 & 0 & b & c & 0 & 0 \\ 0 & 0 & 0 & 0 & a & b \\ 0 & 0 & 0 & 0 & b & c \end{pmatrix}$

5 ☐ $\begin{pmatrix} a & 0 & 0 & 0 & 0 & 0 \\ 0 & b & 0 & 0 & 0 & 0 \\ 0 & 0 & c & 0 & 0 & 0 \\ 0 & 0 & 0 & a & 0 & 0 \\ 0 & 0 & 0 & 0 & b & 0 \\ 0 & 0 & 0 & 0 & 0 & c \end{pmatrix}$

6 ☐ Don't know.

Problem 9.

The figure shows a scatterplot of a three-variable dataset.



Question 9.1.

Which of the correlation matrices below is most likely for the data given in the scatterplot?

1 ☐ $\begin{pmatrix} 1 & -0.7 & 0 \\ -0.7 & 1 & 0.7 \\ 0 & 0.7 & 1 \end{pmatrix}$

2 ☐ $\begin{pmatrix} 1 & 0.7 & -0.7 \\ 0.7 & 1 & 0 \\ -0.7 & 0 & 1 \end{pmatrix}$

3 ☐ $\begin{pmatrix} 1 & -0.7 & 0.7 \\ -0.7 & 1 & 0 \\ 0.7 & 0 & 1 \end{pmatrix}$

4 ☐ $\begin{pmatrix} 1 & 0.7 & 0 \\ 0.7 & 1 & -0.7 \\ 0 & -0.7 & 1 \end{pmatrix}$

5 ☐ $\begin{pmatrix} 1 & 0 & 0.7 \\ 0 & 1 & -0.7 \\ 0.7 & -0.7 & 1 \end{pmatrix}$

6 ☐ Don't know.

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EnclA.sas

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```
/* encla.sas   Crted: 10-11-10 15:23 by BKE. Updt: 27-11-10 18:02 */
/* Purpose:   */

title1 'Enclosure A - data on engineering students and pilots';

title2 'Print of data';
proc print data=stat2.engpilot;
var type x1 x2 x3 x4 x5 x6;
run;

title2 'Output from proc princomp';
proc princomp cov data=stat2.engpilot;
var x1 x2 x3 x4 x5 x6;
run;

title2 'Output from proc factor';
proc factor corr eigenvectors nfactor=3 rotate=varimax data=stat2.engpilot;
var x1 x2 x3 x4 x5 x6;
run;
```

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EnclA.III

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Enclosure A - data on engineering students and pilots
Print of data

1

Obs	type	x1	x2	x3	x4	x5	x6
1	1	22	74	223	54	254	121
2	1	30	80	175	40	300	108
3	1	49	87	266	41	223	122
4	1	37	66	178	80	209	77
5	1	35	71	175	38	261	140
6	1	37	57	241	59	245	108
7	1	39	52	194	72	242	124
8	1	34	89	200	85	242	130
9	1	55	91	198	50	277	149
10	1	38	72	162	47	268	129
11	1	37	87	170	60	244	154
12	1	33	88	208	51	228	145
13	1	40	60	232	29	279	112
14	1	39	73	159	39	233	120
15	1	21	83	152	88	233	118
16	1	42	80	195	36	241	141
17	1	49	73	152	42	249	135
18	1	37	76	223	74	268	151
19	1	46	83	164	31	243	97
20	1	42	82	188	57	267	109
21	2	17	77	232	50	249	132
22	2	32	79	192	64	315	123
23	2	31	96	250	55	319	129
24	2	23	67	291	48	310	131
25	2	24	96	239	42	268	110
26	2	22	87	231	40	217	47
27	2	32	87	227	30	324	125
28	2	29	102	234	58	300	129
29	2	26	104	256	58	270	130
30	2	47	82	240	30	322	147
31	2	37	80	227	58	317	159
32	2	41	83	216	39	306	135
33	2	35	83	183	57	242	100
34	2	37	94	227	30	240	149
35	2	38	78	258	42	271	149
36	2	27	89	283	66	291	153
37	2	31	83	257	31	311	136
38	2	36	100	252	30	225	97
39	2	37	105	250	27	243	141
40	2	32	76	187	30	264	164

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EnclA.III

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Enclosure A - data on engineering students and pilots
Output from proc princomp

2

The PRINCOMP Procedure

Observations	40
Variables	6

Simple Statistics

	x1	x2	x3
Mean	34.90000000	81.80000000	214.6750000
StD	8.30137462	12.04947068	36.9652557

Simple Statistics

	x4	x5	x6
Mean	48.95000000	265.2500000	126.9000000
StD	16.25904088	32.6973299	22.9824905

Covariance Matrix

	x1	x2	x3
x1	68.912821	#####	-81.751282
x2	#####	145.189744	128.882051
x3	-81.751282	128.882051	1366.430128
x4	-33.005128	-30.856410	-113.580769
x5	-18.282051	29.384615	395.185897
x6	35.989744	27.979487	104.428205

Covariance Matrix

	x4	x5	x6
x1	-33.005128	-18.282051	35.989744
x2	-30.856410	29.384615	27.979487
x3	-113.580769	395.185897	104.428205
x4	264.356410	-79.858974	-20.030769
x5	-79.858974	1069.115385	291.153846
x6	-20.030769	291.153846	528.194872

Total Variance 3442.199359

Eigenvalues of the Covariance Matrix

	Eigenvalue	Difference	Proportion	Cumulative
1	1722.04238	843.68455	#####	#####
2	878.35783	476.91927	#####	#####
3	#####	#####	#####	#####
4	#####	#####	#####	#####
5	#####	#####	#####	#####
6	#####	#####	#####	#####

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Enclosure A - data on engineering students and pilots
Output from proc factor

6

The FACTOR Procedure
Rotation Method: Varimax

Orthogonal Transformation Matrix

	1	2	3
1	0.72707	0.68234	0.07603
2	-0.41710	0.35103	0.83833
3	-0.54534	0.64124	-0.53983

Rotated Factor Pattern

	Factor1	Factor2	Factor3
x1	-0.35783	0.10052	0.81720
x2	0.72378	-0.02628	0.06826
x3	0.73913	0.29475	-0.19222
x4	-0.48338	-0.01266	-0.72932
x5	0.23898	0.80017	-0.06863
x6	-0.06347	0.83447	0.16939

Variance Explained by Each Factor

Factor1	Factor2	Factor3
1.4930209	1.4344375	1.2747357

Final Communality Estimates: Total = 4.202194

x1	x2	x3	x4	x5	x6
0.80595890	0.52921335	0.67014233	0.76573408	0.70208857	0.72905685

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EnclB.sas

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```
/* enclb.sas   Crted: 10-11-10 16:30 by BKE. Updt: 27-11-10 18:04 */  
/* Purpose:   */
```

```
title1 'Enclosure B - data on engineering students and pilots';
```

```
title2 'Print of data';  
proc print data=stat2.engpilot;  
var type x1 x2 x3 x4 x5 x6;  
run;
```

```
title2 'Output from proc discrim';  
proc discrim simple pool=yes data=stat2.engpilot;  
class type;  
var x1 x2 x3 x4 x5;  
run;
```

```
title2 'Output from 1st proc reg';  
proc reg data=stat2.engpilot;  
model type=x1 / influence r covb;  
run;
```

```
title2 'Output from 2nd proc reg';  
proc reg data=stat2.engpilot;  
model type=x2;  
run;
```

```
title2 'Output from 3rd proc reg';  
proc reg data=stat2.engpilot;  
model type=x3;  
run;
```

```
title2 'Output from 4th proc reg';  
proc reg data=stat2.engpilot;  
model type=x4;  
run;
```

```
title2 'Output from 5th proc reg';  
proc reg data=stat2.engpilot;  
model type=x5;  
run;
```

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Enclosure B - data on engineering students and pilots
Print of data

1

Obs	type	x1	x2	x3	x4	x5	x6
1	1	22	74	223	54	254	121
2	1	30	80	175	40	300	108
3	1	49	87	266	41	223	122
4	1	37	66	178	80	209	77
5	1	35	71	175	38	261	140
6	1	37	57	241	59	245	108
7	1	39	52	194	72	242	124
8	1	34	89	200	85	242	130
9	1	55	91	198	50	277	149
10	1	38	72	162	47	268	129
11	1	37	87	170	60	244	154
12	1	33	88	208	51	228	145
13	1	40	60	232	29	279	112
14	1	39	73	159	39	233	120
15	1	21	83	152	88	233	118
16	1	42	80	195	36	241	141
17	1	49	73	152	42	249	135
18	1	37	76	223	74	268	151
19	1	46	83	164	31	243	97
20	1	42	82	188	57	267	109
21	2	17	77	232	50	249	132
22	2	32	79	192	64	315	123
23	2	31	96	250	55	319	129
24	2	23	67	291	48	310	131
25	2	24	96	239	42	268	110
26	2	22	87	231	40	217	47
27	2	32	87	227	30	324	125
28	2	29	102	234	58	300	129
29	2	26	104	256	58	270	130
30	2	47	82	240	30	322	147
31	2	37	80	227	58	317	159
32	2	41	83	216	39	306	135
33	2	35	83	183	57	242	100
34	2	37	94	227	30	240	149
35	2	38	78	258	42	271	149
36	2	27	89	283	66	291	153
37	2	31	83	257	31	311	136
38	2	36	100	252	30	225	97
39	2	37	105	250	27	243	141
40	2	32	76	187	30	264	164

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Enclosure B - data on engineering students and pilots
Output from proc discrim

2

The DISCRIM Procedure

Observations	40	DF Total	39
Variables	5	DF Within Classes	38
Classes	2	DF Between Classes	1

Class Level Information

type	Variable Name	Frequency	Weight	Proportion	Prior Probability
1	_1	20	20.0000	0.500000	0.500000
2	_2	20	20.0000	0.500000	0.500000

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Enclosure B - data on engineering students and pilots
Output from proc discrim

3

The DISCRIM Procedure
Simple Statistics

Total-Sample

Variable	N	Sum	Mean	Variance	Standard Deviation
x1	40	1396	34.90000	68.91282	8.3014
x2	40	3272	81.80000	145.18974	12.0495
x3	40	8587	214.67500	1366	36.9653
x4	40	1958	48.95000	264.35641	16.2590
x5	40	10610	265.25000	1069	32.6973

type = 1

Variable	N	Sum	Mean	Variance	Standard Deviation
x1	20	762.00000	38.10000	68.20000	8.2583
x2	20	1524	76.20000	121.11579	11.0053
x3	20	3855	192.75000	1000	31.6259
x4	20	1073	53.65000	322.45000	17.9569
x5	20	5006	250.30000	470.22105	21.6846

type = 2

Variable	N	Sum	Mean	Variance	Standard Deviation
x1	20	634.00000	31.70000	51.69474	7.1899
x2	20	1748	87.40000	110.88421	10.5302
x3	20	4732	236.60000	792.56842	28.1526
x4	20	885.00000	44.25000	173.67105	13.1784
x5	20	5604	280.20000	1254	35.4083

Pooled Covariance Matrix Information

Covariance Matrix Rank Natural Log of the
Determinant of the
Covariance Matrix

5

27.65931

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Enclosure B - data on engineering students and pilots
Output from proc discrim

4

The DISCRIM Procedure

Pairwise Generalized Squared Distances Between Groups

$$D^2(i|j) = (\bar{X}_i - \bar{X}_j)' \text{COV}^{-1} (\bar{X}_i - \bar{X}_j)$$

Generalized Squared Distance to type

From type	1	2
1	0	6.64206
2	6.64206	0

Linear Discriminant Function

$$\text{Constant} = -.5 \bar{X}_j' \text{COV}^{-1} \bar{X}_j \quad \text{Coefficient Vector} = \text{COV}^{-1} \bar{X}_j$$

Linear Discriminant Function for type

Variable	1	2
Constant	-112.26431	-133.74843
x1	0.75467	0.55634
x2	0.77518	0.90258
x3	0.20390	0.24634
x4	0.39634	0.32364
x5	0.30420	0.35107

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Enclosure B - data on engineering students and pilots
Output from proc discrim

5

The DISCRIM Procedure
Classification Summary for Calibration Data: STAT2.ENGPILOT
Resubstitution Summary using Linear Discriminant Function

Generalized Squared Distance Function

$$D_j^2(X) = (X - \bar{X}_j)' \text{COV}_j^{-1} (X - \bar{X}_j)$$

Posterior Probability of Membership in Each type

$$\Pr(j|X) = \exp(-.5 D_j^2(X)) / \sum_k \exp(-.5 D_k^2(X))$$

Number of Observations and Percent Classified into type

From type	1	2	Total
1	18 90.00	2 10.00	20 100.00
2	2 10.00	18 90.00	20 100.00
Total	20 50.00	20 50.00	40 100.00
Priors	0.5	0.5	

Error Count Estimates for type

	1	2	Total
Rate	0.1000	0.1000	0.1000
Priors	0.5000	0.5000	

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Enclosure B - data on engineering students and pilots
Output from 1st proc reg

6

The REG Procedure
Model: MODEL1
Dependent Variable: type

Number of Observations Read 40
Number of Observations Used 40

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	1.52404	1.52404	6.83	0.0128
Error	38	8.47596	0.22305		
Corrected Total	39	10.00000			

Root MSE 0.47228 R-Square 0.1524
Dependent Mean 1.50000 Adj R-Sq 0.1301
Coeff Var 31.48556

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t
Intercept	1	2.33108	0.32659	7.14	<.0001
x1	1	-0.02381	0.00911	-2.61	0.0128

Covariance of Estimates

Variable	Intercept	x1
Intercept	0.1066624549	-0.002896452
x1	-0.002896452	0.0000829929

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Enclosure B - data on engineering students and pilots
Output from 1st proc reg

7

The REG Procedure
Model: MODEL1
Dependent Variable: type

Output Statistics

Obs	Dependent Variable	Predicted Value	Std Error Mean Predict	Residual	Std Error Residual	Student Residual
1	1.0000	1.8072	0.1392	-0.8072	0.451	-1.789
2	1.0000	1.6167	0.0870	-0.6167	0.464	-1.328
3	1.0000	1.1642	0.1486	-0.1642	0.448	-0.366
4	1.0000	1.4500	0.0771	-0.4500	0.466	-0.966
5	1.0000	1.4976	0.0747	-0.4976	0.466	-1.067
6	1.0000	1.4500	0.0771	-0.4500	0.466	-0.966
7	1.0000	1.4024	0.0835	-0.4024	0.465	-0.866
8	1.0000	1.5214	0.0751	-0.5214	0.466	-1.118
9	1.0000	1.0214	0.1978	-0.0214	0.429	-0.0498
10	1.0000	1.4262	0.0798	-0.4262	0.465	-0.916
11	1.0000	1.4500	0.0771	-0.4500	0.466	-0.966
12	1.0000	1.5452	0.0767	-0.5452	0.466	-1.170
13	1.0000	1.3786	0.0879	-0.3786	0.464	-0.816
14	1.0000	1.4024	0.0835	-0.4024	0.465	-0.866
15	1.0000	1.8310	0.1470	-0.8310	0.449	-1.852
16	1.0000	1.3309	0.0988	-0.3309	0.462	-0.717
17	1.0000	1.1642	0.1486	-0.1642	0.448	-0.366
18	1.0000	1.4500	0.0771	-0.4500	0.466	-0.966
19	1.0000	1.2357	0.1257	-0.2357	0.455	-0.518
20	1.0000	1.3309	0.0988	-0.3309	0.462	-0.717
21	2.0000	1.9263	0.1794	0.0737	0.437	0.169
22	2.0000	1.5691	0.0792	0.4309	0.466	0.926
23	2.0000	1.5929	0.0827	0.4071	0.465	0.876
24	2.0000	1.7834	0.1316	0.2166	0.454	0.478
25	2.0000	1.7596	0.1242	0.2404	0.456	0.528
26	2.0000	1.8072	0.1392	0.1928	0.451	0.427
27	2.0000	1.5691	0.0792	0.4309	0.466	0.926
28	2.0000	1.6405	0.0920	0.3595	0.463	0.776
29	2.0000	1.7119	0.1102	0.2881	0.459	0.627
30	2.0000	1.2119	0.1331	0.7881	0.453	1.739
31	2.0000	1.4500	0.0771	0.5500	0.466	1.180
32	2.0000	1.3547	0.0931	0.6453	0.463	1.394
33	2.0000	1.4976	0.0747	0.5024	0.466	1.077
34	2.0000	1.4500	0.0771	0.5500	0.466	1.180
35	2.0000	1.4262	0.0798	0.5738	0.465	1.233
36	2.0000	1.6881	0.1037	0.3119	0.461	0.677
37	2.0000	1.5929	0.0827	0.4071	0.465	0.876
38	2.0000	1.4738	0.0753	0.5262	0.466	1.129
39	2.0000	1.4500	0.0771	0.5500	0.466	1.180
40	2.0000	1.5691	0.0792	0.4309	0.466	0.926

Output Statistics

Obs	-2-1 0 1 2	Cook's D	RStudent	Hat Diag H	Cov Ratio	DFFITS
1	***	0.152	-1.8443	0.0869	0.9689	-0.5690
2	**	0.031	-1.3424	0.0339	0.9928	-0.2516
3		0.007	-0.3621	0.0990	1.1624	-0.1200
4	*	0.013	-0.9649	0.0266	1.0311	-0.1596
5	**	0.015	-1.0691	0.0250	1.0180	-0.1712
6	*	0.013	-0.9649	0.0266	1.0311	-0.1596
7	*	0.012	-0.8627	0.0313	1.0463	-0.1550

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Enclosure B - data on engineering students and pilots
Output from 1st proc reg

8

The REG Procedure
Model: MODEL1
Dependent Variable: type

Output Statistics

Obs	-2-1 0 1 2	Cook's D	RStudent	Hat Diag H	Cov Ratio	DFFITS
8	**	0.016	-1.1221	0.0253	1.0121	-0.1808
9		0.000	-0.0491	0.1753	1.2789	-0.0227
10	*	0.012	-0.9136	0.0286	1.0384	-0.1567
11	*	0.013	-0.9649	0.0266	1.0311	-0.1596
12	**	0.019	-1.1759	0.0263	1.0067	-0.1934
13	*	0.012	-0.8121	0.0347	1.0547	-0.1539
14	*	0.012	-0.8627	0.0313	1.0463	-0.1550
15	***	0.184	-1.9154	0.0969	0.9667	-0.6274
16	*	0.012	-0.7119	0.0438	1.0734	-0.1523
17		0.007	-0.3621	0.0990	1.1624	-0.1200
18	*	0.013	-0.9649	0.0266	1.0311	-0.1596
19	*	0.010	-0.5126	0.0708	1.1193	-0.1416
20	*	0.012	-0.7119	0.0438	1.0734	-0.1523
21		0.002	0.1666	0.1442	1.2307	0.0684
22	*	0.012	0.9238	0.0281	1.0369	0.1572
23	*	0.012	0.8728	0.0307	1.0447	0.1552
24		0.010	0.4727	0.0777	1.1299	0.1372
25	*	0.010	0.5226	0.0692	1.1167	0.1425
26		0.009	0.4226	0.0869	1.1441	0.1304
27	*	0.012	0.9238	0.0281	1.0369	0.1572
28	*	0.012	0.7719	0.0380	1.0619	0.1533
29	*	0.011	0.6222	0.0545	1.0926	0.1493
30	***	0.131	1.7890	0.0795	0.9707	0.5257
31	**	0.019	1.1867	0.0266	1.0056	0.1963
32	**	0.039	1.4117	0.0388	0.9881	0.2838
33	**	0.015	1.0796	0.0250	1.0168	0.1729
34	**	0.019	1.1867	0.0266	1.0056	0.1963
35	**	0.022	1.2415	0.0286	1.0007	0.2129
36	*	0.012	0.6720	0.0482	1.0817	0.1513
37	*	0.012	0.8728	0.0307	1.0447	0.1552
38	**	0.017	1.1328	0.0255	1.0110	0.1831
39	**	0.019	1.1867	0.0266	1.0056	0.1963
40	*	0.012	0.9238	0.0281	1.0369	0.1572

Output Statistics

-----DFBETAS-----
Obs Intercept x1

1	-0.5373	0.4803
2	-0.1751	0.1291
3	0.0872	-0.1038
4	0.0032	-0.0396
5	-0.0371	-0.0021
6	0.0032	-0.0396
7	0.0358	-0.0693
8	-0.0603	0.0197
9	0.0185	-0.0210
10	0.0204	-0.0554
11	0.0032	-0.0396
12	-0.0856	0.0437
13	0.0493	-0.0813
14	0.0358	-0.0693

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Enclosure B - data on engineering students and pilots
Output from 1st proc reg

9

The REG Procedure
Model: MODEL1
Dependent Variable: type

Output Statistics

Obs	Intercept	x1
15	-0.5990	0.5404
16	0.0707	-0.0997
17	0.0872	-0.1038
18	0.0032	-0.0396
19	0.0916	-0.1139
20	0.0707	-0.0997
21	0.0671	-0.0622
22	0.0849	-0.0524
23	0.0970	-0.0667
24	0.1278	-0.1130
25	0.1305	-0.1139
26	0.1231	-0.1100
27	0.0849	-0.0524
28	0.1156	-0.0896
29	0.1301	-0.1098
30	-0.3563	0.4352
31	-0.0039	0.0487
32	-0.1129	0.1694
33	0.0375	0.0021
34	-0.0039	0.0487
35	-0.0278	0.0753
36	0.1271	-0.1050
37	0.0970	-0.0667
38	0.0178	0.0243
39	-0.0039	0.0487
40	0.0849	-0.0524

Sum of Residuals 0
Sum of Squared Residuals 8.47596
Predicted Residual SS (PRESS) 9.31711

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Enclosure B - data on engineering students and pilots
Output from 2nd proc reg

10

The REG Procedure
Model: MODEL1
Dependent Variable: type

Number of Observations Read 40
Number of Observations Used 40

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	2.21532	2.21532	10.81	0.0022
Error	38	7.78468	0.20486		
Corrected Total	39	10.00000			

Root MSE 0.45261 R-Square 0.2215
Dependent Mean 1.50000 Adj R-Sq 0.2010
Coeff Var 30.17432

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t
Intercept	1	-0.11797	0.49720	-0.24	0.8137
x2	1	0.01978	0.00601	3.29	0.0022

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Enclosure B - data on engineering students and pilots 11
Output from 3rd proc reg

The REG Procedure
Model: MODEL1
Dependent Variable: type

Number of Observations Read 40
Number of Observations Used 40

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	3.60817	3.60817	21.45	<.0001
Error	38	6.39183	0.16821		
Corrected Total	39	10.00000			

Root MSE	0.41013	R-Square	0.3608
Dependent Mean	1.50000	Adj R-Sq	0.3440
Coeff Var	27.34195		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t
Intercept	1	-0.26644	0.38687	-0.69	0.4952
x3	1	0.00823	0.00178	4.63	<.0001

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Enclosure B - data on engineering students and pilots 12
Output from 4th proc reg

The REG Procedure
Model: MODEL1
Dependent Variable: type

Number of Observations Read 40
Number of Observations Used 40

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	0.85704	0.85704	3.56	0.0668
Error	38	9.14296	0.24060		
Corrected Total	39	10.00000			

Root MSE	0.49051	R-Square	0.0857
Dependent Mean	1.50000	Adj R-Sq	0.0616
Coeff Var	32.70095		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t
Intercept	1	1.94630	0.24886	7.82	<.0001
x4	1	-0.00912	0.00483	-1.89	0.0668

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Enclosure B - data on engineering students and pilots 13
Output from 5th proc reg

The REG Procedure
Model: MODEL1
Dependent Variable: type

Number of Observations Read 40
Number of Observations Used 40

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	2.14414	2.14414	10.37	0.0026
Error	38	7.85586	0.20673		
Corrected Total	39	10.00000			

Root MSE	0.45468	R-Square	0.2144
Dependent Mean	1.50000	Adj R-Sq	0.1937
Coeff Var	30.31194		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t
Intercept	1	-0.40212	0.59499	-0.68	0.5032
x5	1	0.00717	0.00223	3.22	0.0026