Solutions for Exam 2011, Problem 4-5

ANYM, 20190923

4.1 – 4

We consider Theorem 6.12:

$$\text{(i)} \quad \begin{bmatrix} -\varrho_r \boldsymbol{\Sigma}_{yy} & \boldsymbol{\Sigma}_{yx} \\ \boldsymbol{\Sigma}_{xy} & -\varrho_r \boldsymbol{\Sigma}_{xx} \end{bmatrix} \begin{bmatrix} \boldsymbol{a}_r \\ \boldsymbol{b}_r \end{bmatrix} = \mathbf{0}$$

$$\text{(ii)} \quad \boldsymbol{a}_r^T \boldsymbol{\Sigma}_{yy} \boldsymbol{a}_r = 1$$

$$\text{(iii)} \quad \boldsymbol{b}_r^T \boldsymbol{\Sigma}_{xx} \boldsymbol{b}_r = 1$$

We have the canonical variable coefficient $\boldsymbol{a}_1, \boldsymbol{b}_1$, and can thus use (i) to find the first canonical correlation, which is also the largest, see Definition 6.11.

Inserting:

$$\begin{bmatrix} -\varrho \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \begin{matrix} \rho & \rho \\ \rho & \rho \end{matrix} \\ \begin{matrix} \rho & \rho \\ \rho & \rho \end{matrix} & -\varrho \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix} = 0$$

We see that $\varrho = 2\rho$

5.1 – 4

The amount (fraction) of variance described is the multiple correlation coefficient (R-square). See section 1.3.2 and remark 3.6

The R-square can be read directly in the SAS output in Enclosure C.

5.2 – 4

We use Theorem 1.37:

$$\frac{R}{\sqrt{1-R^2}} \sqrt{n-2-(p-m)} = \frac{0.3807}{\sqrt{1-0.3807^2}} \sqrt{198-2-(5-2)} = 5.7195$$

5.3 – 1

We use Theorem 1.37:

$$t\big(198-2-(5-2)\big) = t(193)$$

5.4 – 3

Reading definition 6.11: "*The maximum correlation ρ_1 is the **first canonical correlation**.*"

We simply find the first canonical correlation in the output.

5.5 – 1

Can be read directly in the SAS-output. Use the standardized as they are easier to interpret.

Standardized Canonical Coefficients for the VAR Variables

	V1	V2	V3
STRGTH3	0.9256	-2.0481	-0.4605
STRGTH7	0.0619	2.6915	-0.4684
STRGTH28	0.0295	-0.6434	1.3898

We see that the first strength variable **STRGTH3** dominates **V1.**