Technical University of Denmark

Written examination, date: 9. December 2011 *Page 1 of 18 pages* *Enclosure: 12 pages*
Course name: Multivariate Statistics *Course number:* 02409
Aids allowed: All usual ones
Exam duration: 4 hours
Weighting: The questions are given equal weight

This exam is answered by:

____________________________ ____________________________ ____________
(name) (signature) (study no.)

There is a total of 30 questions for the 6 problems. The answers to the 30 questions must be written into the table below.

Problem	1	1	1	1	1	1	2	2	2	2
Question	1.1	1.2	1.3	1.4	1.5	1.6	2.1	2.2	2.3	2.4
Answer										

Problem	2	2	3	3	3	3	3	3	4	5
Question	2.5	2.6	3.1	3.2	3.3	3.4	3.5	3.6	4.1	5.1
Answer										

Problem	5	5	5	5	6	6	6	6	6	6
Question	5.2	5.3	5.4	5.5	6.1	6.2	6.3	6.4	6.5	6.6
Answer										

The possible answers for each question are numbered from 1 to 6. If you enter a wrong number, you may correct it by crossing the wrong number in the table and writing the correct answer immediately below. If there is any doubt about the meaning of a correction then the question will be considered not answered.

Only the front page must be returned. The front page must be returned even if you do not answer any of the questions or if you leave the exam prematurely. Drafts and/or comments are not considered, only the numbers entered above are registered.

A correct answer gives 5 points, a wrong answer gives – 1 point. Unanswered questions or a 6 (corresponding to “don’t know”) give 0 points. The total number of points needed for a satisfactorily answered exam is determined at the final evaluation of the exam. Especially note that the grade 10 may be given even if only one answer is wrong or unanswered.

Remember to write your name, signature, and study number on the front page.

Please note, that there is one and only one correct answer to each question. Furthermore, some of the possible alternative answers may not make sense. When the text refers to SAS-output the values may be rounded to fewer decimal places than in the output itself. Please check that all pages of the exam paper and the enclosure are present.

Problem 1

We consider a simple regression model

$$\boldsymbol{Y} = \boldsymbol{x\theta} + \boldsymbol{\varepsilon} = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 0 \\ 1 & 1 \\ 1 & 10 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} + \boldsymbol{\varepsilon}$$

$$\boldsymbol{\varepsilon} \in \boldsymbol{N}(\boldsymbol{0}, \sigma^2\, \boldsymbol{I})$$

and assume that we have observed

$$\boldsymbol{y}' = [y_1 , \cdots , y_5] = [1, 1, -1, -1, 2]$$

We estimate the parameters and obtain

$$\widehat{\boldsymbol{\theta}} = \begin{bmatrix} 0.06 \\ 0.17 \end{bmatrix}$$

$$\hat{\sigma}^2 = 1.60$$

$$\widehat{\boldsymbol{D}}(\widehat{\boldsymbol{\theta}}) = \begin{bmatrix} 0.40 & -0.04 \\ -0.04 & 0.02 \end{bmatrix}$$

Question 1.1.

The value of the t-test statistic for testing the hypothesis that the intercept is equal to 0 versus all alternatives is:

1 ☐ $\frac{0.06}{1.60}$

2 ☐ $\frac{0.06}{\sqrt{1.60}}$

3 ☐ $\frac{0.06}{\sqrt{0.40 \times 0.02 - 0.04 \times 0.04}}$

4 ☐ $\frac{0.06}{\sqrt{0.40}}$

5 ☐ $\frac{0.06}{0.40}$

6 ☐ Don’t know.

The problem continues on the next page

Question 1.2.
The estimated mean value for the fifth observation is:

1 ☐ $1 + 1.96\sqrt{1.60}$

2 ☐ $1 + \sqrt{1.60}$

3 ☐ 2

4 ☐ 0.23

5 ☐ 1.76

6 ☐ Don't know.

Question 1.3.
Then the 5th leverage

$$h_5 = \boldsymbol{x}_5(\mathbf{x}'\mathbf{x})^{-1}\mathbf{x}_5'$$

is approximately:

1 ☐ 1

2 ☐ 2

3 ☐ 3

4 ☐ 4

5 ☐ 5

6 ☐ Don't know.

The problem continues on the next page

Question 1.4.

If we repeat the analysis without the fifth observation, the estimated value of the intercept is:

1 ☐ -2

2 ☐ -1

3 ☐ 0

4 ☐ 1

5 ☐ 2

6 ☐ Don't know.

Question 1.5.

We continue the considerations from the previous question. The estimated value of the slope is:

1 ☐ -2

2 ☐ -1

3 ☐ 0

4 ☐ 1

5 ☐ 2

6 ☐ Don't know.

Question 1.6.

When solving the next problem it may be convenient to note that

$$[0.06\ 1.17]\begin{bmatrix} 5 & 10 \\ 10 & 102 \end{bmatrix}\begin{bmatrix} 0.06 \\ 1.17 \end{bmatrix} = 141.05$$

The value of Cook's D for the fifth observation is:

1 ☐ 141.05

2 ☐ -1/141.05

3 ☐ 1.60×141.05

4 ☐ 141.05/3.2

5 ☐ 141.05/1.60

6 ☐ Don't know.

Problem 2

Enclosure A with SAS-program and SAS-output belongs to this problem.

The data are taken from I.T. Jolliffe: Principal Component Analysis, 2nd ed., Springer 2002. The 10 variables analyzed give the strength of the reflexes for the right and left triceps (TriR, TriL), biceps (BicR, BicL), wrist (WriR, WriL), knee (KneR, KneL), and ankle (AnkR, AnkL). The biceps and triceps are muscles in the upper arm. Based on measurements on 143 persons the correlations between the different variables were found, and they form the basis for the following questions.

Question 2.1.

Consider the correlation matrix. The number of principal components needed to account for at least 90% of the total variation is:

1 ☐ 1

2 ☐ 2

3 ☐ 3

4 ☐ 4

5 ☐ 5

6 ☐ Don't know.

Question 2.2.

The ususal test statistic for testing whether the smallest 7 eigenvalues of the correlation matrix may be assumed to be equal has the following degrees of freedom

1 ☐ 142

2 ☐ 140

3 ☐ 71

4 ☐ 27

5 ☐ 17

6 ☐ Don't know.

The problem continues on the next page

Question 2.3.

The principle component that best expresses the difference between the reflexes in the left and the right ankle is no.:

1 ☐ 2

2 ☐ 4

3 ☐ 6

4 ☐ 8

5 ☐ 10

6 ☐ Don't know.

Question 2.4.

If we consider the initial 3-factor model, the amount of variation in the left ankle reflex that is explained by the 3 factors is:

1 ☐ 2.035

2 ☐ 0.78

3 ☐ 0.97

4 ☐ 0.43

5 ☐ 0.42

6 ☐ Don't know.

The problem continues on the next page

Question 2.5.

The maximal correlation between the right wrist reflex and a rotated factor is

1 ☐ 0.7450

2 ☐ 0.6800

3 ☐ 0.3405

4 ☐ 0.6679

5 ☐ 0.8575

6 ☐ Don't know.

Question 2.6.

Which of the following statements is true?

1 ☐ Rotated factor 1 is mainly an average of the arm reflexes, rotated factor 2 the average of the knee reflexes, and rotated factor 3 the average of the ankle reflexes.

2 ☐ Unrotated factor 1 is basically the overall average, unrotated factor 2 is a contrast between the arm and the leg reflexes, and unrotated factor 3 is the average ankle reflex.

3 ☐ Rotated factor 1 is basically the overall average, rotated factor 2 is a contrast between knee and ankle reflexes, and rotated factor 3 is the average of the ankle reflexes.

4 ☐ The unrotated factor 3 show the differences between the left and the right reflex measurements.

5 ☐ Unrotated factor 1 is an overall average, unrotated factor 2 is the average leg reflex, and unrotated factor 3 is a contrast between the leg and the arm reflexes.

6 ☐ Don't know.

Problem 3

Enclosure B with SAS-program and SAS-output belongs to this problem.

The data are observations from the Landsat Thematic Mapper satellite. Data are from Eastern Greenland, more specifically from the north-western part of Ymer Ø.

The problem continues on the next page

Each observation consists of values of reflected light from 6 spectral bands shown in the table below:

Variable	Spectral band (in μm)	Description
b1	0.45 – 0.52	visible blue
b2	0.52 – 0.60	visible green
b3	0.63 – 0.69	visible red
b4	0.76 – 0.90	near infrared
b5	1.55 – 1.75	near infrared
b6	2.08 – 2.35	near infrared

We are analyzing data from different geological units corresponding to values 10, 11, 12, and 13 of a variable called ‘mask’. In the following discriminant analyses we are assuming that the dispersion (variance-covariance) matrices are the same for the different geological units.

Question 3.1.

Consider the analysis based on all 6 variables and the geological units 10 and 13. Then the observed value of Hotellings T^2 for testing equality of the two mean vectors is

1 ☐ 74.36

2 ☐ 8×74.36

3 ☐ $4\sqrt{74.36}$

4 ☐ 95.24

5 ☐ $\sqrt{95.24}$

6 ☐ Don’t know.

The problem continues on the next page

Question 3.2.

If we consider all 4 geological units, the usual test statistic will – if the hypothesis that all mean values are the same is true – follow the following distribution

1 ☐ U(6, 3, 127)

2 ☐ U(6, 3, 25)

3 ☐ U(4, 3, 125)

4 ☐ U(4, 2, 25)

5 ☐ U(5, 2, 122)

6 ☐ Don't know.

Question 3.3.

The usual test statistic for testing whether the near infrared channels b4, b5, and b6 contribute significantly to the discrimination between the two units 10 and 13 is

1 ☐ $\frac{25}{3}\frac{256(74.36-55.29)}{32\times 30+256\times 55.29}$

2 ☐ $\frac{74.36}{55.29}$

3 ☐ $\frac{(74.36-55.29)^2}{74.36^2}$

4 ☐ $\frac{15.30586}{7.06698}$

5 ☐ $\frac{256(74.36-55.29)}{960+256\times 74.36}$

6 ☐ Don't know.

The problem continues on the next page

Question 3.4.

If the hypothesis in Question 3.3 is true, the test statistic follows the following distribution:

1 ☐ F(6, 122)

2 ☐ F(3,127)

3 ☐ F(3, 25)

4 ☐ F(1,127)

5 ☐ F(1, 24)

6 ☐ Don't know.

Question 3.5.

By omitting the three infrared channels the total number of misclassified observations (for all geological units) is increased by

1 ☐ 32

2 ☐ 21

3 ☐ 20

4 ☐ 11

5 ☐ 9

6 ☐ Don't know.

The problem continues on the next page

Question 3.6.

When we consider all 6 variables, the estimate of the generalized variance is:

1 ☐ 6

2 ☐ $e^{15.31}$

3 ☐ $e^{-7.06698}$

4 ☐ $35.99 \times 4.33 \times 74.36$

5 ☐ e^{6}

6 ☐ Don’t know.

Problem 4

We consider a random variable

$$\begin{bmatrix} \boldsymbol{X} \\ \boldsymbol{Y} \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \\ Y_1 \\ Y_2 \end{bmatrix}$$

with dispersion (variance-covariance) matrix

$$\begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \begin{pmatrix} \rho & \rho \\ \rho & \rho \end{pmatrix} \\ \begin{pmatrix} \rho & \rho \\ \rho & \rho \end{pmatrix} & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix}$$

where we assume that ρ is so small that the matrix is positive definite. It may be shown that the first set of canonical variables between the $\boldsymbol{X}$ and the $\boldsymbol{Y}$ variables are

$$V_1 = \frac{\sqrt{2}}{2}(X_1 + X_2)$$

$$W_1 = \frac{\sqrt{2}}{2}(Y_1 + Y_2)$$

The problem continues on the next page

Question 4.1
From this follows that the largest canonical correlation coefficient is:

1 ☐ $-\rho$

2 ☐ 0

3 ☐ ρ

4 ☐ 2ρ

5 ☐ 3ρ

6 ☐ Don't know.

Problem 5

Enclosure C with SAS-program and SAS-output belongs to this problem.

We aim to analyze the relation between cement strength after 3, 7, and 28 days (variables STRGTH3, STRGTH7, and STRGTH28) and physical (as opposed to chemical) measurements from the cement mill, namely two measurements of fineness (specific surface) LSR and BLAINE, and the amount of added gypsum (XSO3). The data consists of the measured means, standard deviations, and correlations based on 198 samples.

Question 5.1.
If we want to predict the 3 strength variables based on the mill data, the amount of variation described lies between

1 ☐ 60.8% - 65.0%

2 ☐ 60.8% - 89.6%

3 ☐ 38.7% - 63.3%

4 ☐ 26.7% - 65.0%

5 ☐ 5.80% - 63.3%

6 ☐ Don't know

The problem continues on the next page

Question 5.2.

The usual test statistic for testing whether the partial correlation between STRGTH3 and STRGTH28 given the mill variables LSR, BLAINE, and XSO3 is 0 is:

1 ☐ -1.72

2 ☐ 1.72

3 ☐ 3.72

4 ☐ 5.72

5 ☐ 7.72

6 ☐ Don’t know

Question 5.3.

The test statistic in Question 5.2 should be compared with a quantile in the following distribution:

1 ☐ t(193)

2 ☐ t(3)

3 ☐ F(3,195)

4 ☐ $\chi^2(198)$

5 ☐ $\chi^2(6)$

6 ☐ Don’t know.

The problem continues on the next page

Question 5.4.

We are now looking on uncorrelated linear combinations of the strength variables and the mill variables.
The maximum correlation between a linear combination of the strength variables and the mill variables is

1 ☐ 0.8004

2 ☐ 0.7364

3 ☐ 0.8071

4 ☐ 0.2421

5 ☐ 0.5141

6 ☐ Don’t know.

Question 5.5.

The first canonical variable corresponding to the strength variables is basically:

1 ☐ The first strength variable.

2 ☐ The difference between the first and the second strength variable.

3 ☐ The difference between the two first and the third strength variable.

4 ☐ The average of the three strength variables.

5 ☐ The last strength variable.

6 ☐ Don’t know.

Problem 6

Enclosure D with SAS-program and SAS-output belongs to this problem.

The data we analyze are coming from a Landsat Thematic Mapper Satellite (Landsat 5), and we refer to spectral band information provided in Problem 3. The data are collected from an area around a water reservoir in Hindustan in India. The data set contains 12 variables of which 6 were taken on 29. March 1998 (MR1, MR2, MR3, MR4, MR5, and MR6) and 6 (MJ1, MJ2, MJ3, MJ4, MJ5, and MJ6) were taken on 18. May 1998.

The problem continues on the next page

The images are aligned so that we have matching observations from the two time periods and we want to see how well band 1 from May (MJ1) may be predicted by means of the six bands from March (MR1, MR2, MR3, MR4, MR5, and MR6). Some of the results obtained by the SAS procedure REG are shown in Enclosure D.

Question 6.1.

If we instead of using the model selection principle RSquare had used the backward elimination method, the variables would have been eliminated in the following sequence (ending with two variables in the equation):

1 ☐ MR1, MR2, MR4, MR5

2 ☐ MR6, MR3, MR5, MR4

3 ☐ MR6, MR5, MR4, MR3

4 ☐ MR6, MR5, MR4, MR2

5 ☐ MR1, MR3, MR4, MR2

6 ☐ Don’t know.

Question 6.2.

The percentage of the total variation in MJ1 that is explained by MR1,⋯, MR6 is

1 ☐ 65.13%

2 ☐ 62.99%

3 ☐ 64.92%

4 ☐ 34.87%

5 ☐ 37.01%

6 ☐ Don’t know.

The problem continues on the next page

Question 6.3.
If we in the model

$$\mathrm{MJ1} = \alpha + \sum_{i=1}^{6} \beta_i \mathrm{MR}i$$

want to test the hypothesis that

$$\beta_1 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = 0$$

against all alternatives, the usual test statistic is:

1 ☐ $\frac{0.6513-0.5984}{1-0.6513}$

2 ☐ $\frac{1-0.5984}{0.5984}\frac{98}{1}$

3 ☐ $\frac{(917.17-796.33)/5}{796.33/93}$

4 ☐ $\frac{1-0.6513}{0.6513}\frac{93}{6}$

5 ☐ $\frac{917.17/98}{796.33/93}$

6 ☐ Don’t know.

Question 6.4.
If the above hypothesis is true, the test statistic has the following degrees of freedom:

1 ☐ (1, 98)

2 ☐ (5, 93)

3 ☐ (6, 93)

4 ☐ (98, 93)

5 ☐ (1, 93)

6 ☐ Don’t know.

The problem continues on the next page

Question 6.5.

We now look on predictions in the model

$$\text{MJ1} = \alpha + \beta_2 \text{MR2} + \varepsilon$$

We put

$$\boldsymbol{A} = \begin{pmatrix} 63.84580 & 5.92881 \\ 0.83660 & 0.06923 \end{pmatrix}$$

$$\boldsymbol{B} = \begin{pmatrix} 35.15 & -0.40988 \\ -0.40988 & 0.00479 \end{pmatrix}$$

$$\boldsymbol{C} = \begin{pmatrix} 63.84580 \\ 0.83660 \end{pmatrix}$$

$$\boldsymbol{D} = \begin{pmatrix} 1 & 80 \end{pmatrix}$$

Furthermore we assume that the relevant t-quantile is

$$t_{0.975} \cong 1.98$$

The width of a 95% confidence interval for the mean corresponding to an MR2 value of 80 is:

1 ☐ $3.96\sqrt{63.84580 + 80}$

2 ☐ $1.98\sqrt{\boldsymbol{DAD'}}$

3 ☐ $3.96\sqrt{\boldsymbol{DBD'}}$

4 ☐ $1.98\sqrt{\boldsymbol{DC}}$

5 ☐ $3.96\sqrt{\boldsymbol{DAD'}}$

6 ☐ Don’t know.

The problem continues on the next page

Question 6.6.

The degrees of freedom for the t-distribution used in Question 6.5 is:

1 ☐ 99

2 ☐ 98

3 ☐ 97

4 ☐ 96

5 ☐ 95

6 ☐ Don’t know.

Enclosure A - SAS PROGRAM

```
data Reflexes (type=corr);
infile cards missover;
_type_ = 'corr';
input _type_ $ _name_ $ TriR TriL BicR BicL WriR WriL KneR KneL AnkR AnkL;
cards;
N    .          143  143  143  143  143  143  143  143  143  143
CORR TriR   1.00
CORR TriL   0.98 1.00
CORR BicR   0.60 0.62 1.00
CORR BicL   0.71 0.73 0.88 1.00
CORR WriR  0.55 0.57 0.61 0.68 1.00
CORR WriL   0.55 0.57 0.56 0.68 0.97 1.00
CORR KneR  0.38 0.40 0.48 0.53 0.33 0.33 1.00
CORR KneL  0.25 0.28 0.42 0.47 0.27 0.27 0.90 1.00
CORR AnkR  0.22 0.21 0.19 0.23 0.16 0.19 0.40 0.41 1.00
CORR AnkL  0.20 0.19 0.18 0.21 0.13 0.16 0.39 0.40 0.94 1.00
;
proc print data=Reflexes;
proc princomp data=Reflexes;
run;
ods graphics on;
proc factor data=Reflexes(type=corr)
  rotate = VARIMAX
  plots=(scree initloadings loadings);
run;
ods graphics off;
```


Enclosure A - SAS output

Reflexes data

Obs	_type_	_name_	TriR	TriL	BicR	BicL	WriR	WriL	KneR	KneL	AnkR	AnkL
1	N		143.00	143.00	143.00	143.00	143.00	143.00	143.00	143.00	143.00	143
2	CORR	TriR	1.00	.	.	.	.	.	.	.	.	.
3	CORR	TriL	0.98	1.00	.	.	.	.	.	.	.	.
4	CORR	BicR	0.60	0.62	1.00	.	.	.	.	.	.	.
5	CORR	BicL	0.71	0.73	0.88	1.00	.	.	.	.	.	.
6	CORR	WriR	0.55	0.57	0.61	0.68	1.00	.	.	.	.	.
7	CORR	WriL	0.55	0.57	0.56	0.68	0.97	1.00	.	.	.	.
8	CORR	KneR	0.38	0.40	0.48	0.53	0.33	0.33	1.00	.	.	.
9	CORR	KneL	0.25	0.28	0.42	0.47	0.27	0.27	0.90	1.00	.	.
10	CORR	AnkR	0.22	0.21	0.19	0.23	0.16	0.19	0.40	0.41	1.00	.
11	CORR	AnkL	0.20	0.19	0.18	0.21	0.13	0.16	0.39	0.40	0.94	1

Reflexes data

The PRINCOMP Procedure

Observations	143
Variables	10

Eigenvalues of the Correlation Matrix

	Eigenvalue	Difference	Proportion	Cumulative
1	5.22298654	3.18739392	0.5223	0.5223
2	2.03559262	0.94146709	0.2036	0.7259
3	1.09412553	0.23707467	0.1094	0.8353
4	0.85705086	0.36134577	0.0857	0.9210
5	0.49570509	0.38812846	0.0496	0.9705
6	0.10757663	0.02125229	0.0108	0.9813
7	0.08632433	0.02701308	0.0086	0.9899
8	0.05931125	0.03670948	0.0059	0.9959
9	0.02260177	0.00387639	0.0023	0.9981
10	0.01872538		0.0019	1.0000

Eigenvectors

	Prin1	Prin2	Prin3	Prin4	Prin5	Prin6	Prin7	Prin8	Prin9	Prin10
TriR	0.350352	-.183988	0.184670	-.490826	0.270932	0.060954	0.051002	-.004851	-.102747	0.691439
TriL	0.357806	-.188528	0.154215	-.467074	0.265585	0.023004	0.127124	0.011620	0.126581	-.700792
BicR	0.356108	-.129558	-.136654	-.044036	-.709875	0.497504	0.218329	-.030288	0.185693	0.035898
BicL	0.392263	-.137316	-.092028	-.047993	-.407999	-.701519	-.346880	0.024450	-.190600	-.025641
WriR	0.340469	-.241415	0.138392	0.512555	0.164068	0.211901	0.125058	-.011970	-.667870	-.101394
WriL	0.339271	-.222051	0.167625	0.517218	0.229419	-.105982	-.077120	0.032746	0.674829	0.117034
KneR	0.301552	0.293467	-.499763	-.017276	0.243538	0.348538	-.624786	-.019864	-.010051	-.035193
KneL	0.268308	0.345724	-.539056	0.067452	0.179565	-.275359	0.633323	0.022605	0.018011	0.057930
AnkR	0.198068	0.534487	0.405725	0.034376	-.071116	-.032852	-.001551	-.709150	0.005094	-.015438
AnkL	0.188062	0.544107	0.403440	0.020780	-.097527	0.040016	-.014153	0.702361	-.026727	-.006821

Enclosure A - SAS output

Reflexes data

The FACTOR Procedure

Initial Factor Method: Principal Components

Prior Communality Estimates: ONE

Eigenvalues of the Correlation Matrix: Total = 10 Average = 1

	Eigenvalue	Difference	Proportion	Cumulative
1	5.22298654	3.18739392	0.5223	0.5223
2	2.03559262	0.94146709	0.2036	0.7259
3	1.09412553	0.23707467	0.1094	0.8353
4	0.85705086	0.36134577	0.0857	0.9210
5	0.49570509	0.38812846	0.0496	0.9705
6	0.10757663	0.02125229	0.0108	0.9813
7	0.08632433	0.02701308	0.0086	0.9899
8	0.05931125	0.03670948	0.0059	0.9959
9	0.02260177	0.00387639	0.0023	0.9981
10	0.01872538		0.0019	1.0000

3 factors will be retained by the MINEIGEN criterion.

Factor Pattern

	Factor1	Factor2	Factor3
TriR	0.80069	-0.26250	0.19317
TriL	0.81772	-0.26898	0.16131
BicR	0.81384	-0.18485	-0.14294
BicL	0.89647	-0.19591	-0.09626
WriR	0.77810	-0.34444	0.14476
WriL	0.77536	-0.31681	0.17534
KneR	0.68916	0.41870	-0.52275
KneL	0.61319	0.49326	-0.56385
AnkR	0.45266	0.76258	0.42439
AnkL	0.42980	0.77630	0.42200

Variance Explained by Each Factor

Factor1	Factor2	Factor3
5.2229865	2.0355926	1.0941255

Final Communality Estimates: Total = 8.352705

TriR	TriL	BicR	BicL	WriR	WriL	KneR	KneL	AnkR	AnkL
0.747325	0.767044	0.716942	0.851312	0.745035	0.732301	0.923529	0.937236	0.966532	0.965450

Enclosure A - SAS output

Orthogonal Transformation Matrix

	1	2	3
1	0.84662	0.44963	0.28472
2	-0.48297	0.42441	0.76590
3	0.22353	-0.78595	0.57648

Rotated Factor Pattern

	Factor1	Factor2	Factor3
TriR	0.84784	0.09678	0.13828
TriL	0.85827	0.12673	0.11980
BicR	0.74634	0.39982	0.00774
BicL	0.83208	0.39558	0.04970
WriR	0.85747	0.08990	0.04119
WriL	0.84865	0.07636	0.07920
KneR	0.26439	0.89842	0.21555
KneL	0.15487	0.92821	0.22733
AnkR	0.10979	0.19363	0.95759
AnkL	0.08327	0.19105	0.96022

Variance Explained by Each Factor

Factor1	Factor2	Factor3
4.2731796	2.0984124	1.9811127

Final Communality Estimates: Total = 8.352705

TriR	TriL	BicR	BicL	WriR	WriL	KneR	KneL	AnkR	AnkL
0.747325	0.767044	0.716942	0.851312	0.745035	0.732301	0.923529	0.937236	0.966532	0.965450

Enclosure B - SAS program

```
data Ymerfull;
set sasuser.ymer6t;
if mask=10 or mask=11 or mask=12 or mask=13 then output Ymerfull;
run;
data Ymertest;
set Ymerfull;
rand=ranuni(1098);
if rand<0.1 then output Ymertest;
run;
Title 'Satellite Data from Ymer Ø';
Title2 'The first 40 observations';
proc print data=Ymertest (obs=40);
   var mask b1-b6;
   run;
Title2;
proc sort data=Ymertest;
by mask;
proc means data=Ymertest;
   var b1-b6;
   by mask;
run;
proc discrim data=Ymertest distance listerr pool=yes;
   class mask;
   var b1-b6;
run;
proc discrim data=Ymertest distance listerr pool=yes;
   class mask;
   var b1-b3;
run;
```


Enclosure B - SAS output

Satellite Data from Ymer Ø

The first 40 observations

Obs	mask	b1	b2	b3	b4	b5	b6
1	13	94	50	75	73	114	72
2	13	102	58	79	75	116	79
3	13	97	55	77	75	117	78
4	13	98	53	76	74	119	74
5	13	94	50	75	71	118	77
6	13	98	54	78	75	126	78
7	13	99	52	76	71	96	66
8	13	94	48	68	65	97	61
9	13	99	58	79	77	120	76
10	13	99	60	85	80	128	84
11	13	95	51	72	69	110	72
12	13	102	62	87	82	149	89
13	13	89	49	69	67	107	68
14	13	95	53	73	69	118	73
15	13	101	59	82	76	131	83
16	13	97	53	74	71	102	66
17	10	74	33	42	38	66	40
18	10	79	37	44	40	70	43
19	10	69	31	34	37	68	38
20	10	67	30	34	34	67	39
21	10	78	33	38	39	75	42
22	11	87	47	67	66	119	71
23	10	79	35	43	41	71	48
24	10	74	34	41	38	68	42
25	10	75	33	39	38	67	38
26	11	112	62	81	71	138	84
27	11	113	62	80	69	128	76
28	10	77	36	44	42	65	42
29	12	87	40	46	41	81	43
30	12	80	39	43	37	74	41
31	10	71	32	37	40	68	37
32	12	83	37	43	42	78	42
33	12	83	40	46	45	80	42
34	12	84	36	41	43	82	48
35	12	91	36	42	47	84	45
36	12	88	42	47	42	80	47
37	12	87	40	46	38	65	42
38	12	87	39	46	41	75	44
39	12	89	39	43	40	70	42
40	12	89	40	47	41	73	46

Enclosure B - SAS output

Satellite Data from Ymer Ø

The MEANS Procedure

mask=10

Variable	N	Mean	Std Dev	Minimum	Maximum
b1	16	70.1250000	6.5510813	61.0000000	79.0000000
b2	16	29.9375000	4.9459579	23.0000000	37.0000000
b3	16	33.8750000	8.2613558	22.0000000	44.0000000
b4	16	35.6250000	4.8287334	26.0000000	42.0000000
b5	16	62.9375000	8.1524536	49.0000000	75.0000000
b6	16	35.9375000	7.2614852	25.0000000	48.0000000

mask=11

Variable	N	Mean	Std Dev	Minimum	Maximum
b1	11	102.7272727	8.0633853	87.0000000	113.0000000
b2	11	54.3636364	6.0707945	46.0000000	62.0000000
b3	11	68.3636364	8.8574571	53.0000000	81.0000000
b4	11	63.4545455	4.9872565	56.0000000	71.0000000
b5	11	121.9090909	8.0430659	108.0000000	138.0000000
b6	11	71.0000000	6.4342832	62.0000000	84.0000000

mask=12

Variable	N	Mean	Std Dev	Minimum	Maximum
b1	88	79.9886364	7.4917492	57.0000000	93.0000000
b2	88	35.2613636	5.2116014	23.0000000	44.0000000
b3	88	39.3977273	7.1124624	22.0000000	50.0000000
b4	88	38.2613636	5.4149523	19.0000000	47.0000000
b5	88	67.2613636	11.6248785	22.0000000	86.0000000
b6	88	38.2272727	7.6349578	13.0000000	53.0000000

mask=13

Variable	N	Mean	Std Dev	Minimum	Maximum
b1	16	97.0625000	3.4538626	89.0000000	102.0000000
b2	16	54.0625000	4.2185108	48.0000000	62.0000000
b3	16	76.5625000	5.1635743	68.0000000	87.0000000
b4	16	73.1250000	4.5588741	65.0000000	82.0000000
b5	16	116.7500000	13.3191591	96.0000000	149.0000000
b6	16	74.7500000	7.3439317	61.0000000	89.0000000

Enclosure B - SAS output

Satellite Data from Ymer Ø

The DISCRIM Procedure

Total Sample Size	131	DF Total	130
Variables	6	DF Within Classes	127
Classes	4	DF Between Classes	3

Number of Observations Read	131
Number of Observations Used	131

Class Level Information

mask	Variable Name	Frequency	Weight	Proportion	Prior Probability
10	10	16	16.0000	0.122137	0.250000
11	11	11	11.0000	0.083969	0.250000
12	12	88	88.0000	0.671756	0.250000
13	13	16	16.0000	0.122137	0.250000

Pooled Covariance Matrix Information

Covariance Matrix Rank	Natural Log of the Determinant of the Covariance Matrix
6	15.30586

Satellite Data from Ymer Ø

The DISCRIM Procedure

Squared Distance to mask

From mask	10	11	12	13
10	0	35.99468	4.32708	74.35995
11	35.99468	0	29.06112	36.86522
12	4.32708	29.06112	0	82.37583
13	74.35995	36.86522	82.37583	0

F Statistics, NDF=6, DDF=122 for Squared Distance to mask

From mask	10	11	12	13
10	0	37.56575	9.37928	95.24319
11	37.56575	0	45.49433	38.47428
12	9.37928	45.49433	0	178.55581
13	95.24319	38.47428	178.55581	0

Prob > Mahalanobis Distance for Squared Distance to mask

From mask	10	11	12	13
10	1.0000	<.0001	<.0001	<.0001
11	<.0001	1.0000	<.0001	<.0001
12	<.0001	<.0001	1.0000	<.0001
13	<.0001	<.0001	<.0001	1.0000

Generalized Squared Distance to mask				
From mask	10	11	12	13
10	0	35.99468	4.32708	74.35995
11	35.99468	0	29.06112	36.86522
12	4.32708	29.06112	0	82.37583
13	74.35995	36.86522	82.37583	0

Linear Discriminant Function for mask				
Variable	10	11	12	13
Constant	-93.40039	-165.00369	-119.87304	-154.30306
b1	3.10612	3.46920	3.51010	3.10499
b2	0.32445	0.03541	0.86505	-3.51414
b3	-1.87184	-1.60859	-2.25654	0.89833
b4	1.63165	2.19217	1.66187	2.93783
b5	0.76093	1.24570	0.87185	0.65163
b6	-2.31900	-2.94782	-2.74280	-2.17355

Satellite Data from Ymer Ø

The DISCRIM Procedure

Classification Results for Calibration Data: WORK.YMERTEST

Resubstitution Results using Linear Discriminant Function

Posterior Probability of Membership in mask							
Obs	From mask	Classified into mask		10	11	12	13
17	11	13	*	0.0000	0.0000	0.0000	1.0000
41	12	10	*	0.6294	0.0007	0.3699	0.0000
57	12	10	*	0.7154	0.0000	0.2846	0.0000
73	12	10	*	0.5657	0.0000	0.4342	0.0000
76	12	10	*	0.7533	0.0000	0.2467	0.0000
91	12	10	*	0.9923	0.0000	0.0077	0.0000
92	12	10	*	0.8453	0.0000	0.1547	0.0000
114	12	10	*	0.6521	0.0000	0.3479	0.0000
115	12	10	*	0.9939	0.0000	0.0061	0.0000

* Misclassified observation

Satellite Data from Ymer Ø

The DISCRIM Procedure

Classification Summary for Calibration Data: WORK.YMERTEST

Resubstitution Summary using Linear Discriminant Function

Number of Observations and Percent Classified into mask					
From mask	10	11	12	13	Total
10	16	0	0	0	16
	100.00	0.00	0.00	0.00	100.00
11	0	10	0	1	11
	0.00	90.91	0.00	9.09	100.00
12	8	0	80	0	88
	9.09	0.00	90.91	0.00	100.00
13	0	0	0	16	16
	0.00	0.00	0.00	100.00	100.00
Total	24	10	80	17	131
	18.32	7.63	61.07	12.98	100.00
Priors	0.25	0.25	0.25	0.25	

Error Count Estimates for mask					
	10	11	12	13	Total
Rate	0.0000	0.0909	0.0909	0.0000	0.0455
Priors	0.2500	0.2500	0.2500	0.2500	

Satellite Data from Ymer Ø

The DISCRIM Procedure

Total Sample Size	131	DF Total	130
Variables	3	DF Within Classes	127
Classes	4	DF Between Classes	3

Number of Observations Read	131
Number of Observations Used	131

Class Level Information					
mask	Variable Name	Frequency	Weight	Proportion	Prior Probability
10	10	16	16.0000	0.122137	0.250000
11	11	11	11.0000	0.083969	0.250000
12	12	88	88.0000	0.671756	0.250000
13	13	16	16.0000	0.122137	0.250000

Pooled Covariance Matrix Information	
Covariance Matrix Rank	Natural Log of the Determinant of the Covariance Matrix
3	7.06698

Enclosure B - SAS output

Satellite Data from Ymer Ø

The DISCRIM Procedure

Squared Distance to mask				
From mask	10	11	12	13
10	0	24.54514	2.91205	55.29380
11	24.54514	0	16.88005	26.62660
12	2.91205	16.88005	0	59.89058
13	55.29380	26.62660	59.89058	0

F Statistics, NDF=3, DDF=125 for Squared Distance to mask				
From mask	10	11	12	13
10	0	52.49277	12.93460	145.12808
11	52.49277	0	54.15007	56.94422
12	12.93460	54.15007	0	266.01914
13	145.12808	56.94422	266.01914	0

Prob > Mahalanobis Distance for Squared Distance to mask				
From mask	10	11	12	13
10	1.0000	<.0001	<.0001	<.0001
11	<.0001	1.0000	<.0001	<.0001
12	<.0001	<.0001	1.0000	<.0001
13	<.0001	<.0001	<.0001	1.0000

Generalized Squared Distance to mask				
From mask	10	11	12	13
10	0	24.54514	2.91205	55.29380
11	24.54514	0	16.88005	26.62660
12	2.91205	16.88005	0	59.89058
13	55.29380	26.62660	59.89058	0

Linear Discriminant Function for mask				
Variable	10	11	12	13
Constant	-65.01804	-112.96242	-84.65805	-98.14227
b1	2.29002	2.60506	2.52151	2.33452
b2	1.30217	1.22813	2.00335	-2.45684
b3	-2.05271	-1.58639	-2.61479	1.33895

Enclosure B - SAS output

Satellite Data from Ymer Ø

The DISCRIM Procedure

Classification Results for Calibration Data: WORK.YMERTEST

Resubstitution Results using Linear Discriminant Function

Posterior Probability of Membership in mask							
Obs	From mask	Classified into mask		10	11	12	13
5	10	12	*	0.4512	0.0001	0.5487	0.0000
17	11	13	*	0.0000	0.0000	0.0000	1.0000
25	11	12	*	0.0066	0.1780	0.8154	0.0000
41	12	10	*	0.7210	0.0000	0.2790	0.0000
57	12	10	*	0.5904	0.0003	0.4093	0.0000
76	12	10	*	0.6232	0.0006	0.3763	0.0000
91	12	10	*	0.9611	0.0000	0.0389	0.0000
92	12	10	*	0.9361	0.0000	0.0639	0.0000
96	12	10	*	0.8201	0.0000	0.1799	0.0000
97	12	10	*	0.6957	0.0005	0.3038	0.0000
102	12	10	*	0.8243	0.0000	0.1757	0.0000
103	12	10	*	0.6836	0.0000	0.3164	0.0000
106	12	10	*	0.7903	0.0000	0.2097	0.0000
107	12	10	*	0.7532	0.0000	0.2468	0.0000
108	12	10	*	0.5701	0.0000	0.4299	0.0000
111	12	10	*	0.7937	0.0000	0.2063	0.0000
112	12	10	*	0.5403	0.0000	0.4597	0.0000
113	12	10	*	0.6681	0.0000	0.3319	0.0000
114	12	10	*	0.9315	0.0000	0.0685	0.0000
115	12	10	*	0.8955	0.0000	0.1045	0.0000

* Misclassified observation

Enclosure B - SAS output

Satellite Data from Ymer Ø

The DISCRIM Procedure

Classification Summary for Calibration Data: WORK.YMERTEST

Resubstitution Summary using Linear Discriminant Function

Number of Observations and Percent Classified into mask					
From mask	10	11	12	13	Total
10	15	0	1	0	16
	93.75	0.00	6.25	0.00	100.00
11	0	9	1	1	11
	0.00	81.82	9.09	9.09	100.00
12	17	0	71	0	88
	19.32	0.00	80.68	0.00	100.00
13	0	0	0	16	16
	0.00	0.00	0.00	100.00	100.00
Total	32	9	73	17	131
	24.43	6.87	55.73	12.98	100.00
Priors	0.25	0.25	0.25	0.25	

Error Count Estimates for mask					
	10	11	12	13	Total
Rate	0.0625	0.1818	0.1932	0.0000	0.1094
Priors	0.2500	0.2500	0.2500	0.2500	

Enclosure C - SAS program

```
data cementmill(type=corr);
infile cards missover;
input _type_ $ _name_ $  BLAINE XSO3 LSR STRGTH3 STRGTH7 STRGTH28 ;
cards;
N    .              198      198      198      198       198      198
MEAN .             3095.71717 1.94126 40.29268  263.89394 382.28788 515.27778
STD  .             234.22485 0.25448 8.55410  36.88332 36.70451 32.67124
CORR BLAINE     1
CORR XSO3        0.49077  1
CORR LSR        -0.19604 -0.52069  1
CORR STRGTH3     0.80042  0.47398 -0.17769  1
CORR STRGTH7     0.73643  0.37344 -0.07175  0.89557  1
CORR STRGTH28    0.51413  0.27931 -0.08501  0.60826  0.74561  1
;
Title 'Cement Mill Data';
proc print data=cementmill(type=corr);
run;
proc princomp data=cementmill(type=corr);

partial XSO3 LSR BLAINE;
var STRGTH3 STRGTH7 STRGTH28;
run;
proc cancorr data=cementmill(type=corr);
var STRGTH3 STRGTH7 STRGTH28 ;
with XSO3 LSR BLAINE;
run;
```

Enclosure C - SAS output

Cement Mill Data

Obs	_type_	_name_	BLAINE	XSO3	LSR	STRGTH3	STRGTH7	STRGTH28
1	N		198.00	198.000	198.000	198.000	198.000	198.000
2	MEAN		3095.72	1.941	40.293	263.894	382.288	515.278
3	STD		234.22	0.254	8.554	36.883	36.705	32.671
4	CORR	BLAINE	1.00	.	.	.	.	.
5	CORR	XSO3	0.49	1.000	.	.	.	.
6	CORR	LSR	-0.20	-0.521	1.000	.	.	.
7	CORR	STRGTH3	0.80	0.474	-0.178	1.000	.	.
8	CORR	STRGTH7	0.74	0.373	-0.072	0.896	1.000	.
9	CORR	STRGTH28	0.51	0.279	-0.085	0.608	0.746	1.000

Cement Mill Data

The PRINCOMP Procedure

Observations	198
Variables	3
Partial Variables	3

Regression Statistics

	STRGTH3	STRGTH7	STRGTH28
R-Square	0.6501844058	0.5512118819	0.2666052674
RMSE	21.982740718	24.778348332	28.194653345

Standardized Regression Coefficients

	STRGTH3	STRGTH7	STRGTH28
XSO3	0.1259033534	0.0770949762	0.0593892723
LSR	0.0339733769	0.1095553672	0.0426278300
BLAINE	0.7452905521	0.7200713327	0.4933402866

Partial Correlation Matrix

	STRGTH3	STRGTH7	STRGTH28
STRGTH3	1.0000	0.7625	0.3807
STRGTH7	0.7625	1.0000	0.6330
STRGTH28	0.3807	0.6330	1.0000

Eigenvalues of the Partial Correlation Matrix

	Eigenvalue	Difference	Proportion	Cumulative
1	2.19749509	1.56906593	0.7325	0.7325
2	0.62842917	0.45435343	0.2095	0.9420
3	0.17407574		0.0580	1.0000

Eigenvectors

	Prin1	Prin2	Prin3
STRGTH3	0.570479	-.607006	0.553261
STRGTH7	0.637205	-.097904	-.764451
STRGTH28	0.518192	0.788644	0.330935

Enclosure C - SAS output

Cement Mill Data

The CANCORR Procedure

Canonical Correlation Analysis

	Canonical Correlation	Adjusted Canonical Correlation	Approximate Standard Error	Squared Canonical Correlation	Eigenvalues of Inv(E)*H = CanRsq/(1-CanRsq) Eigenvalue	Difference	Proportion	Cumulative	Test of H0: The canonical correlations in the current row and all that follow are zero Likelihood Ratio	Approx. F Value	Num DF	Den DF	Pr > F
1	0.8071	0.8034	0.02483	0.6515	1.8692	1.807	0.9677	0.9677	0.3280	30.17	9	467.43	<.0001
2	0.2421	0.2236	0.06707	0.0586	0.0623	0.062	0.0322	1.0000	0.9413	2.96	4	386	0.0197
3	0.0092	.	0.07124	0.0001	0.0001		0.0000	1.0000	0.9999	0.02	1	194	0.8986

Multivariate Statistics and F Approximations

S=3 M=-0.5 N=95

Statistic	Value	F Value	Num DF	Den DF	Pr > F
Wilks' Lambda	0.32807447	30.17	9	467.43	<.0001
Pillai's Trace	0.71017295	20.06	9	582	<.0001
Hotelling-Lawley Trace	1.93151671	41.05	9	298.55	<.0001
Roy's Greatest Root	1.86915884	120.87	3	194	<.0001

NOTE: F Statistic for Roy's Greatest Root is an upper bound.

Cement Mill Data

The CANCORR Procedure

Canonical Correlation Analysis

Raw Canonical Coefficients for the VAR Variables

	V1	V2	V3
STRGTH3	0.0250949371	-0.055529541	-0.012485742
STRGTH7	0.0016859572	0.0733301742	-0.012762037
STRGTH28	0.0009044423	-0.019693279	0.0425387843

Raw Canonical Coefficients for the WITH Variables

	W1	W2	W3
XSO3	0.599124626	-1.437480845	-4.958007106
LSR	0.0057188045	0.0955344974	-0.098543797
BLAINE	0.0039617207	0.0016619274	0.0023892113

Enclosure C - SAS output

Cement Mill Data

The CANCORR Procedure

Canonical Correlation Analysis

Standardized Canonical Coefficients for the VAR Variables	V1	V2	V3
STRGTH3	0.9256	-2.0481	-0.4605
STRGTH7	0.0619	2.6915	-0.4684
STRGTH28	0.0295	-0.6434	1.3898

Standardized Canonical Coefficients for the WITH Variables	W1	W2	W3
XSO3	0.1525	-0.3658	-1.2617
LSR	0.0489	0.8172	-0.8430
BLAINE	0.9279	0.3893	0.5596

Cement Mill Data

The CANCORR Procedure

Canonical Structure

Correlations Between the VAR Variables and Their Canonical Variables	V1	V2	V3
STRGTH3	0.9990	-0.0290	-0.0347
STRGTH7	0.9128	0.3776	0.1554
STRGTH28	0.6387	0.1177	0.7604

Correlations Between the WITH Variables and Their Canonical Variables	W1	W2	W3
XSO3	0.5824	-0.6003	-0.5482
LSR	-0.2124	0.9314	-0.2957
BLAINE	0.9932	0.0495	0.1057

Correlations Between the VAR Variables and the Canonical Variables of the WITH Variables	W1	W2	W3
STRGTH3	0.8063	-0.0070	-0.0003
STRGTH7	0.7368	0.0914	0.0014
STRGTH28	0.5155	0.0285	0.0070

Correlations Between the WITH Variables and the Canonical Variables of the VAR Variables	V1	V2	V3
XSO3	0.4701	-0.1453	-0.0050
LSR	-0.1714	0.2255	-0.0027
BLAINE	0.8016	0.0120	0.0010

Enclosure D - SAS program


```
data trainprel;
set sasuser.mergelandsat;
*Selecting app. 20%;
rand=ranuni(1234);
if rand>0.8 then output trainprel;
*Taking the first 100 observations of those;
data train;
set trainprel;
if _N_ LE 100;
run;
Title 'First 25 out of 100 randomly chosen observations';
proc print data=train (obs=25);
var MJ1 MR1-MR6;
run;
Title 'Full model';
proc reg data=train;
model MJ1 = MR1-MR6;
run;
Title 'Model selected by Rsquare';
proc reg data=train;
model MJ1 = MR1-MR6/selection=Rsquare;
run;
Title 'Model Band2';
 proc reg data=train;
model MJ1 = MR2/cov;
run;
```

Enclosure D - SAS output

First 25 out of 100 randomly chosen observations

Obs	MJ1	MR1	MR2	MR3	MR4	MR5	MR6
1	137	162	89	135	105	210	144
2	134	161	88	136	104	210	143
3	143	165	92	142	109	212	148
4	143	165	93	146	111	216	152
5	139	164	93	144	109	215	152
6	133	164	90	138	106	216	147
7	140	166	93	145	111	218	153
8	132	158	82	123	96	201	135
9	136	161	83	124	97	204	135
10	131	157	82	125	97	206	137
11	134	160	83	127	99	206	137
12	131	156	83	125	97	206	137
13	137	164	89	135	106	215	147
14	134	165	89	137	107	215	147
15	131	158	82	124	96	205	135
16	130	159	82	124	97	204	137
17	130	159	82	124	96	204	133
18	126	156	81	122	95	202	132
19	127	150	76	112	88	196	119
20	130	151	78	115	91	195	123
21	134	156	80	121	95	202	131
22	127	150	77	117	91	193	124
23	131	155	79	118	91	195	128
24	129	157	79	121	94	196	130
25	129	156	79	120	93	197	131

Enclosure D - SAS output

Full model

The REG Procedure

Model: MODEL1

Dependent Variable: MJ1

Number of Observations Read	100
Number of Observations Used	100

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	6	1487.66894	247.94482	28.96	<.0001
Error	93	796.33106	8.56270		
Corrected Total	99	2284.00000			

Root MSE	2.92621	R-Square	0.6513
Dependent Mean	135.40000	Adj R-Sq	0.6288
Coeff Var	2.16116		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	14.08344	16.53430	0.85	0.3965
MR1	1	0.49664	0.17416	2.85	0.0054
MR2	1	0.78758	0.44952	1.75	0.0831
MR3	1	-0.07149	0.24942	-0.29	0.7750
MR4	1	-0.29987	0.24562	-1.22	0.2252
MR5	1	0.07008	0.13056	0.54	0.5927
MR6	1	-0.00169	0.17887	-0.01	0.9925

Enclosure D - SAS output

Model selected by Rsquare

The REG Procedure

Model: MODEL1

Dependent Variable: MJ1

R-Square Selection Method

Number of Observations Read	100
Number of Observations Used	100

Number in Model	R-Square	Variables in Model
1	0.6299	MR1
1	0.5984	MR2
1	0.5433	MR3
1	0.5230	MR6
1	0.5092	MR4
1	0.4705	MR5
2	0.6408	MR1 MR2
2	0.6358	MR1 MR6
2	0.6345	MR1 MR3
2	0.6325	MR1 MR5
2	0.6312	MR1 MR4
2	0.6126	MR2 MR3
2	0.6122	MR2 MR4
2	0.5996	MR2 MR6
2	0.5984	MR2 MR5
2	0.5479	MR3 MR5
2	0.5479	MR3 MR6
2	0.5438	MR3 MR4
2	0.5329	MR4 MR6
2	0.5242	MR5 MR6
2	0.5166	MR4 MR5
3	0.6492	MR1 MR2 MR4
3	0.6455	MR1 MR2 MR3
3	0.6408	MR1 MR2 MR5
3	0.6408	MR1 MR2 MR6
3	0.6374	MR1 MR3 MR4
3	0.6370	MR1 MR4 MR6
3	0.6360	MR1 MR5 MR6
3	0.6359	MR1 MR3 MR6
3	0.6347	MR1 MR3 MR5
3	0.6325	MR1 MR4 MR5
3	0.6167	MR2 MR4 MR5
3	0.6162	MR2 MR3 MR4

Enclosure D - SAS output

3	0.6135	MR2 MR3 MR5
3	0.6127	MR2 MR3 MR6
3	0.6122	MR2 MR4 MR6
3	0.6006	MR2 MR5 MR6
3	0.5515	MR3 MR4 MR5
3	0.5491	MR3 MR5 MR6
3	0.5490	MR3 MR4 MR6
3	0.5330	MR4 MR5 MR6
4	0.6510	MR1 MR2 MR4 MR5
4	0.6496	MR1 MR2 MR3 MR4
4	0.6496	MR1 MR2 MR4 MR6
4	0.6458	MR1 MR2 MR3 MR6
4	0.6456	MR1 MR2 MR3 MR5
4	0.6408	MR1 MR2 MR5 MR6
4	0.6395	MR1 MR3 MR4 MR6
4	0.6392	MR1 MR3 MR4 MR5
4	0.6370	MR1 MR4 MR5 MR6
4	0.6361	MR1 MR3 MR5 MR6
4	0.6202	MR2 MR3 MR4 MR5
4	0.6188	MR2 MR4 MR5 MR6
4	0.6165	MR2 MR3 MR4 MR6
4	0.6136	MR2 MR3 MR5 MR6
4	0.5520	MR3 MR4 MR5 MR6
5	0.6513	MR1 MR2 MR3 MR4 MR5
5	0.6510	MR1 MR2 MR4 MR5 MR6
5	0.6503	MR1 MR2 MR3 MR4 MR6
5	0.6458	MR1 MR2 MR3 MR5 MR6
5	0.6398	MR1 MR3 MR4 MR5 MR6
5	0.6209	MR2 MR3 MR4 MR5 MR6
6	0.6513	MR1 MR2 MR3 MR4 MR5 MR6

Enclosure D - SAS output

Model Band2

The REG Procedure

Model: MODEL1

Dependent Variable: MJ1

Number of Observations Read	100
Number of Observations Used	100

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	1366.83331	1366.83331	146.05	<.0001
Error	98	917.16669	9.35884		
Corrected Total	99	2284.00000			

Root MSE	3.05922	**R-Square**	0.5984
Dependent Mean	135.40000	**Adj R-Sq**	0.5943
Coeff Var	2.25940		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	63.84580	5.92881	10.77	<.0001
MR2	1	0.83660	0.06923	12.09	<.0001

Covariance of Estimates

Variable	Intercept	MR2
Intercept	35.150763105	-0.409881617
MR2	-0.409881617	0.0047922555