Exercise 6.2

In some attempts of geological studies by means of "remote sensing" one has used the measurements from the American Landsat-4 satellite. This satellite measures simultaneously the values of 7 spectral bands in the electromagnetic spectrum of square areas (so-called pixels) of $35m \times 35m$ on the surface of the earth.

There is special interest in differentiating between two types of geology:

bed group 10 (a type of slate) and
bed group 13 (a type of quartsite).

One assumes that 3 of the 7 bands of the satellite are especially suited to differentiate between the two bed groups. They are:

$X_1 =$ Band 1 $\sim 0.45 - 0.52\,\mu m$
$X_2 =$ Band 3 $\sim 0.63 - 0.69\,\mu m$
$X_3 =$ Band 4 $\sim 0.76 - 0.90\,\mu m$

From an area on Ymer Island in East Greenland one has measured 493 pixels from bed group 10 and 73 pixels from bed group 13.

The estimated mean vectors, where μ_1 is from: bed group 10 and μ_2 from bed group 13 and the common variance-covariance matrix are given below. Furthermore other measures are given as a help.

$$\hat{\boldsymbol{\mu}}_1 = \begin{pmatrix} 75.9 \\ 45.3 \\ 41.8 \end{pmatrix}, \qquad \hat{\boldsymbol{\mu}}_2 = \begin{pmatrix} 78.2 \\ 39.5 \\ 44.1 \end{pmatrix}$$

$$\hat{\boldsymbol{\Sigma}} = \begin{pmatrix} 26.9 & 13.5 & 3.9 \\ 13.5 & 17.5 & 12.2 \\ 3.9 & 12.2 & 16.6 \end{pmatrix}$$

$$\hat{\boldsymbol{\Sigma}}^{-1} = \begin{pmatrix} 0.0788 & -0.0987 & 0.0543 \\ -0.0987 & 0.2415 & -0.1547 \\ 0.0543 & -0.1547 & 0.1615 \end{pmatrix}$$

$$\hat{\boldsymbol{\Sigma}}^{-1}(\hat{\boldsymbol{\mu}}_1 - \hat{\boldsymbol{\mu}}_2) = \begin{pmatrix} -0.88 \\ 1.98 \\ -1.39 \end{pmatrix}$$

$$\hat{\boldsymbol{\mu}}_1'\hat{\boldsymbol{\Sigma}}^{-1}\hat{\boldsymbol{\mu}}_1 = 310.4\,, \qquad \hat{\boldsymbol{\mu}}_2'\hat{\boldsymbol{\Sigma}}^{-1}\hat{\boldsymbol{\mu}}_2 = 396.5$$

Exercise 6.2

1) Are these measurements reasonable to use in an attempt to differentiate between *bed group* 10 and *bed group* 13?

2) Would it be reasonable to drop bands 1 and 3 and use only band 4 in an attempt to differentiate between *bed group* 10 and *bed group* 13?

3) If one uses all the information from all 3 bands, what is then the Bayes-classification rule, assuming equal priors?

4) What is the Bayes-classification rule corresponding to using prior probabilities which are proportional to the number of observations?

5) Using the rule found under 3), what will

$$\boldsymbol{x} = \begin{pmatrix} 74 \\ 43 \\ 40 \end{pmatrix}$$

be classified as?