Solution for Exam 2001-1 Problem 2

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The key here is to identify which type of problem we are facing. We are asked for a test of equality of means in multivariate normal distributions. If we only had two groups, we would use Hotellings T^2, but as we have three here, me instead use a *one-sided MANOVA,* section 4.3.1.

Q 2.1

The usual test-statistic for equality of means.

We go to section 4.3.1 in the book page 302, and find the test given in theorem 4.25 (p. 304). We now need to identify **w** and **t** as they are denoted in the theorem.

They are:

$$\begin{aligned}\mathbf{T} &= \sum_{i=1}^{k}\sum_{j=1}^{n_i}(\mathbf{Y}_{ij}-\bar{\mathbf{Y}})(\mathbf{Y}_{ij}-\bar{\mathbf{Y}})^T \\ \mathbf{W} &= \sum_{i=1}^{k}\sum_{j=1}^{n_i}(\mathbf{Y}_{ij}-\bar{\mathbf{Y}}_i)(\mathbf{Y}_{ij}-\bar{\mathbf{Y}}_i)^T\end{aligned}$$

Where

$$\begin{aligned}\bar{\boldsymbol{Y}}_i &= \frac{1}{n_i}\sum_{j=1}^{n_i}\boldsymbol{Y}_{ij} \\ \bar{\boldsymbol{Y}} &= \frac{1}{n}\sum_{i=1}^{k}\sum_{j=1}^{n_i}\boldsymbol{Y}_{ij}.\end{aligned}$$

We identify the grand mean $\bar{Y}$:

$$\underline{A} = \frac{1}{n_X+n_Y+n_Z}\left(\sum_{i=1}^{n_X}\underline{X}_i+\sum_{j=1}^{n_Y}\underline{Y}_j+\sum_{k=1}^{n_Z}\underline{Z}_k\right)$$

And the group mean $\bar{Y}_i$:

$$\underline{A}_X = \frac{1}{n_X}\sum_{i=1}^{n_X}\underline{X}_i \quad ; \quad \underline{A}_Y = \frac{1}{n_Y}\sum_{j=1}^{n_Y}\underline{Y}_j \quad ; \quad \underline{A}_Z = \frac{1}{n_Z}\sum_{k=1}^{n_Z}\underline{Z}_k$$

It is then apparent that **w** is given by:

$$\underline{\underline{U}}_1 = \sum_{i=1}^{n_X}(\underline{X}_i-\underline{A}_X)(\underline{X}_i-\underline{A}_X)' + \sum_{j=1}^{n_Y}(\underline{Y}_j-\underline{A}_Y)(\underline{Y}_j-\underline{A}_Y)' + \sum_{k=1}^{n_Z}(\underline{Z}_k-\underline{A}_Z)(\underline{Z}_k-\underline{A}_Z)'$$

And that **t** is given by:

$$\underline{\underline{U}}_0 = \sum_{i=1}^{n_X}(\underline{X}_i-\underline{A})(\underline{X}_i-\underline{A})' + \sum_{j=1}^{n_Y}(\underline{Y}_j-\underline{A})(\underline{Y}_j-\underline{A})' + \sum_{k=1}^{n_Z}(\underline{Z}_k-\underline{A})(\underline{Z}_k-\underline{A})'$$

The correct answer is thus 3.

Q 2.2

The U distribution is given in theorem 4.25 page 304:

$$U(p, k-1, n-k)_\alpha$$

Where *p* is the dimensionality of our observations, in this case *p*=4.

k is the number of groups, in this case *k*=3.

n is the number of observations.

This leads us to: $U(p, k-1, n-k) = U(4,3-1, n-3)$ this is answer 1.