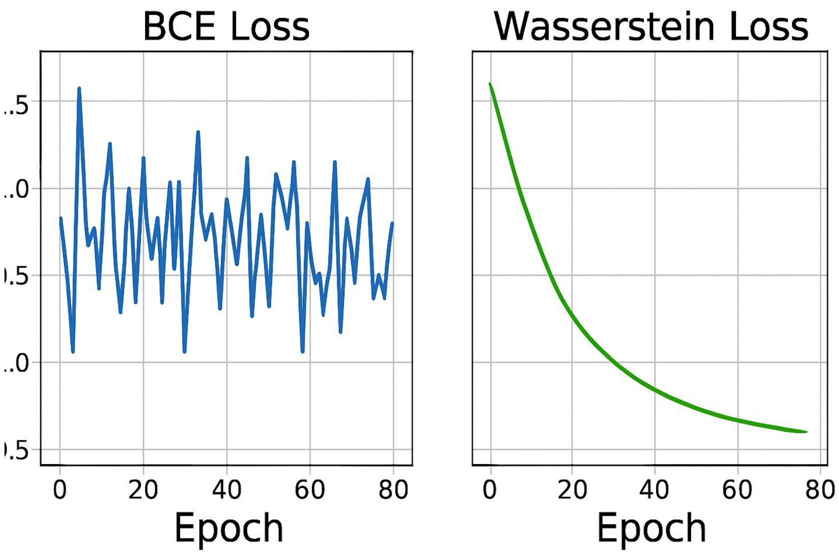
Wasserstein GANs (WGAN)

A Stable Approach to Generative Modeling

Two Big Problems with Classic GANs

Loss function is not **meaningful** (doesn’t track progress).

Training is **unstable** (mode collapse, oscillations).

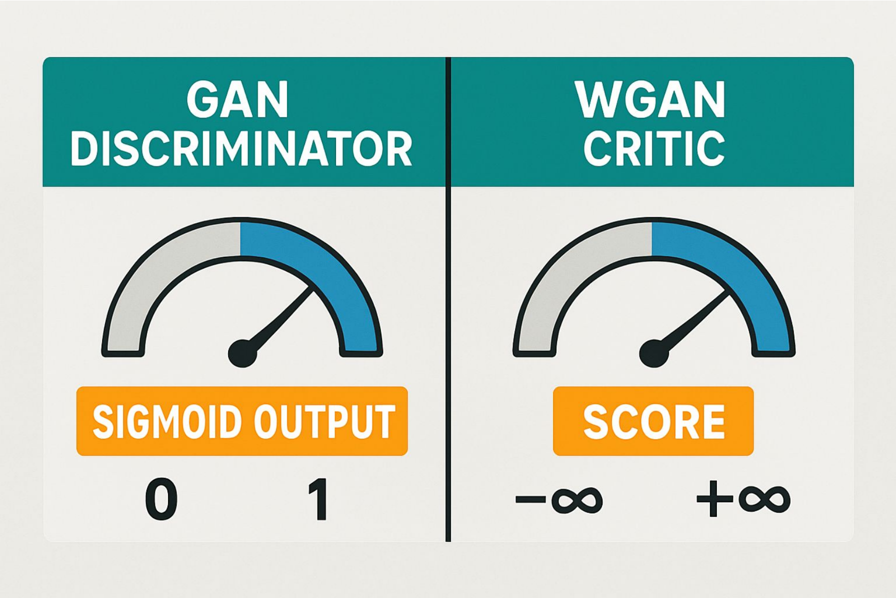


Wasserstein Loss Intuition

Classic GAN: **Critic outputs probability (0 = fake, 1 = real).**

WGAN: **Critic outputs a score (real number, any range).**

Goal: **Maximize distance between real and fake scores.**

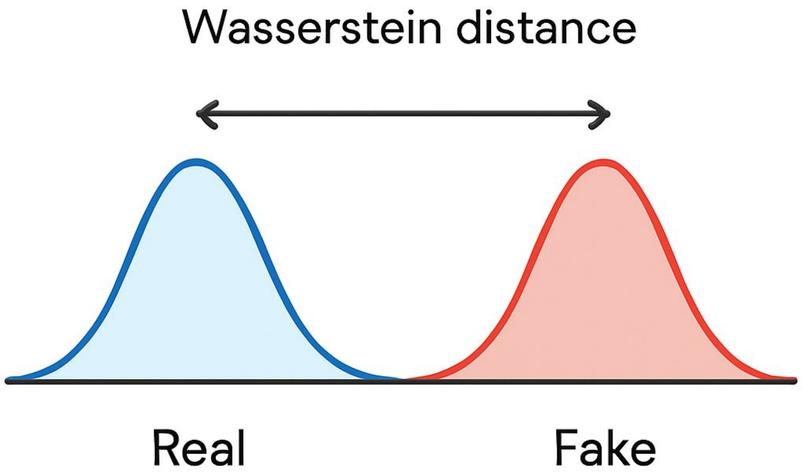


WGAN Loss Functions

Critic Loss: **Maximize** (Real score – Fake score).

Generator Loss: **Maximize** Fake score.

Critic **outputs scores** (not probabilities).



Classic GAN Loss vs. WGAN Loss

Classic GAN Loss:

- Discriminator:

$$\min_D -\left(\mathbb{E}_{x\sim p_X}[\log D(x)] + \mathbb{E}_{z\sim p_Z}[\log(1 - D(G(z)))]\right)$$

- Generator:

$$\min_G -\mathbb{E}_{z\sim p_Z}[\log D(G(z))]$$

Problems:

Outputs probabilities in [0,1][0,1][0,1].
Loss not always meaningful → unstable training.

Wasserstein Loss Idea

- Remove sigmoid → outputs real-valued scores $(-\infty, \infty)$.
- Labels: $+1$ (real), -1 (fake).
- Loss measures **distance** between real and fake distributions.

WGAN Loss Functions

- **Critic Loss:**

$$\min_{D} -\left(\mathbb{E}_{x\sim p_X}[D(x)] - \mathbb{E}_{z\sim p_Z}[D(G(z))]\right)$$

→ Critic tries to **maximize score difference** (real – fake).

- **Generator Loss:**

$$\min_{G} -\mathbb{E}_{z\sim p_Z}[D(G(z))]$$

→ Generator tries to **maximize fake scores** (make fakes look real).

The Lipschitz Constraint

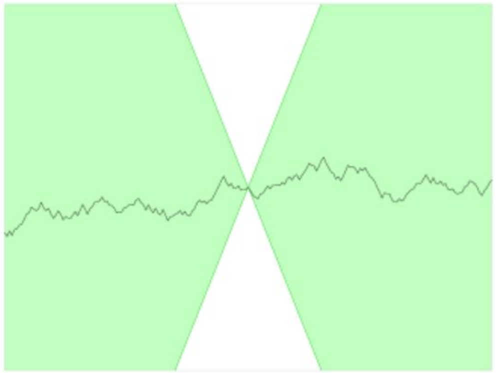
- In WGAN, the critic outputs **real numbers** in $(-\infty, \infty)$ instead of $[0, 1]$.
- This means the Wasserstein loss can become **unbounded**.
- To make it mathematically valid, the critic must satisfy the **1-Lipschitz condition**.
- This ensures the critic's predictions **don't change too abruptly** between two inputs.

Understanding 1-Lipschitz Functions

- A function D is **1-Lipschitz** if:

$$\frac{|D(x_1) - D(x_2)|}{|x_1 - x_2|} \leq 1$$

- Interpretation: The slope (rate of change) is **never greater than 1.**
- This means the critic's output changes **gradually**, not sharply.
- Geometric view: The function always stays **outside a 45° cone** placed anywhere on the curve.



Key Takeaways

✅ Loss = meaningful distance metric

✅ Training = stable & avoids collapse

✅ Critic outputs scores, not probabilities

✅ Lipschitz constraint ensures fairness

Quiz Time!

Q1. In WGAN, the discriminator is replaced by a:

a) Classifier

b) Critic

c) Generator

d) Encoder

Quiz Time!

Q2. What is the main role of the critic in WGAN?

a) To classify images as real or fake

b) To output probabilities in range [0,1]

c) To provide a score that measures how real or fake an image looks

d) To generate realistic images

Quiz Time!

Q3. The generator in WGAN is trained to:

a) Maximize the critic’s loss

b) Minimize the critic’s score

c) Produce images that receive a high score from the critic

d) Directly minimize Wasserstein distance

Quiz Time!

Q4. Why is the Lipschitz constraint applied to the WGAN critic?

a) To ensure gradient clipping

b) To prevent the Wasserstein distance from becoming unstable

c) To force critic outputs to stay between 0 and 1

d) To reduce computation cost

Quiz Time!

Q5. What is a key advantage of WGAN over traditional GANs?

a) It eliminates the need for a generator

b) It avoids mode collapse entirely

c) It provides a more stable training process with meaningful loss values

d) It removes the need for backpropagation