Industrial Symbiosis: Operational Impact and Firms' Willingness to Implement

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Abstract

Industrial Symbiosis or *By-Product Synergy* is defined as a resource-sharing strategy that engages traditionally separate industries in a collective approach that involves a physical exchange of materials, water, energy, and by-products. Inspired by a real-world example of a paper-sugar symbiotic complex, we study the impact of competition on a firm's willingness to implement an industrial symbiotic system. Sugar and paper firms are symbiotically connected, in the sense that the biomass from the manufacture of one product is used as a raw material for the second product, and vice-versa. We characterize the firm's operational optimal/equilibrium decisions for its two products – both in the presence and absence of a symbiotic system – under monopoly as well as under competition. Our models capture the supply-side (e.g., a fixed production cost and changes in the variable production costs) as well as the demand-side ("green" consumers who value the nature-friendly production process) impact of implementing industrial symbiosis. Our results indicate that firms are more willing to implement industrial symbiosis when (a) the proportion of the green consumers is high; or (b) consumers' appreciation for the green variants is high; or (c) variable production costs after implementation are lower. For a firm, competition from firms that only produce regular products encourages implementation of industrial symbiosis, whereas competition from firms that produce both regular and green products discourages it.

Key words and phrases: industrial symbiosis, biomass utilization, sharing of resources, consumer welfare.

"*Industrial Ecosystem is an integrated model wherein the consumption of energy and materials is optimized, waste generation is minimized and the effluents of one process serve as a raw material for another process.*" – Frosch and Gallopoulos, "Strategies for Manufacturing," Scientific American, 1989.

1 Introduction: Industrial Symbiotic Systems

The principle of industrial ecology (Ehrenfeld 1995) proposes to reorganize the industrial system so that it evolves towards a mode of operation that is compatible with the biosphere over the long term. Industrial ecology suggests the idea of an industrial food chain in which companies can be linked in some form of network, in order to exploit unutilized resources or by-products and thereby increase resource utilization. The two main elements under this concept are (a) the need to optimize the use of materials and energy, and (b) to close material loops while minimizing emissions.

Industrial Symbiosis, a subfield of industrial ecology, can be defined as engaging "traditionally separate industries in a collective approach to competitive advantage involving physical exchange of material, energy, water, and by-products" (Chertow 2000). The term *By-Product Synergy* (BPS) has

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also been used synonymously with industrial symbiosis. BPS can offer true business opportunities beyond cost reduction, if wastes are viewed as raw materials for other industries. As BPS networks develop, industry goals may shift from reducing waste generation towards producing near-zero waste and finally to producing 100% product, while emissions are lowered and energy use is minimized (Mangan and Olivetti 2010). The economic activity created in a BPS network creates new businesses and jobs, under the premise that turning biomass output from one organization into a product stream for another organization can generate revenue while reducing emissions and the need for virgin materials.

To focus on the operational issues involved in the implementation of an industrial symbiotic system, it is beneficial to first discuss a specific example.

1.1 A Paper-Sugar Industrial Symbiotic System

Seshasayee Paper and Boards (SPB; http://www.spbltd.com) was set up as a public enterprise in 1961 in the southern state of Tamil Nadu, India. The company's main products include Bristol boards, Manila boards, colored bank paper, colored poster paper, and writing paper. The paper industry is capital intensive and one that is highly dependent on easy access to high-quality raw materials. In particular, the paper industry in India has traditionally faced considerable difficulty with the availability of raw materials. The continued efforts by the Government of India to minimize de-forestation have progressively increased the scarcity of wood. Consequently, paper manufacturers have been forced to look for alternate sources of raw materials. One such alternative for the paper production process is *bagasse*, a fibrous mass remaining after the extraction process of juice from sugarcane. The use of bagasse in paper production is energy efficient and also has a lesser impact on the environment, relative to wood.

Traditionally, bagasse has been used as an important fuel input in the sugar industry and can meet the requirement of fuel for the industry. Since alternative fuels such as coal or furnace oil are relatively costly, the sugar industry has been reluctant to sell the bagasse to the paper industry. Not surprisingly, following the steep increase in the prices of furnace oil and coal over the years, SPB was unable to make any arrangement to obtain a regular supply of bagasse from the sugar industry. The company also faces several environmental challenges. In addition to the emission of non-condensible gases, a paper mill also releases chlorinated compounds, dioxins, and furans. The waste water (effluent) carries high levels of Biochemical Oxygen Demand, Chemical Oxygen Demand, and other suspended solids. Furthermore, the problem of solid waste disposal is also a major concern in the face of local environmental pressures.

SPB responded to these challenges by creating a revolutionary industrial complex. The company got involved in sugar production by locating a sugar mill (Ponni Sugars; http://www.ponnisugars.com) close to its paper production facility, so that the entire output of bagasse from the production of sugar can be used as a biomass for the production of paper. This, however, led to another hurdle – availability of sugarcane that is a key ingredient for sugar production. This was a difficult problem as there was very little cultivation of sugarcane in the immediate neighborhood due to poor availability of water. The land around the nearby river Cauvery was elevated and dry, and local farmers could not afford building facilities to pump water from the river (20 feet lower) to the surrounding areas. In 1991, the company entered into a tripartite agreement with the local farmers and its sugar mill, and decided to irrigate about 1500 acres of dry land with SPB's treated effluent for the cultivation of sugarcane. The sugar mill, in turn, purchases the sugarcane from these farmers and supplies its by-product bagasse to the paper mill of SPB. This "symbiotic triangle" is illustrated in Figure 1 and has thrived since its inception: SPB recently announced a new Mill Development Plan with an estimated investment of 3.15 billion Indian Rupees to increase its paper production capacity from 132,000 tons to 165,000 tons (SPB Annual Report 2018-19). Despite the recurring failure of the monsoon rains, the sugarcane yield in the surrounding area has been enhanced (Ponni Sugars Annual Report 2018-19).

1.2 Differences from Traditional Process Innovation

There are several features of this implementation of the paper-sugar industrial complex that distinguish it from a traditional process innovation. We discuss a few below. Later, in Sections 2 and 3, our models exploit these properties.

1. *Simultaneous Change in the Production Processes of Multiple Products*: It is important to note the fundamental change in the use of bagasse in the symbiotic system. While bagasse has been

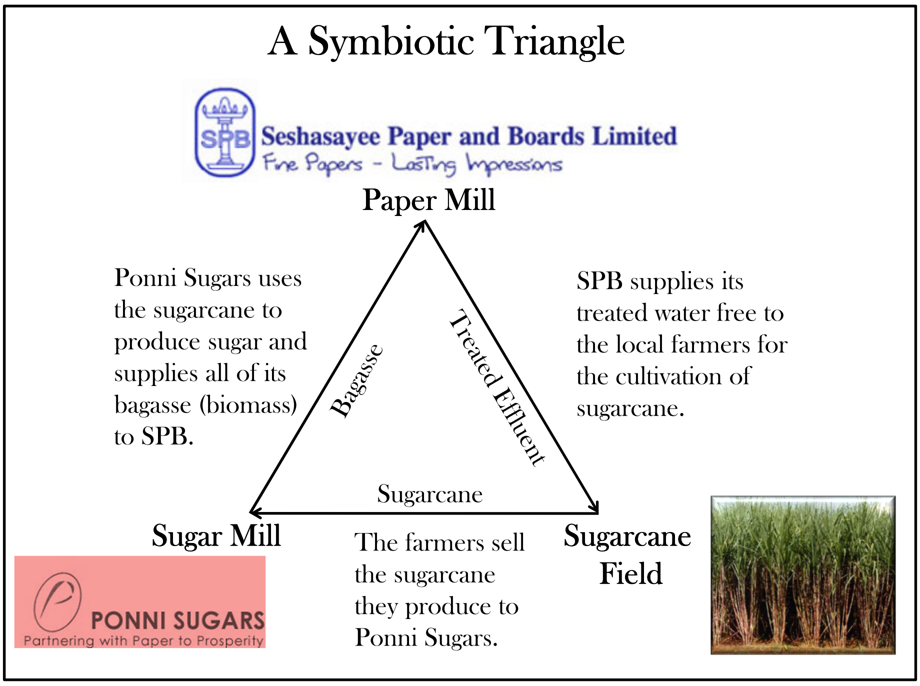

Figure 1: A Pictorial Illustration of the Paper-Sugar Symbiotic Industrial Complex

traditionally used by sugar mills as a source of fuel, its use in paper production (in the symbiotic system) is as a core raw material. This affects the production processes of both sugar and paper. On the one hand, the sugar mill needs to source alternate fuel since bagasse is no longer available. On the other hand, the production process of paper needs to be appropriately altered to use bagasse instead of wood. Also, in the larger symbiotic system, there are other implications: (a) sugar production benefits from the use of reliable and relatively cheap supply of sugarcane from the local farmers, (b) the effluent from paper production needs to be treated and delivered to the farmers, which is an additional expense for the paper mill, and (c) after implementation, the need to treat the waste (biomass) from the production of both paper and sugar reduces significantly relative to that before implementation. Clearly, these changes together imply advantages as well as disadvantages for both production processes.

2. *A Common Fixed Cost*: Since the process changes are symbiotic, the fixed investments required for the implementation naturally constitute one common cost for the entire implementation. This common fixed cost includes (i) acquiring the capability for the large-scale processing of bagasse in paper production, (ii) acquiring the capability to appropriately treat the effluent from the production processes, so that it can be used by sugarcane farmers, and (iii) building the transportation infrastructure for distributing the treated effluent to the sugarcane farmers.

3. *Linked Production Costs and Production Quantities*: Using the output (bagasse) of the sugar mill as raw material reduces the procurement cost of paper production. Therefore, the production quantities and marginal costs of the two products get inter-connected. While the production quantities of paper and sugar do not necessarily depend on each other (since, if required, the paper mill can obtain wood and the sugar mill can get additional sugarcane from other sources), their reciprocal influences on the cost economies in the symbiotic system are clear.

Finally, it should be emphasized that implementing the symbiotic system is a decision that implies accepting the (different) impacts on both the products together. In other words, it is not possible to change the production process of only one product while leaving the other unaffected. It is also clear from the discussion above that the cost-benefit tradeoff for the symbiotic system as a whole is not straightforward.

There are several other implementations of symbiotic systems that have been reported in the literature. We briefly mention two examples.

- Londonderry Eco-Industrial Park, New Hampshire (Block 1998, Bermejo 2014): The 100-acre park near the New Hampshire airport is one of the nation's prime examples of eco-industrial synergies.

A plastics recycling company (a tenant at the park) purchases waste plastic from Stonyfield Farms Yogurt (a firm that is located close to the park). AES Corporation (Puerto Rico) commissioned and built a 720 MW combined natural gas power plant on site that uses treated waste water, which is pumped from the city of Manchester's sewage treatment facility, in its cooling towers.

- Industrial Symbiosis Park, Kalundborg, Denmark (Ehrenfeld and Gertler 1997, Herczeg et al. 2016, Herczeg et al. 2018): The park consists of a web of material and energy exchanges that occur amongst several diverse companies and the local community. There are five core partners – a power station, an oil refinery, a gypsum board facility, a pharmaceutical plant, and the city of Kalundborg – who have all developed a series of bilateral exchanges. The partners share ground water, surface water, waste water, steam, electricity, and also exchange a variety of residues that become feedstock for other processes.

1.3 Our Goals and Contributions

Our discussion thus far raises a variety of important research issues concerning symbiotic systems. This paper focuses on the following:

- **Characterization of the Impact of a Symbiotic System:** The differences with traditional process innovation (Section 1.2) imply non-trivial cost-side (operational) tradeoffs of implementing a symbiotic system. The implementation provides a firm an option of offering "green" (environment-friendly) variants of its products. Accordingly, the demand-side impact (to be soon discussed in Section 2.3) depends on the nature of the consumer population. Two other factors affecting the demand-side influence are the presence of a competing firm and the nature of the products offered by the competition. Motivated by the real-world system discussed in Section 1.1, we analyze a firm's production decisions for its two products – both in the presence and absence of a symbiotic system – under monopoly as well as under competition in Section 3.

- **Understanding the "Willingness" to Implement a Symbiotic System:** For a given setting (supply- and demand-side parameters; nature of competition, if any), the difference in a firm's total profits (from all the products connected by a symbiotic system) before and after the implementation of the system is a good measure to capture its willingness to implement the system. Section 4 exploits the corresponding optimal/equlibrium solutions to examine the behavior of this metric both under monopoly and competition, with respect to changes in several important parameters, including the variable production cost after implementation, consumers' appreciation of the green variant, and the proportion of green consumers.

- **Increased Willingness under Competition:** SPB's decision to implement the symbiotic system in 1991 came at an interesting time. Around the same time, the Government of India initiated the process of economic liberalization, which opened the country's markets to foreign firms in several sectors, including paper and pulp. This coincidence raises an interesting question: Can the presence of competition increase a firm's willingness (relative to that under a monopoly) to implement the symbiotic system? We address this issue in Section 4.3 and identify some situations under which the answer is in the affirmative.

- **Benefiting Both the Firm and the Consumers:** While the benefits of a symbiotic system for the society at large are clear, a stronger motivation for a firm to implement the system results when both the firm and its consumers benefit. In Section 4.4, we identify some conditions under which such a simultaneous improvement occurs. We also illustrate a stronger case where a firm's willingness to implement is negative in a monopoly, but positive under competition and consumer welfare too increases after the implementation.

Technically, the challenges in our analysis are twofold: Under a monopoly setting, when the symbiotic system is implemented, the flexible green consumers' (defined in Section 2.1) choice of a utility-maximizing variant leads to a tricky pricing problem for the regular and green variants. In a competitive setting, when the symbiotic system is implemented, the products from the two firms interact under a Cournot competition, which coupled with aforementioned pricing problem generates a previously-unexplored analytical challenge.

1.4 Literature Review

Our research is related to closed-loop supply chain management. The primary focus of the closed-loop supply chain literature has been on taking back products from consumers and recovering added value by using all or part of the products. Souza (2013) offers a comprehensive review of reverse logistics and closed-loop supply chain research, including remanufacturing, recycling, reselling, and disposal. Easwaran and Uster (2010) and Uster and Hwang (2017) both study the issue of designing an integrated closed-loop supply chain network with two physical flow channels: forward and reverse. The existing work on closed-loop supply chains primarily addresses post-consumer waste, where the source of the waste is the consumer. In contrast, our paper considers the simultaneous utilization of pre-consumer wastes from multiple products.

Our research is also related to the design of supply chains that involve the usage of biomass/biofuel. Malladi and Sowlati (2018) conduct a comprehensive review of the key features of biomass logistics. The authors discuss several optimization models to capture these features, and suggest interesting directions for future work. To address the issue of uncertain yield rates in the biomass production process, Uster and Memisoglu (2018) develop a two-stage stochastic integer program to determine both the design of the network and biomass pricing. Schroder et al. (2019) develop an integrated model for a wood-based biorefinery in Canada to consider both the production planning and the availability of the input. The use of bagasse as a biomass for paper production is also reported in Zafar (2018). In our paper, we consider utilizing the biomass generated in both production processes of the industrial symbiotic system.

Operations managers now need to increasingly focus on how to address environmental challenges, while maintaining competitiveness (Corbett and Klassen 2006). Ehrenfeld (1995) suggests simple design principles, including closing material loops, dematerialization, toxic elimination, and pollution prevention, that can serve as emerging models for operationalizing industrial ecology. Lee and Tongarlak (2017) study a retail grocer's operating strategies when it implements a by-product synergy avoid waste disposal and reduce costs. Cambero and Sowlati (2016) propose an indicator to quantify the social benefits related to the implementation of bioenergy and biofuel supply chains. Our attempt has been to characterize both the cost-side and demand-side implications of implementing an industrial symbiotic system. In particular, we show that these considerations generate rich tradeoffs and provide valuable insights on the decision to implement the environment-friendly initiative.

One consequence of a symbiotic system is the ability to manufacture different grades of a product. Lin et al. (2019) study a firm using a co-production technology to produce both traditional and green products. They also assume that producing green products requires an additional production cost. They consider two customer classes: traditional consumers and green consumers. The authors focus on impacts of resource scarcity and green production efficiency on the firm's product line design decisions, including the products' prices and quantities, and the raw material quantity. Chen et al. (2013) study a co-production system in which output differs in quality, and characterize optimal prices, quality grade specifications, and production quantities. The authors show that in co-product technologies, quality availability (i.e., the firm's ability to supply quality levels) replaces the cost of quality as a fundamental driver of product line design. Due to this difference, a firm may benefit from a more variable output distribution.

We now proceed to describe the setting of our analysis.

2 Model Setting

Given the complex nature of the real-world examples in Section 1 and the reasoning that led to their existence, our attempt is only to capture the basic operational changes resulting from such implementations. In particular, to address the research issues raised in Section 1.3, we develop several models to capture the operations of a domestic firm (Firm 1) that produces two products, P and S, under various settings. These models represent the scenarios both before and after the implementation of an industrial symbiotic system, and will be described in detail in Section 3. As justified later in Section 2.3, under an assumption that typically holds in practice, the firm's operational decisions for the two products can be made separately. Thus, for the purpose of analyzing the firm's optimal decisions (under monopoly) and equilibrium decisions (under competition), a single-product model suffices here. Subsequently, in Section 4, when evaluating the firm's strategic decision of whether or not to implement the symbiotic system, we consider the combined impact on the two products.

This section is organized as follows: in Section 2.1, we introduce the products and consumers studied in our models. Next, the structure of demand is described in Section 2.2. The cost-side and demand-side impacts of implementing the symbiotic system are formalized in Section 2.3.

2.1 Types of Product Variants and Consumers

There are two possible variants of the product: *regular* and *green*. If a firm does not implement the symbiotic system, its products can only be labeled as regular. On the other hand, if the system is implemented, then the firm has the option of labeling its products as green, which represents the use of an environment-friendly production process. Note that, in the latter case, the firm could still label its products as regular and we include this option in our analysis. Agrawal and Lee (2019) discuss a setting in which the manufacturer could offer either conventional (i.e., *regular* in our setting), or sustainable (i.e., *green* in our setting), or both types of products, depending on the parts provided by the supplier. In our model, the prices are fixed during the planning horizon. Sen (2013a, 2013b) compare the performances of fixed and dynamic pricing policies. We now introduce the primary notation used in our analysis. Additional notation is introduced later, as required.

Notation:

a	The appreciation of the green variant, defined by the ratio of a green consumer's valuation for a unit of the green variant to that for a unit of the regular variant, $a \geq 1$.
p_i	The market price of variant i, $i \in \{r, g\}$, r (resp., g) represents regular (resp., green).
v_i	The consumer's valuation of variant i, $i \in \{r, g\}$, r (resp., g) represents regular (resp., green).
Q_i	The total demand of variant i, $i \in \{r, g\}$, r (resp., g) represents regular (resp., green).

Several studies (e.g., Moser 2016, Guyader et al. 2017, Dai et al. 2018) have indicated the existence of environment-conscious consumers who are willing to pay a premium for variants produced by an ecologically friendly process. European Commission (2013) reports that 77% of European consumers are willing to pay the "green premium," the price difference between eco-friendly and regular products. Li et al. (2016) show that even in developing countries, consumers are willing to pay a higher price for green products. They model the premium for a green variant as a percentage over customers' valuation of the regular variant. We consider three types of consumers: *regular*, *flexible green* and *dedicated green*.

On the one extreme, we have regular consumers who do not have any special appreciation for the "green" label. Accordingly, the valuation of these consumers for the green variant is the same as that for the regular variant. On the other extreme, dedicated green consumers are devoted to the green variant, if it is available. Thus, the valuation of dedicated green consumers for the regular variant becomes 0 if the green variant is available. In general, these consumers have a relatively higher valuation for the green variant. Flexible green consumers represent the intermediate kind: they simply have a higher valuation for the green variant, relative to the regular variant. These consumers buy the regular variant, if it results in a higher utility. The precise valuations of these three types of consumers are defined in Table 1.

Consumer's Type ↓ Valuation →	Regular Variant, Green Variant (v_r, v_g)
Flexible Green	(v, av)

Table 1: Valuations of the Three Types of Consumers for Regular and Green Variants, for an Arbitrary Value of v, $0 \leq v \leq 1$.

We assume that an individual consumer's valuation (v) for a unit of regular variant is uniformly distributed between 0 and 1.

2.2 Structure of Demand

To obtain the aggregate demands of the regular variant and the green variant, we need to derive the demand of each type (regular/green) of variant from consumers. We assume that $0 \leq p_r \leq 1$, $0 \leq p_g \leq a$. We now discuss these demands under three scenarios.

1. Only the regular variant is available.

   All the consumers who have a valuation higher than or equal to the market price will purchase the variant. Thus, we have
   $$Q_r = 1 - p_r.$$

2. Both regular and green variants are available.

   A consumer's utility of buying a regular product is $v - p_r$, while his utility of buying a green product is $av - p_g$. There are three possible price settings: $p_r \leq p_g \leq ap_r$, $ap_r \leq p_g \leq a - 1 + p_r$, and $a - 1 + p_r \leq p_g \leq a$. The choices of consumers and the total market demand for each variant under these three price settings are summarized in the following table. For brevity, we avoid providing the derivations of the expressions in the table.

3. Only the green variant is available.

   In this case, consumers either buy the green variant or buy nothing. Thus,
   $$Q_g = 1 - \frac{p_g}{a}.$$

$\rightarrow$ *Price Setting* $\downarrow$ *Demand*	$p_r \leq p_g \leq ap_r$	$ap_r \leq p_g \leq (a-1)+p_r$	$(a-1)+p_r \leq p_g \leq a$
Q_r	0	$\frac{p_g - ap_r}{a-1}$	$1 - p_r$
Q_g	$1 - \frac{p_g}{a}$	$1 - \frac{p_g - p_r}{a-1}$	0

Table 2: The Market Demand when Both Regular and Green Variants are Available

2.3 Cost-Side and Demand-Side Impacts of Implementing a Symbiotic System

Like traditional process innovation, the implementation of the symbiotic system affects a firm's production cost. Furthermore, it is clear from the discussion above that implementing the system also influences the market demand of a firm's variants. To describe these two impacts, we need some additional notation. For brevity, we only define the notation for Product P. The corresponding notation for Product S will then be clear.

Notation:

π_1	The profit of Firm 1 before the implementation of the symbiotic system.
$\pi_1^{'}$	The profit of Firm 1 after the implementation of the symbiotic system.
K	The fixed cost of implementing the symbiotic system.
p_p	The market price of Product P.
$q_{1,p}$	Firm 1's production quantity of Product P.
$c_{1,p}$	Firm 1's per unit production cost before the implementation of the symbiotic system.
$b_1^{'}$	The increment of the per unit waste treatment cost after the implementation of the symbiotic system.
$c_{1,p}^{'}$	Firm 1's per unit production cost after the implementation of the symbiotic system.
β_p	The amount of waste generated from the production of one unit of Product P.
γ_p	The amount of waste of Product P needed to produce one unit of Product S.
s_p	The cost saving for using one unit of the waste of Product P to produce Product S.
$\bar{s}_p$	The revenue from selling one unit of the additional waste of Product P in the open market.

- **The Cost-Side Impact**

Before the implementation of a symbiotic system, Firm 1's total profit is:

$$\pi_1 = p_p q_{1,p} - c_{1,p} q_{1,p} + p_s q_{1,s} - c_{1,s} q_{1,s}.$$

The implementation of a symbiotic system affects the cost of production in three ways:

1. The additional fixed cost of implementing the symbiotic system

   The firm incurs an additional fixed cost K.

2. The increase in waste treatment cost

   After the implementation, the firm introduces a more sophisticated treatment process of the wastes of both the products. Thus, it incurs an additional per unit cost $b_1^{'}$.

3. The savings in procurement cost of raw material

We assume that β_p units of waste are generated from the production of one unit of Product P. One unit of Product S can be produced by using γ_p units of this waste. The firm saves an amount of s_p for using every unit of the waste of Product P to produce Product S. Thus, the firm saves $(s_p \min\{\beta_p q_{1,p}, \gamma_p q_{1,s}\})$ from using the waste of Product P to produce Product S. If any waste of Product P remains after satisfying the required production of Product S, then we assume that the firm can sell it in the open market at a per-unit price $\bar{s}_p$; refer to Figure 2.

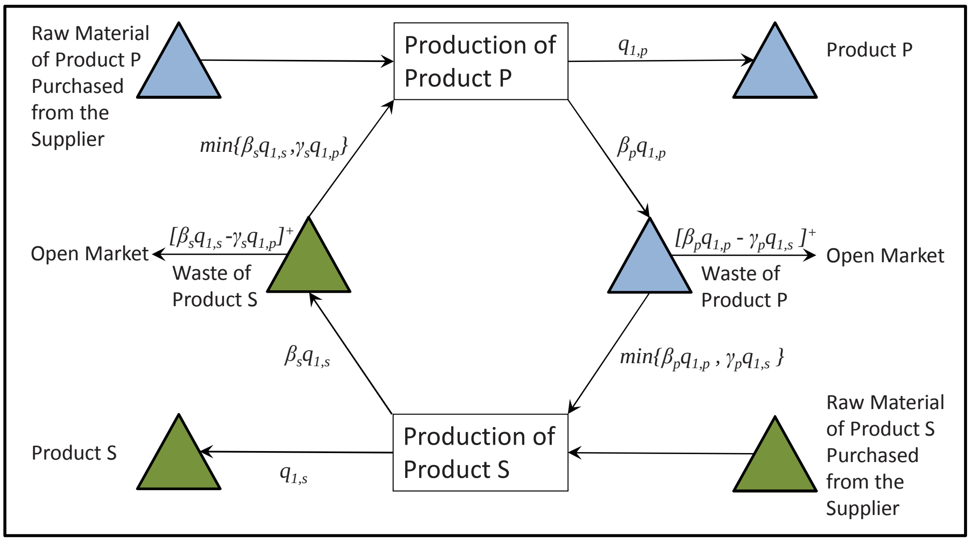

Figure 2: The Mutually Beneficial Use of Waste in a Symbiotic System. The Figure Shows the Production Cycle of Products P and S. The Notation is as Defined at the Start of Section 2.3.

Then, Firm 1's total profit after the implementation of the system is:

$$\begin{aligned} \pi_1' &= p_p q_{1,p} - b_1' q_{1,p} - c_{1,p} q_{1,p} + s_p \min\{\beta_p q_{1,p}, \gamma_p q_{1,s}\} + \bar{s}_p[\beta_p q_{1,p} - \gamma_p q_{1,s}]^+ \\ &\quad + p_s q_{1,s} - b_1' q_{1,s} - c_{1,s} q_{1,s} + s_s \min\{\beta_s q_{1,s}, \gamma_s q_{1,p}\} + \bar{s}_s[\beta_s q_{1,s} - \gamma_s q_{1,p}]^+ - K. \end{aligned}$$

Our analysis in this paper assumes that the difference between (i) the cost saving achieved by using one unit of the waste of Product P (resp., S) to produce Product S (resp., P) and (ii) the revenue from selling one unit of the additional waste of Product P (resp., S), is negligible. We offer two arguments to justify this assumption in our context. First, in practical implementations, it is often the case that the waste generated from one product does not contribute a *dominant* supply of raw material required for the production of the other product. For instance, our extensive discussions with the senior administrators and operations managers at SPB revealed that while a substantial amount of bagasse needed for paper comes from the sugar mill, it is far from sufficient to meet demand. Therefore, the company sources additional bagasse and wood from both domestic and international sources. Thus, all the waste of one product is used in the production of the other product, and vice-versa. Consequently, there is no additional waste of these products to be sold in the open market. This is a natural outcome in an "ideal" symbiotic system, since the useful utilization of all wastes is a cornerstone of the concept. So, in this case, we do not need the assumption. Second, even if additional waste remains, its quantity is typically small enough. Thus, there is sufficient demand in the open market for the firm to realize a per-unit revenue similar to the savings achieved if the waste were to be used for its own production. Under this assumption, Firm 1's total profit after implementation can be rewritten as follows:

$$\pi_1^{'} = p_p q_{1,p} - b_1^{'} q_{1,p} - c_{1,p} q_{1,p} + s_p \beta_p q_{1,p} + p_s q_{1,s} - b_1^{'} q_{1,s} - c_{1,s} q_{1,s} + s_s \beta_s q_{1,s} - K.$$

Denote $c_{1,p}^{'} = b_1^{'} + c_{1,p} - s_p \beta_p$, $c_{1,s}^{'} = b_1^{'} + c_{1,s} - s_s \beta_s$. Then, we have

$$\pi_1^{'} = p_p q_{1,p} - c_{1,p}^{'} q_{1,p} + p_s q_{1,s} - c_{1,s}^{'} q_{1,s} - K.$$

Note that depending on the value of $b_1^{'} - s_p \beta_p$, the coefficient $c_{1,p}^{'}$ of the production cost after the implementation could be either higher or lower than $c_{1,p}$.

- **The Demand-Side Impact**

Prior to the implementation of the symbiotic system, the firm can only label Products P and S as regular. Implementation of the system enables the firm to introduce the "green" (environment-friendly) variants of the two products, in an attempt to capture the consumers who have a relatively higher valuation for these variants and take advantage of their higher willingness to pay. We emphasize that, if the firm so chooses, it can also label a green variant as regular. One of the key characteristics of the symbiotic system is that both Products P and S can be labeled as green *simultaneously*. This symbolizes the "mutually dependant" relationship of the two products. The reactions of the three types of consumers (regular, flexible green, and dedicated green) to these two variants (see Section 2.2) then constitute the demand-side impact of the implementation.

In the next section, we analyze a firm's production decisions for the two products – both in the presence and absence of a symbiotic system – under monopoly as well as under competition.

3 Analyzing the Impact of a Symbiotic System under Monopoly and Competition

Section 3.1 considers the scenario in which the firm is the only supplier of both Products P and S in their respective markets. The justification of our chosen setting for analyzing competition and some related assumptions is provided in Section 3.2. In Section 3.3, we describe Model CR, in which Firm 1 faces a competitor (Firm 2) that only produces the regular variants of the products. Model CG, in which Firm 1 faces a competitor that has the ability to produce the green variants, is discussed in Section 3.4. As argued in Section 2.3, the operational decisions about Product P and Product S can be made separately. Therefore, our discussion in this section is for a single product, say Product P.

Accordingly, we simplify the notation by avoiding the subscript that represents the product (e.g., $c_{1,p}$ is simply denoted as c_1).

3.1 Monopoly Setting (Model M)

Consider the situation where Firm 1 is the only supplier of Product P in the market. We assume that the firm can decide the price of the product directly, and then produce to meet the generated demand. We also assume $c_1 < 1$ and $c_1^{'} < 1$. In Section 3.1.1 (resp., Section 3.1.2), we discuss the scenario when the firm does not (resp., does) implement the symbiotic system.

3.1.1 No Symbiotic System For Firm 1

If Firm 1 does not implement the symbiotic system, then it can only produce the regular variant. Since the firm is the only supplier, only the regular variant is available in the market. The optimal price, obtained by solving the firm's profit maximization problem, is given in Proposition 1. Since this is a standard result under a monopoly setting, we avoid providing a proof.

Proposition 1 *In Model M, if Firm 1 does not implement the symbiotic system, then the optimal price of the regular variant is* $p_r^* = \frac{1+c_1}{2}$.

3.1.2 Symbiotic System For Firm 1

In this case, the firm can produce both the regular and green variants. The optimal prices and quantities for the variants are summarized in the following theorem. The proof is in Section A of the Appendix.

Theorem 1 *In Model M, if Firm 1 implements the symbiotic system, then the optimal price of the regular (resp., green) variant is* $p_r^* = \frac{1+c_1^{'}}{2}$ *(resp.,* $p_g^* = \frac{a+c_1^{'}}{2}$*) and the optimal quantity of the regular (resp., green) variant is* $q_r^* = \frac{\theta_1(1-c_1^{'})}{2}$ *(resp.,* $q_g^* = \frac{(1-\theta_1)(a-c_1^{'})}{2a}$*).*

The argument in the proof of Theorem 1 shows that the optimal setting of the prices corresponds to the scenario when regular consumers buy the regular variant, while all the flexible and dedicated green consumers buy the green variant. We note the following two implications.

Corollary 1 *Under a monopoly, if Firm 1 implements the symbiotic system, then it will avoid charging a price for the green variant that is high enough to drive (some) flexible green consumers to the regular variant. Under optimal prices for the regular and green variants, no flexible green consumer switches to the regular variant.*

Consequently, under the optimal prices, the behavior of flexible green consumers and dedicated green consumers is the same.

Corollary 2 *Under a monopoly, green consumers' type (either flexible or dedicated) does not affect Firm 1's optimal decisions.*

Next, we discuss the situation when Firm 1 faces a competitor who also produces both Products P and S. We first justify our choice of the setting to analyze competition and then specify two assumptions on the production amounts in equilibrium.

3.2 The Challenges in the Model Setting under Competition

The first challenge is to choose the type of competition. There are two possible settings: price and quantity. If we assume that the two firms compete on price, then under the assumption of a homogenous product, the firm with a lower variable production cost sets an equilibrium price just below the other firm's variable production cost. Thus, the sales for the high-cost firm become 0, while the low-cost firm captures the entire market. This is hardly the case in practice. Bougette (2010) observes "Cournot competition fits a significant number of industries in which capacities or outputs are the long-run variables, prices being set in the short run." Christensen and Caves (1997) choose Cournot Competition to study the North American Pulp and Paper Industry, while Genesove and Mullin (1998) use Cournot Model to analyze the Sugar Industry in the United States. Therefore, we choose to use Cournot competition and let the firms compete on quantity. If a firm is the only one to provide a given variant, we assume that it can decide the price of that variant directly.

The next challenge is to determine the market price of the regular variant. In our setting, we have three types of consumers and two variants of the product. The flexible green consumers may purchase the regular variant, when the price of the green variant is high enough. Thus, the information about the prices of the two variants is needed to determine the demand from the flexible green consumers for the regular variant. However, under Cournot competition, the price of the regular variant is determined by the total demand of that variant. To break this deadlock, we make the following assumption: the market price of the regular variant is determined by the Cournot inverse demand function of the regular consumers.

The third challenge is to assign the flexible green consumers' demand of the regular variant (if any) to the firms. There are two general approaches: (a) both firms share this demand (either equally or based on market share), and (b) one firm (e.g., the powerful one) exclusively supplies this demand. We choose the latter approach. Note that there is no technical difficulty to pursue the former approach. When Firm 1 can produce the green variant and Firm 2 cannot, each firm decides its production quantity of the regular variant. The market price of the regular variant is determined by the combined production quantity of both firms. Thus, each firm only partially influences the market price of the regular variant. Meanwhile, Firm 1 decides the price of the green variant directly. Thus, Firm 1 is relatively more influential in deciding the market prices of the two variants. We emphasize this advantage by allocating the demand of the regular variant from the flexible green consumers to Firm 1.

To focus our analysis on realistic situations under competition, we make two assumptions as follows:

1. If a firm has the option to label its product as the green variant, then it will produce a positive amount of the green variant. Thus, the firm has two choices: (i) providing only the green variant, or (ii) providing both regular and green variants.

   The theoretically possible scenario, in which a firm provides only the regular variant even if it can label its product as green, is rarely seen in practice. A firm will naturally be reluctant to forgo the opportunity to exploit the more-profitable green market. Under Cournot competition, since the price of a variant is determined by the total production quantities of two firms, neither firm can directly decide the prices of both variants. Thus, a firm's action of providing both regular and green variants is legal from the viewpoint of anti-price discrimination regulations (e.g., the Robinson-Patman Act).

2. The total amount of the product (i.e., the sum of the regular and green variants of the product, as applicable) produced by each firm is positive.

   As mentioned earlier, we avoid considering the situation where a firm drives its competitor out of the market.

The Chosen Setting under Competition

We assume two firms, Firm 1 and Firm 2, in the market. Competition is realized via the following 2-stage game.

- Stage 1. Each firm indicates the types of variants it will produce. If a firm has the option of labeling its product as the green variant, then it has two choices: (1) providing only the green variant, or (2) providing both the regular and green variants. Each firm decides its choice and makes this decision public.

- Stage 2. If there are two firms who produce a given variant, then the firms decide their individual production quantities simultaneously. The market price of this variant is then determined by the corresponding Cournot inverse demand function. If a firm is the only one to provide a given variant, then it decides the price of that variant directly.

For consistency, we continue to use the notation defined in Section 2. We need the following additional notation.

Notation:

q_2	The production quantity of Firm 2.
$C_2(q_2)$	The production cost of Firm 2 to produce q_2 units of product; $C_2(q_2) = c_2 q_2$.
π_2	The profit of Firm 2.

3.3 Competition for the Regular Variant (Model CR)

In this section, we assume the competitor only produces the regular variant. We first consider the case when Firm 1 decides not to implement the symbiotic system in Section 3.3.1. Later we discuss the case when Firm 1 implements the system. Although equilibrium results can be obtained in the presence of both dedicated and flexible green consumers, (i.e., $\theta_2 > 0$, $\theta_3 > 0$), for the purpose of deriving managerial insights, we restrict our analysis to two special cases: (i) $\theta_2 = 0$ (all green consumers are dedicated), (ii) $\theta_3 = 0$ (all green consumers are flexible). The case when all green consumers are flexible is considered in Section 3.3.2. The case in which all green consumers are dedicated is analyzed in Section 3.3.3.

3.3.1 No Symbiotic System For Firm 1

In the absence of the symbiotic system, only the regular variant is available in the market. Thus, both firms compete only in the regular market, where all consumers have the same distribution of valuation. The following result states the equilibrium solution. Since the proof is straightforward, we avoid providing it here.

Proposition 2 *In Model CR, if Firm 1 does not implement the symbiotic system, then the equilibrium price of the regular variant is* $p_r^* = \frac{1+c_1+c_2}{3}$*, and the equilibrium production quantities of the two firms are:* $q_1^* = \frac{1-2c_1+c_2}{3}$ *(Firm 1),* $q_2^* = \frac{1+c_1-2c_2}{3}$ *(Firm 2).*

3.3.2 Symbiotic System For Firm 1 When All Green Consumers Are Flexible

In this case, depending on the prices of the green and regular variants, the green consumers may choose to buy either one. As described in Section 2.2, consumers' demand for the regular and green variants has multiple possible forms. Recall our assumption that both firms produce positive amounts of the product. Since Firm 2 can only produce the regular variant, a necessary condition for this to occur is that the price of the regular variant is lower than that of the green variant but higher than Firm 2's variable production cost ($c_2 < p_r^* < p_g^*$). If Firm 1's variable production cost after implementation is less than Firm 2's variable production cost ($c_1^{'} < c_2$), then the former is lower than the price of the regular variant as well ($c_1^{'} < p_r^*$). Thus, Firm 1 only producing the green variant is not an equilibrium, since Firm 1 can always improve its profit by providing a nonzero amount of the regular variant, and gain additional profit. Thus, in equilibrium, Firm 1 provides both regular and green variants, while

Firm 2 only provides the regular variant. The following result summarizes the equilibrium prices for Model CR under three possible parameter settings. The proof is in Section B of the Appendix.

Theorem 2 *When all green consumers are flexible,* $a > 1$ *and* $c_1^{'} < c_2 < \frac{1}{2}[1 + \frac{c_1^{'}}{a(1-\theta_1)+\theta_1}]$*, if Firm 1 implements the symbiotic system, then there are three possible equilibria (depending on parametric relationships). These are described below and pictorially illustrated in Figure 3.*

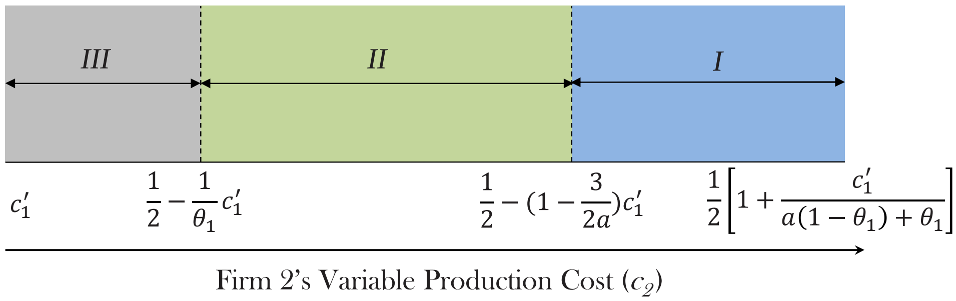

Figure 3: Categorization of Equilibria Based on the Competitor's Variable Production Cost (c_2), when $a > 1$, $c_1^{'} < \frac{\theta_1}{2(1+\theta_1)}$, and $c_1^{'} < c_2 < \frac{1}{2}[1 + \frac{c_1^{'}}{a(1-\theta_1)+\theta_1}]$.

1. *If* $c_1^{'} < \frac{\theta_1}{2(1+\theta_1)}$*, then*

   - *Type I: If* $[\frac{1}{2} - (1 - \frac{3}{2a})c_1^{'}] < c_2 < \frac{1}{2}[1 + \frac{c_1^{'}}{a(1-\theta_1)+\theta_1}]$*, then we have*

   $$p_r^I = \frac{1 + c_1^{'} + c_2}{3}, \quad p_g^I = \frac{a + c_1^{'}}{2}.$$

   - *Type II: If* $(\frac{1}{2} - \frac{1}{\theta_1}c_1^{'}) \leq c_2 \leq [\frac{1}{2} - (1 - \frac{3}{2a})c_1^{'}]$*, then we have*

   $$p_r^{II} = \frac{(a + \theta_1 - a\theta_1) + c_1^{'} + \theta_1 c_2}{2a(1-\theta_1) + 3\theta_1}, \quad p_g^{II} = \frac{a(a + \theta_1 - a\theta_1) + ac_1^{'} + a\theta_1 c_2}{2a(1-\theta_1) + 3\theta_1}.$$

   - *Type III: If* $c_1^{'} < c_2 < (\frac{1}{2} - \frac{1}{\theta_1}c_1^{'})$*, then we have*

   $$p_r^{III} = \frac{1 + c_1^{'} + \theta_1 c_2}{2 + \theta_1}, \quad p_g^{III} = \frac{(2a + a\theta_1 - \theta_1) + 2c_1^{'} + 2\theta_1 c_2}{2(2 + \theta_1)}.$$

2. *If* $\frac{\theta_1}{2(1+\theta_1)} \leq c_1^{'} < \frac{a}{4a-3}$ *and* $c_1^{'} < c_2 < \frac{1}{2}[1 + \frac{c_1^{'}}{a(1-\theta_1)+\theta_1}]$*, then the region representing Type III vanishes.*

3. *If* $\frac{a}{4a-3} \leq c_1^{'} < \frac{1}{2 - \frac{1}{a(1-\theta_1)+\theta_1}}$ *and* $c_1^{'} < c_2 < \frac{1}{2}[1 + \frac{c_1^{'}}{a(1-\theta_1)+\theta_1}]$*, then the regions representing Type II and III vanish.*

Note that in both Type I and Type II equilibria, all the flexible green consumers buy either the green variant or nothing. In the Type III equilibrium, some flexible green consumers switch to the regular variant. As a consequence, we have the following corollary.

Corollary 3 *When the competitor's cost disadvantage is marginal (c_2 is in Region III of Figure 3), Firm 1 chooses to price in a manner so that some low-end flexible green consumers switch to the regular variant. When the competitor's cost disadvantage increases (c_2 is in either Region I or Region II), Firm 1 avoids this switching behavior.*

3.3.3 Symbiotic System For Firm 1 When All Green Consumers Are Dedicated

When all green consumers are dedicated, they purchase either the green variant or nothing. Thus, the demand of the green variant is unaffected by the price of the regular variant. Accordingly, only the price of the green variant determines its demand. Recall our discussion in Section 2.2 of the aggregate market demand under various scenarios. Due to the absence of the flexible green consumers, the aggregate market demand now has only one form when both regular and green variants are available. When $p_g > p_r$, the regular consumers purchase either the regular variant or nothing. Meanwhile, the green consumers purchase either the green variant or nothing. The aggregate market demand is as follows:

$$Q_r = (1 - p_r)\theta_1, \quad Q_g = (1 - \frac{p_g}{a})(1 - \theta_1).$$

Since our goal is to avoid exploring all theoretical possibilities and, instead, focus on practically-relevant scenarios in which the two firms enjoy healthy competition, we impose the following assumption: The two competing firms have similar production economies. In other words, both firms' variable production costs are of the same magnitude, both before and after Firm 1's implementation of the symbiotic system. Specifically, we assume that $2c_1 - 1 < c_2 < \frac{1+c_1}{2}$ and $2c_1^{'} - 1 < c_2 < \frac{1+c_1^{'}}{2}$. This assumption is reasonable in practice, since a significant difference in the production costs of the two firms will typically result in the weaker firm being driven out of the market. The following result provides the production quantities of each variant for each firm in equilibrium. Since the proof is similar to that of Theorem 2, we avoid presenting it.

Proposition 3 *When all green consumers are dedicated, if Firm 1 implements the symbiotic system, then it will provide both regular and green variants. The equilibrium production quantities are:* $q_{1,r}^* = \frac{\theta_1(1-2c_1^{'}+c_2)}{3}$, $q_{2,r}^* = \frac{\theta_1(1+c_1^{'}-2c_2)}{3}$, *and* $q_{1,g}^* = \frac{(1-\theta_1)(a-c_1^{'})}{2a}$. *The equilibrium prices are:* $p_r^* = \frac{1+c_1^{'}+c_2}{3}$ *and* $p_g^* = \frac{a+c_1^{'}}{2}$.

In the next section, we consider the case in which a competitor has the ability to produce the green variant.

3.4 Competition for both Regular and Green Variants (Model CG)

Under Model CG, we consider the scenario in which the competitor (Firm 2) has the ability to produce the green variant. Firm 2 can choose one of the following two strategies: (1) Strategy g: to provide only the green variant or (2) Strategy rg: to provide both the regular and green variants. If Firm 1 implements the symbiotic system, then it also needs to choose one of the above two strategies. Thus, each firm has to make the following two decisions: (i) the strategy to choose, and (ii) the corresponding production quantities. Pun and DeYong (2017) use a game theoretical model to examine a manufacturer's decisions when competing with a copycat firm to attract strategic customers. In Section 3.4.1 (resp., Section 3.4.2), we consider the case when Firm 1 does not implement (resp., implements) the symbiotic system.

3.4.1 No Symbiotic System For Firm 1 Under Model CG

If Firm 1 does not implement the symbiotic system, then only Firm 2 has the option to label its product as the green variant. By using a reasoning similar to that in the proof of Proposition 3, we obtain the equilibrium result.

Proposition 4 *In Model CG, if Firm 1 does not implement the symbiotic system, then Firm 2 provides both regular and green variants. The equilibrium production quantities are:* $q_{1,r}^* = \frac{\theta_1(1-2c_1+c_2)}{3}$, $q_{2,r}^* = \frac{\theta_1(1+c_1-2c_2]}{3}$, *and* $q_{2,g}^* = \frac{(1-\theta_1)(a-c_2)}{2a}$. *The equilibrium prices are:* $p_g^* = \frac{a+c_2}{2}$ *and* $p_r^* = \frac{1+c_1+c_2}{3}$.

3.4.2 Symbiotic System For Firm 1 Under Model CG

Under this setting, both firms are capable of offering both variants. Each firm has two strategies. We denote $\pi_i^{j,k}$ as Firm i's profit if Firm 1 chooses Strategy j and Firm 2 chooses Strategy k; $i \in \{1, 2\}$, $j, k \in \{g, rg\}$. The following result shows that both firms provide both regular and green variants in equilibrium and derives the corresponding production quantities. The proof is in Section C of the Appendix. The payoffs of the two firms under the four scenarios are listed in Table 3.

		Firm 2's Strategy		
		Only Green		*Regular & Green*
Firm 1's	*Only Green*	$(\pi_1^{g,g}, \pi_2^{g,g})$	$\rightarrow$	$(\pi_1^{g,rg}, \pi_2^{g,rg})$
		$\downarrow$		$\downarrow$
Strategy	*Regular & Green*	$(\pi_1^{rg,g}, \pi_2^{rg,g})$	$\rightarrow$	$(\pi_1^{rg,rg}, \pi_2^{rg,rg})$

Table 3: The Two Firms' Payoffs when Both Can Provide Regular and Green Variants

Theorem 3 *In Model CG, if Firm 1 implements the symbiotic system, then in equilibrium, both firms provide both the regular and the green variants. The equilibrium prices and production quantities are:* $p_r^* = \frac{1+c_1'+c_2}{3}$, $p_g^* = \frac{a+c_1'+c_2}{3}$, $q_{1,r}^* = \frac{\theta_1(1-2c_1'+c_2)}{3}$, $q_{1,g}^* = \frac{(1-\theta_1)(a-2c_1'+c_2)}{3a}$, $q_{2,r}^* = \frac{\theta_1(1+c_1'-2c_2)}{3}$, *and* $q_{2,g}^* = \frac{(1-\theta_1)(a+c_1'-2c_2)}{3a}$.

Next, we use the results obtained in this section to derive some useful managerial insights.

4 Understanding the Willingness to Implement the Symbiotic System

In the previous section, we derived Firm 1's optimal (under monopoly) and equilibrium (under competition) decisions, under various settings. These results enable us to assess the firm's profits both before and after the implementation of a symbiotic system. We interpret the difference between these two profits as the firm's "willingness" to implement the system. Note that this difference may not always be positive. Accordingly, our interest is in understanding the forces that influence willingness and the impact on their relative strengths with respect to changes in operational parameters, consumer characteristics, and competition. Section 4.1 (resp., Section 4.2) discusses the situation under Model M (resp., Model CR). We also briefly comment on the main differences under Model CG. Next, in Section 4.3, we compare the firm's willingness under monopoly with that under competition. Our focus is on analyzing the impact of the nature of competition (the competitor offering only the regular variants vs. both variants) and on identifying scenarios where competition improves the firm's willingness to implement. Finally, Section 4.4 discusses the impact of the implementation of a symbiotic system on consumer welfare. Here, our aim is to identify situations in which the firm's willingness to implement is positive and consumers (as a whole) benefit from the implementation as well.

For simplicity, when evaluating willingness, we assume that green consumers are dedicated. The case when green consumers are flexible can also be analyzed, but is technically more cumbersome. To calculate the firm's profits before and after implementation, we need to consider the decisions for both Product P and Product S. Therefore, we introduce the corresponding subscripts (p and s) to the notation defined for the single-product models in earlier sections. Let π_i^b (resp., π_i^a) denote Firm 1's maximum total profit before (resp., after) implementing the symbiotic system under Model i, $i \in \{M, CR, CG\}$.

4.1 Willingness in a Monopoly

Recall from Section 3.1 the optimal decisions under monopoly, obtained under the assumptions $\max\{c_{1,p}, c_{1,p}^{'}\} < 1$ and $\max\{c_{1,s}, c_{1,s}^{'}\} < 1$. Before implementing the symbiotic system, Firm 1's total profit from products P and S is: $\pi_M^b = \frac{(1-c_{1,p})^2}{4} + \frac{(1-c_{1,s})^2}{4}$. The total profit after implementation is: $\pi_M^a = \frac{\theta_1(1-c_{1,p}^{'})^2}{4} + \frac{(1-\theta_1)(a_p-c_{1,p}^{'})^2}{4a_p} + \frac{\theta_1(1-c_{1,s}^{'})^2}{4} + \frac{(1-\theta_1)(a_s-c_{1,s}^{'})^2}{4a_s} - K$.

To capture the firm's willingness to implement, let $\Delta_M = \pi_M^a - \pi_M^b$. To better understand the impact of parametric changes, it is convenient to partition the expression for Δ_M into three terms:

$$\Delta_M = \underbrace{\frac{(1-\theta_1)(a_p-1)(a_p-c_{1,p}^{'2})}{4a_p} + \frac{(1-\theta_1)(a_s-1)(a_s-c_{1,s}^{'2})}{4a_s}}_{T_1^M:\text{ The gain from exploiting the green market segment}}$$
$$+ \underbrace{\frac{(c_{1,p}-c_{1,p}^{'})(2-c_{1,p}-c_{1,p}^{'})}{4} + \frac{(c_{1,s}-c_{1,s}^{'})(2-c_{1,s}-c_{1,s}^{'})}{4}}_{T_2^M:\text{ The gain/loss from the changes in the variable production costs}} - \underbrace{K.}_{T_3^M:\text{ The fixed cost}}$$

These three terms can be further categorized into two types: Demand-side and Cost-side.

- Demand-side Influence: $T_1^M = \frac{(1-\theta_1)(a_p-1)(a_p-c_{1,p}^{'2})}{4a_p} + \frac{(1-\theta_1)(a_s-1)(a_s-c_{1,s}^{'2})}{4a_s}$

This represents the additional revenue for Firm 1 from providing the green variants of the two products (instead of the regular variants) to the green consumers. Since the green consumers have higher valuation for the green variants (relative to their regular counterparts), this term is always non-negative. If either (i) the green consumers do not exist (i.e., $\theta_1 = 1$), or (ii) the green consumers value the green variants the same as the regular variants (i.e., $a_p = 1$; $a_s = 1$), then we have $T_1^M = 0$.

- Cost-side Influence:

1. $T_2^M = \frac{(c_{1,p}-c_{1,p}^{'})(2-c_{1,p}-c_{1,p}^{'})}{4} + \frac{(c_{1,s}-c_{1,s}^{'})(2-c_{1,s}-c_{1,s}^{'})}{4}$

   This represents the gain/loss from changes in the variable production costs. If the production costs of both products reduce after the implementation of the symbiotic system (i.e., $c_{1,p}^{'} \leq c_{1,p}$ and $c_{1,s}^{'} \leq c_{1,s}$), then this term represents a benefit for Firm 1. Otherwise, if both production costs increase, then the term reflects a loss. In general, T_2^M can be either a benefit or a loss. If the variable production costs of both products remain the same after implementation (i.e., $c_{1,p} = c_{1,p}^{'}$ and $c_{1,s} = c_{1,s}^{'}$), then we have $T_2^M = 0$.

2. $T_3^M = K$

   This is the fixed cost incurred by Firm 1 in implementing the symbiotic system.

Our interest is in gaining insights on the conditions under which $\Delta_M \geq 0$, since this would indicate the firm's willingness to implement the system. The classification above implies the following remark.

Remark 1 In a monopoly, if the combined benefit of (i) exploiting the green market segment (T_1^M) and (ii) lowering the variable production cost (T_2^M, if $T_2^M > 0$) is more than the sum of the (1) higher variable production costs ($-T_2^M$, if $T_2^M < 0$) and (2) fixed cost (T_3^M), then the firm's willingness (Δ_M) to implement the symbiotic system is nonnegative. ■

Figure 4(a) illustrates Firm 1's combined benefit and combined cost from implementation with a change in the variable production cost after implementation ($c_{1,p}^{'}$), while keeping all other parameters fixed. To allow us to focus on one product (say Product P), we assume $c_{1,s} = c_{1,s}^{'}$ for simplicity. When $c_{1,p}^{'}$ is less than the value of the variable production cost $c_{1,p}$ before implementation, we have $T_2^M \geq 0$. Thus, when $c_{1,p}^{'} \leq c_{1,p}$, the combined benefit consists of two terms: T_1^M and T_2^M, while the combined cost is T_3^M. When $c_{1,p}^{'} > c_{1,p}$, the term T_2^M is negative. Thus, the combined benefit now consists of only one term: T_1^M, while the combined cost has two terms: $-T_2^M$ and T_3^M. As shown in the figure, the curve representing the combined benefit (resp., combined cost) of implementing the system decreases (resp., increases) with an increase in $c_{1,p}^{'}$. The willingness measure Δ_M changes sign at the threshold

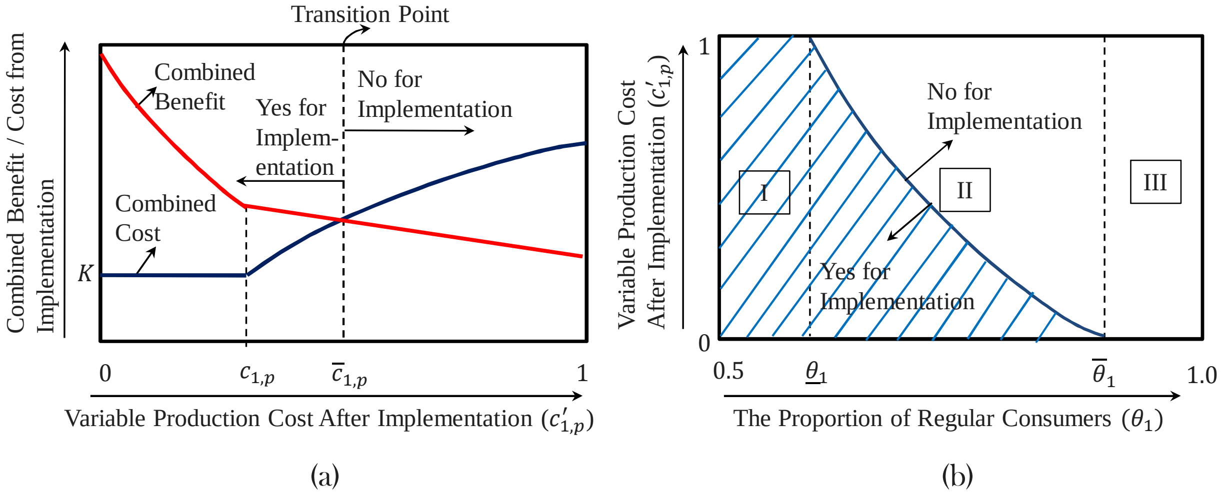

Figure 4: Willingness in a Monopoly: Influence on the Decision to Implement of the Change in the (a) Variable Production Cost after Implementation ($c'_{1,p}$), and (b) Proportion of Regular Consumers (θ_1)

production cost $\bar{c}_{1,p}$. As long as $c'_{1,p}$ is smaller than this threshold, the combined benefit exceeds the combined cost.

In Figure 4(b), we illustrate the impact of the proportion of regular consumers of Product P (θ_1) on the firm's decision to implement the system. Note that (i) $c'_{1,p}$ is between 0 and 1 and (ii) the value of Δ_M decreases with an increase in $c'_{1,p}$.

- First, consider the (worst-case) situation when $c'_{1,p}$ is at its upper bound 1. In this case, the gain from exploiting the green market is positive (i.e., $T_1^M > 0$), while the firm incurs a loss from changes in the variable product costs (i.e., $T_2^M < 0$). In Region I of Figure 4(b), where θ_1 is less than a threshold $\underline{\theta}_1$, the benefit of T_1^M dominates the combined loss of $-T_2^M$ and T_3^M. Consequently, we have $\Delta_M > 0$. Since Δ_M increases with a decrease in $c'_{1,p}$, we continue to have $\Delta_M > 0$ as $c'_{1,p}$ reduces from 1 to 0.

- Next, consider the (best-case) situation when $c'_{1,p}$ is at its lower bound 0. Here, the firm benefits from changes in the variable product costs (i.e., $T_2^M > 0$). Thus, the combined benefit consists of two terms T_1^M and T_2^M. However, the contribution of T_1^M decreases with an increase in θ_1. Therefore, in Region III of Figure 4(b), where θ_1 is greater than a threshold $\overline{\theta}_1$, the cost T_3^M dominates the combined benefit of T_1^M and T_2^M. Consequently, we have $\Delta_M < 0$. Again, as Δ_M decreases with an increase in $c'_{1,p}$, we continue to have $\Delta_M < 0$ as $c'_{1,p}$ increases from 0 to 1.

- In Region II, when the value of θ_1 is between the two thresholds $\underline{\theta}_1$ and $\overline{\theta}_1$, there is a healthier tradeoff between the combined cost and the combined benefit. For any value of θ_1 in this region, there exists a threshold of $c'_{1,p}$ (represented by the curve) below which the decision for implementation is in the affirmative.

In the next section, we consider the firm's willingness under competition.

4.2 Willingness under Competition

We first consider the firm's willingness under Model CR. Recall the optimal decisions under competition (Section 3.3.3), obtained under the assumptions $\max\{2c_{1,p}-1, 2c'_{1,p}-1, 0\} < c_{2,p} < \min\{\frac{1+c_{1,p}}{2}, \frac{1+c'_{1,p}}{2}\}$ and $\max\{2c_{1,s}-1, 2c'_{1,s}-1, 0\} < c_{2,s} < \min\{\frac{1+c_{1,s}}{2}, \frac{1+c'_{1,s}}{2}\}$. Firm 1's total profit from the two products before implementation is: $\pi^b_{CR} = \frac{(1-2c_{1,p}+c_{2,p})^2}{9} + \frac{(1-2c_{1,s}+c_{2,s})^2}{9}$. After implementation, the total profit is: $\pi^a_{CR} = \frac{\theta_1(1-2c'_{1,p}+c_{2,p})^2}{9} + \frac{(1-\theta_1)(a_p-c'_{1,p})^2}{4a_p} + \frac{\theta_1(1-2c'_{1,s}+c_{2,s})^2}{9} + \frac{(1-\theta_1)(a_s-c'_{1,s})^2}{4a_s} - K$. Let $\Delta_{CR} = \pi^a_{CR} - \pi^b_{CR}$. As before, we partition Δ_{CR} into four terms that can be conveniently interpreted:

$$\begin{aligned}\Delta_{CR} \;=\; & \underbrace{\frac{(1-\theta_1)(a_p-1)(a_p-c'^2_{1,p})}{4a_p} + \frac{(1-\theta_1)(a_s-1)(a_s-c'^2_{1,s})}{4a_s}}_{T_1^R:\text{ The gain from exploring the green market segment}} \\ & \underbrace{+\frac{(1-\theta_1)(1+c'_{1,p}-2c_{2,p})(5-7c'_{1,p}+2c_{2,p})}{36} + \frac{(1-\theta_1)(1+c'_{1,s}-2c_{2,s})(5-7c'_{1,s}+2c_{2,s})}{36}}_{T_2^R:\text{ The benefit of being the exclusive supplier of the green market segment}} \\ & + \underbrace{\frac{4(c_{1,p}-c'_{1,p})(1-c_{1,p}-c'_{1,p}+c_{2,p})}{9} + \frac{4(c_{1,s}-c'_{1,s})(1-c_{1,s}-c'_{1,s}+c_{2,s})}{9}}_{T_3^R:\text{ The gain/loss from the changes in the variable production costs}} - \underbrace{K}_{T_4^R:\text{ The fixed cost}}.\end{aligned}$$

T_1^R, T_3^R, and T_4^R have similar corresponding terms (T_1^M, T_2^M, and T_3^M, respectively) in the expression of Δ_M in Section 4.1. The only term unique to Δ_{CR} is T_2^R. We, therefore, discuss this expression here. T_2^R represents the additional revenue of Firm 1 for being the *exclusive supplier to the green consumers*. It can be shown that $T_2^R \geq 0$, with equality holding only when green consumers do not exist (i.e., $\theta_1 = 1$). Under competition for the regular variants, the demand-side impact is represented by T_1^R and T_2^R while the cost-side influence is reflected in T_3^R and T_4^R. This implies our next remark. The subsequent discussion offers insights on the willingness to implement.

Remark 2 Under competition for the regular variants of the two products (Model CR), if the combined benefit of (i) exploiting the green consumers (T_1^R), (ii) being the exclusive supplier of the green market segment (T_2^R), and (iii) lowering the variable production cost (T_3^R, if $T_3^R > 0$) exceeds the sum of the (1) higher variable production cost ($-T_3^R$, if $T_3^R < 0$) and (2) fixed cost (T_4^R), then the firm's willingness (Δ_{CR}) to implement the symbiotic system is nonnegative. ■

Figure 5 illustrates the impact of the appreciation a_p of the green variant of Product P on the firm's decision to implement the system. Again, note that (a) $c'_{1,p}$ is between $c'_{lb} = \max\{2c_{2,p}-1, 0\}$ and $c'_{ub} = \frac{1+c_{2,p}}{2}$ and (b) the value of Δ_{CR} decreases with an increase in $c'_{1,p}$. When $c'_{1,p}$ is at its lower bound c'_{lb}, the firm benefits from changes in the variable product costs (i.e., $T_3^R > 0$). Thus, the combined benefit consists of three terms T_1^R, T_2^R, and T_3^R. However, the contributions of T_1^R and T_2^R are marginal when a_p is close to its lower bound 1. Therefore, in Region I of Figure 5, where a_p is less than a threshold $\underline{a}_p$, the cost T_4^R dominates the combined benefit of T_1^R, T_2^R, and T_3^R. Consequently, we have $\Delta_{CR} < 0$.

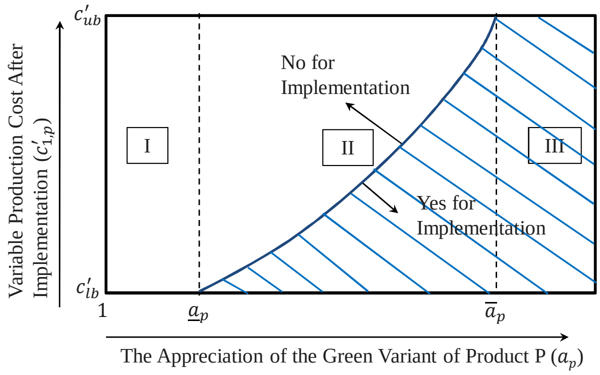

Figure 5: Willingness under Competition for Regular Variants: Influence on the Decision to Implement of the Change in the Appreciation of the Green Variant of Product P (a_p)

Since Δ_{CR} decreases with an increase in $c'_{1,p}$, we continue to have $\Delta_{CR} < 0$ as $c'_{1,p}$ increases from c'_{lb} to c'_{ub}. Now consider the situation when $c'_{1,p}$ is at its upper bound c'_{ub}. The firm incurs a loss from changes in the variable product costs (i.e., $T_3^R < 0$). However, the contributions of T_1^R and T_2^R increase with an increase of a_p. In Region III of Figure 5, where a_p is more than a threshold $\overline{a}_p$, the combined benefit of T_1^R and T_2^R dominates the combined loss of $-T_3^R$ and T_4^R. Consequently, we have $\Delta_{CR} > 0$ and (since $\Delta_{CR} > 0$ increases as $c'_{1,p}$ reduces) this continues to hold as $c'_{1,p}$ reduces from c'_{ub} to c'_{lb}. As in Figure 5, in the intermediate region (Region II), where the value of a_p is between the two thresholds $\underline{a}_p$ and $\overline{a}_p$, there exists a threshold of $c'_{1,p}$ (represented by the curve) below which the willingness to implement the symbiotic system is positive.

When the competitor can produce both regular and green variants (Model CG), the behavior of the the firm's willingness to implement the symbiotic system can be analyzed in a similar manner. We, therefore, avoid providing a detailed description here and briefly list the main change in the expression of the willingness Δ_{CG} under Model CG over Δ_{CR}.

In the willingness under Model CG, there exists an additional term reflecting the access to the green market. Before implementation of the system, under Model CR, both Firm 1 and Firm 2 provide the regular variant. Since the green variant is unavailable, Firm 1 provides the regular variant to the entire market which includes the dedicated green consumers. Under Model CG, Firm 2 produces the green variant. Since the dedicated green consumers only purchase the green variant, Firm 1 cannot access the green market before the implementation. After implementation, under Model CR, Firm 1 exclusively supplies the green variant to the green consumers. Under Model CG, Firm 1 gains access to the green market after implementation.

In the next section, our aim is to investigate how a change in the nature of competition affects the firm's willingness to implement the symbiotic system.

4.3 Shift in Willingness under Competition

In general, with the introduction of competition, Firm 1 produces less quantities of the two products and earns less profit. Competition also hurts Firm 1's market share. For example, if the competitor produces the green variant, then Firm 1 shares the green market after the implementation of the symbiotic system. Since one of the benefits of the implementation is to exploit the green market, competition reduces this benefit. Thus, an intuitive projection would be that Firm 1 would be less willing to implement the system under competition than under monopoly. A natural question arises: *Can the firm's willingness to implement the symbiotic system increase under competition, relative to that under a monopoly?* In Theorems 4, 5, 6 and their corresponding corollaries, our effort is to (a) identify and interpret scenarios where competition results in either an increase or a decrease in willingness and (b) understand the relative impact of the nature of the competition. The proofs are in Sections D, E, and F of the Appendix.

Theorem 4 *If $c_{2,p} \leq c'_{1,p} = c_{1,p}$, $c_{2,s} \leq c'_{1,s} = c_{1,s}$, then we have $\Delta_{CR} \geq \Delta_M$.*

Corollary 4 *If Firm 1's variable production costs of both products remain the same after the implementation, but the competitor continues to have a cost advantage over Firm 1, then competing with a firm who only produces regular variants encourages Firm 1 to implement the symbiotic system.*

Theorem 5 *If $c'_{1,p} = c_{1,p}$ and $c'_{1,s} = c_{1,s}$, then*

(a) If $a_p = 1$ and $a_s = 1$, we have $\Delta_{CG} \geq \Delta_M$.

(b) If $c_{2,p} \leq c_{1,p}$, $c_{2,s} \leq c_{1,s}$, $a_p \geq \frac{9}{5}$, and $a_s \geq \frac{9}{5}$, we have $\Delta_{CG} \leq \Delta_M$.

Corollary 5 *If the implementation of the symbiotic system does not change the variable production costs of both products, then*

(a) Even if the dedicated green consumers' appreciation for the green variants is marginal, the firm is more willing (than in a monopoly) to implement the symbiotic system when competing with firms who produce green variants.

(b) Even if the dedicated green consumers' appreciation for the green variants is higher than a threshold, competing with a firm who produces green variants discourages the firm to implement the symbiotic system if the competitor has cost advantages over Firm 1 for both products.

Theorem 6 *If $c_{2,p} \leq c_{1,p} = c'_{1,p}$, and $c_{2,s} \leq c_{1,s} = c'_{1,s}$, then $\Delta_{CG} < \Delta_{CR}$.*

Corollary 6 *If we suppress the savings in the variable production costs of both products, and assume that the competitor has cost advantages over Firm 1 for both products, then Firm 1 is more willing to implement the symbiotic system when the competitor can only produce regular variants as compared to the situation when the competitor produces green variants.*

4.4 Simultaneously Benefiting the Firm and Its Consumers

It is important to note that our comparison of willingness in Section 4.3 across the different settings was purely a relative one. In other words, we did not impose that the value of willingness be non-negative. Clearly, one desirable outcome in support of implementation would be that the willingness be positive. Furthermore, the motivation for implementing the system is higher if consumers (as a whole) benefit from it. In this section, our goal is to identify situations under which the above two conditions are simultaneously satisfied. We need the following additional notation.

Notation:

- W_i^b Consumer welfare before the implementation of symbiotic system under Model i, $i \in \{CR, CG\}$.
- W_i^a Consumer welfare after the implementation of symbiotic system under Model i, $i \in \{CR, CG\}$.
- $p_{i,j}^b$ Market price of variant i of Product j before the implementation of symbiotic system, $i \in \{r, g\}$, r (resp., g) represents regular (resp., green); $j \in \{p, s\}$.
- $p_{i,j}^a$ Market price of variant i of Product j after the implementation of symbiotic system, $i \in \{r, g\}$, r (resp., g) represents regular (resp., green); $j \in \{p, s\}$.

We first consider competition only for the regular variants (Model CR). Recall from Section 2 that consumers' valuation of the regular variant is uniformly distributed between 0 and 1. Therefore, we have

$$W_{CR}^b = \int_{p_{r,p}^b}^1 (v_p - p_{r,p}^b)\,\mathrm{d}v_p + \int_{p_{r,s}^b}^1 (v_s - p_{r,s}^b)\,\mathrm{d}v_s = \frac{(1-p_{r,p}^b)^2}{2} + \frac{(1-p_{r,s}^b)^2}{2} = \frac{(2-c_{1,p}-c_{2,p})^2}{18} + \frac{(2-c_{1,s}-c_{2,s})^2}{18}.$$

Similarly, we have

$$\begin{aligned} W_{CR}^a &= \int_{p_{r,p}^a}^1 [\theta_1(v_p - p_{r,p}^a)]\,\mathrm{d}v_p + \int_{\frac{p_{g,p}^a}{a_p}}^1 [(1-\theta_1)(a_p v_p - p_{g,p}^a)]\,\mathrm{d}v_p + \int_{p_{r,s}^a}^1 [\theta_1(v_s - p_{r,s}^a)]\,\mathrm{d}v_s + \int_{\frac{p_{g,s}^a}{a_s}}^1 [(1-\theta_1)(a_s v_s - p_{g,s}^a)]\,\mathrm{d}v_s \\ &= \frac{\theta_1(2-c_{1,p}^{'}-c_{2,p})^2}{18} + \frac{(1-\theta_1)(a_p-c_{1,p}^{'})^2}{8a_p} + \frac{\theta_1(2-c_{1,s}^{'}-c_{2,s})^2}{18} + \frac{(1-\theta_1)(a_s-c_{1,s}^{'})^2}{8a_s}. \end{aligned}$$

The following result identifies conditions under which consumer welfare improves after the implementation of the system; the proof is in Section G of the Appendix.

Theorem 7 *If $a_p \geq 2$, $a_s \geq 2$, $c_{1,p}^{'} \leq c_{1,p}$, and $c_{1,s}^{'} \leq c_{1,s}$, then we have $W_{CR}^a > W_{CR}^b$.*

Corollary 7 *When Firm 1 competes with Firm 2 who only produces regular variants, if the green consumers' appreciation for the green variant is relatively high, and the implementation of the symbiotic system does not increase the variable production costs of both products, then consumer welfare increases after the implementation.*

Theorem 7 can be used to identify a special case when the willingness of Firm 1 is negative in a monopoly but positive under competition, and consumers benefit from the implementation of the symbiotic system as well. The proof of the following result is in Section H of the Appendix.

Theorem 8 *If* $a_p = 2$, $a_s = 2$, $c_{2,p} \leq c_{1,p} = c'_{1,p}$, $c_{2,s} \leq c_{1,s} = c'_{1,s}$, *and* $K = \frac{(1-\theta_1)}{8}[(2-c_{1,p})^2 - (1-2c_{1,p}+c_{2,p})^2 + (2-c_{1,s})^2 - (1-2c_{1,s}+c_{2,s})^2]$, *then we have* $W^a_{CR} > W^b_{CR}$, $\Delta_{CR} > 0 > \Delta_M$.

Corollary 8 *If (i) green consumers appreciate the green variant about two times the regular variant, (ii) implementation of the symbiotic system does not change the variable production costs of both products, (iii) the competitor has a cost advantage over Firm 1, and (iv) the fixed cost of implementation of the symbiotic system is modest, then (a) willingness of Firm 1 to implement is negative in a monopoly, but positive under competition for the regular variants and (b) consumer welfare increases after the implementation.*

Next, we consider competition for both regular and green variants (Model CG). We first derive consumer welfare before and after the implementation of a symbiotic system.

$$\begin{aligned} W^b_{CG} &= \frac{\theta_1(1-p^b_{r,p})^2}{2} + \frac{(1-\theta_1)(a_p-p^b_{g,p})^2}{2a_p} + \frac{\theta_1(1-p^b_{r,s})^2}{2} + \frac{(1-\theta_1)(a_s-p^b_{g,s})^2}{2a_s}, \\ W^a_{CG} &= \frac{\theta_1(1-p^a_{r,p})^2}{2} + \frac{(1-\theta_1)(a_p-p^a_{g,p})^2}{2a_p} + \frac{\theta_1(1-p^a_{r,s})^2}{2} + \frac{(1-\theta_1)(a_s-p^a_{g,s})^2}{2a_s}. \end{aligned}$$

As with Model CR, our effort here is to identify conditions under which both Firm 1 and consumers benefit from the implementation of the symbiotic system. Theorem 9 identifies a case in which consumers benefit from the implementation; the proof is in Section I of the Appendix.

Theorem 9 *If* $c'_{1,p} \leq c_{1,p}$ *and* $c'_{1,s} \leq c_{1,s}$, *then we have* $W^a_{CG} > W^b_{CG}$.

Corollary 9 *When the competitor produces both regular and green variants, if the implementation of the symbiotic system reduces the variable production costs of both products, then consumer welfare increases after the implementation.*

The following result extends Theorem 9 by identifying a special case when the willingness of Firm 1 changes from negative (in a monopoly) to positive (under competition), and consumers benefit from the implementation of the symbiotic system as well. The proof is in Section J of the Appendix.

Theorem 10 *If* $a_p = 1$, $a_s = 1$, $c'_{1,p} = c_{1,p}$, $c'_{1,s} = c_{1,s}$, *and* $0 < K \leq \frac{(1-\theta_1)(1-2c_{1,p}+c_{2,p})^2}{9} + \frac{(1-\theta_1)(1-2c_{1,s}+c_{2,s})^2}{9}$, *then we have* $W^a_{CG} > W^b_{CG}$ *and* $\Delta_{CG} > 0 > \Delta_M$.

Corollary 10 *If (i) the green consumers' appreciation for the green variant is negligible, and (ii) the implementation of the symbiotic system does not change the variable production costs of both products,* *then Firm 1 is more willing to implement the symbiotic system under the competition with Firm 2 who produces both regular and green variants. The consumer welfare also increases after the implementation.*

Together, Theorems 8 and 10 and Corollaries 8 and 10 suggest that competition for either the regular variant or both regular and green variants can indeed shift a firm's willingness from negative to positive, and improve consumer welfare as well.

5 Directions for Future Work

Going back to the real-world implementation described in Section 1.1, the unique symbiotic model followed by SPB has transformed the livelihood of the local farming community and has provided a reliable supply of raw material for both SPB and Ponni Sugars. The rain shadow region around the paper-sugar complex has witnessed a green revolution, through the irrigation of more than 1500 acres of dry and fallow land with treated effluent. For the government, the positive societal impact of this implementation provides an ideal case study to identify and incentivize similar symbiotic systems. Some examples of possible incentives structures that can be studied include (a) providing a one-time subsidy, (b) providing long-term tax relief, (c) subsidizing the supply of electricity and water, and (d) improving societal consciousness of symbiotic initiatives. Chen et al. (2019) consider government subsidy in the form of a research joint venture.

In the models analyzed in this paper, we considered two products that are symbiotically connected. A much-larger example of Burnside Industrial Park, which involves about 1300 businesses, is described in Noronha (1999). The businesses within the industrial park are involved in (a) scavenger roles (reuse, remanufacture, refurbish, repair and recover), (b) decomposer roles (recycling), (c) producing/selling environment-friendly products, and (d) providing environmental management services. These companies deal in a diverse range of materials and have established complex relationships within the park. This example motivates the notion of *Design for Symbiosis*: problems of designing the layout of a large-scale symbiotic system, coordinating the material exchanges between the firms, and scheduling the logistics activities involved.

For the two-product symbiotic system analyzed in this paper, we assumed that the production decisions of both the products are taken by a common firm interested in maximizing its total profit. Under this centralized setting, even if the implementation of the system hurts the profit from one product, the firm may be willing to implement if total profit improves. Once the system is implemented, the complete exchange of wastes between the two production processes is guaranteed. However, if the products are manufactured by two independent firms, then participation in a symbiotic relationship becomes voluntary. Thus, in a decentralized setting, the problem of pricing the wastes becomes relevant.

On the one hand, if the price of one type of waste is too low, then the firm that generates the waste may be less willing to supply it to the other firm (that uses this waste as a raw material). On the other hand, if this price is excessive, then the firm that can potentially purchase the waste may switch to an alternative source of raw material. Thus, the pricing of the wastes affects not only the revenues of the firms involved but also their operations.

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Appendix: Technical Statements and Proofs

A Proof of Theorem 1

We consider four scenarios.
Scenario M1: $0 < p_g \le p_r$

Since Firm 1 charges a lower price for the green variant, all consumers choose to buy either a green variant or nothing. The aggregate market demand is as follows: $Q_r = 0, Q_g = (1-p_g)\theta_1 + (1-\frac{p_g}{a})(1-\theta_1)$. Thus, Firm 1 faces the following profit-maximization problem:

$$\max_{p_r, p_g} \quad \pi_1 = (p_g - c_1^{'})Q_g = (p_g - c_1^{'})[(1-p_g)\theta_1 + (1-\frac{p_g}{a})(1-\theta_1)].$$

The optimal price and optimal profit are as follows:

$$p_r^* \ge p_g^* = \frac{a + (a-1)\theta_1 c_1^{'} + c_1^{'}}{2[(a-1)\theta_1 + 1]}, \pi_1^{M1} = \frac{[a - (a-1)\theta_1 c_1^{'} - c_1^{'}]^2}{4a[(a-1)\theta_1 + 1]}.$$

Scenario M2: $p_r \le p_g \le ap_r$

Under this scenario, regular consumers purchase either the regular variant or nothing. Meanwhile, flexible and dedicated green consumers purchase either the green variant or nothing. The aggregate market demand is as follows: $Q_r = (1-p_r)\theta_1, Q_g = (1-\frac{p_g}{a})(1-\theta_1)$. Thus, Firm 1 faces the following profit-maximization problem.

$$\begin{aligned}\max_{p_r, p_g} \quad \pi_1 &= (p_r - c_1^{'})Q_r + (p_g - c_1^{'})Q_g \\ &= (p_r - c_1^{'})(1-p_r)\theta_1 + (p_g - c_1^{'})(1-\frac{p_g}{a})(1-\theta_1).\end{aligned}$$

The optimal solution is $p_r^* = \frac{1+c_1^{'}}{2}$, $p_g^* = \frac{a+c_1^{'}}{2}$. Since $a \ge 1$, we have $p_r^* \le p_g^* \le ap_r^*$. Thus, Firm 1's maximum profit under Scenario M2 is

$$\pi_1^{M2} = \frac{\theta_1(1-c_1^{'})^2}{4} + \frac{(1-\theta_1)(a-c_1^{'})^2}{4a}.$$

We compare the profits under scenarios M1 and M2.

$$\pi_1^{M2} - \pi_1^{M1} = \frac{\theta_1(1-c_1^{'})^2}{4} + \frac{(1-\theta_1)(a-c_1^{'})^2}{4a} - \frac{[a-(a-1)\theta_1 c_1^{'} - c_1^{'}]^2}{4a[(a-1)\theta_1+1]} = \frac{\theta_1(1-\theta_1)(a-1)^2}{4[(a-1)\theta_1+1]} \ge 0.$$

Scenario M3: $(a-1) + p_r \le p_g \le a$

Under this scenario, regular and flexible green consumers purchase either the regular variant or nothing. Dedicated green consumers purchase either the green variant or nothing. The aggregate market demand is as follows: $Q_r = (1-p_r)(\theta_1+\theta_2), Q_g = (1-\frac{p_g}{a})\theta_3$. Thus, Firm 1 faces the following profit-maximization problem:

$$\begin{aligned}\max_{p_r, p_g} \quad \pi_1 &= (p_r - c_1^{'})Q_r + (p_g - c_1^{'})Q_g = (p_r - c_1^{'})(1-p_r)(\theta_1+\theta_2) + (p_g - c_1^{'})(1-\frac{p_g}{a})\theta_3 \\ s.t. \quad & \quad (a-1) + p_r \le p_g \le a.\end{aligned}$$

We first solve the unconstrained problem; the corresponding optimal solution is as follows:

$$p_r^* = \frac{1+c_1^{'}}{2}, p_g^* = \frac{a+c_1^{'}}{2}, \pi_{1,r}^* = \frac{(\theta_1+\theta_2)(1-c_1^{'})^2}{4}, \pi_{1,g}^* = \frac{\theta_3(a-c_1^{'})^2}{4a}.$$

Clearly, Firm 1's maximum profit is bounded from above by that of the unconstrained problem. Thus, $\pi_1^{M3} \le \frac{(\theta_1+\theta_2)(1-c_1^{'})^2}{4} + \frac{\theta_3(a-c_1^{'})^2}{4a}$. We can then compare the profits under Scenarios M2 and M3.

$$\begin{aligned}\pi_1^{M2} - \pi_1^{M3} &\ge \frac{\theta_1(1-c_1^{'})^2}{4} + \frac{(1-\theta_1)(a-c_1^{'})^2}{4a} - \frac{(\theta_1+\theta_2)(1-c_1^{'})^2}{4} - \frac{\theta_3(a-c_1^{'})^2}{4a} \\ &= \theta_2[\frac{(a-c_1^{'})^2}{4a} - \frac{(1-c_1^{'})^2}{4}] = \frac{\theta_2(a-1)[a^2-(c_1^{'})^2]}{4a} \ge 0.\end{aligned}$$

Scenario M4: $ap_r \le p_g \le (a-1) + p_r$

Under this scenario, regular consumers purchase the regular variant or nothing and dedicated green consumers purchase the green variant or nothing. Among flexible green consumers, some buy the regular variant, some others buy the green variant, and the remaining buy nothing. The aggregate market demand is as follows: $Q_r = (1-p_r)\theta_1 + \frac{p_g - ap_r}{a-1}\theta_2, Q_g = (1-\frac{p_g}{a})\theta_3 + (1-\frac{p_g-p_r}{a-1})\theta_2$.

The firm's total profit can be written as follows:

$$\begin{aligned}
\max_{p_r,p_g} \quad \pi_1 &= Q_r p_r + Q_g p_g - c_1^{'}(Q_r+Q_g) \\
&= -(\theta_1+\frac{a\theta_2}{a-1})p_r^2+\frac{2\theta_2}{a-1}p_g p_r+[\theta_1+c_1^{'}(\theta_1+\theta_2)]p_r-(\frac{\theta_3}{a}+\frac{\theta_2}{a-1})p_g^2+[(\theta_2+\theta_3)+c_1^{'}\frac{\theta_3}{a}]p_g-c_1^{'} \\
s.t. \quad & p_g - ap_r \geq 0, \\
& (a-1)+p_r-p_g \geq 0.
\end{aligned}$$

The Lagrangean and the Karush-Kuhn-Tucker optimality conditions are:

$$\begin{aligned}
L(p_r,p_g) &= \pi_1+\lambda_1(p_g-ap_r)+\lambda_2[(a-1)+p_r-p_g] \\
\frac{\partial L}{\partial p_r} &= -2(\theta_1+\frac{a\theta_2}{a-1})p_r+\frac{2\theta_2}{a-1}p_g+[\theta_1+c_1^{'}(\theta_1+\theta_2)]-a\lambda_1+\lambda_2=0 \\
\frac{\partial L}{\partial p_g} &= -2(\frac{\theta_3}{a}+\frac{\theta_2}{a-1})p_g+\frac{2\theta_2}{a-1}p_r+[(\theta_2+\theta_3)+c_1^{'}\frac{\theta_3}{a}]+\lambda_1-\lambda_2=0 \\
\lambda_1(p_g-ap_r) &= 0, \\
\lambda_2[(a-1)+p_r-p_g] &= 0.
\end{aligned}$$

- **Scenario M4.1:** $\lambda_1=0$ and $\lambda_2=0$

Solving the above system, we have $p_g^*=\frac{a+c_1^{'}}{2}, p_r^*=\frac{1+c_1^{'}}{2}$. However, since $p_g^*-ap_r^*=\frac{(1-a)c_1^{'}}{2}<0$, the solution is invalid.

- **Scenario M4.2:** $p_g=ap_r$

We can rewrite the demand functions as follows: $Q_r=(1-p_r)\theta_1, Q_g=(1-\frac{p_g}{a})(1-\theta_1)$. Thus, the objective function has the same form as in Scenario M2. However, the optimization problem here has an additional constraint $p_g=ap_r$. Thus, the optimal value obtained in Scenario M2 is at least as good as that under Scenario M4.2.

- **Scenario M4.3:** $p_g=(a-1)+p_r$

We can rewrite the demand functions as follows: $Q_r=(1-p_r)(\theta_1+\theta_2), Q_g=(1-\frac{p_g}{a})\theta_3$. Thus, the objective function has the same form as in Scenario M3. However, the optimization problem here has an additional constraint $p_g=(a-1)+p_r$. Thus, the optimal value obtained in Scenario M3 is at least as good as that under Scenario M4.3.

Combining the analysis above, we can conclude that the decisions obtained under Scenario M2 are optimal. This completes the proof. ■

B Proof of Theorem 2

In equilibrium, Firm 1 provides both regular and green variants. Firm 2 only provides the regular variant. Recall that among the multiple price settings described in Section 2.2, only $p_r<p_g\leq ap_r$ and $ap_r\leq p_g<(a-1)+p_r$ guarantee the existence of both regular and green variants in equilibrium. Thus, following our assumptions in Section 3.2, we consider only these two price settings in our analysis.

1. If $c_1^{'}<\frac{\theta_1}{2(1+\theta_1)}$ and $c_1^{'}<c_2<\frac{1}{2}[1+\frac{c_1^{'}}{a(1-\theta_1)+\theta_1}]$

We first consider the values of the lower and upper bounds of c_2. Since $a>1$, $a(1-\theta_1)+\theta_1$ decreases with an increase in θ_1. Thus, for $0\leq\theta_1\leq1$, we have $1\leq a(1-\theta_1)+\theta_1\leq a$. Therefore, we have $\frac{1}{2}+\frac{c_1^{'}}{2a}\leq\frac{1}{2}[1+\frac{c_1^{'}}{a(1-\theta_1)+\theta_1}]\leq\frac{1}{2}+\frac{c_1^{'}}{2}$. Thus, $c_2<\frac{1}{2}[1+\frac{c_1^{'}}{a(1-\theta_1)+\theta_1}]\leq(\frac{1}{2}+\frac{c_1^{'}}{2})$.

Since $a>1$, $0\leq\theta_1\leq1$, and $c_1^{'}<\frac{\theta_1}{2(1+\theta_1)}$, we have

$$\begin{aligned}
&(\frac{1}{2}+\frac{c_1^{'}}{2a})-[\frac{1}{2}-(1-\frac{3}{2a})c_1^{'}]=(1-\frac{1}{a})c_1^{'}>0, \\
&[\frac{1}{2}-(1-\frac{3}{2a})c_1^{'}]-(\frac{1}{2}-\frac{1}{\theta_1}c_1^{'})=(\frac{3}{2a}+\frac{1-\theta_1}{\theta_1})c_1^{'}>0, \\
&(\frac{1}{2}-\frac{1}{\theta_1}c_1^{'})-c_1^{'}=\frac{1}{2}-(1+\frac{1}{\theta_1})c_1^{'}>\frac{1}{2}-(1+\frac{1}{\theta_1})\frac{\theta_1}{2(1+\theta_1)}=0.
\end{aligned}$$

Thus, $c_1^{'}<(\frac{1}{2}-\frac{1}{\theta_1}c_1^{'})<[\frac{1}{2}-(1-\frac{3}{2a})c_1^{'}]<(\frac{1}{2}+\frac{c_1^{'}}{2a})\leq\frac{1}{2}[1+\frac{c_1^{'}}{a(1-\theta_1)+\theta_1}]$. Therefore, all the three regions, which represent the corresponding three types of equilibria, have positive lengths.

We now consider three scenarios:

Scenario D1: $p_r<p_g<ap_r$

Under this scenario, regular consumers only purchase regular products and flexible green consumers only purchase green products. The aggregate market demand is as follows: $Q_r=(1-p_r)\theta_1, Q_g=(1-\frac{p_g}{a})(1-\theta_1)$. Thus, we have $p_r=1-\frac{q_{1,r}+q_{2,r}}{\theta_1}$. Firm 1's total profit can be written as follows:

$$\max_{q_{1,r},p_g} \quad \pi_1 = q_{1,r}(p_r-c_1^{'})+Q_g(p_g-c_1^{'})=q_{1,r}(1-\frac{q_{1,r}+q_{2,r}}{\theta_1}-c_1^{'})+(1-\frac{p_g}{a})(1-\theta_1)(p_g-c_1^{'}).$$

The first-order conditions are

$$\frac{\partial \pi_1}{\partial q_{1,r}} = 1 - \frac{q_{1,r}+q_{2,r}}{\theta_1} - c_1^{'} - \frac{q_{1,r}}{\theta_1} = 0, \tag{1}$$

$$\frac{\partial \pi_1}{\partial p_g} = (1-\frac{p_g}{a})(1-\theta_1) - \frac{(1-\theta_1)(p_g - c_1^{'})}{a} = 0. \tag{2}$$

Next we find the Hessian for $\pi_1(q_{1,r}, p_g)$

$$H(q_{1,r}, p_g) = \begin{bmatrix} -\frac{2}{\theta_1} & 0 \\ 0 & -\frac{2(1-\theta_1)}{a} \end{bmatrix}$$

Since $H_1(q_{1,r}, p_g) = -\frac{2}{\theta_1} < 0$, $H_2(q_{1,r}, p_g) = (-\frac{2}{\theta_1})(-\frac{2(1-\theta_1)}{a}) > 0$, the first-order conditions are both necessary and sufficient.
Firm 2's profit is $\max_{q_{2,r}} \pi_2 = (p_r - c_2)q_{2,r} = (1 - \frac{q_{1,r}+q_{2,r}}{\theta_1} - c_2)q_{2,r}$. The first-order condition is

$$\frac{\partial \pi_2}{\partial q_{2,r}} = 1 - \frac{q_{1,r}+q_{2,r}}{\theta_1} - c_2 - \frac{q_{2,r}}{\theta_1} = 0 \tag{3}$$

Solving (1), (2), and (3), we obtain the following solution:

	Regular Market	Green Market
q_1^*	$\frac{\theta_1(1-2c_1^{'}+c_2)}{3}$	$\frac{(1-\theta_1)(a-c_1^{'})}{2a}$
q_2^*	$\frac{\theta_1(1+c_1^{'}-2c_2)}{3}$	-
p^*	$\frac{1+c_1^{'}+c_2}{3}$	$\frac{a+c_1^{'}}{2}$

Verifying Validity: We now verify that the quantities derived above are all positive. Since $c_1^{'} < c_2 < \frac{1+c_1^{'}}{2}$, we have $1 - 2c_1^{'} + c_2 = (1-c_1^{'}) + (c_2 - c_1^{'}) > 0$, and $1 + c_1^{'} - 2c_2 > 0$. Thus, $q_{1,r}^* > 0$ and $q_{2,r}^* > 0$. Also, since $c_1^{'} < \frac{1+c_1^{'}}{2}$, we have $c_1^{'} < 1 < a$. Thus, $q_{1,g}^* > 0$. We also need to verify that the optimal prices satisfy the constraint $p_r < p_g < ap_r$. We have $p_g - p_r = \frac{(3a-2)+c_1^{'}-2c_2}{6} \geq \frac{1+c_1^{'}-2c_2}{6} > 0$. We also have $ap_r - p_g = \frac{-a+(2a-3)c_1^{'}+2ac_2}{6}$. Therefore, if $c_2 > \frac{1}{2} - (1-\frac{3}{2a})c_1^{'}$, then we have $ap_r^* > p_g^*$. Thus, the above solution is valid.

Scenario D2: $p_g = ap_r$
The aggregate market demand is the same as that in scenario D1. We have $p_r = 1 - \frac{q_{1,r}+q_{2,r}}{\theta_1}$ and $p_g = ap_r$. Firm 1's total profit can be written as follows:

$$\begin{aligned}\max_{q_{1,r}} \quad \pi_1 &= q_{1,r}(p_r - c_1^{'}) + Q_g(ap_g - c_1^{'}) = q_{1,r}(p_r - c_1^{'}) + (1-p_r)(1-\theta_1)(ap_r - c_1^{'}) \\ &= q_{1,r}(1 - \frac{q_{1,r}+q_{2,r}}{\theta_1} - c_1^{'}) + \frac{(q_{1,r}+q_{2,r})}{\theta_1}(1-\theta_1)[a(1-\frac{q_{1,r}+q_{2,r}}{\theta_1}) - c_1^{'}].\end{aligned}$$

The first-order condition is

$$\frac{\partial \pi_1}{\partial q_{1,r}} = 1 - \frac{q_{1,r}+q_{2,r}}{\theta_1} - c_1^{'} - \frac{q_{1,r}}{\theta_1} + \frac{(1-\theta_1)}{\theta_1}[a(1-\frac{q_{1,r}+q_{2,r}}{\theta_1}) - c_1^{'}] - \frac{a(q_{1,r}+q_{2,r})}{\theta_1^2}(1-\theta_1) = 0. \tag{4}$$

Firm 2's profit is $\max_{q_{2,r}} \pi_2 = (p_r - c_2)q_{2,r} = (1 - \frac{q_{1,r}+q_{2,r}}{\theta_1} - c_2)q_{2,r}$. The first-order condition is

$$\frac{\partial \pi_2}{\partial q_{2,r}} = 1 - \frac{q_{1,r}+q_{2,r}}{\theta_1} - c_2 - \frac{q_{2,r}}{\theta_1} = 0 \tag{5}$$

Solving (4) and (5), we obtain the following solution:

	Regular Market	Green Market
q_1^*	$\frac{\theta_1[\theta_1 - 2c_1^{'} + c_2(2a-2a\theta_1+\theta_1)]}{2a(1-\theta_1)+3\theta_1}$	$\frac{(1-\theta_1)[(a-a\theta_1+2\theta_1)-c_1^{'}-c_2\theta_1]}{2a(1-\theta_1)+3\theta_1}$
q_2^*	$\frac{\theta_1[(a-a\theta_1+\theta_1)+c_1^{'}-2c_2(a-a\theta_1+\theta_1)]}{2a(1-\theta_1)+3\theta_1}$	-
p^*	$\frac{(a+\theta_1-a\theta_1)+c_1^{'}+c_2\theta_1}{2a(1-\theta_1)+3\theta_1}$	$\frac{a[(a+\theta_1-a\theta_1)+c_1^{'}+c_2\theta_1]}{2a(1-\theta_1)+3\theta_1}$

Verifying Validity: We now verify that the quantities derived above are all positive. Since $c_1^{'} < c_2$, we have $\theta_1 - 2c_1^{'} + c_2(2a - 2a\theta_1 + \theta_1) = \theta_1(1-c_1^{'}) + (2-\theta_1)(c_2 - c_1^{'}) + 2(a-1)(1-\theta_1)c_2 > 0$. Thus, $q_{1,r}^* > 0$. Since $c_2 < \frac{1}{2}[1 + \frac{c_1^{'}}{a(1-\theta_1)+\theta_1}]$, we have $(a - a\theta_1 + \theta_1) + c_1^{'} - 2c_2(a - a\theta_1 + \theta_1) > 0$. Thus, $q_{2,r}^* > 0$. Also, since $c_1^{'} < 1$ and $c_2 < 1$, we have $(a - a\theta_1 + 2\theta_1) - c_1^{'} - c_2\theta_1 > (a - a\theta_1 + 2\theta_1) - 1 - \theta_1 = (a-1)(1-\theta_1) \geq 0$. Thus, $q_{1,g}^* > 0$.

Scenario D3: $ap_r < p_g < (a-1) + p_r$

Under this scenario, regular consumers purchase the regular variant or nothing. Among flexible green consumers, some buy the regular variant, some others buy the green variant, and the remaining buy nothing. The aggregate market demand is as follows: $Q_r = (1-p_r)\theta_1 + \frac{p_g - ap_r}{a-1}(1-\theta_1), Q_g = (1 - \frac{p_g - p_r}{a-1})(1-\theta_1)$.

Recall our assumption (from Section 3.2) that the market price of the regular variant is determined by the Cournot inverse demand function of the regular consumers. Thus, we have $p_r = 1 - \frac{q_{1,r}+q_{2,r}}{\theta_1}$. Firm 1's total profit can be written as follows:

$$\begin{aligned}\max_{q_{1,r},p_g} \quad \pi_1 &= [q_{1,r} + \frac{p_g - ap_r}{a-1}(1-\theta_1)](p_r - c_1^{'}) + (1 - \frac{p_g - p_r}{a-1})(1-\theta_1)(p_g - c_1^{'}) \\ &= [q_{1,r} + \frac{p_g - a(1 - \frac{q_{1,r}+q_{2,r}}{\theta_1})}{a-1}(1-\theta_1)](1 - \frac{q_{1,r}+q_{2,r}}{\theta_1} - c_1^{'}) + (1 - \frac{p_g - (1 - \frac{q_{1,r}+q_{2,r}}{\theta_1})}{a-1})(1-\theta_1)(p_g - c_1^{'}).\end{aligned}$$

The first-order conditions are

$$\frac{\partial \pi_1}{\partial q_{1,r}} = [1 + \frac{a(1-\theta_1)}{(a-1)\theta_1}](1 - \frac{q_{1,r}+q_{2,r}}{\theta_1} - c_1^{'}) - \frac{1}{\theta_1}[q_{1,r} + \frac{p_g - a(1 - \frac{q_{1,r}+q_{2,r}}{\theta_1})}{a-1}(1-\theta_1)] - \frac{(1-\theta_1)(p_g - c_1^{'})}{\theta_1(a-1)} = 0, \tag{6}$$

$$\frac{\partial \pi_1}{\partial p_g} = \frac{1-\theta_1}{a-1}(1 - \frac{q_{1,r}+q_{2,r}}{\theta_1} - c_1^{'}) + (1 - \frac{p_g - (1 - \frac{q_{1,r}+q_{2,r}}{\theta_1})}{a-1})(1-\theta_1) - \frac{1-\theta_1}{a-1}(p_g - c_1^{'}) = 0. \tag{7}$$

Next, we consider the Hessian for $\pi_1(q_{1,r}, p_g)$

$$H(q_{1,r}, p_g) = \begin{bmatrix} -\frac{2(a-\theta_1)}{\theta_1^2(a-1)} & -\frac{2(1-\theta_1)}{\theta_1(a-1)} \\ -\frac{2(1-\theta_1)}{\theta_1(a-1)} & -\frac{2(1-\theta_1)}{a-1} \end{bmatrix}$$

Since $H_1(q_{1,r}, p_g) = -\frac{2(a-\theta_1)}{\theta_1^2(a-1)} < 0$, $H_2(q_{1,r}, p_g) = \frac{4(1-\theta_1)}{\theta_1^2(a-1)} > 0$, the first-order conditions are both necessary and sufficient.

Firm 2's profit is $\max_{q_{2,r}} \pi_2 = (p_r - c_2)q_{2,r} = (1 - \frac{q_{1,r}+q_{2,r}}{\theta_1} - c_2)q_{2,r}$.

The first-order condition is

$$\frac{\partial \pi_2}{\partial q_{2,r}} = 1 - \frac{q_{1,r}+q_{2,r}}{\theta_1} - c_2 - \frac{q_{2,r}}{\theta_1} = 0 \tag{8}$$

Solving (6), (7), (8), we obtain the following solution:

	Regular Market	Green Market
q_1^*	$\frac{\theta_1[\theta_1 - 2c_1^{'} + c_2(2-\theta_1)]}{2+\theta_1}$	$\frac{(1-\theta_1)}{2}$
q_2^*	$\frac{\theta_1(1+c_1^{'}-2c_2)}{2+\theta_1}$	-
p^*	$\frac{1+c_1^{'}+c_2\theta_1}{2+\theta_1}$	$\frac{(2a+a\theta_1-\theta_1)+2c_1^{'}+2c_2\theta_1}{2(2+\theta_1)}$

Verifying Validity: Since $c_1^{'} < c_2$ and $c_1^{'} < 1$, we have $\theta_1 - 2c_1^{'} + c_2(2-\theta_1) = \theta_1(1-c_1^{'}) + (2-\theta_1)(c_2 - c_1^{'}) > 0$. Thus, $q_{1,r}^* > 0$. Since $c_2 < (\frac{1}{2} + \frac{c_1^{'}}{2})$, we have $q_{2,r}^* > 0$. Also, $q_{1,g}^* > 0$.

We also need to verify that the optimal prices satisfy the constraint $p_g \geq ap_r$. We have $p_g - ap_r = \frac{(a-1)(\theta_1 - 2c_1^{'} - 2\theta_1 c_2)}{2(2+\theta_1)}$. Thus, if $c_2 < (\frac{1}{2} - \frac{1}{\theta_1}c_1^{'})$, then we have $p_g - ap_r > 0$.

We consider the two interior solutions in scenario D1 and scenario D3 (given that each satisfies the corresponding constraint) and one boundary solution in scenario D2. The equilibrium prices in these three scenarios are categorized and validated as follows:

- Type I: If $p_r < p_g < ap_r$, we have $p_r^* = \frac{1+c_1^{'}+c_2}{3}, p_g^* = \frac{a+c_1^{'}}{2}$. Under the imposed condition, $[\frac{1}{2} - (1 - \frac{3}{2a}c_1^{'})] < c_2 < \frac{1}{2}[1 + \frac{c_1^{'}}{a(1-\theta_1)+\theta_1}]$, we indeed have have $p_r^* < p_g^* < ap_r^*$.
- Type II: If $ap_r = p_g$, we have $p_r^* = \frac{(a+\theta_1-a\theta_1)+c_1^{'}+\theta_1 c_2}{2a(1-\theta_1)+3\theta_1}, p_g^* = \frac{a(a+\theta_1-a\theta_1)+ac_1^{'}+a\theta_1 c_2}{2a(1-\theta_1)+3\theta_1}$.
- Type III: If $ap_r < p_g < (a-1) + p_r$, we have $p_r^* = \frac{1+c_1^{'}+\theta_1 c_2}{2+\theta_1}, p_g^* = \frac{(2a+a\theta_1-\theta_1)+2c_1^{'}+2\theta_1 c_2}{2(2+\theta_1)}$. Under the imposed condition, $c_1^{'} < c_2 < (\frac{1}{2} - \frac{1}{\theta_1}c_1^{'})$, we have $ap_r^* < p_g^* < (a-1) + p_r^*$.

Since $[\frac{1}{2} - (1 - \frac{3}{2a}c_1^{'})] > [\frac{1}{2} - \frac{1}{\theta_1}c_1^{'}]$, Type I and Type III solutions cannot both be valid. When either of these two solutions is valid, it is straightforward to show that the valid solution is also better than the Type II solution. When neither is valid, the Type II solution is optimal.

2. If $\frac{\theta_1}{2(1+\theta_1)} \leq c_1^{'} < \frac{a}{4a-3}$ and $c_1^{'} < c_2 < \frac{1}{2}[1 + \frac{c_1^{'}}{a(1-\theta_1)+\theta_1}]$

We have $(\frac{1}{2} - \frac{1}{\theta_1} c_1^{'}) \leq c_1^{'} < [\frac{1}{2} - (1 - \frac{3}{2a} c_1^{'})] < (\frac{1}{2} + \frac{c_1^{'}}{2a})$. Therefore, only the two regions corresponding to Types I and II equilibria have positive lengths.

3. If $\frac{a}{4a-3} \leq c_1^{'} < \frac{1}{2 - \frac{1}{a(1-\theta_1)+\theta_1}}$ and $c_1^{'} < c_2 < \frac{1}{2}[1 + \frac{c_1^{'}}{a(1-\theta_1)+\theta_1}]$

We have $[\frac{1}{2} - (1 - \frac{3}{2a}) c_1^{'}] \leq c_1^{'} < (\frac{1}{2} + \frac{c_1^{'}}{2a})$. Only the region corresponding to the Type I equilibrium exists.

This completes the proof. ■

C Proof of Theorem 3

Proof: Since each firm has two strategies, there are four possible combinations. We derive the payoffs of both firms under each combination in the following analysis.

1. Combination (g, g): If both firms only provide the green variant

   All consumers choose to buy either a green variant or nothing. The aggregate market demand is as follows:

$$q_{1,g} + q_{2,g} = Q_g = (1 - p_g)\theta_1 + (1 - \frac{p_g}{a})(1 - \theta_1).$$

   Thus, $p_g = \frac{1 - q_{1,g} - q_{2,g}}{\theta_1 + \frac{1-\theta_1}{a}}$. The two firms face the following profit-maximization problems:

$$\begin{aligned} \max_{q_{1,g}} \quad \pi_1 &= (p_g - c_1^{'}) q_{1,g}, \\ \max_{q_{2,g}} \quad \pi_2 &= (p_g - c_2) q_{2,g}. \end{aligned}$$

   By using the method similar to that used in the proof of Proposition 3, we obtain the equilibrium results as follows:

$$p_g^* = \frac{1}{3}[c_1^{'} + c_2 + \frac{a}{1 + (a-1)\theta_1}],$$

$$q_{1,g}^* = \frac{a - 2c_1^{'} + c_2 - (a-1)(2c_1^{'} - c_2)\theta_1}{3a}, q_{2,g}^* = \frac{a + c_1^{'} - 2c_2 + (a-1)(c_1^{'} - 2c_2)\theta_1}{3a},$$

$$\pi_1^* = \frac{[a - 2c_1^{'} + c_2 - (a-1)(2c_1^{'} - c_2)\theta_1]^2}{9a[1 + (a-1)\theta_1]}, \pi_2^* = \frac{[a + c_1^{'} - 2c_2 + (a-1)(c_1^{'} - 2c_2)\theta_1]^2}{9a[1 + (a-1)\theta_1]}.$$

   We check whether the production quantities derived are positive.

   - If $(2c_1^{'} - c_2) \geq 0$, then $[a - 2c_1^{'} + c_2 - (a-1)(2c_1^{'} - c_2)\theta_1]$ reaches its minimum when θ_1 reaches its upper bound 1. Thus, $a - 2c_1^{'} + c_2 - (a-1)(2c_1^{'} - c_2)\theta_1 \geq a - 2c_1^{'} + c_2 - (a-1)(2c_1^{'} - c_2) = a(1 - 2c_1^{'} + c_2)$. Since $c_2 > 2c_1^{'} - 1$, we have $a(1 - 2c_1^{'} + c_2) > 0$. Thus, $q_{1,g}^* > 0$.
   - If $(2c_1^{'} - c_2) < 0$, then $[a - 2c_1^{'} + c_2 - (a-1)(2c_1^{'} - c_2)\theta_1]$ reaches its minimum when θ_1 reaches its lower bound 0. Thus, $a - 2c_1^{'} + c_2 - (a-1)(2c_1^{'} - c_2)\theta_1 \geq a - 2c_1^{'} + c_2 \geq (1 - 2c_1^{'} + c_2) > 0$. Thus, $q_{1,g}^* > 0$.

   Similarly, we can show $q_{2,g}^* > 0$. Thus, the solution above is valid. These two firms' profits are as follows.

$$\pi_1^{g,g} = \frac{[a - 2c_1^{'} + c_2 - (a-1)(2c_1^{'} - c_2)\theta_1]^2}{9a[1 + (a-1)\theta_1]}, \pi_2^{g,g} = \frac{[a + c_1^{'} - 2c_2 + (a-1)(c_1^{'} - 2c_2)\theta_1]^2}{9a[1 + (a-1)\theta_1]}.$$

2. Combination (rg, rg): If both firms provide both regular and green variants

   Under this scenario, both firms compete in both the regular and the green markets. We obtain the equilibrium results as follows:

$$p_r^* = \frac{1 + c_1^{'} + c_2}{3}, p_g^* = \frac{a + c_1^{'} + c_2}{3}, q_{1,r}^* = \frac{\theta_1(1 - 2c_1^{'} + c_2)}{3},$$

$$q_{2,r}^* = \frac{\theta_1(1 + c_1^{'} - 2c_2)}{3}, q_{1,g}^* = \frac{(1-\theta_1)(a - 2c_1^{'} + c_2)}{3a}, q_{2,g}^* = \frac{(1-\theta_1)(a + c_1^{'} - 2c_2)}{3a}.$$

   We have $p_g^* - p_r^* = \frac{(a-1)}{3} > 0$. We can show all four production quantities are positive. These two firms' profits are as follows.

$$\pi_1^{rg,rg} = \frac{\theta_1(1 - 2c_1^{'} + c_2)^2}{9} + \frac{(1-\theta_1)(a - 2c_1^{'} + c_2)^2}{9a}, \quad \pi_2^{rg,rg} = \frac{\theta_1(1 + c_1^{'} - 2c_2)^2}{9} + \frac{(1-\theta_1)(a + c_1^{'} - 2c_2)^2}{9a}.$$

   If we compare the profits under this setting with those under (g, g), we have

$$\pi_1^{rg,rg} - \pi_1^{g,g} = \frac{\theta_1(1-\theta_1)(a-1)^2}{9[1 + (a-1)\theta_1]} \geq 0, \quad \pi_2^{rg,rg} - \pi_2^{g,g} = \frac{\theta_1(1-\theta_1)(a-1)^2}{9[1 + (a-1)\theta_1]} \geq 0.$$

3. Combination (g, rg): If Firm 1 only provides the green variant, Firm 2 provides both regular and green variants
We now compare these two firms' profits under this setting and those under the setting (rg, rg). First, both firms obtain the same amount of profits from the green market under both settings. Second, Firm 1 produces the regular variant under the setting (rg, rg) but not under the setting (g, rg). Thus, in the regular market, Firm 1 obtains positive profit under the setting (rg, rg) but 0 under the setting (g, rg). Third, in the regular market, Firm 2 competes with Firm 1 under the setting (rg, rg) but is the exclusive supplier under the setting (g, rg). Thus, Firm 2 obtains more profit in the regular market under the setting (rg, rg) than under the setting (g, rg). Therefore, we have the following results:
$$\pi_1^{rg,rg} > \pi_1^{g,rg}, \quad \pi_2^{g,rg} > \pi_2^{rg,rg}.$$

4. Combination (rg, g): If Firm 1 provides both regular and green variants, Firm 2 only provides the green variant
This scenario is similar to Combination (g, rg). If we compare these two firms' profits under this setting and under the setting (rg, rg), we have the following results:
$$\pi_1^{rg,g} > \pi_1^{rg,rg}, \quad \pi_2^{rg,rg} > \pi_2^{rg,g}.$$

Now we derive the Nash equilibrium. Since we have $\pi_1^{rg,g} > \pi_1^{rg,rg} > \pi_1^{g,g}$ and $\pi_1^{rg,rg} > \pi_1^{g,rg}$, Firm 1's dominating strategy is rg no matter which strategy Firm 2 chooses. Similarly, we have $\pi_2^{rg,rg} > \pi_2^{rg,g}$ and $\pi_2^{g,rg} > \pi_2^{rg,rg} > \pi_2^{g,g}$. Thus, Firm 2's dominating strategy is rg as well. Therefore, (rg, rg) is the only Nash equilibrium. Both Firms provide both the regular and the green variants. This completes the proof. ∎

D Proof of Theorem 4

Since $c'_{1,p} = c_{1,p}$ and $c'_{1,s} = c_{1,s}$, then we have

$$\Delta_M - \Delta_{CR} = [\frac{(1-c_{1,p})^2}{4} - \frac{(1-2c_{1,p}+c_{2,p})^2}{9}](\theta_1 - 1) + [\frac{(1-c_{1,s})^2}{4} - \frac{(1-2c_{1,s}+c_{2,s})^2}{9}](\theta_1 - 1)$$

Since $\frac{(1-c_{1,p})}{2} - \frac{(1-2c_{1,p}+c_{2,p})}{3} = \frac{(1+c_{1,p}-2c_{2,p})}{6} > 0$, we have $\frac{(1-c_{1,p})^2}{4} - \frac{(1-2c_{1,p}+c_{2,p})^2}{9} > 0$. Similarly, we have $\frac{(1-c_{1,s})^2}{4} - \frac{(1-2c_{1,s}+c_{2,s})^2}{9} > 0$. Since $\theta_1 \leq 1$, we have $\Delta_M \leq \Delta_{CR}$. ∎

E Proof of Theorem 5

If $c'_{1,p} = c_{1,p}$ and $c'_{1,s} = c_{1,s}$, then

$$\begin{aligned}
&\Delta_{CG} - \Delta_M \\
= \; & \frac{(1-\theta_1)(a_p - 2c_{1,p} + c_{2,p})^2}{9a_p} + \frac{(1-\theta_1)(1-c_{1,p})^2}{4} - \frac{(1-\theta_1)(a_p - c_{1,p})^2}{4a_p} \\
& + \frac{(1-\theta_1)(a_s - 2c_{1,s} + c_{2,s})^2}{9a_s} + \frac{(1-\theta_1)(1-c_{1,s})^2}{4} - \frac{(1-\theta_1)(a_s - c_{1,s})^2}{4a_s}
\end{aligned}$$

(a) If $a_p = 1$ and $a_s = 1$, then

$$\Delta_{CG} - \Delta_M = \frac{(1-\theta_1)(1-2c_{1,p}+c_{2,p})^2}{9} + \frac{(1-\theta_1)(1-2c_{1,s}+c_{2,s})^2}{9} \geq 0.$$

(b) If $c_{2,p} \leq c_{1,p}$ and $c_{2,s} \leq c_{1,s}$, then

$$\begin{aligned}
\Delta_{CG} - \Delta_M \leq \; & \frac{(1-\theta_1)(a_p - c_{1,p})^2}{9a_p} + \frac{(1-\theta_1)(1-c_{1,p})^2}{4} - \frac{(1-\theta_1)(a_p - c_{1,p})^2}{4a_p} \\
& + \frac{(1-\theta_1)(a_s - c_{1,s})^2}{9a_s} + \frac{(1-\theta_1)(1-c_{1,s})^2}{4} - \frac{(1-\theta_1)(a_s - c_{1,s})^2}{4a_s} \\
= \; & \frac{(1-\theta_1)}{36a_p}[(-5a_p^2 + 9a_p) - 8a_p c_{1,p} + (9a_p - 5)c_{1,p}^2] + \frac{(1-\theta_1)}{36a_s}[(-5a_s^2 + 9a_s) - 8a_s c_{1,s} + (9a_s - 5)c_{1,s}^2]
\end{aligned}$$

If $a_p \geq \frac{9}{5}$, then $(-5a_p^2 + 9a_p) \leq 0$. Since $1 > c_{1,p}$, we have

$$\begin{aligned}
& \frac{(1-\theta_1)}{36a_p}[(-5a_p^2 + 9a_p) - 8a_p c_{1,p} + (9a_p - 5)c_{1,p}^2] \\
\leq \; & \frac{(1-\theta_1)}{36a_p}[(-5a_p^2 + 9a_p)c_{1,p}^2 - 8a_p c_{1,p}^2 + (9a_p - 5)c_{1,p}^2] = \frac{-5(1-\theta_1)(a_p - 1)^2 c_{1,p}^2}{36a_p} \leq 0.
\end{aligned}$$

Similarly, we have $\frac{(1-\theta_1)}{36a_s}[(-5a_s^2 + 9a_s) - 8a_s c_{1,s} + (9a_s - 5)c_{1,s}^2] \leq 0$. Therefore, we have $\Delta_{CG} \leq \Delta_M$.

∎

F Proof of Theorem 6

If $c'_{1,p} = c_{1,p}$, and $c'_{1,s} = c_{1,s}$, then we have

$$\begin{aligned} & \Delta_{CG} - \Delta_{CR} \\ = \ & (1-\theta_1)[\frac{(a_p - 2c_{1,p} + c_{2,p})^2}{9a_p} - \frac{(a_p - c_{1,p})^2}{4a_p} + \frac{(1 - 2c_{1,p} + c_{2,p})^2}{9}] \\ & +(1-\theta_1)[\frac{(a_s - 2c_{1,s} + c_{2,s})^2}{9a_s} - \frac{(a_s - c_{1,s})^2}{4a_s} + \frac{(1 - 2c_{1,s} + c_{2,s})^2}{9}] \end{aligned}$$

Since $\frac{\partial \frac{(a_p-2c_{1,p}+c_{2,p})^2}{9a_p}}{\partial a_p} = \frac{(a_p-2c_{1,p}+c_{2,p})(a_p+2c_{1,p}-c_{2,p})}{9a_p^2} > 0$, then $\frac{(a_p-2c_{1,p}+c_{2,p})^2}{9a_p}$ increases with an increase in a_p. Thus, for $a_p \geq 1$, we have

$$\frac{(a_p - 2c_{1,p} + c_{2,p})^2}{9a_p} \geq \frac{(1 - 2c_{1,p} + c_{2,p})^2}{9}.$$

Therefore,

$$\frac{(a_p - 2c_{1,p} + c_{2,p})^2}{9a_p} - \frac{(a_p - c_{1,p})^2}{4a_p} + \frac{(1 - 2c_{1,p} + c_{2,p})^2}{9} \leq \frac{2(a_p - 2c_{1,p} + c_{2,p})^2}{9a_p} - \frac{(a_p - c_{1,p})^2}{4a_p}.$$

Since $c_{2,p} \leq c_{1,p}$, we have

$$\frac{2(a_p - 2c_{1,p} + c_{2,p})^2}{9a_p} - \frac{(a_p - c_{1,p})^2}{4a_p} \leq \frac{2(a_p - c_{1,p})^2}{9a_p} - \frac{(a_p - c_{1,p})^2}{4a_p} = \frac{-(a_p - c_{1,p})^2}{36a_p} < 0.$$

Similarly, we can show $\frac{(a_s-2c_{1,s}+c_{2,s})^2}{9a_s} - \frac{(a_s-c_{1,s})^2}{4a_s} + \frac{(1-2c_{1,s}+c_{2,s})^2}{9} < 0$. Thus, we have $\Delta_{CG} < \Delta_{CR}$. ■

G Proof of Theorem 7

Since $\frac{\partial \frac{(1-\theta_1)(a_p-c'_{1,p})^2}{8a_p}}{\partial a_p} = \frac{(1-\theta_1)(a_p-c'_{1,p})(a_p+c'_{1,p})}{8a_p^2} > 0$, we have $\frac{(1-\theta_1)(a_p-c'_{1,p})^2}{8a_p}$ increases with an increase in a_p. Thus, for $a_p \geq 2$, we have $\frac{(1-\theta_1)(a_p-c'_{1,p})^2}{8a_p} \geq \frac{(1-\theta_1)(2-c'_{1,p})^2}{16} > \frac{(1-\theta_1)(2-c'_{1,p}-c_{2,p})^2}{18}$. Therefore, $\frac{\theta_1(2-c'_{1,p}-c_{2,p})^2}{18} + \frac{(1-\theta_1)(a_p-c'_{1,p})^2}{8a_p} > \frac{\theta_1(2-c'_{1,p}-c_{2,p})^2}{18} + \frac{(1-\theta_1)(2-c'_{1,p}-c_{2,p})^2}{18} = \frac{(2-c'_{1,p}-c_{2,p})^2}{18}$.

Since $c'_{1,p} \leq c_{1,p}$, we have $\frac{(2-c_{1,p}-2c_{2,p})^2}{18} \leq \frac{(2-c'_{1,p}-2c_{2,p})^2}{18}$. Thus, we have $\frac{\theta_1(2-c'_{1,p}-c_{2,p})^2}{18} + \frac{(1-\theta_1)(a_p-c'_{1,p})^2}{8a_p} > \frac{(2-c_{1,p}-c_{2,p})^2}{18}$. Similarly, we have $\frac{\theta_1(2-c'_{1,s}-c_{2,s})^2}{18} + \frac{(1-\theta_1)(a_s-c'_{1,s})^2}{8a_s} > \frac{(2-c_{1,s}-c_{2,s})^2}{18}$. Thus, $W^a_{CR} - W^b_{CR} = \frac{\theta_1(2-c'_{1,p}-c_{2,p})^2}{18} + \frac{(1-\theta_1)(a_p-c'_{1,p})^2}{8a_p} + \frac{\theta_1(2-c'_{1,s}-c_{2,s})^2}{18} + \frac{(1-\theta_1)(a_s-c'_{1,s})^2}{8a_s} - \frac{(2-c_{1,p}-c_{2,p})^2}{18} - \frac{(2-c_{1,s}-c_{2,s})^2}{18} > 0$. This completes the proof. ■

H Proof of Theorem 8

- Since $a_p = 2$, $a_s = 2$, $c'_{1,p} = c_{1,p}$, and $c'_{1,s} = c_{1,s}$, we have $W^a_{CR} > W^b_{CR}$ from Theorem 7.
- If $a_p = 2$, $c'_{1,p} = c_{1,p}$, we have

$$\begin{aligned} & \frac{\theta_1(1 - 2c'_{1,p} + c_{2,p})^2}{9} + \frac{(1-\theta_1)(a_p - c'_{1,p})^2}{4a_p} - \frac{(1 - 2c_{1,p} + c_{2,p})^2}{9} \\ = \ & \frac{\theta_1(1 - 2c_{1,p} + c_{2,p})^2}{9} + \frac{(1-\theta_1)(2 - c_{1,p})^2}{8} - \frac{(1 - 2c_{1,p} + c_{2,p})^2}{9} \\ = \ & \frac{(1-\theta_1)(2 - c_{1,p})^2}{8} - \frac{(1-\theta_1)(1 - 2c_{1,p} + c_{2,p})^2}{9} \geq \frac{(1-\theta_1)(2 - c_{1,p})^2}{8} - \frac{(1-\theta_1)(1 - 2c_{1,p} + c_{2,p})^2}{8} \\ = \ & \frac{(1-\theta_1)}{8}[(2 - c_{1,p})^2 - (1 - 2c_{1,p} + c_{2,p})^2] = \frac{(1-\theta_1)}{8}[(1 + c_{1,p} - c_{2,p})(3 - 3c_{1,p} + c_{2,p})] > 0. \end{aligned}$$

Similarly, we have $\frac{\theta_1(1-2c'_{1,s}+c_{2,s})^2}{9} + \frac{(1-\theta_1)(a_s-c'_{1,s})^2}{4a_s} - \frac{(1-2c_{1,s}+c_{2,s})^2}{9} \geq \frac{(1-\theta_1)}{8}[(2 - c_{1,s})^2 - (1 - 2c_{1,s} + c_{2,s})^2]$.

Since $K = \frac{(1-\theta_1)}{8}[(2 - c_{1,p})^2 - (1 - 2c_{1,p} + c_{2,p})^2] + \frac{(1-\theta_1)}{8}[(2 - c_{1,s})^2 - (1 - 2c_{1,s} + c_{2,s})^2]$, we have $\Delta_{CR} \geq 0$.

- Since $a_p = 2$, $a_s = 2$, $c'_{1,p} = c_{1,p}$, $c'_{1,s} = c_{1,s}$, we have

$$\begin{aligned} \Delta_M \ = \ & \frac{\theta_1(1 - c'_{1,p})^2}{4} + \frac{(1-\theta_1)(a_p - c'_{1,p})^2}{4a_p} - \frac{(1 - c_{1,p})^2}{4} + \frac{\theta_1(1 - c'_{1,s})^2}{4} + \frac{(1-\theta_1)(a_s - c'_{1,s})^2}{4a_s} - \frac{(1 - c_{1,s})^2}{4} - K \\ = \ & \frac{\theta_1(1 - c_{1,p})^2}{4} + \frac{(1-\theta_1)(2 - c_{1,p})^2}{8} - \frac{(1 - c_{1,p})^2}{4} + \frac{\theta_1(1 - c_{1,s})^2}{4} + \frac{(1-\theta_1)(2 - c_{1,s})^2}{8} - \frac{(1 - c_{1,s})^2}{4} - K \\ = \ & \frac{(1-\theta_1)}{8}[(2 - c_{1,p})^2 - 2(1 - c_{1,p})^2] + \frac{(1-\theta_1)}{8}[(2 - c_{1,s})^2 - 2(1 - 2c_{1,s})^2] - K. \end{aligned}$$

Since $K = \frac{(1-\theta_1)}{8}[(2-c_{1,p})^2 - (1-2c_{1,p}+c_{2,p})^2] + \frac{(1-\theta_1)}{8}[(2-c_{1,s})^2 - (1-2c_{1,s}+c_{2,s})^2]$, we have

$\Delta_M = \frac{(1-\theta_1)}{8}[(1-2c_{1,p}+c_{2,p})^2 - 2(1-c_{1,p})^2] + \frac{(1-\theta_1)}{8}[(1-2c_{1,s}+c_{2,s})^2 - 2(1-2c_{1,s})^2]$. Since $c_{2,p} \leq c_{1,p}$, $c_{2,s} \leq c_{1,s}$, we have

$$\begin{aligned}
\Delta_M &= \frac{(1-\theta_1)}{8}[(1-2c_{1,p}+c_{2,p})^2 - 2(1-c_{1,p})^2] + \frac{(1-\theta_1)}{8}[(1-2c_{1,s}+c_{2,s})^2 - 2(1-2c_{1,s})^2] \\
&< \frac{(1-\theta_1)}{8}[(1-2c_{1,p}+c_{2,p})^2 - (1-c_{1,p})^2] + \frac{(1-\theta_1)}{8}[(1-2c_{1,s}+c_{2,s})^2 - (1-2c_{1,s})^2] \\
&= \frac{(1-\theta_1)(c_{2,p}-c_{1,p})}{8}[(1-2c_{1,p}+c_{2,p}) + (1-c_{1,p})] + \frac{(1-\theta_1)(c_{2,s}-c_{1,s})}{8}[(1-2c_{1,s}+c_{2,s}) + (1-2c_{1,s})] \leq 0.
\end{aligned}$$

The result follows. ■

I Proof of Theorem 9

We first compare the prices of the regular variants of Product P before and after the implementation.

$$p^b_{r,p} - p^a_{r,p} = \frac{1+c_{1,p}+c_{2,p}}{3} - \frac{1+c'_{1,p}+c_{2,p}}{3} = \frac{c_{1,p}-c'_{1,p}}{3}.$$

Since $c'_{1,p} \leq c_{1,p}$, we have $p^b_{r,p} - p^a_{r,p} \geq 0$. Next, we compare the prices of the green variants of Product P before and after the implementation.

$$p^b_{g,p} - p^a_{g,p} \quad = \quad \frac{a_p + c_{2,p}}{2} - \frac{a_p + c'_{1,p} + c_{2,p}}{3} = \frac{a_p - 2c'_{1,p} + c_{2,p}}{6} \geq 0.$$

Similarly, for Product S, we have $p^b_{r,s} \geq p^a_{r,s}$ and $p^b_{g,s} \geq p^a_{g,s}$. Since $p^b_{r,p} \geq p^a_{r,p}$, $p^b_{g,p} \geq p^a_{g,p}$, $p^b_{r,s} \geq p^a_{r,s}$, and $p^b_{g,s} \geq p^a_{g,s}$, then we have $W^a_{CG} > W^b_{CG}$. ■

J Proof of Theorem 10

- Since $c'_{1,p} = c_{1,p}$, $c'_{1,s} = c_{1,s}$, then from Theorem 9, we have $W^a_{CG} \geq W^b_{CG}$.
- If $a_p = 1$, $a_s = 1$, $c'_{1,p} = c_{1,p}$, $c'_{1,s} = c_{1,s}$, we have

$$\Delta_M = \frac{\theta_1(1-c_{1,p})^2}{4} + \frac{(1-\theta_1)(1-c_{1,p})^2}{4} - \frac{(1-c_{1,p})^2}{4} + \frac{\theta_1(1-c_{1,s})^2}{4} + \frac{(1-\theta_1)(1-c_{1,s})^2}{4} - \frac{(1-c_{1,s})^2}{4} - K = -K < 0.$$

- We also have

$$\begin{aligned}
\Delta_{CG} &= \frac{\theta_1(1-2c'_{1,p}+c_{2,p})^2}{9} + \frac{(1-\theta_1)(a_p-2c'_{1,p}+c_{2,p})^2}{9a_p} - \frac{\theta_1(1-2c_{1,p}+c_{2,p})^2}{9} \\
&\quad + \frac{\theta_1(1-2c'_{1,s}+c_{2,s})^2}{9} + \frac{(1-\theta_1)(a_s-2c'_{1,s}+c_{2,s})^2}{9a_s} - \frac{\theta_1(1-2c_{1,s}+c_{2,s})^2}{9} - K \\
&= \frac{(1-\theta_1)(1-2c_{1,p}+c_{2,p})^2}{9} + \frac{(1-\theta_1)(1-2c_{1,s}+c_{2,s})^2}{9} - K.
\end{aligned}$$

Since $K \leq \frac{(1-\theta_1)(1-2c_{1,p}+c_{2,p})^2}{9} + \frac{(1-\theta_1)(1-2c_{1,s}+c_{2,s})^2}{9}$, we have $\Delta_{CG} \geq 0$.
This completes the proof. ■