№68

a) $\sum\limits_{k=1}^{n} k^2 C_n^k$

$$kC_n^k = k \cdot \frac{n!}{k!(n-k)!} = \frac{n!}{(k-1)!(n-k)!} =$$

$$= \frac{n(n-1)!}{(k-1)!(n-k)!} = nC_{n-1}^{k-1}$$

$$\sum_{k=1}^{n} k^2 C_n^k = \sum_{k=1}^{n} kn C_{n-1}^{k-1} = \sum_{k=2}^{n} (k-1)n C_{n-1}^{k-1} + \sum_{k=1}^{n} nC_{n-1}^{k-1} =$$

$$= \sum_{k=2}^{n} (n-1)n C_{n-2}^{k-2} + \sum_{k=1}^{n} nC_{n-1}^{k-1} =$$

$$= (n-1)n \sum_{k=2}^{n} C_{n-2}^{k-2} + n\sum_{k=1}^{n} C_{n-1}^{k-1} = (n-1)n \cdot 2^{n-2} + n2^{n-1}$$

б) $\sum_{k=1}^{n}(2k^2-5k+4)C_n^k =$

$=2\sum_{k=1}^{n}k^2C_n^k+(-5)\sum_{k=1}^{n}kC_n^k+4\sum_{k=1}^{n}C_n^k=$

$=2n(n-1)2^{n-2}+2n2^{n-1}-5n^{2n-1}+4\sum_{k=1}^{n}C_n^k=$

$=2n(n-1)2^{n-2}-3n2^{n-1}+4(2^n-C_n^0)=$

$=(2n^2-2)2^{n-1}-3n\cdot2^{n-1}+4(2^n-1)$

в) $\sum_{k=1}^{n}(3k^2+4k-3)C_n^k=$

$=3\sum_{k=1}^{n}k^2C_n^k+4\sum_{k=1}^{n}kC_n^k-3\sum_{k=1}^{n}C_n^k=$

$\cancel{=3(n^2-n)2^{n-2}+4n2^{n-1}}$

$=3(n^2-n)2^{n-2}+3n2^{n-1}+4n2^{n-1}-3(2^n-1)=$

$=3(n^2-n)2^{n-2}+7n\cdot2^{n-1}-3(2^n-1)$