Solution to exercise 5.1.

We use theorem 4.9 and obtain

$$\boldsymbol{S} = \frac{1}{2}\left\{\begin{bmatrix} 6 & -2 \\ -2 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}\right\} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$\overline{\boldsymbol{X}} - \overline{\boldsymbol{Y}} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$$

$$(\overline{\boldsymbol{X}} - \overline{\boldsymbol{Y}})^T \boldsymbol{S}^{-1} (\overline{\boldsymbol{X}} - \overline{\boldsymbol{Y}}) = [-3 \ \ -2] \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{5} \end{bmatrix} \begin{bmatrix} -3 \\ -2 \end{bmatrix} = \frac{9}{5} + \frac{4}{5} = 2.6$$

$$T^2 = \frac{10 \times 10}{10 + 10} \times 2.6 = 13$$

$$\frac{n+m-p-1}{(n+m-2)p} t^2 = \frac{17}{18 \times 2} \times 13 = 6.1389$$

If we assume, that X and Y are distributed identically, Hotellings T^2 will follow an F-distribution. We thus simple compute the P-value, i.e. the probability of getting the same or more extreme result, given that the nul-hypothesis is true.

This can be done in SAS by evaluating an F(2,17) distribution:

```
data test;
	pval = 1 - cdf('F',6.1389,2,17);
run;
```

Since the P-value is 0.0098, we see that the difference between the two averages $\overline{\boldsymbol{X}}$ and $\overline{\boldsymbol{Y}}$ is just significant at the 1% level, i.e. we assume that the expectations $E(\boldsymbol{X})$ and $E(\boldsymbol{Y})$ are different.

■

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