

Q1) What do you mean by minimum spanning tree? What are the application of MST?

A minimum spanning tree (MST) is a subset of edges of a connected undirected graph that connect all the vertices together, without any cycle and without any cycles and with the minimum possible total edge weight. It is a way to connect all the vertices in a graph in way that minimize the total weight of the edge in a tree.

Application of MST.

- 1) Network design.
- 2) Cluster analysis.
- 3) Approximation Algorithm.
- 4) Routing algorithm.
- 5) Image Segmentation.

Q2) Analyse the time and space complexity of Prim, Kruskal, Dijkstra and Bellman Ford Algorithm.

Time complexity

Space Complexity

Prim

$O(V^2)$

$O(E \log V)$

Kruskal

$O(E \log E)$

$O(E+V)$

Dijkstra

$O(V^2)$

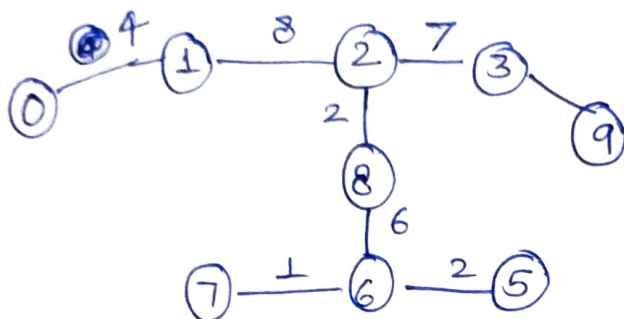
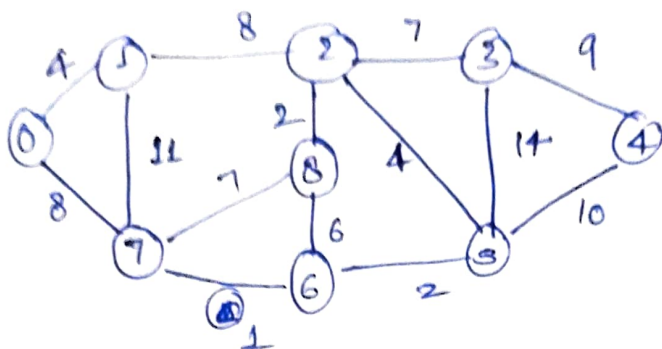
$O(V+E)$

Bellman ford

$O(VE)$

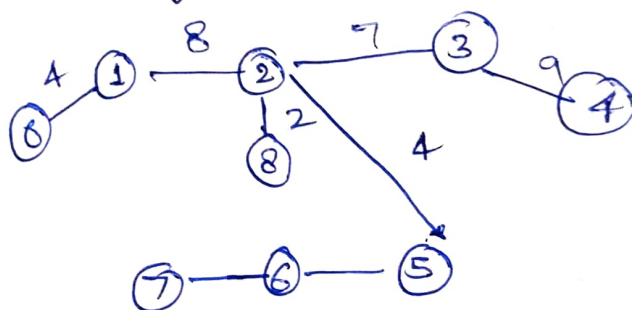
$O(V+E)$

Q. Apply Kruskal and Prim's Algorithm on given graph to compute MST and its weight.



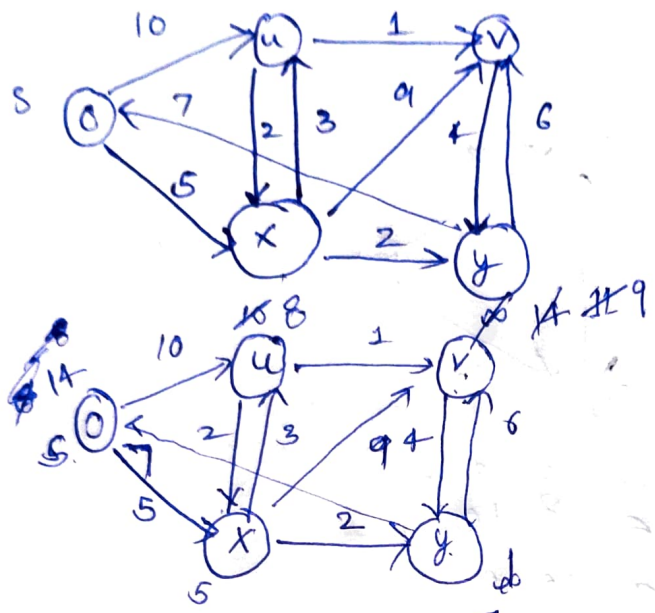
MST.

Prim Algorithm.



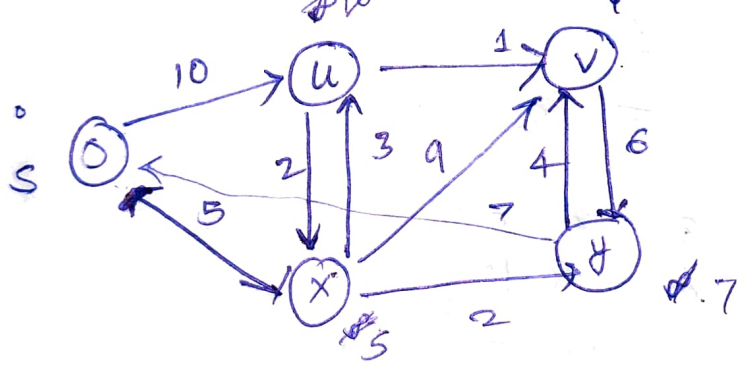
wt	u	v
1	6	7
2	6	5
2	8	2
2	2	8
4	0	1
4	5	2
6	8	6
7	7	8
7	2	3
8	0	7
8	1	2
9	3	4
10	2	5
11	1	7

⑤ Apply Dijkstra and Bellman algorithm on given graph to compute the shortest path to all nodes from node S.



u	d(u)
x	5
u	8
v	9
y	7

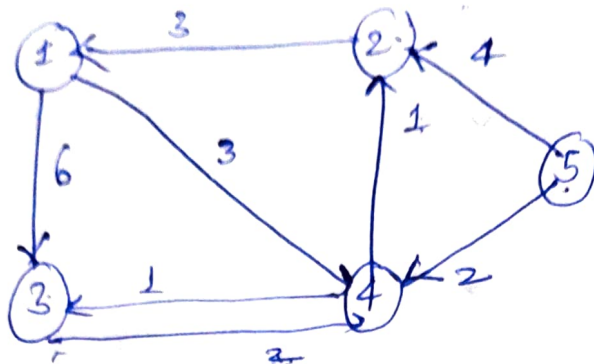
Dijkstra Algorithm



$(0, u)$ $(0, x)$ (u, x) (u, v) (x, y) (y, v) (x, v) (x, u) (x, y) $(0, y)$

a	d(a)
u	8
x	5
v	9
y	7

⑥ Apply all pair shortest path algorithm - Floyd Warshall on below mentioned graph and also analyse time and complexity of an algorithm



$A^0 =$

	1	2	3	4	5
1	0	∞	6	8	10
2	3	0	9	10	∞
3	∞	3	0	2	7
4	4	1	1	0	∞
5	7	4	3	2	0

$A^1 =$

	1	2	3	4	5
1	0	∞	6	8	10
2	3	0	9	10	13
3	∞	3	0	2	7
4	4	1	1	0	4
5	7	4	3	2	0

$A^2 =$

	1	2	3	4	5
1	0	∞	6	8	10
2	3	0	9	10	13
3	6	3	0	2	7
4	4	1	1	0	4
5	7	4	3	2	0

$$A^B = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 9 & 6 & 8 & 10 \\ 3 & 0 & 9 & 10 & 13 \\ 6 & 3 & 0 & 2 & 7 \\ 4 & 1 & 1 & 0 & 8 \\ 7 & 4 & 3 & 2 & 0 \end{bmatrix} \end{matrix}$$

$$A^4 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 9 & 6 & 8 & 10 \\ 3 & 0 & 9 & 10 & 13 \\ 6 & 3 & 0 & 2 & 7 \\ 4 & 1 & 1 & 0 & 8 \\ 6 & 3 & 3 & 2 & 0 \end{bmatrix} \end{matrix}$$

Time complexity = $O(V^3)$
 Space complexity = $O(V^2)$

$$A^5 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 9 & 6 & 8 & 10 \\ 3 & 0 & 9 & 10 & 13 \\ 6 & 3 & 0 & 2 & 7 \\ 4 & 1 & 1 & 0 & 8 \\ 6 & 3 & 3 & 2 & 0 \end{bmatrix} \end{matrix}$$