Positioning and Location Based Services

Laboratory 01 – *Reference Systems*

Angelly Pugliese & Juan Pablo Duque

Objetive

Compute geodetic coordinates (latitude, longitude and geodetic height) of BRUN, 0001, 0002, 0003 in ETRF at February 2nd, 2019. Compute also the standard deviations of the geodetic coordinates.

Working Flow

1. Compute Geocentric Cartesian (GC) coordinates of COMO at 2019/02/02.

They are computed according to the initial position and the velocity given in the EUREF Permanent Network webpage and calculated using distributed estimates for permanent stations:

$$x(t) = x(t_0) + x(t - t_0)$$

So, replacing in the formula the values for the x, y and z coordinates:

```
COMOx = 4398306.209 + (-0.0145)*(2019.09041096 - 2010.0);
COMOy = 704149.948 + 0.0181*(2019.09041096 - 2010.0);
COMOz = 4550154.733 + 0.0113*(2019.09041096 - 2010.0);
```

The results are the following:

$$COMO_{GC} = \begin{bmatrix} 4398306.07718904130160808563232 42 \\ 704150.11253643839154392480850 22 \\ 4550154.83572164364159107208251 95 \end{bmatrix}$$

2. Compute BRUN coordinates in ITRF.

Having the Como coordinates in ITRF from (1) and knowing the difference between Como and Brunate in ITRF:

$$\Delta X_{COMO-BRUN(ITRF)} = \begin{bmatrix} -1040.168 \\ -72.970 \\ 1631.398 \end{bmatrix}$$

$$X_{BRUN} = \Delta X_{COMO-BRUN(ITRF)} + X_{COMO}$$

$$BRUN_{GC} = \begin{bmatrix} 4397265.909189041703939437866 2109 \\ 704077.142536438419483602046966 55 \\ 4551786.233721643686294555664 0625 \end{bmatrix}$$

3. Compute Local Cartesian coordinates of 0001 with respect to BRUN.

Given a Global Cartesian (GC) known point (BRUN), the rotation matrix can be computed using φ and λ coordinates, and with the difference between the known and unknown points in GC, the unknown Local Cartesian (LC) coordinates can be computed.

$$R_0 = \begin{bmatrix} -sin\lambda_0 & cos\lambda_0 & 0 \\ -sin\varphi_0 cos\lambda_0 & -sin\varphi_0 sin\lambda_0 & cos\varphi_0 \\ cos\varphi_0 cos\lambda_0 & cos\varphi_0 sin\lambda_0 & sin\varphi_0 \end{bmatrix}$$

$$\begin{bmatrix} E \\ N \\ U \end{bmatrix}_{0001} = R_0 * \Delta X_{\substack{BRUN \\ 0001}}$$

Where,

$$\Delta X_{\substack{BRUN \\ 0001}} = [-51.130 \;\; 76.749 \;\; 38.681]^T$$

```
P0001_LC = [ 83.8675      54.4621       1.0108]
```

4. Convert LC of 0001 to pseudo Local Level with respect to BRUN.

Pseudo Local Level means to compute a LL coordinate tuple only with respect to the X and Y components using the already given ξ and η angles (north and east components of vertical deflection), in order to compute the Z rotation matrix.

$$\xi \; = \; 10.23'' \qquad \eta \; = \; 9.5''$$

So, the pseudo or temporary LL coordinates of point 0001 would be:

$$X_{temp} = R_y(\eta) * R_x(-\xi) * X_{p1,LC}$$

Where,

$$R_x(-\xi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\xi & -sin\xi \\ 0 & sin\xi & cos\xi \end{bmatrix} \cong \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\xi \\ 0 & \xi & 1 \end{bmatrix}$$

$$R_y(\eta) = \begin{bmatrix} cos\eta & 0 & -sin\eta \\ 0 & 1 & 0 \\ sin\eta & 0 & cos\eta \end{bmatrix} \cong \begin{bmatrix} 1 & 0 & -\eta \\ 0 & 1 & 0 \\ \eta & 0 & 1 \end{bmatrix}$$

```
xtemp = [83.8675      54.4621       1.0174]
```

5. Compute alpha rotation between LC and LL in BRUN.

The angle alpha can be computed using the X and Y components of the pseudo LL point 0001:

$$\alpha = tan^{-1}\left(\frac{Y_{pseudoLL0001}}{X_{pseudoLL0001}}\right)$$

```
alpha =      0.5759
```

Then, the alpha rotation (Z axis rotation) can be computed:

$$R_z(\alpha) = \begin{bmatrix} cos\alpha & sin\alpha & 0 \\ -sin\alpha & cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

```
Rot =
    0.8387    0.5446   -0.0001
   -0.5446    0.8387   -0.0000
    0.0000    0.0000    1.0000
```

6. Compute LC of 0002 and 0003.

Local Level coordinates can be computed using the inverse of the already computed rotation matrix.

$$LC = R^T * LL$$

$0001LL \ = \ [99.9994, 0.0, 1.0174]m$

$0002LL \ = \ [80.0, 85.0, 2.0] \ m$

$0003LL \ = \ [-25.0; \ 10.0; \ -3.0] \ m$

```
P0002_LC = [   20.8014    114.8579      1.9933]
P0003_LC = [  -26.4134     -5.2290     -2.9985]
```

7. Compute GC of 0002 and 0003.

Having the GC coordinates of BRUN and using the rotation matrix computed in (3), the LC coordinates of 0002 and 0003 can be transformed to GC:

$$R^T * X_{LC} + X_{BRUN} = X_{GC}$$

```
P0001_GC = [ 4397214.7791890418156981468200684
             704153.89153643837198615074157715
            4551824.9147216435521841049194336]

P0002_GC = [ 4397182.6586554059758782386779785
            704084.87909476342611014842987061
             4551867.7106775762513279914855957]
```

```
P0003_GC = [ 4397271.7245083237066864967346191
            704051.32384068646933883428573608
             4551780.4391541089862585067749023]
```

8. Convert through EPN website ITRF GC coordinates of all the stations to ETRF GC coordinates at the same epoch.

The results were the following:

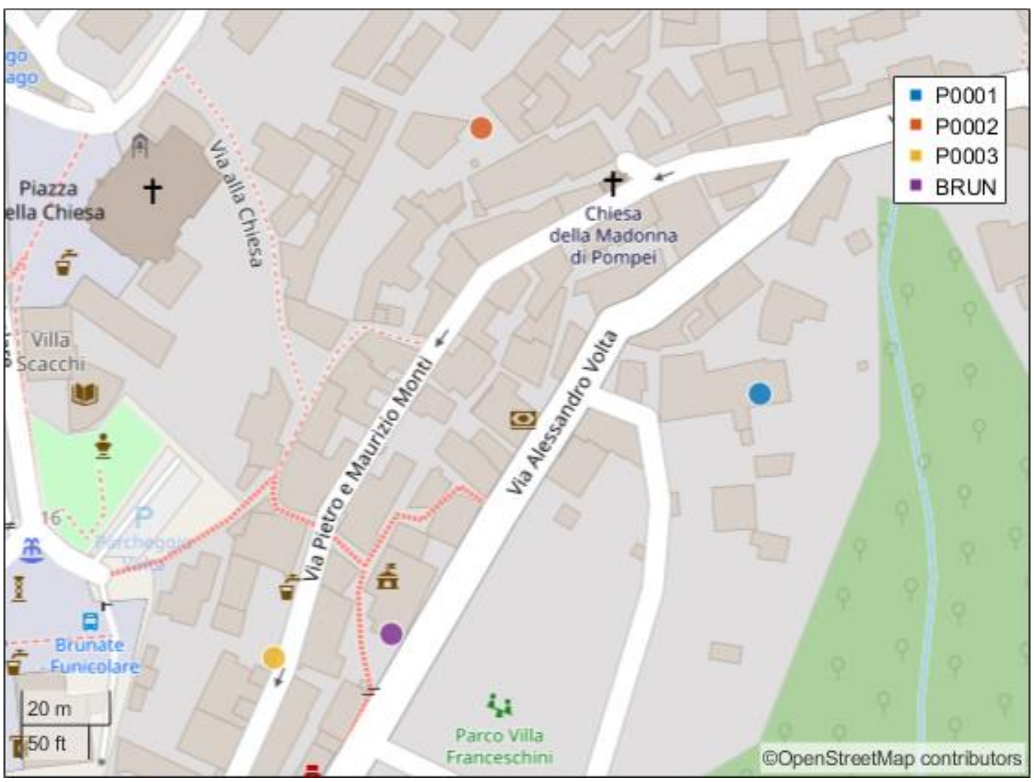
```
% From ITRF2014 2019.33 to ETRF2014 2019.33
```

```
P0001_GC_ETRF = [4397215.21430  704153.33680 4551824.58020];
P0002_GC_ETRF = [4397183.09380  704084.32430 4551867.37610];
P0003_GC_ETRF = [4397272.15960  704050.76910 4551780.10460];
BRUN_GC_ETRF = [4397266.34430  704076.58780 4551785.89920];
```

9. Compute ETRF geodetic coordinates of all the stations.

The obtained results were:

```
P0001_GEO = [ 45.8196  9.0979  739.1276]
P0002_GEO = [45.8201  9.0971  740.1104]
P0003_GEO = [45.8190  9.0965  735.1175]
BRUN_GEO = [45.8191  9.0968  738.1160]
```



10. Propagate accuracies from LL in BRUN to LC in BRUN

As it is possible to compute a permanent station position according to initial position, epoch and velocity in ITRF, it is possible to compute its covariance in the same way, for Como:

$$C_c(t) = C_0 + (t - t_0)^2 C_x$$

```
covComoInd =    1.8264e-06
covComo =    1.0e-05 *
    0.1826         0         0
         0    0.1826         0
         0         0    0.1826
```

Given the covariance matrix of Como, it can be propagated to Brunate:

$$C_B(t) = C_c + C_{\Delta XC \to B}$$

$$C_{\Delta XC \to B} = \begin{bmatrix} 2.0e^{-6} & 0.5e^{-6} & 0.5e^{-6} \\ 0.5e^{-6} & 1.0e^{-6} & 0.5e^{-6} \\ 0.5e^{-6} & 0.5e^{-6} & 2.0e^{-6} \end{bmatrix}$$

```
covBrun =    1.0e-05 *
    0.3826    0.0500    0.0500
    0.0500    0.2826    0.0500
    0.0500    0.0500    0.3826
```

From As the difference of covariances between Brunate and point 0001, the covariance of the last one can be propagated the same way as before.

```
cov_11_ITRF =    1.0e-05 *
    0.5326    0.0800    0.0800
    0.0800    0.3826    0.0700
    0.0800    0.0700    0.5826
```

For the two remaining points 0002 and 0003, a different path must be followed in order to compute their covariances in ITRF. The covariances of the points X, Y and Z components are given for Local Level:

$$\sigma_{XLL} \;=\; 10\ cm\ \sigma_{YLL} \;=\; 10\ cm\ \sigma_{ZLL} \;=\; 15\ cm$$

Then, the two covariance matrices will be:

$$C_{22LL} = C_{33LL} = \begin{bmatrix} 0.1^2 & 0 & 0 \\ 0 & 0.1^2 & 0 \\ 0 & 0 & 0.15^2 \end{bmatrix} m^2$$

Using the rotation matrix computed in point (6), as for the coordinates, it is possible to transform covariances from Local Level to Local Cartesian:

$$C_{LC} = R * C_{LL} * R^T$$

```
cov_22_LC =
    0.0100    0.0000   -0.0000
    0.0000    0.0100   -0.0000
   -0.0000   -0.0000    0.0225
cov_33_LC =
    0.0100    0.0000   -0.0000
    0.0000    0.0100   -0.0000
   -0.0000   -0.0000    0.0225
```

11. Propagate accuracies from LC in BRUN to ETRF geodetic coordinates

    As the Local Cartesian coordinates and covariances are with respect to the Brunate point, it is possible to compute the rotation matrix to transform Local Cartesian to Geocentric Cartesian coordinates as it was done in point (7). But, in this case, we want to obtain the difference of covariances between Brunate and the two points so their covariances can be propagated as before.

$$\Delta X_{BRUN-000X} = R^T * C_{LC} * R$$

```
RotLC2GC =
   -0.1581    0.9874         0
   -0.7081   -0.1134    0.6969
    0.6882    0.1102    0.7171
```

```
deltaBrun_22 =
    0.0159    0.0009    0.0062
    0.0009    0.0102    0.0010
    0.0062    0.0010    0.0164
cov_22_ITRF =
    0.0159    0.0009    0.0062
    0.0009    0.0102    0.0010
    0.0062    0.0010    0.0164
```

```
deltaBrun_33 =
    0.0159    0.0009    0.0062
    0.0009    0.0102    0.0010
    0.0062    0.0010    0.0164

cov_33_ITRF =
    0.0159    0.0009    0.0062
    0.0009    0.0102    0.0010
    0.0062    0.0010    0.0164
```

Having all the covariances in ITRF (geocentric), they can be propagated to local baselines:

$$C = R * C_{ITRF} * R^T$$

```
cov_Brun_ENU =

   1.0e-05 *

    0.2695    0.0060    0.0520
    0.0060    0.3321   -0.0082
    0.0520   -0.0082    0.4463
```

And so, the standard deviations ENU for points 0001, 0002 and 0003 can be computed:

```
SIGMA_E_11 =

    0.0019

SIGMA_N_11 =

    0.0022

SIGMA_U_11 =

    0.0026

 SIGMA_E_22 =

    0.1000

 SIGMA_N_22 =

    0.1000

 SIGMA_U_22 =

    0.1500

 SIGMA_E_33 =

    0.1000

SIGMA_N_33 =

    0.1000

 SIGMA_U_33 =

    0.1500
```