

Assignment No : 1.

Title :- Calculate Fibonacci number and its step Count

Problem Statement:- Develop a program in C++ to create a Fibonacci Series.

Prerequisite:- Design and analysis Algorithm

Objectives:- Implement recursive and non-recursive program to calculate Fibonacci number and step count.

Outcome:- To understand the implementation of Fibonacci and analysis and find its step count.

Theory Fibonacci Series:-

Fibonacci Series is defined as a sequence of numbers in which the first two numbers are 1 and 1 or 0 and 1 depending on the selected beginning point of sequence and each subsequent number is sum of previous two. So, in this series the  $n^{th}$  term is the sum of  $(n-1)^{th}$  term and  $(n-2)^{th}$  term.

Fibonacci Series in C++ - In case of Fibonacci Series, next no. is the sum of previous two numbers for example: 0, 1, 1, 2, 3, 5, 8, 13, 21 etc.

The first two no. of Fibonacci Series are 0 and 1.

There are two ways to which write Fibonacci Series program.

i) without Recursion.

ii) with Recursion.

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Generation of Fibonacci Series:-

mathematically, the  $n^{\text{th}}$  term of the Fibonacci Series can be

$$T_n = T_{n-1} + T_{n-2}$$

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

5, 8, 13, 21, 34, 55

facilities : Linux operating system, awk

Algorithm :-

Step 1 : Start

Step 2 : Declare Variable i, a, b, Show

Step 3 : Initialize the Variable , a=0, b=1 and Show=0.

Step 4 : Enter the no. of terms of series to be printed, p  
two no. of series.

Step 5 : Use the following steps.

$$\Rightarrow Show = a + b$$

$\Rightarrow ab$

$$\Rightarrow b = Show$$

$\Rightarrow$  increase value of i each time by 1.

$\Rightarrow$  print the value of show.

Step 6 : End.

SOP : take the term no. as an input . say it is 10.

SOP : Enter the no. of elements : 14.

Conclusion : Hence we studied the concept of Fibonacci Series.

### Assignment No: 2

Title:- Solve a Fractional knapsack problem using a greedy method.

problem statement:- Given a set of  $n$  items; each having value  $v_i$  with weight  $w_i$  and the total capacity of a knapsack. The task is to find the maximum value of fractions of items that can fit into the knapsack.

objective:- The objective is to fill the knapsack to some given volume with different sub parts that value of selected item is maximized.

outcome:- To understand the implementation of greedy method using fraction knapsack problem.

Theory:- Given a lot of items; each with weight and value, determine a subset of volume to include in a collection so that the total weight is less or equal to a given limit and the total value is large as possible. The knapsack problem is a Combinatorial optimization problem. It appears as a subproblem in many more complex mathematical model of real world problems.

Greedy Approach:- In this, we calculate the profit / weight and accordingly, we will select the item with the highest ratio would be selected first.

Fractional knapsack problem can be solved in C++ and Java.

- We have fixed size array of structure named item.
- Each item has value and weight.
- We are calculating density = value / weight for each item in order of decreasing density.

~~Find Approach:-~~

Objective	Profit	Weight	Remaining Weight
3	15	5	$15 - 5 = 10$
2	10	3	$10 - 3 = 7$
6	9	3	$7 - 3 = 4$
5	8	1	$4 - 1 = 3$
7	$7 \times \frac{3}{4} = 5.25$	.3	$3 - .3 = 2.7$

The total profit would be equal to

$$(15 + 10 + 9 + 8 + 5.25) = 47.25$$

~~Second Approach:-~~

Objective	Profit	Weight	Remaining weight
1	5	1	$15 - 1 = 14$
5	7	1	$14 - 1 = 13$
7	4	2	$13 - 2 = 11$
2	10	3	$11 - 3 = 8$

$$(5 + 7 + 4 + 10 + 9 + 7 + 3) = 46$$

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(third approach) In this Calc we will calculate the ratio profit/weight

Object :- 1 2 3 4 5 6 7

Profit (P) :- 5 10 10 15 7 8 4

Weight :- 1 3 5 4 1 3 2

$$\text{object 1} \therefore 5/1 = 5$$

$$\text{object 2} \therefore 10/3 = 3.3$$

$$\text{object 3} \therefore 15/5 = 3$$

$$\text{object 4} \therefore 15/4 = 3.75$$

$$\text{object 5} \therefore 7/1 = 7$$

$$\text{object 6} \therefore 8/3 = 2.67$$

$$\text{object 7} \therefore 4/2 = 2$$

$$P/W : 5 \ 3.3 \ 3 \ 3.75 \ 7 \ 2.67 \ 2$$

In this approach we still select object based on maximum Profit/weight ratio since the P/W of object 5 is maximum so we select object 5.

Algorithm:

$(W[1-n], P[1-n], w)$

for i=1 to n

do  $x[i] = 0$

weight = 0

for i=1

if weight + w[i]  $\leq w$  then  
 $x[i] = 1$

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$\text{weight} = \text{weight} + w[i]$

+15e

$x[i] = [v[i] - \text{weight}] / w[i]$

$\text{weight} = w$

break

return x

I/p:  $A[7] = \{ \{2, 15\}, \{100, 20\}, \{120, 30\} \}$

Total capacity = 50

O/p: Enter total & item value and weight.

Enter 1 value 60

Enter 1 weight 10

Enter 2 value 100

Enter 2 weight 20

Enter 3 value 120

Enter 3 weight 30

Enter data: value 60 100 120

weight 10 20 30

Enter knapsack weight 50

Total value in bag 50

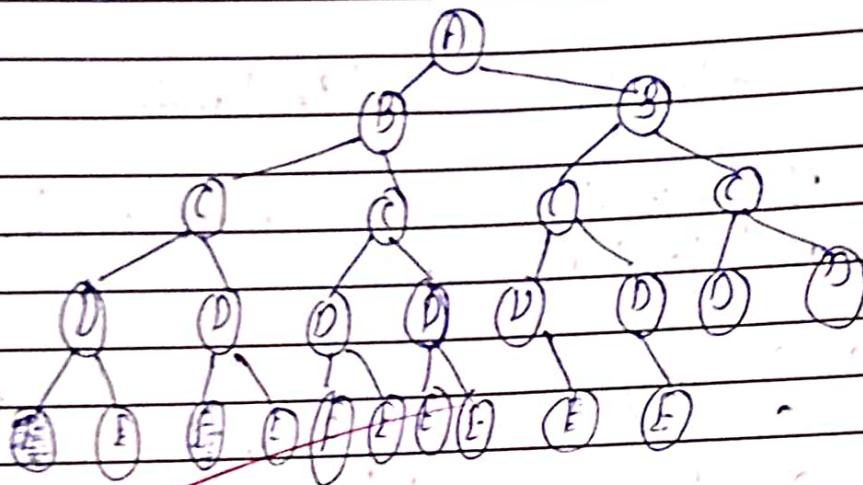
... max value for 50 weight is 640

Process turned O(0.10) execution

Time: 17.876 . 56

Conclusion: Hence, we have successfully studied Concept problem using greedy method.

The branch and bound still work better than brute force -  
by ignoring infeasible solutions.



~~(A)~~ ignored because best solution is infeasible ~~(B)~~ ignored because best solution is worse than current best.

Branch and Bound is very useful techniques for searching a soln but in worst case.

Algorithm

Step 1: Sort all items in decreasing order of ratio of value per unit weight so that an upper bound can be computed using Greedy algorithm.

Step 2: Initialize max profit, max profit = 0.

Step 3: Create an empty queue, Q.

Step 4: Create a dummy node of decision tree and enque it. Its profit, weight of dummy node are 0.

Step 5: Do following while Q is not empty

Step 6: Extract an item type Q. Set the extracted

Step 7: compute profit of next level node.

Step 8: If the profit is more than max. profit, then  
next level node to Q.

Step 9: Consider the case where next level node  
considered as part of sum and add a node  
with level as max, but weight and profit will  
considering next level nodes.

Input: Input the number of items 4.

Enter the weight and profit of the item 1: 2  
enter the weight and profit of the item 2: 1  
enter the weight and profit of the item 3: 3  
enter the weight and profit of the item 4: 2

Enter the capacity of knapsack: 5

Output: The table is.

0	0	0	0	0	0
0	0	12	12	12	12
0	10	12	22	22	22
0	10	12	22	30	32
0	10	15	25	30	37

Assignment No. 3.

Title: Write a program to solve a 0-1 knapsack problem using dynamic programming or branch and bound strategy.

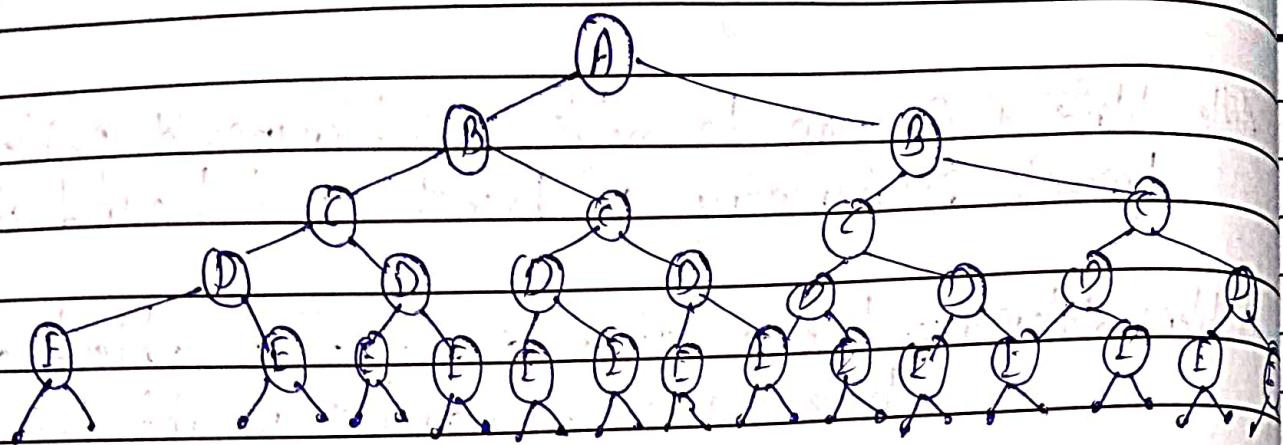
Problem Statement: We are given a set of n objects which have a value  $v_i$  and a weight  $w_i$ .

Objective: Implement a knapsack problem using dynamic programming or branch and bound strategy.

Theory: Branch and bound is an algorithm design paradigm which is generally used for solving combinatorial optimization problems. These problems typically exponential in terms of time complexity and may require exploring all possible permutations in worst case.

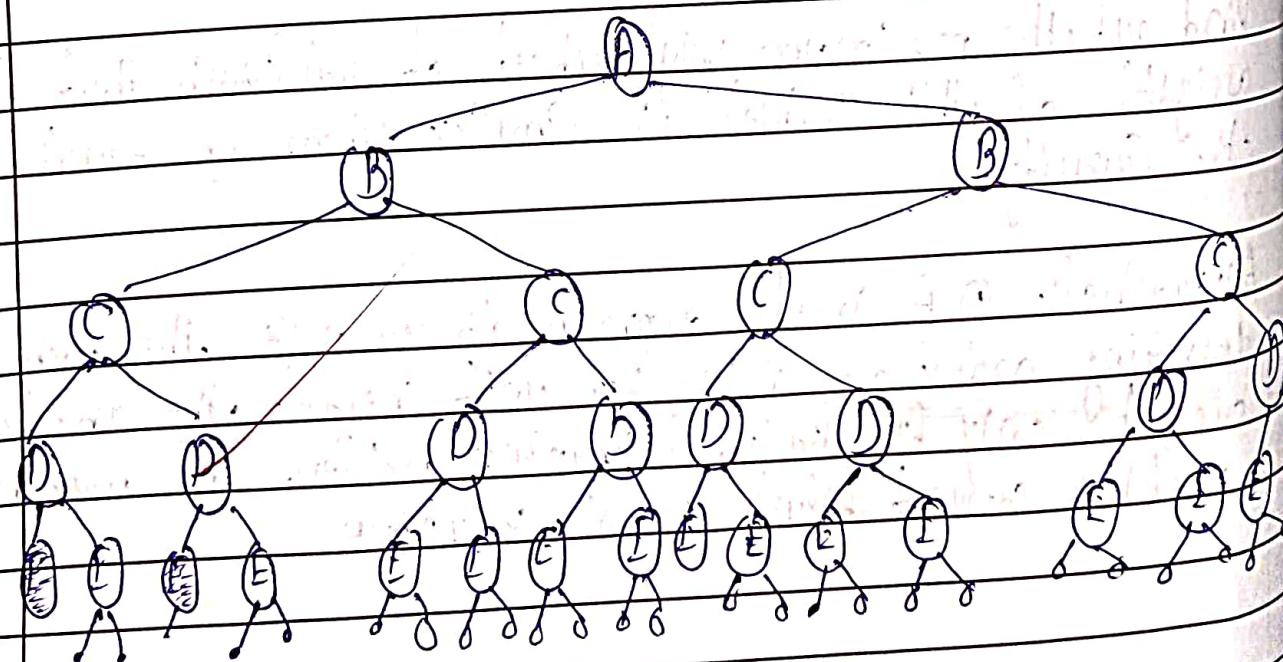
Find out the maximum value subset of  $v_i$  &  $w_i$  such that sum of weight of this subset is smaller than or equal to knapsack capacity.

Approaches: ① A greedy approach is to pick the items in decreasing order of value per unit weight, the greedy approach works only for fractional knapsack problem and may not produce correct result for 0-1 knapsack.



We can use backtracking to optimize the brute force approach.  
In the tree representation, we can do DFS tree. If we reach a point where a gain no longer is feasible, there is no need to continue exploring. In the given example, backtracking will be much more effective if we had even more items with smaller knapsack capacity.

### Backtracking



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The maximum profit is : 37

The most valuable subset is:  
(item 4: item 2: item 1: )

Conclusion: We can see that branch and bound method will give the best to the problem by exploring just 1 path in the best case.

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Assignment no: 5

Title: Design a queen matrix having first queen placed. Use backtracking to place remaining queens to generate the final 8 Queen matrix.

Problem Statement: Find an arrangement of n-queens on a chess board, such that no queen can attack any other queen on the board.

Objective: Implement 8 queen using backtracking.

Outcome: To understand the implementation of 8 queen using backtracking.

Theory: In backtracking we start with one possible move out of many available moves. We then try to solve the problem with selective moves and print the soln.

This problem is commonly seen for n=4 and n=8. Let us look at an example where n=8 before solving the problem you need to know about the movement of the queen in chess. A. Queen can move any no. of steps in any direction. The only constraint is that we can't change its direction while it is moving.

$$i - j = k - l \quad \text{--- (1)}$$

$$\text{or } i + j = k + l \quad \text{--- (2)}$$

From (1) and (2) imply  $j - l = i - k$  and  $j - l = k - i$

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- A  $n \times n$  queen is on the same diagonal if  $|i-j| = |j-k|$ .  
So if  $n$  Queen Problem can be obtained to  $n$  Queen problem.

$$x_1 = 3 \quad x_2 = 6 \quad x_3 = 2 \quad x_4 = 7 \quad x_5 = 1 \quad x_6 = 4 \quad x_7 = 8 \quad x_8 = 5$$

- place  $(k, i)$  returns a boolean value that is true if  $k$ th queen placed in column  $i$  lefts both whether  $i$  is distinct to all previous last  $x_1, x_2, \dots, x_{k-1}$  and whether there is no other queen on same diagonal ceiling place we give precise soln to the  $n$ -Queen problem.

The soln can be shown as -

C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	C <sub>5</sub>	C <sub>6</sub>	C <sub>7</sub>	C <sub>8</sub>
Q							
	Q						
		Q					
			Q				
				Q			
					Q		
						Q	

(Time Complexity analysis.

- The possible method like also time.
- For each iteration of loop in  $n$ -Queen helper it runs  $O(n)$  time.

$$T(n) = 0 \cdot (n+2) + n \cdot T(n-1)$$

Algorithm:-

START.

- 1) Begin from the leftmost column.
- 2) If all queen are placed return the true configuation.
- 3) check for all row in current column.
- a) If queen placed mark row and column and recursively check if we approach in the currently configuration do we obtain 64! or not.
- b) If replacing yeild a 63! return false.
- c) If all row tried and so on not obtain return false and backback.

END.

Conclusion:- Hence we learnt about 8-queen algorithm.

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Assignment no: 5.

Title: Write a program for analysis of quick sort by using deterministic and randomized variant.

Problem Statement: Write a fun<sup>n</sup>, that takes two parameters  $n$  and  $k$  and return the value of binomial coefficient  $(n,k)$ .

Objective: Implement deterministic quick sort

Theory:

In deterministic quick sort is applied to an array of length  $n$  whose entries are already sorted, then this algorithm will  $\sim (n^2)$  step.

It comes down to how the pivot are chosen in deterministic quicksort, the pivot are chosen by either always choosing the pivot at same relative index. For instance a common method is to choose the median of first, last and middle element as pivot.

array [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 9, 8, 6, 5, 4, 3, 2, 1].

partition the array then recursively sort the piece before and after the pivot element.

Quick sort ( $A, p, r$ )

if  $p < r$  then

$q = \text{Partition } (A, p, r);$  // Rearrange  $A[p..r]$  in place:

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Quicksort ( $A, p, q-1$ ) ~~selection of pivot in first~~  
Quicksort ( $A, p+1, r$ ) ~~selection of pivot in last~~

divide and conquer.

- Divide - partition the array  $A[p..r]$  into two subarrays:
  - $A[p..q-1] \leq A[q+1, r]$  such that
  - $A[x] \leq A[1]$  for all  $x$  in  $\{p, \dots, q-1\}$ .
  - $A[x] > A[1]$  for all  $x$  in  $\{q+1, \dots, r\}$
- Conquer: Recursively sort  $A[p..q-1]$  +  $A[q+1..r]$ .
- Combine: nothing to do there.

Randomized Quicksort:

We expect good average time behaviour if all  $i/p$  perm are equally likely, but what if it is not? we tend the algorithm to get the same effect as if the were random. Instead of explicitly permitting randomization can be accomplished trivially by sampling of one of array element as pivot.

Randomized Quicksort ( $A, p, r$ ).

- i) if  $p < q$ .
- ii)  $\rightarrow$  Randomized partition ( $A, p, r$ ).

- iii) Randomized -  $\alpha_{\{i\} \text{RSort}}(A, p, q, r)$
- iv) Randomized -  $\alpha_{\{i\} \text{RSort}}(A, p, q+1, r)$ .

Randomized - Partition ( $A, p, r$ )

- i)  $i = \text{random}(p, r)$
- ii) exchange  $A[i]$  with  $A[r]$ .
- iii) ~~return PARTITION ( $A, p, r$ )~~

~~Randomised Quicksort Analysis~~

The analysis assumed that all elements are unique but with some work can be generalized to remove this assumption.

$$T(n) \leq \text{max } T(q) + T(n-q-1) + O(n)$$

~~$0 \leq q \leq n-1$~~

Based on informal analysis we guess  $T(n) \leq n^2$  for some. Substitute this guess into the recurrence.

$$T(n) \leq \text{max } (q^2 + (n-q-1)^2) + O(n)$$

~~$0 \leq q \leq n-1$~~

$$= C \cdot \text{max } (q^2 + (n-q-1)^2) + O(n).$$

~~$0 \leq q \leq n-1$~~

The maximum value of  $q^2 + (n-q-1)^2$  occurs when  $q$  is either 0 or  $n-1$  and has value  $(n-1)^2$  in either case.

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$$\text{more } (1^2 + (n-1-1)^2) \leq (n-1)^2 \\ 0 \leq q \leq n-1 \quad = n^2 - 2n + 1$$

Substituting this back into the recurrence.

$$T(n) \leq cn^2 - c(2n-1) + \Theta(n) \\ \leq cn^2.$$

We can pick so that  $c(2n-1)$  dominates  $\Theta(n)$ . Hence the worst case running time is  $\Theta(n^2)$  one can also show that recurrence is  $\sim n(n^2)$ , so worst case is

Conclusion:

Here, we have studied and implemented the analysis of quick sorting deterministic and randomized variant.

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