DECOMPOSITION, ABSTRACTION, FUNCTIONS

HOW DO WE WRITE CODE?

- so far…
  - covered language mechanisms
  - know how to write different files for each computation
  - each file is some piece of code
  - each code is a sequence of instructions
- problems with this approach
  - easy for small-scale problems
  - messy for larger problems
  - hard to keep track of details
  - how do you know the right info is supplied to the right part of code

GOOD PROGRAMMING

- more code not necessarily a good thing
- measure good programmers by the amount of functionality
- introduce **functions**
- mechanism to achieve **decomposition** and **abstraction**

EXAMPLE -- PROJECTOR

http://www.myprojectorlamps.com/blog/wp-content/uploads/Dell-1610HD-Projector.jpg

- a projector is a black box
- don't know how it works
- know the interface: input/output
- connect any electronics to it that can communicate with that input
- black box somehow converts image from input source to a wall, magnifying it
- **ABSTRACTION IDEA**: do not need to know how projector works to use it

EXAMPLE -- PROJECTOR

- projecting large image for Olympics decomposed into separate tasks for separate projectors
- each projector takes input and produces separate output
- all projectors work together to produce larger image
- **DECOMPOSITION IDEA**: different devices work together to achieve an end goal

APPLY THESE IDEAS TO PROGRAMMING

- **DECOMPOSITION**
  - Break problem into different, self-contained, pieces
- **ABSTRACTION**
  - Suppress details of method to compute something from use of that computation

CREATE STRUCTURE with DECOMPOSITION

- in example, separate devices
- in programming, divide code into **modules**
  - are **self-contained**
  - used to **break up** code
  - intended to be **reusable**
  - keep code **organized**
  - **keep code coherent**
- this lecture, achieve decomposition with **functions**
- in a few weeks, achieve decomposition with **classes**

SUPPRESS DETAILS with ABSTRACTION

- in example, no need to know how to build a projector
- in programming, think of a piece of code as a **black box**
  - cannot see details
  - do not need to see details
  - do not want to see details
  - hide tedious coding details
- achieve abstraction with **function specifications** or **docstrings**

DECOMPOSITION & ABSTRACTION

- powerful together
- code can be used many times but only has to be debugged once!

FUNCTIONS

- write reusable piece/chunks of code, called **functions**
- functions are not run in a program until they are "**called**" or "**invoked**" in a program
- function characteristics:
  - has a **name**
  - has **parameters** (0 or more)
  - has a **docstring** (optional but recommended)
  - has a **body**

HOW TO WRITE and CALL/INVOKE A FUNCTION

keyword

name

parameters or arguments

```
def is_even( i ):
    """
    Input: i, a positive int
    Returns True if i is even, otherwise False
    """
    print("hi")
    return i%2 == 0

is_even(3)
```

Specification, docstring

body

later in the code, you call the function using its name and values for parameters

IN THE FUNCTION BODY

```
def is_even( i ):
    """
    Input: i, a positive int
    Returns True if i is even, otherwise False
    """
    print("hi”)
    return i%2 == 0
```

evaluate some expressions

keyword

expression to evaluate and return

VARIABLE SCOPE

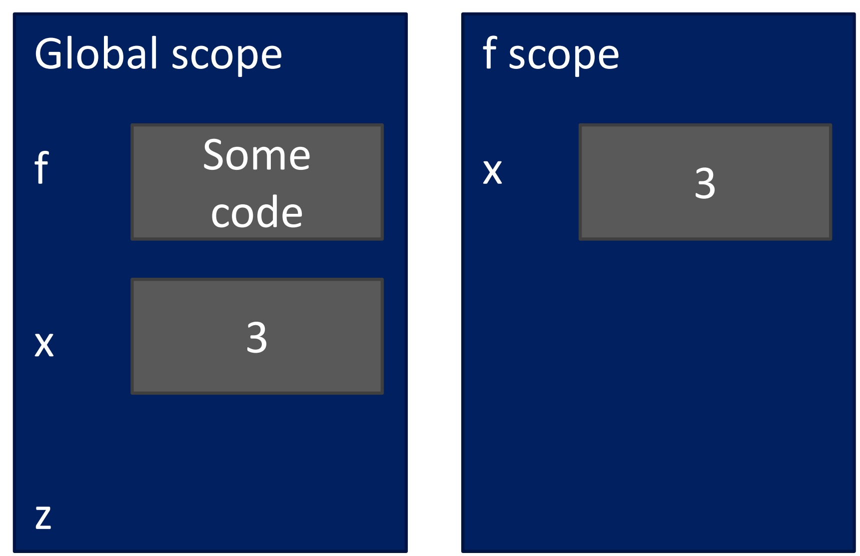
- **formal parameter** gets bound to the value of **actual parameter** when function is called
- new **scope/frame/environment** created when enter a function
- **scope** is mapping of names to objects

formal parameter

```
def f( x ):
    x = x + 1
    print('in f(x): x =', x)
    return x
x = 3
z = f( x )
```

actual parameter

call to f, before body evaluated



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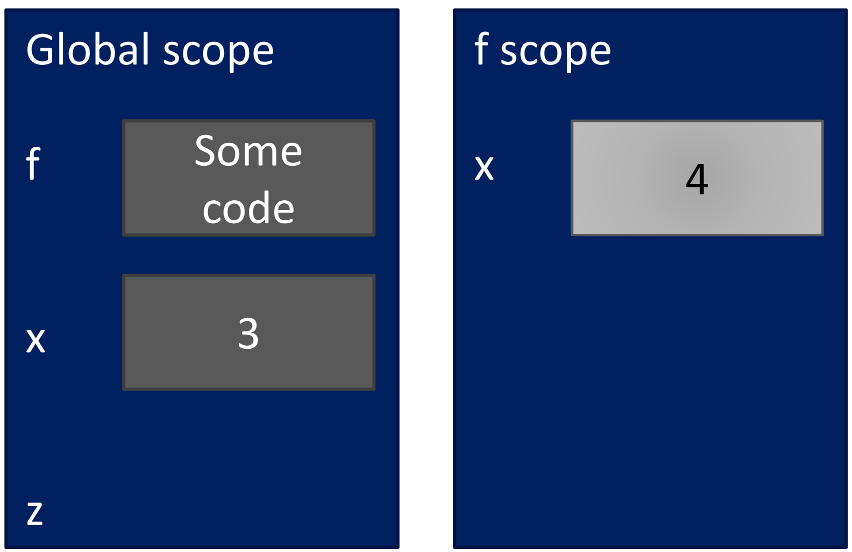

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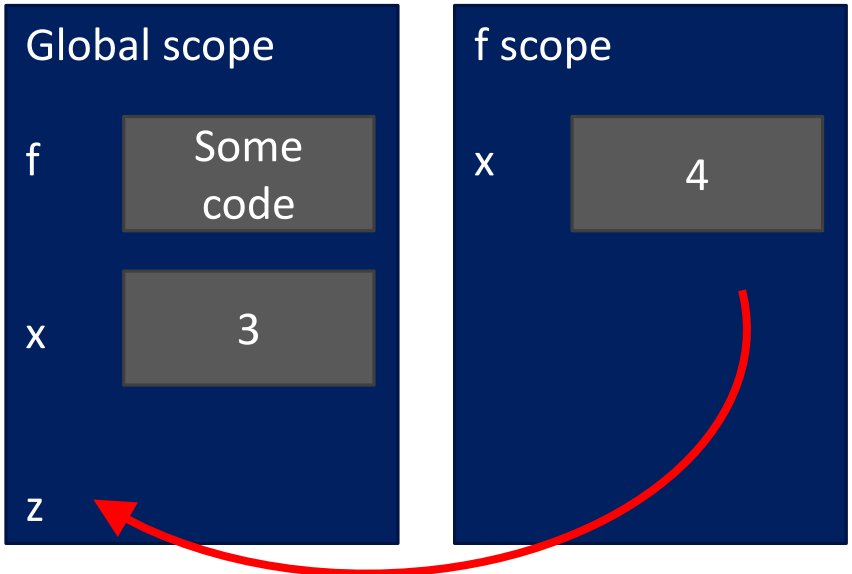

VARIABLE SCOPE

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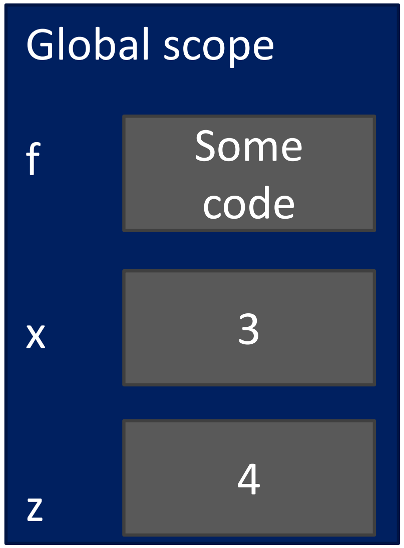
- new **scope/frame/environment** created when enter a function
- **scope** is mapping of names to objects

```
def f( x ):
    x = x + 1
    print('in f(x): x =', x)
    return x
x = 3
z = f( x )
```

binding of returned value to variable z



ONE WARNING IF NO `return` STATEMENT

```
def is_even( i ):
    """
    Input: i, a positive int
    Does not return anything
    """
    i%2 == 0
```

without a return statement

- Python returns the value **None, if no `return` given**
- represents the absence of a value

`return` vs. `print`

- return only has meaning **inside** a function
- only **one** return executed inside a function
- code inside function but after return statement not executed
- has a value associated with it, **given to function caller**

- print can be used **outside** functions
- can execute **many** print statements inside a function
- code inside function can be executed after a print statement
- has a value associated with it, **outputted** to the console

FUNCTIONS AS ARGUMENTS

- arguments can take on any type, even functions

```
def func_a():
    print('inside func_a')
def func_b(y):
    print('inside func_b')
    return y
def func_c(z):
    print('inside func_c')
    return z()
print(func_a())
print(5 + func_b(2))
print(func_c(func_a))
```

call func_a, takes no parameters

call func_b, takes one parameter

call func_c, takes one parameter, another function

SCOPE EXAMPLE

- inside a function, **can access** a variable defined outside
- inside a function, **cannot modify** a variable defined outside

```
def f(y):
    x = 1
    x += 1
    print(x)

x = 5
f(x)
print(x)
```

x is re-defined in scope of f

different x objects

```
def g(y):
    print(x)
    print(x + 1)

x = 5
g(x)
print(x)
```

x from outside g

x inside g is picked up from scope that called function g

```
def h(y):
    x = x + 1

x = 5
h(x)
print(x)
```

UnboundLocalError: local variable 'x' referenced before assignment

SCOPE EXAMPLE

- inside a function, **can access** a variable defined outside
- inside a function, **cannot modify** a variable defined outside

```
def f(y):
    x = 1
    x += 1
    print x

x = 5
f(2)
print x
```

```
def g(y):
    print x

x = 5
g(2)
print x
```

```
def h(y):
    x = x + 1

x = 5
h(2)
print x
```

x from global/main program scope

HARDER SCOPE EXAMPLE

IMPORTANT
and
TRICKY!

Python Tutor is your best friend to help sort this out!

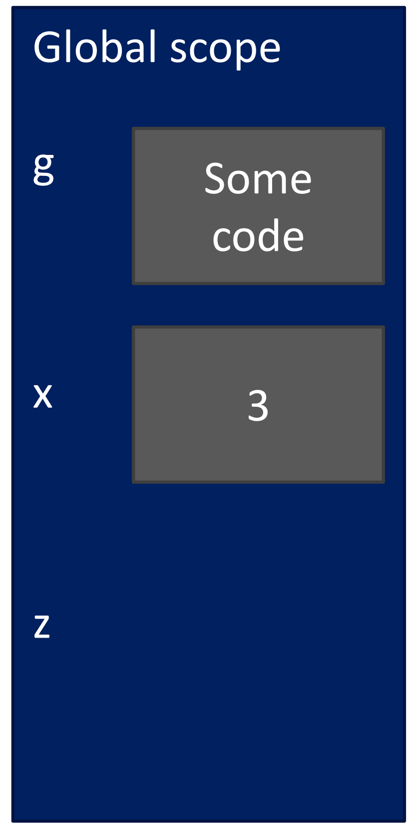
http://www.pythontutor.com/

SCOPE DETAILS

```
def g(x):
    def h():
        x = 'abc'
    x = x + 1
    print('in g(x): x =', x)
    h()
    return x

x = 3
z = g(x)
```

Some code



SCOPE DETAILS

```
def g(x):
    def h():
        x = 'abc'
    x = x + 1
    print('in g(x): x =', x)
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    return x

x = 3
z = g(x)
```
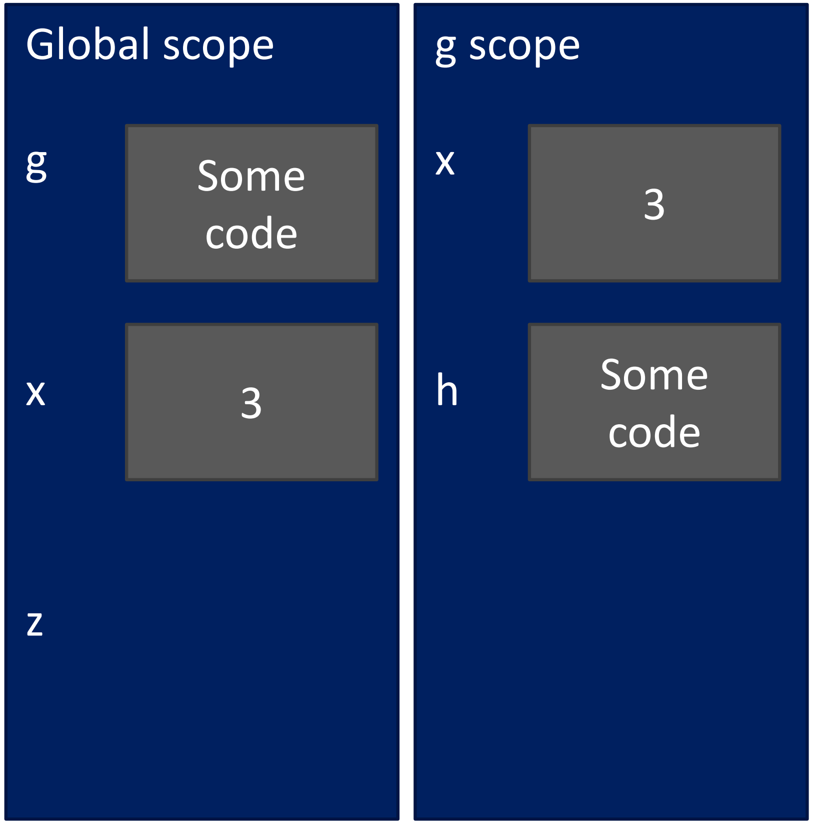
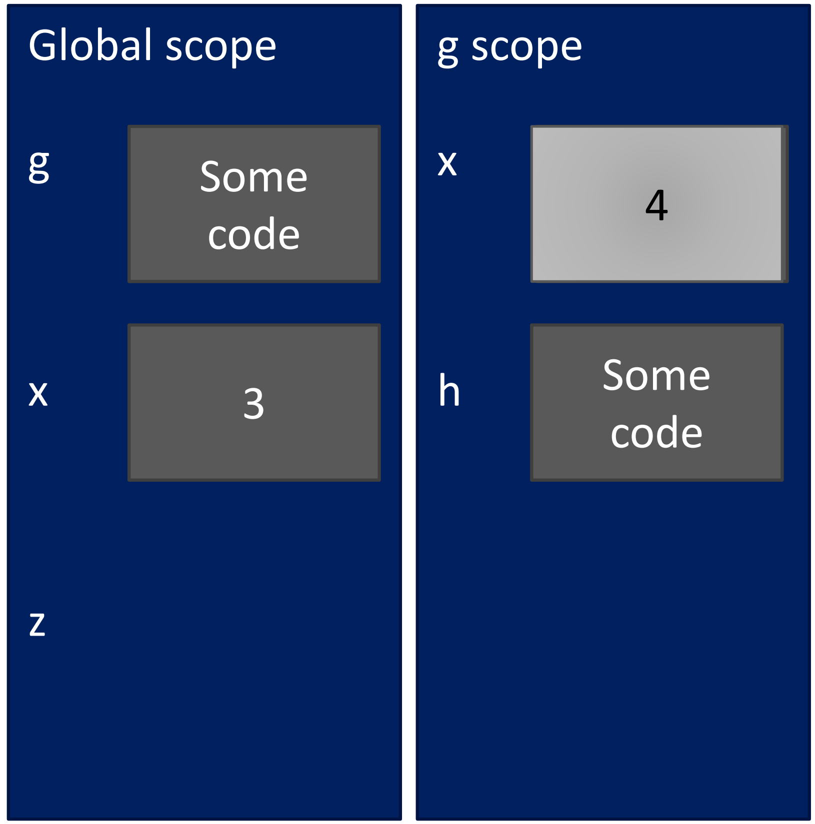



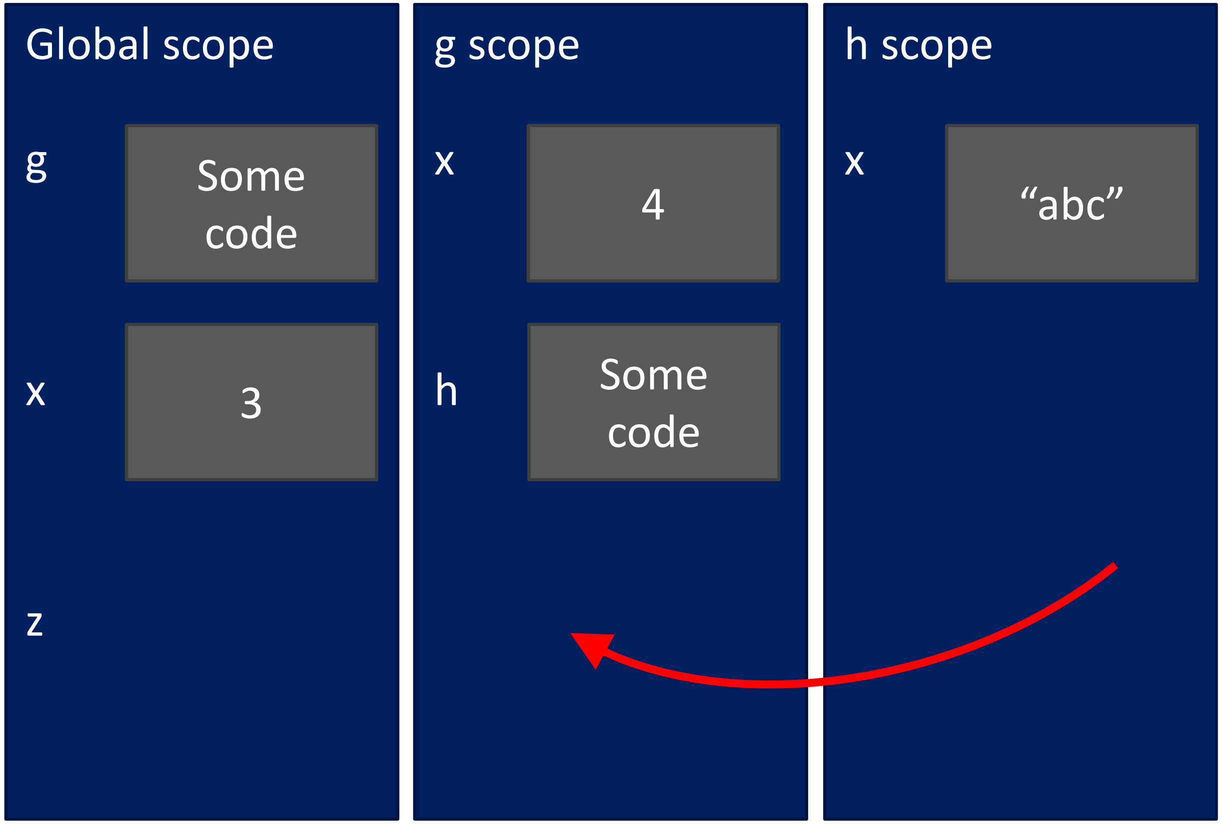
SCOPE DETAILS

```
def g(x):
    def h():
        x = 'abc'
    x = x + 1
    print('in g(x): x =', x)
    h()
    return x

x = 3
z = g(x)
```



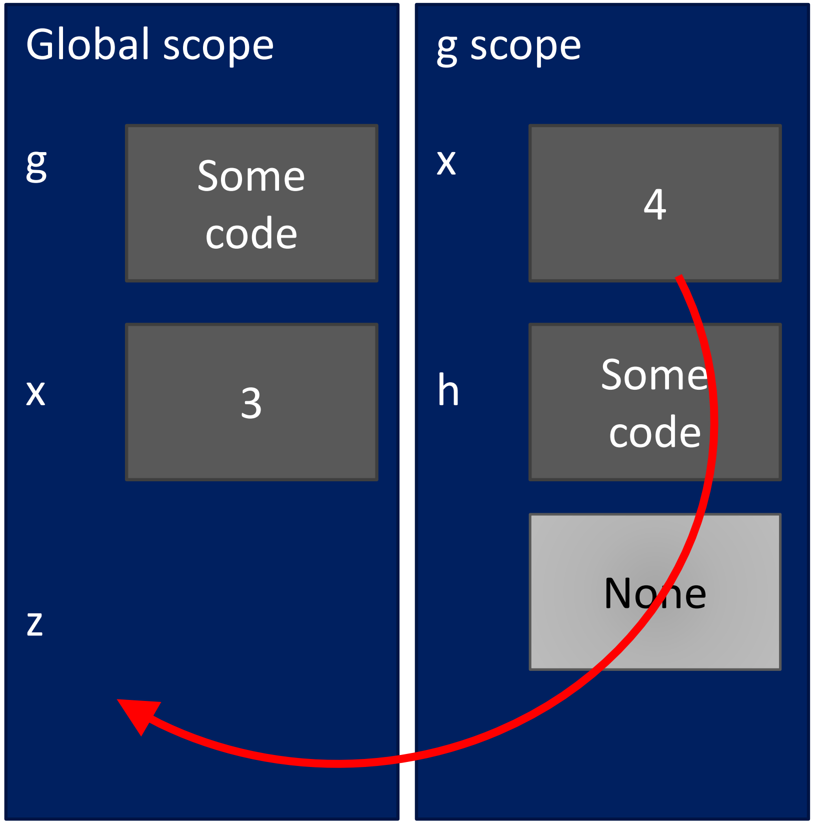
SCOPE DETAILS

```
def g(x):
    def h():
        x = 'abc'
    x = x + 1
    print('in g(x): x =', x)
    h()
    return x

x = 3
z = g(x)
```



SCOPE DETAILS

```
def g(x):
    def h():
        x = 'abc'
    x = x + 1
    print('in g(x): x =', x)
    h()
    return x

x = 3
z = g(x)
```



SCOPE DETAILS

```
def g(x):
    def h():
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    h()
    return x

x = 3
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```
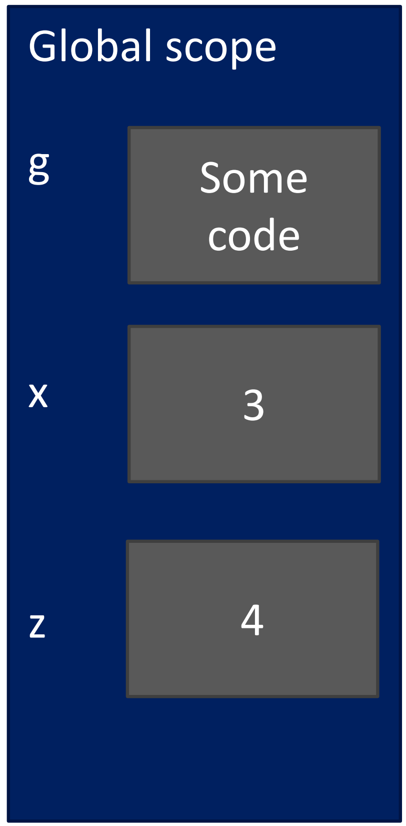



KEYWORD ARGUMENTS AND DEFAULT VALUES

- Simple function definition, if last argument is TRUE, then print lastName, firstName; else firstName, lastName

```
def printName(firstName, lastName, reverse):
    if reverse:
        print(lastName + ', ' + firstName)
    else:
        print(firstName, lastName)
```

KEYWORD ARGUMENTS AND DEFAULT VALUES

- Each of these invocations is equivalent

```
printName('Eric', 'Grimson', False)

printName('Eric', 'Grimson', reverse = False)

printName('Eric', lastName = 'Grimson', reverse = False)

printName(lastName = 'Grimson', firstName = 'Eric',
          reverse = False)
```

KEYWORD ARGUMENTS AND DEFAULT VALUES

- Can specify that some arguments have default values, so if no value supplied, just use that value

```
def printName(firstName, lastName, reverse = False):
    if reverse:
        print(lastName + ', ' + firstName)
    else:
        print(firstName, lastName)


printName('Eric', 'Grimson')
printName('Eric', 'Grimson', True)
```

SPECIFICATIONS

- a **contract** between the implementer of a function and the clients who will use it
  - **Assumptions:** conditions that must be met by clients of the function; typically constraints on values of parameters
  - **Guarantees:** conditions that must be met by function, providing it has been called in manner consistent with assumptions

```
def is_even( i ):
    """
    Input: i, a positive int
    Returns True if i is even, otherwise False
    """
    print "hi"
    return i%2 == 0

is_even(3)
```

WHAT IS RECURSION

- a way to design solutions to problems by **divide-and-conquer or decrease-and-conquer**
- a programming technique where a **function calls itself**
- in programming, goal is to NOT have infinite recursion
  - must have **1 or more base cases** that are easy to solve
  - must solve the same problem on **some other input** with the goal of simplifying the larger problem input

ITERATIVE ALGORITHMS SO FAR

- looping constructs (while and for loops) lead to **iterative** algorithms
- can capture computation in a set of **state variables** that update on each iteration through loop

MULTIPLICATION – ITERATIVE SOLUTION

- "multiply `a * b`" is equivalent to "add `a` to itself `b` times"
- capture **state** by
  - an **iteration** number (`i`) starts at b
    `i` ← `i-1` and stop when 0
  - a current **value of computation** (`result`)
    `result` ← `result + a`

```
def mult_iter(a, b):
    result = 0
    while b > 0:
        result += a
        b -= 1
    return result
```

iteration

current value of computation,
a running sum

current value of iteration variable

MULTIPLICATION – RECURSIVE SOLUTION

- **recursive step**
  - think how to reduce problem to a **simpler/smaller version** of same problem
- **base case**
  - keep reducing problem until reach a simple case that can be **solved directly**
  - when b = 1, a*b = a

$$a*b = \underbrace{a + a + a + a + \ldots + a}_{\text{b times}}$$

$$= a + \underbrace{a + a + a + \ldots + a}_{\text{b-1 times}}$$

$$= a + a * (b-1)$$

```
def mult(a, b):
    if b == 1:
        return a
    else:
        return a + mult(a, b-1)
```

base case

recursive step

FACTORIAL

`n! = n*(n-1)*(n-2)*(n-3)* … * 1`

- what `n` do we know the factorial of?

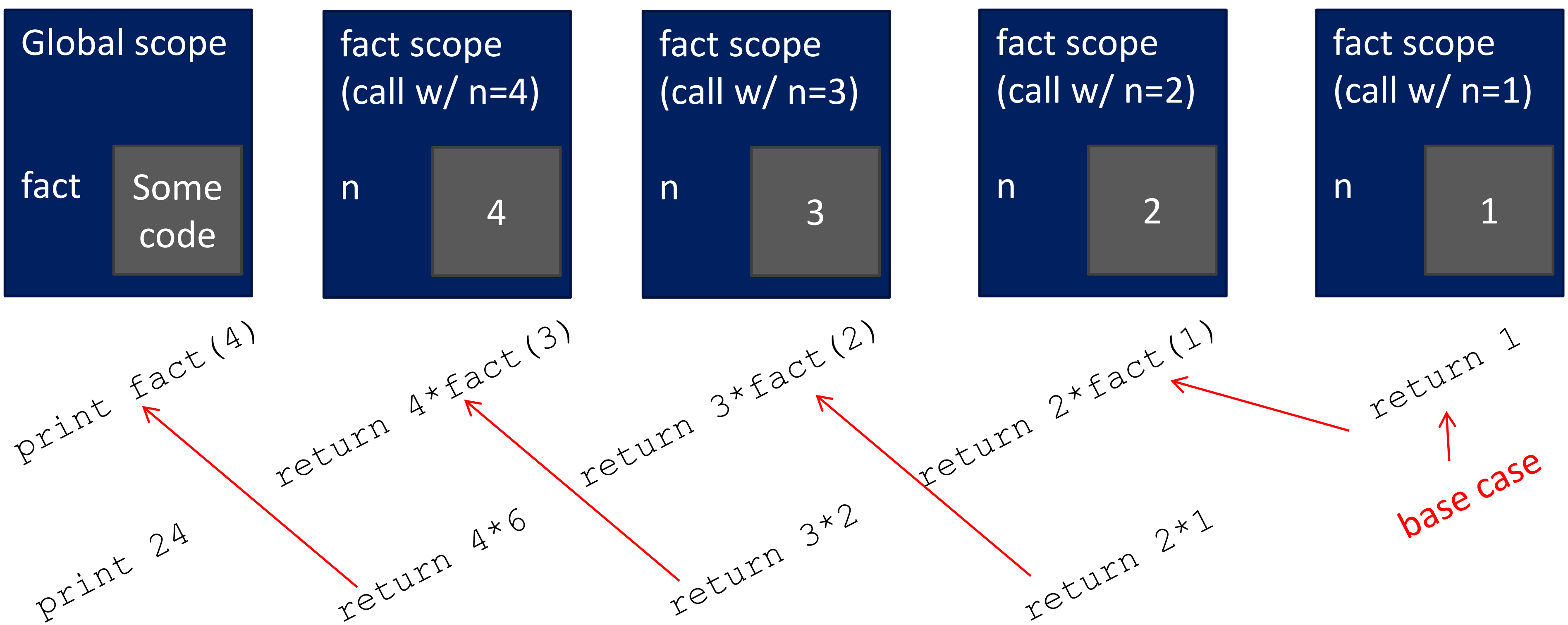
  n = 1 →
  ```
  if n == 1:
      return 1
  ```
  base case

- how to reduce problem? Rewrite in terms of something simpler to reach base case

  n*(n-1)! →
  ```
  else:
      return n*factorial(n-1)
  ```
  recursive step

RECURSIVE FUNCTION SCOPE EXAMPLE

```
def fact(n):
    if n == 1:
        return 1
    else:
        return n*fact(n-1)

print(fact(4))
```



SOME OBSERVATIONS

using the same variable names but they are different objects in separate scopes

- each recursive call to a function creates its **own scope/environment**
- **bindings of variables** in a scope is not changed by recursive call
- flow of control passes back to **previous scope** once function call returns value

ITERATION vs. RECURSION

```
def factorial_iter(n):
    prod = 1
    for i in range(1,n+1):
        prod *= i
    return prod
```

```
def factorial(n):
    if n == 1:
        return 1
    else:
        return n*factorial(n-1)
```

- recursion may be simpler, more intuitive
- recursion may be efficient from programmer POV
- recursion may not be efficient from computer POV

INDUCTIVE REASONING

- How do we know that our recursive code will work?
- `mult_iter` terminates because b is initially positive, and decreases by 1 each time around loop; thus must eventually become less than 1
- `mult` called with b = 1 has no recursive call and stops
- `mult` called with b > 1 makes a recursive call with a smaller version of b; must eventually reach call with b = 1

```
def mult_iter(a, b):
    result = 0
    while b > 0:
        result += a
        b -= 1
    return result


def mult(a, b):
    if b == 1:
        return a
    else:
        return a + mult(a, b-1)
```

MATHEMATICAL INDUCTION

- To prove a statement indexed on integers is true for all values of n:
  - Prove it is true when n is smallest value (e.g. n = 0 or n = 1)
  - Then prove that if it is true for an arbitrary value of n, one can show that it must be true for n+1

LHS - Left Hand Side

RHS - Right Hand Side

Primeiro provamos que é válido para n = 0.
Se n = 0, substintuinddo em (n(n+1))/ 2 => (0(0+1))/2 = 0 portanto válido para n = 0

Agora testando para um caso geral k+1 onde tenho a sequencia 0 + 1 + 2 + 3... k + (k+1) = ((k+1) + (k+2))/2

Substituindo o somatório de 0 até k atribuindo verdadeiro o k(k+1)/2 tem-se k(k+1)/2 + k+1. Através de manipulações algébricas chego ao resultado ((k+1)(k+2))/2

EXAMPLE OF INDUCTION

- 0 + 1 + 2 + 3 + … + n = (n(n+1))/2
- Proof
  - If n = 0, then LHS is 0 and RHS is 0*1/2 = 0, so true
  - Assume true for some k, then need to show that
    - 0 + 1 + 2 + … + k + (k+1) = ((k+1)(k+2))/2
    - LHS is k(k+1)/2 + (k+1) by assumption that property holds for problem of size k
    - This becomes, by algebra, ((k+1)(k+2))/2
  - Hence expression holds for all n >= 0

RELEVANCE TO CODE?

- Same logic applies

```
def mult(a, b):
    if b == 1:
        return a
    else:
        return a + mult(a, b-1)
```

Base code retorna o valor o correto quando b= 1, ou seja para o menor valor ele vai retornar o valor correto. Assumo então que o mult(a,b-1) retorna o valor correto. Essa é a indução.

- Base case, we can show that `mult` must return correct answer
- For recursive case, we can assume that `mult` correctly returns an answer for problems of size smaller than b, then by the addition step, it must also return a correct answer for problem of size b
- Thus by induction, code correctly returns answer

TOWERS OF HANOI

- The story:
  - 3 tall spikes
  - Stack of 64 different sized discs – start on one spike
  - Need to move stack to second spike (at which point universe ends)
  - Can only move one disc at a time, and a larger disc can never cover up a small disc

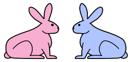
TOWERS OF HANOI

- Having seen a set of examples of different sized stacks, how would you write a program to print out the right set of moves?
- **Think recursively!**
  - Solve a smaller problem
  - Solve a basic problem
  - Solve a smaller problem

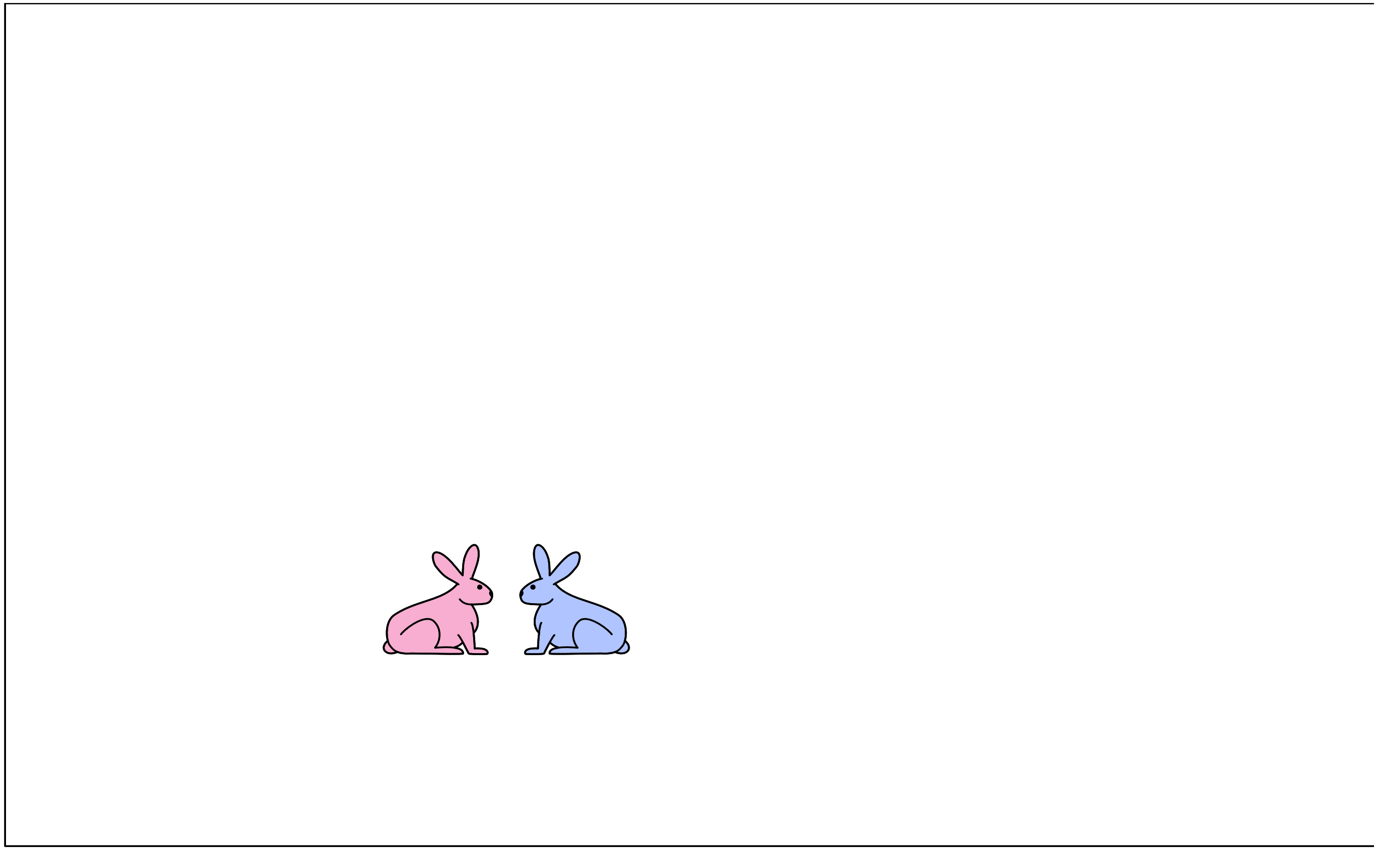
```python
def printMove(fr, to):
    print('move from ' + str(fr) + ' to ' + str(to))

def Towers(n, fr, to, spare):
    if n == 1:
        printMove(fr, to)
    else:
        Towers(n-1, fr, spare, to)
        Towers(1, fr, to, spare)
        Towers(n-1, spare, to, fr)
```

RECURSION WITH MULTIPLE BASE CASES

- Fibonacci numbers
  - Leonardo of Pisa (aka Fibonacci) modeled the following challenge
    - Newborn pair of rabbits (one female, one male) are put in a pen
    - Rabbits mate at age of one month
    - Rabbits have a one month gestation period
    - Assume rabbits never die, that female always produces one new pair (one male, one female) every month from its second month on.
    - How many female rabbits are there at the end of one year?



Demo courtesy of Prof. Denny Freeman and Adam Hartz



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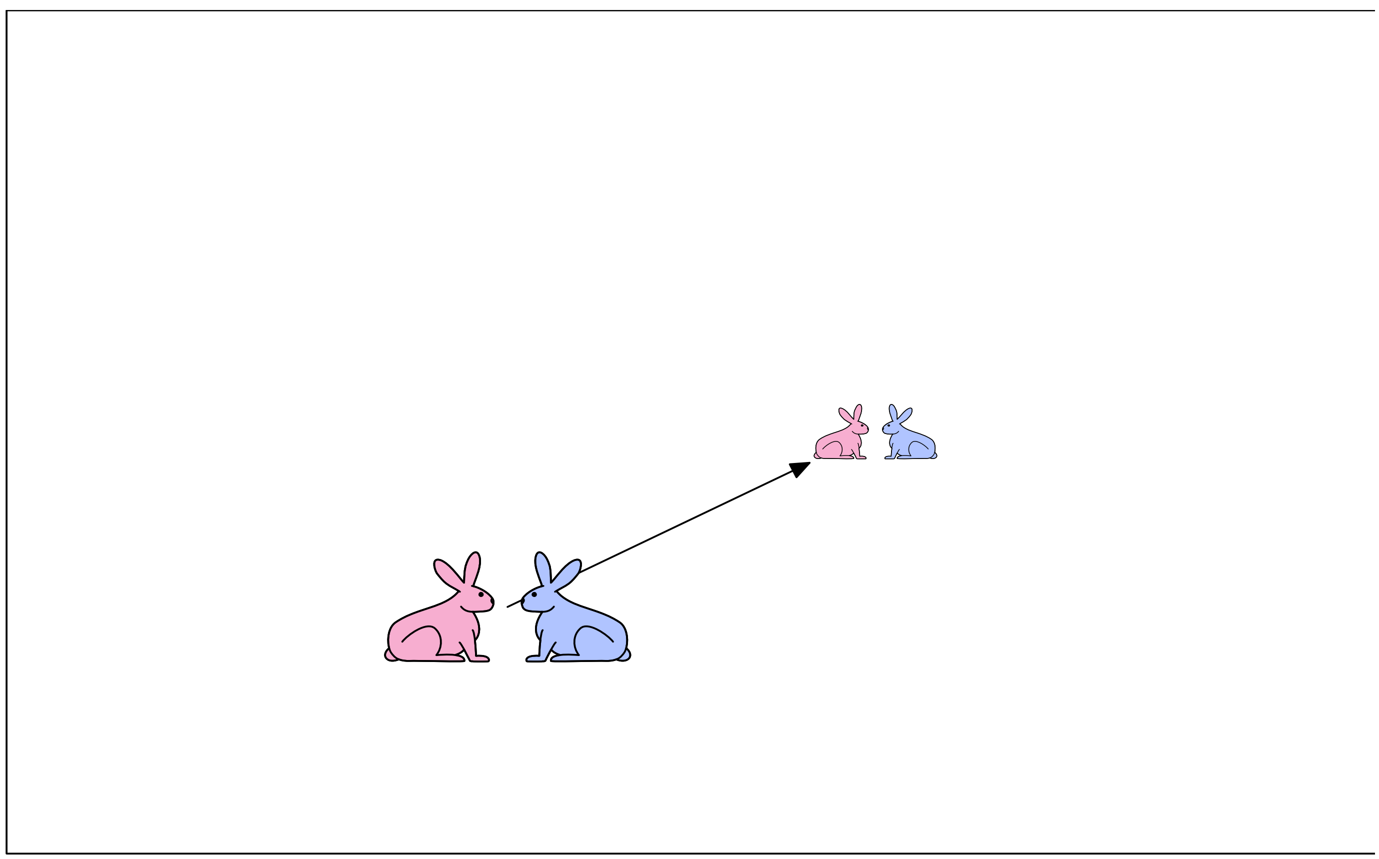

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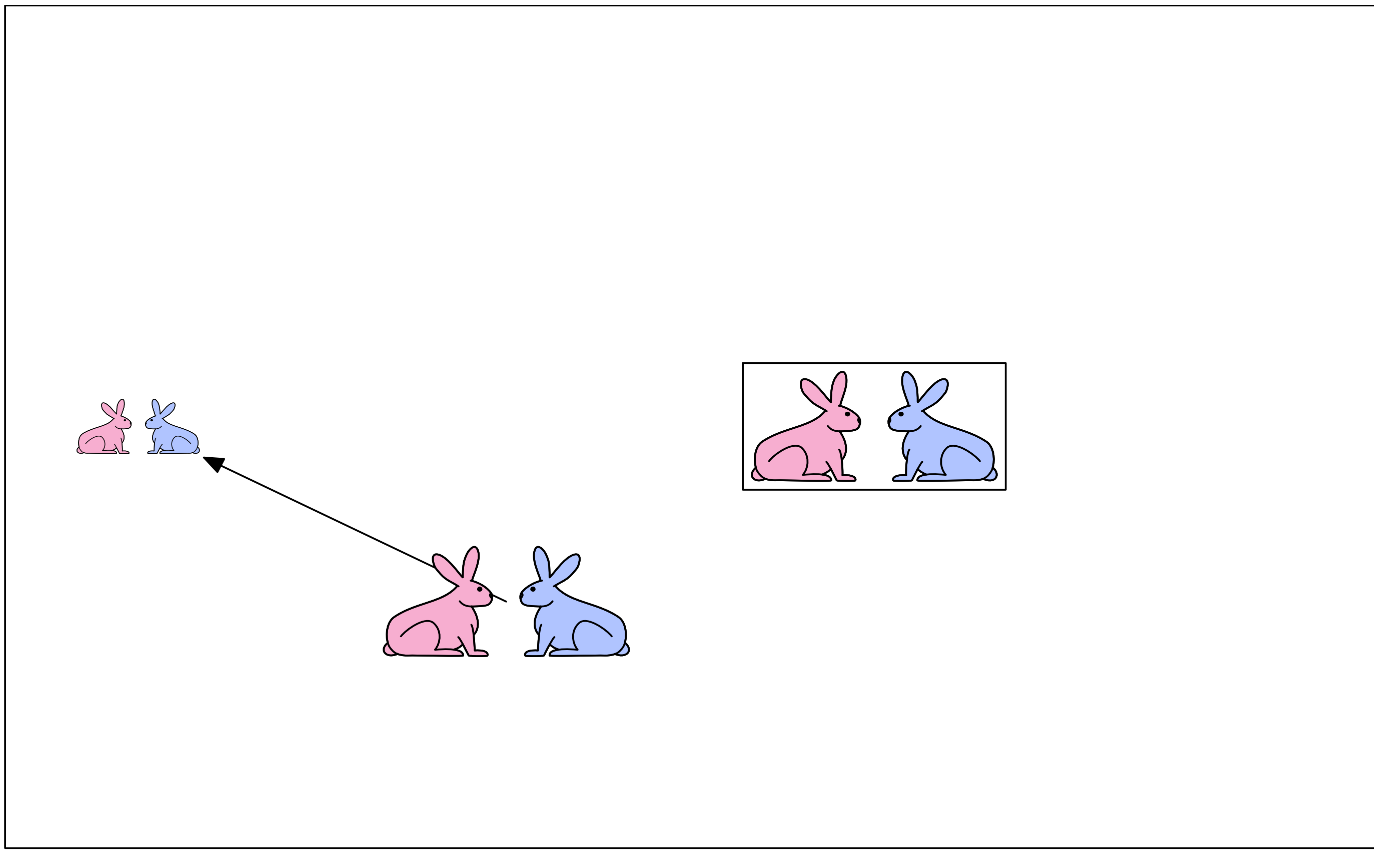

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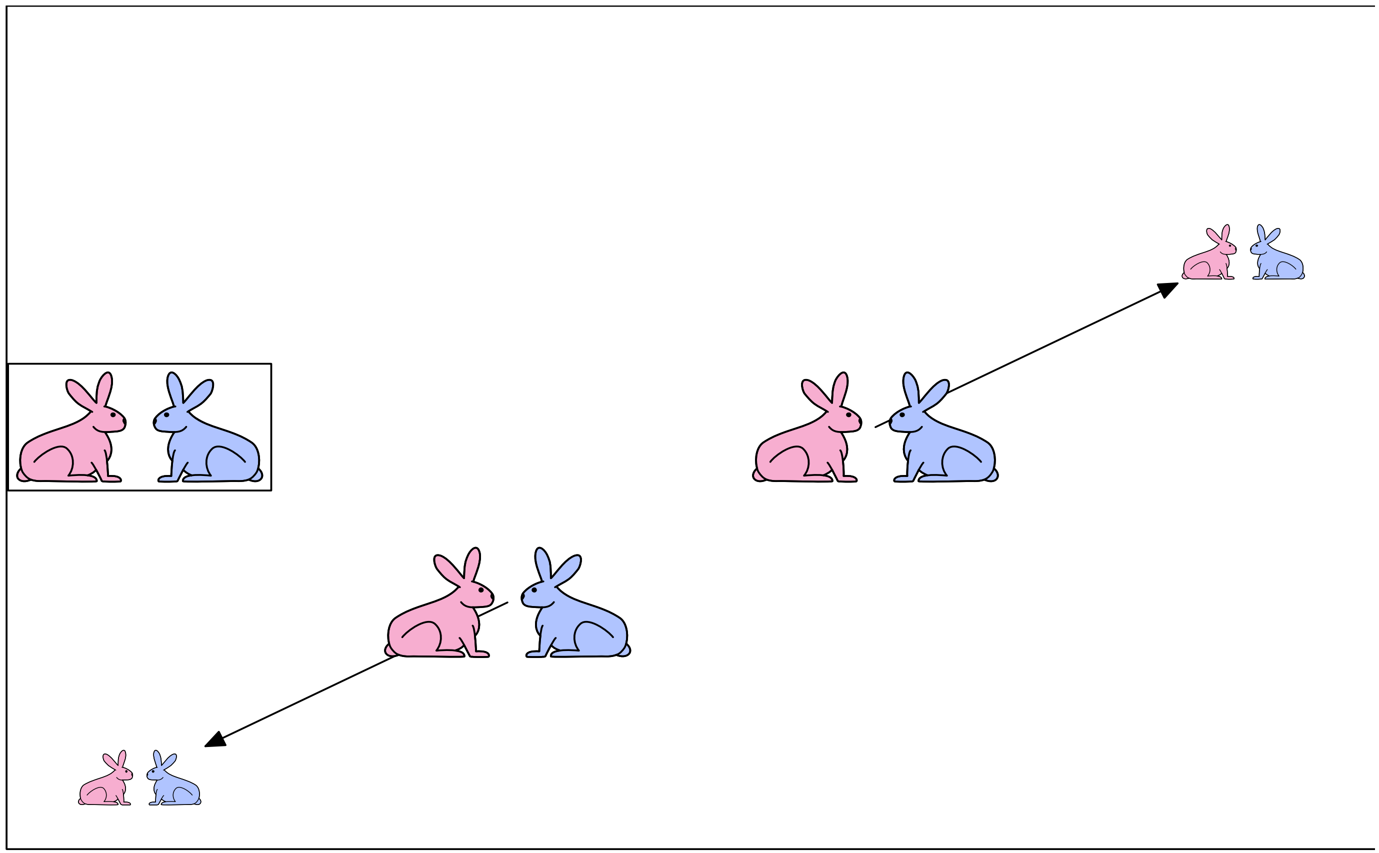

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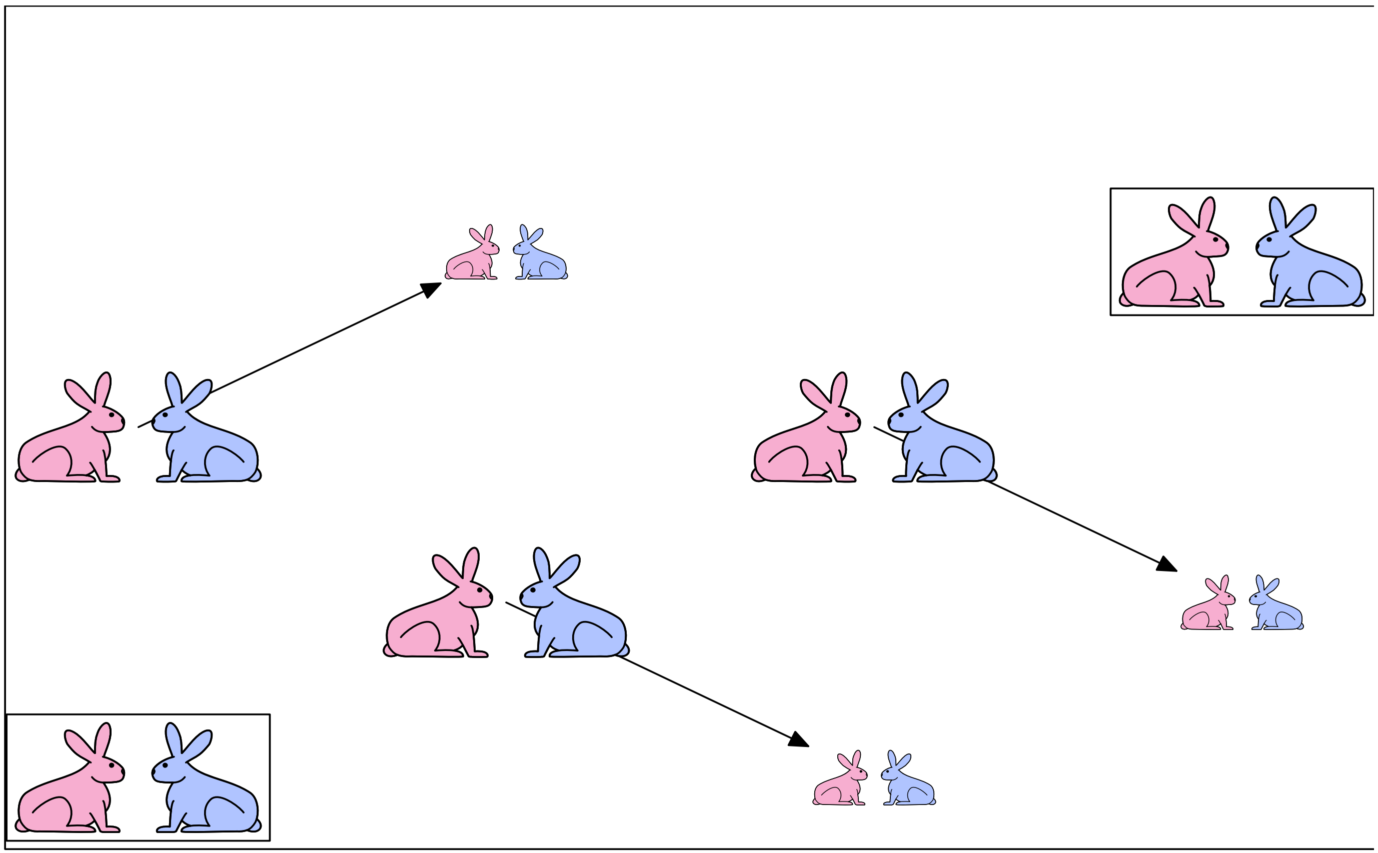

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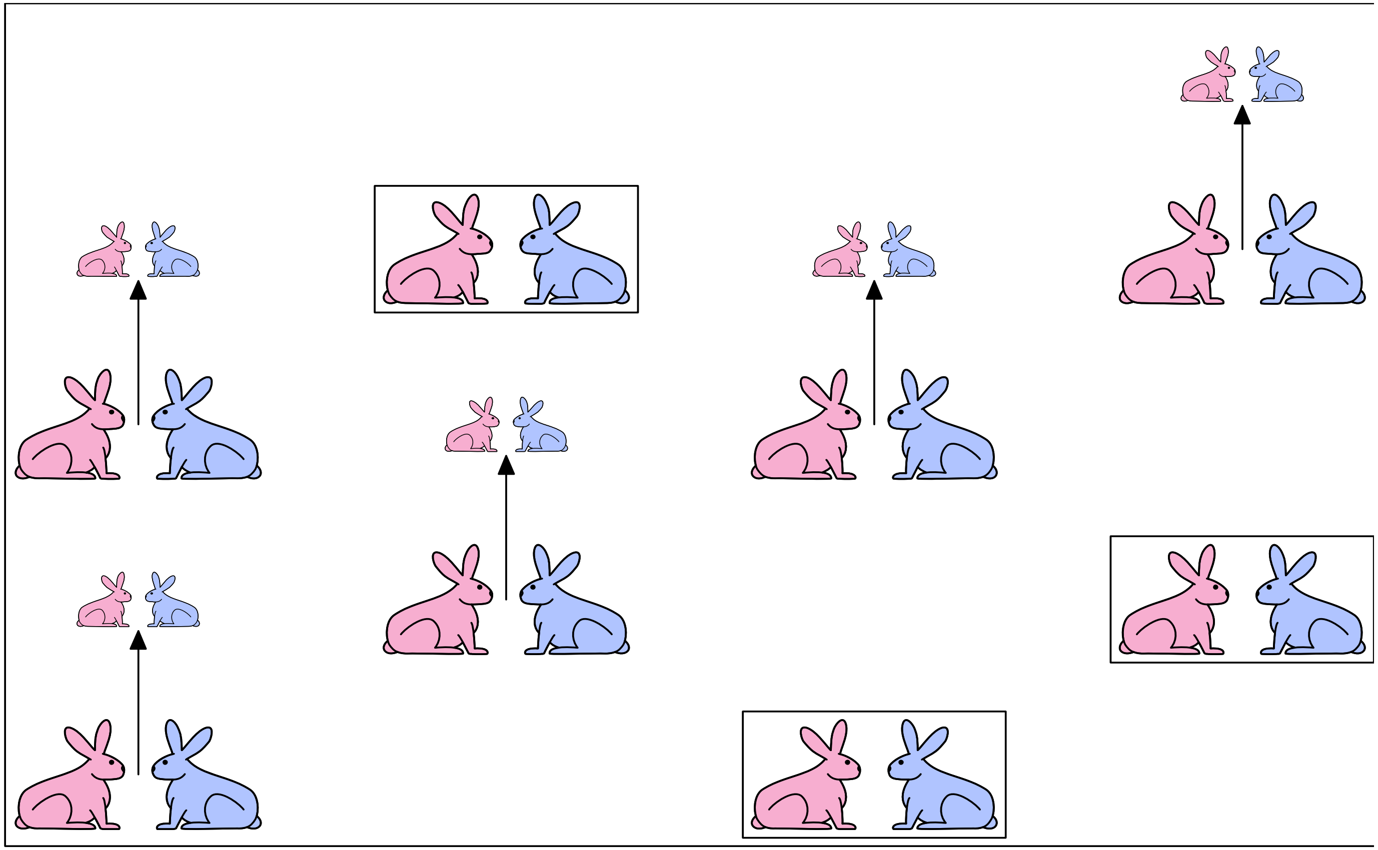

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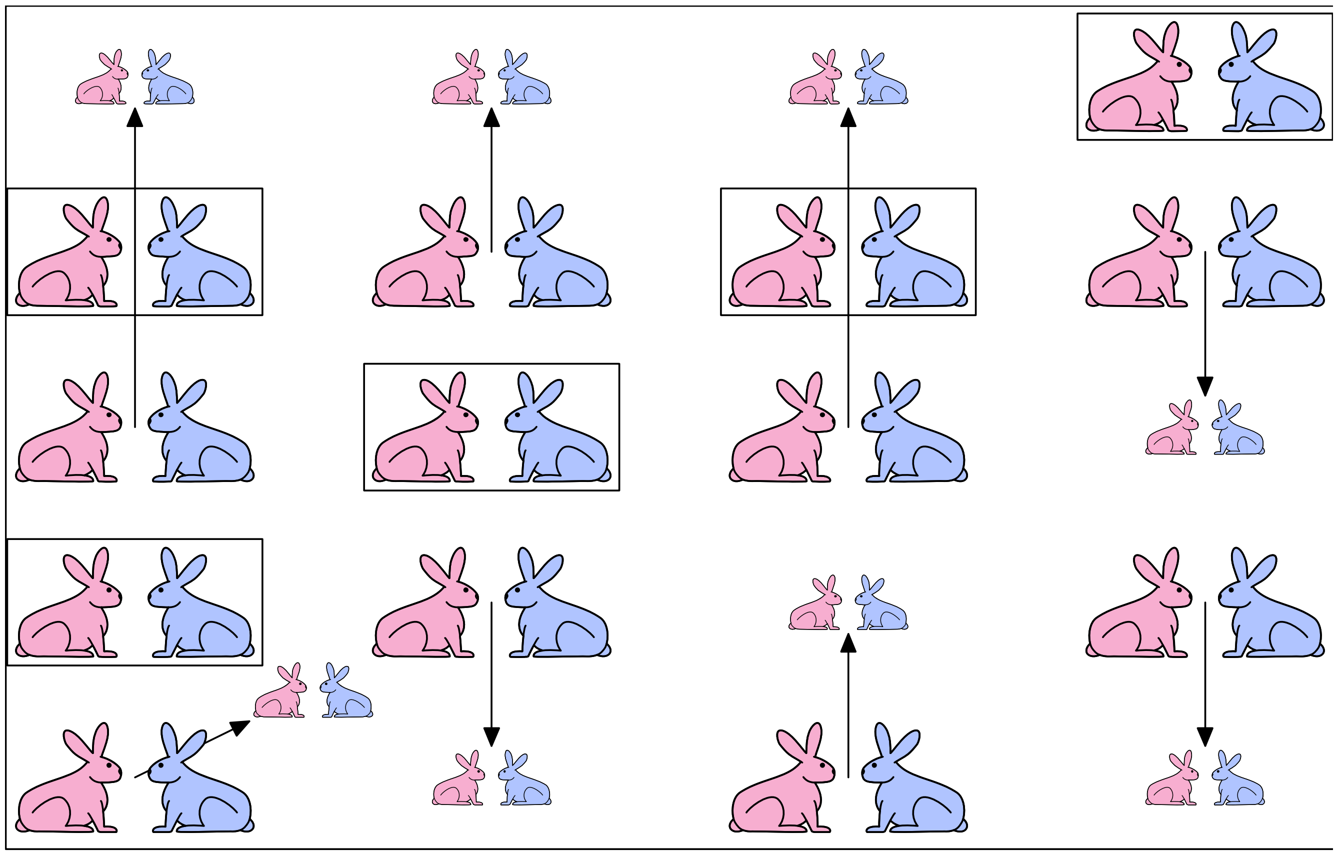

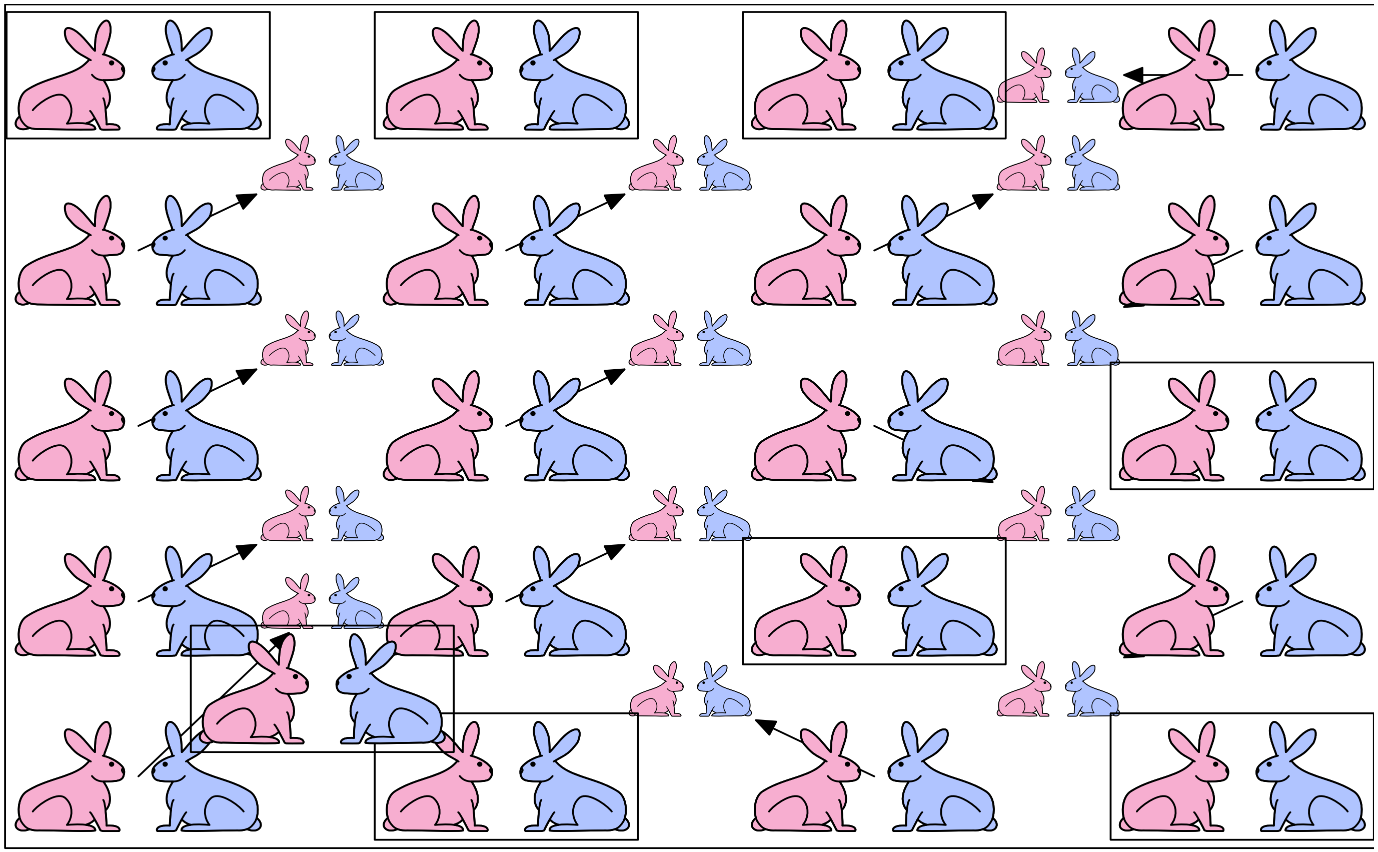

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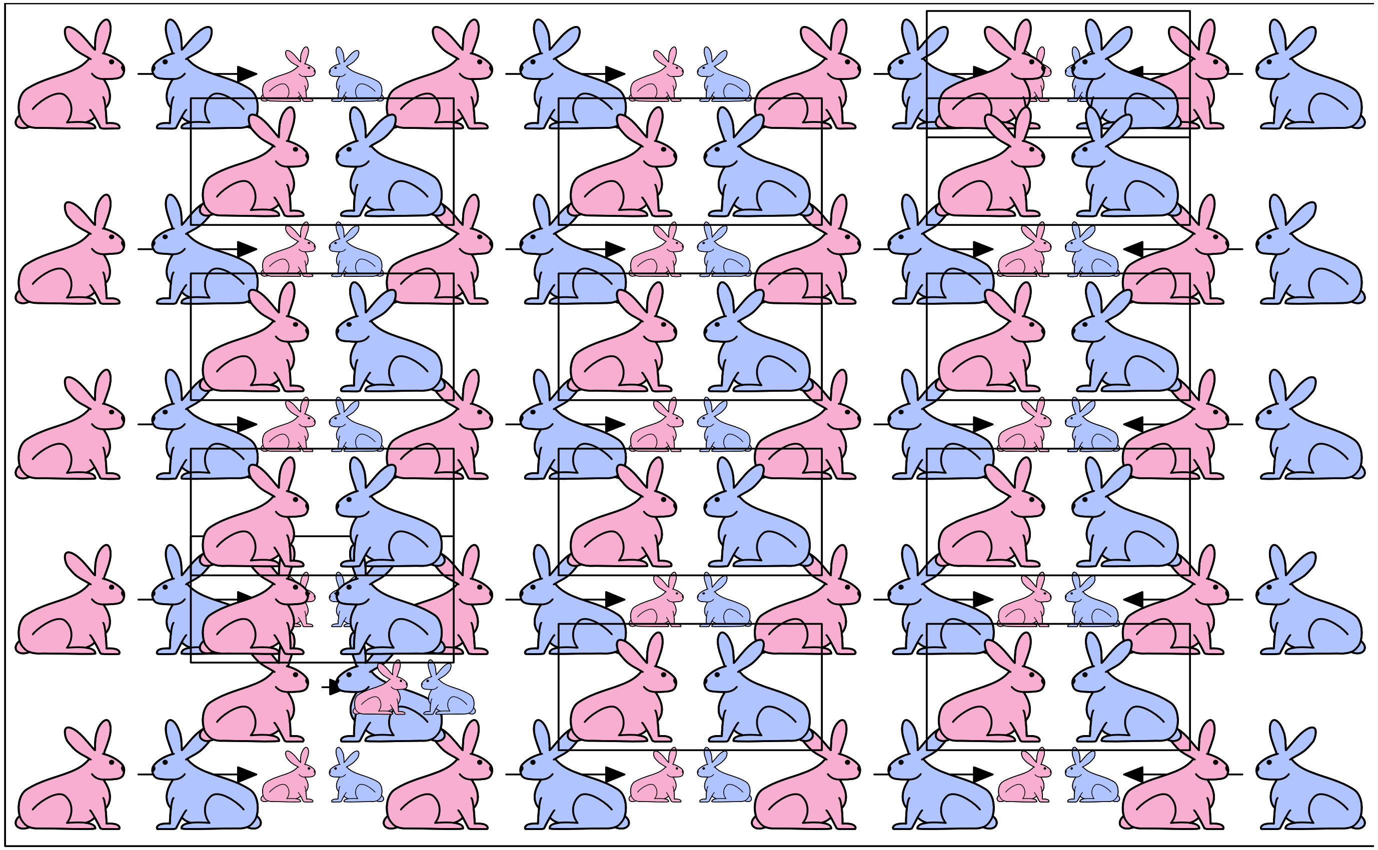

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FIBONACCI

After one month (call it 0) – 1 female

After second month – still 1 female (now pregnant)

After third month – two females, one pregnant, one not

In general, females(n) = females(n-1) + females(n-2)

- Every female alive at month n-2 will produce one female in month n;
- These can be added those alive in month n-1 to get total alive in month n

Month	Females
0	1
1	1
2	2
3	3
4	5
5	8
6	13

FIBONACCI

- Base cases:
  - Females(0) = 1
  - Females(1) = 1
- Recursive case
  - Females(n) = Females(n-1) + Females(n-2)

```
def fib(x):
    """assumes x an int >= 0
       returns Fibonacci of x"""
    if x == 0 or x == 1:
        return 1
    else:
        return fib(x-1) + fib(x-2)
```

RECURSION ON NON-NUMERICS

- how to check if a string of characters is a palindrome, i.e., reads the same forwards and backwards
  - "Able was I, ere I saw Elba" – attributed to Napoleon
  - "Are we not drawn onward, we few, drawn onward to new era?" – attributed to Anne Michaels

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SOLVING RECURSIVELY?

- First, convert the string to just characters, by stripping out punctuation, and converting upper case to lower case
- Then
  - Base case: a string of length 0 or 1 is a palindrome
  - Recursive case:
    - If first character matches last character, then is a palindrome if middle section is a palindrome

EXAMPLE

- ‘Able was I, ere I saw Elba’ → ‘ablewasiereisawleba’
- `isPalindrome(‘ablewasiereisawleba’)` is same as
  - `‘a’ == ‘a’` and `isPalindrome(‘blewasiereisawleb’)`

Transofrma em letras minusculas
Para cada letra na frase ele verifica se a letra esta no abc, caso esteja e adiciona a letra a resposta para formar a palavra sem espaçi ou sinais.

```
def isPalindrome(s):

    def toChars(s):
        s = s.lower()
        ans = ''
        for c in s:
            if c in 'abcdefghijklmnopqrstuvwxyz':
                ans = ans + c
        return ans

    def isPal(s):
        if len(s) <= 1:
            return True
        else:
            return s[0] == s[-1] and isPal(s[1:-1])

    return isPal(toChars(s))
```

DIVIDE AND CONQUER

- an example of a “divide and conquer” algorithm
- solve a hard problem by breaking it into a set of sub-problems such that:
  - sub-problems are easier to solve than the original
  - solutions of the sub-problems can be combined to solve the original

MODULES AND FILES

- have assumed that all our code is stored in one file
- cumbersome for large collections of code, or for code that should be used by many different other pieces of programming
- a **module** is a `.py` file containing a collection Python definitions and statements

EXAMPLE MODULE

- the file `circle.py` contains

```
pi = 3.14159
def area(radius):
    return pi*(radius**2)
def circumference(radius):
    return 2*pi*radius
```

EXAMPLE MODULE

- then we can import and use this module:

```
import circle
pi = 3
print(pi)
print(circle.pi)
print(circle.area(3))
print(circle.circumference(3))
```

- results in the following being printed:

```
3
3.14159
28.27431
18.849539999999998
```

OTHER IMPORTING

- if we don't want to refer to functions and variables by their module, and the names don't collide with other bindings, then we can use:

```
from circle import *
print(pi)
print(area(3))
```

- this has the effect of creating bindings within the current scope for all objects defined within `circle`
- statements within a module are executed only the first time a module is imported

FILES

- need a way to save our work for later use
- every operating system has its own way of handling files; Python provides an operating-system independent means to access files, using a **file handle**

```
nameHandle = open('kids', 'w')
```

- creates a file named `kids` and returns file handle which we can name and thus reference. The `w` indicates that the file is to opened for writing into.

FILES: example

```
nameHandle = open('kids', 'w')
for i in range(2):
    name = input('Enter name: ')
    nameHandle.write(name + '\')
nameHandle.close()
```

FILES: example

```
nameHandle = open('kids', 'r')
for line in nameHandle:
    print(line)
nameHandle.close()
```