Multivariate
Statistics

Anders Stockmarr
DTU Dept. of Applied
Mathematics and
Computer Science

Lyngby, 22/8 2025

Exercise E1.

Let $\mathbb{X}, \mathbb{Y}$ be two real-valued stochastic variables with finite variance. Prove the following relation:

$$Cor(\mathbb{X}, \mathbb{Y}) = 1 \Leftrightarrow V(\mathbb{X}) > 0 \; and \; \exists \; constants \; a, b \; with \; b \neq 0 : \mathbb{X} = a + b\mathbb{Y} \text{ with probability } 1.$$

Interpretation¿If two variables have correlation 1, they essentially contribute with the same information, since one can get one directly from the other.

Hint: Start by showing that a variable with variance 0 is constant with probability 1.