

Resolution in FOL

Algorithm

1. Convert all sentences to CNF

2. Eliminate biconditionals and implications:

- Eliminate \Leftrightarrow , replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$
- Eliminate \Rightarrow , replacing $\alpha \Rightarrow \beta$ with $\neg \alpha \vee \beta$

3. Move \neg inwards

$$\neg(\neg \alpha \vee \beta) \equiv \neg \neg \alpha \wedge \neg \beta$$

$$\neg(\neg \alpha \wedge \beta) \equiv \neg \neg \alpha \vee \neg \beta$$

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$$\neg(\neg \alpha \vee \neg \beta) \equiv \neg \neg \alpha \wedge \neg \neg \beta$$

3. Standardize Variables apart by renaming them: each quantifier should use a different Variable.

4. Skolemize: Each existential Variable is replaced by a Skolem constant or Skolem function of the enclosing universally quantified variables

- For instance, $\exists x \text{ Rich}(x)$ becomes $\text{Rich}(a)$ where a is a new Skolem constant
- "Everyone has a heart" $\forall x \text{ person}(x) \Rightarrow \exists y \text{ heart}(x) \wedge \text{has}(x, y)$ becomes $\forall x \text{ person}(x) \Rightarrow \text{heart}(x, f(x)) \wedge \text{has}(x, f(x))$ where f is a new Symbol

5. Drop universal quantifiers

• For instance $\forall x \text{ person}(x)$ becomes $\text{person}(x)$ 6. Distribute \wedge over \vee :

$$(\alpha \wedge \beta) \vee \gamma \equiv (\alpha \vee \gamma) \wedge (\beta \vee \gamma)$$