but

$ds = L\,d\lambda$

$\Rightarrow L = \sqrt{g_{ij}(x)\frac{dx^i}{d\lambda}\frac{dx^j}{d\lambda}}$

replacing $\frac{dx^i}{d\lambda} = \dot{x}^i$

$\frac{dx^j}{d\lambda} = \dot{x}^j$

to avoid dealing with square root, we can use Lagrangian proportional to square of the interval

$L = \frac{1}{2} g_{ij}(x)\,\dot{x}^i \dot{x}^j$

when λ is affine parameter it leads to geodesic equation

writting the Euler-Lagrange equation

$$\frac{d}{d\lambda}\left(\frac{\partial L}{\partial \dot{x}^m}\right) - \frac{\partial L}{\partial x^m} = 0$$

computing the derivatives

$$\frac{\partial L}{\partial \dot{x}^m} = g_{mj}(x)\,\dot{x}^j$$

$$\frac{d}{d\lambda}\left(\frac{\partial L}{\partial \dot{x}^m}\right) = \frac{d}{d\lambda}\left(g_{mj}\dot{x}^j\right)$$

$$= \frac{\partial g_{mj}}{\partial x^k}\dot{x}^k \dot{x}^j + g_{mj}\ddot{x}^j$$

$$\frac{\partial L}{\partial x^m} = \frac{1}{2}\frac{\partial g_{ij}}{\partial x^m}\dot{x}^i \dot{x}^j$$

substituting back

$$\frac{\partial g_{mj}}{\partial x^k}\dot{x}^k\dot{x}^j + g_{mj}\ddot{x}^j - \frac{1}{2}\frac{\partial g_{ij}}{\partial x^m}\dot{x}^i\dot{x}^j = 0$$

isolating $\ddot{x}^j$

$$g_{mj}\ddot{x}^j = \frac{1}{2}\frac{\partial g_{ij}}{\partial x^m}\dot{x}^i\dot{x}^j - \frac{\partial g_{mj}}{\partial x^k}\dot{x}^k\dot{x}^j$$

multiplying by inverse metric g^{im}

using $g^{im}g_{mj} = \delta^i_j$

$$\ddot{x}^i = g^{im}\left(\frac{1}{2}\frac{\partial g_{jk}}{\partial x^m}\dot{x}^j\dot{x}^k - \frac{\partial g_{mj}}{\partial x^k}\dot{x}^k\dot{x}^j\right)$$

expressing in terms of Christofell symbols Γ^i_{jk}

$$\Gamma^i_{jk} = \frac{1}{2}g^{im}\left(\frac{\partial g_{mj}}{\partial x^k} + \frac{\partial g_{mk}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^m}\right)$$

now no rearranging and using symmetry

$$\ddot{x}^i + \Gamma^i_{jk}\dot{x}^j\dot{x}^k = 0$$

where Γ^i_{jk} is Christofell symbol or Affine Connection

$\therefore$ Geodesic equation

$$\ddot{x}^i + \Gamma^i_{jk}\dot{x}^j\dot{x}^k = 0$$

for a general affine λ

$$p = \frac{dx}{d\lambda}$$

$\rightarrow$ Geodesic equation in terms of momentum is

$$\dot{p}^i + \Gamma^i_{jk} p^j p^k = 0$$

Now substituting

$\dot{x}^i = p^i$

and $\dot{p}^i = -\Gamma^i_{jk} p^j p^k$ in Liouville operator

$$\hat{L}[f(p^\mu, x^\mu)] = p^i\left(\frac{\partial f}{\partial x^i}\right) - \Gamma^i_{jk} p^j p^k\left(\frac{\partial f}{\partial p^i}\right)$$

$\therefore$ Liouville operator is

$$\boxed{\hat{L} = p^i\left(\frac{\partial}{\partial x^i}\right) - \Gamma^i_{jk} p^j p^k\left(\frac{\partial}{\partial p^i}\right)}$$

From Big Bang Model we can say that the Universe is

- expanding
- homogeneous and isotropic

Evidences of expansion

① Hubbles Law and Redshift

Edwin Hubble observed that light from distant galaxies is shifted to longer wavelengths (redshifted) in late 1920's. This indicated that they are moving away from us. The velocity at which galaxies recede is proportional to their distance.

And this observation was similar to the theoritical relationship of velocity and distance, derived from cosmological models considering that the universe is expanding.

② Cosmic Microwave Background (CMB)

The measurements of CMB radiation show patterns of temperature fluctuations consistent with a universe that was once denser and hotter and has been expanding ever since.

CMB confirms the Big Bang frame work and maps the early expansion.

③ Baryon Acoustic Oscillations (BAO)

BAO are periodic fluctuations in the distribution of visible matter and provide a "standard ruler" to measure the expansion history. The data from large galaxy surveys match the theoritical perdictions for an expanding universe.

④ Abundance of light elements

The observed relative amounts of hydrogen, helium and lithium match predictions from Big Bang nucleosythesis, which requires a hot, dense and expanding early universe.

Evidences of universe being homogeneous and isotropic

. Isotropy

① Cosmic Microwave Background Radiation

The CMB is extrodinarly uniform at temperature 2.725K across the sky. with fluctuations as tiny as one part in 100.000. This high degree of istropy suggests the universe looks the same in all directions from our viewpoint.

② Distribution of galaxies and galaxy clusters

Large sky surveys like the Sloan Digital Sky Survey (SDSS) show that on scales of hundreds of millions of light years, galaxy distribution is roughly uniform and lacks preferred directions.

③ X-ray and gamma-ray backgrounds

The diffuse X-ray and gamma-ray backgrounds are observed to be isotropic, supporting isotropy in different wavelengths of radiation.

④ Isotropy of Hubble Expansion

Measurements of expansion rates (redshifts) are consistent in all directions, further conforming isotropy.

• Homogeneity

① Large Scale Structure

On scales greater than about 300 million light-years, the universe's matter distribution smooths out statistically, showing no special locations or clumping beyond what the cosmological principle predicts.

② CMB as a snapshot of early universe

The uniformity of the CMB at the surface of last scattering implies that conditions were very similar everywhere then, implying homogenety in the early universe.

③ No special locations detected.

Observations do not reveal any "centre" or preferred position in the early / current universe, supporting homogeneity.

Now we use the isotropy and homogenity of the universe to build Friedmann - Robertson - Walker metric.

Friedmann - Robertson - Walker metric (FRW):

The standard model of cosmology is built using the FRW metric as the geometric basis, it is often called the Lambda CDM model.

building the metric

general form of space-time metric

$$ds^2 = -dt^2 + g_{ij}(t, \vec{x})\, dx^i dx^j$$

where $t \rightarrow$ cosmic time

$g_{ij} \rightarrow$ spatial metrix tensor

from isotropy (spherically symmetric about every point) and homogeneity (symmetry holds universally) we can say that spatial part is of constant curvature

from differential geometry.

3D spaces with constant curvature K have metrices of the form

$$d\sigma^2 = \frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

where $r, \theta, \phi \rightarrow$ comoving spacial coordinates

$K \rightarrow$ normalized spacial curvature constant

$K = 0$: flat (Eucleadian) space

$K = +1$: positively curved (spherical) space

$K = -1$: negatively curved (hyperbolic) space

using this spatial metric, we get :

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

where $R(t)$ is a scale factor wrt time

we introduced this as universe is expanding / contracting with time

$\therefore$ FRW metric is

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

Representing the above metric in Einstein summation

$$ds^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu} dx^\mu dx^\nu$$

where $g_{\mu\nu} = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & -\frac{R^2(t)}{1-Kr^2} & 0 & 0 \\ 0 & 0 & -R^2(t)r^2 & 0 \\ 0 & 0 & 0 & -R^2(t)r^2\sin^2\theta \end{pmatrix}$ (by comparing)

Now finding values of Christoffel symbols using above $g_{\mu\nu}$

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\alpha}(\partial_\mu g_{\alpha\nu} + \partial_\nu g_{\mu\alpha} - \partial_\alpha g_{\mu\nu})$$

finding $g^{\lambda\alpha}$

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{1-Kr^2}{R^2(t)} & 0 & 0 \\ 0 & 0 & -\frac{1}{R^2(t)r^2} & 0 \\ 0 & 0 & 0 & -\frac{1}{R^2(t)r^2\sin^2\theta} \end{pmatrix}$$

$\therefore \Gamma^{\lambda}_{\mu\nu}$ will be zero for all $\lambda \neq \alpha$

$\because g^{\lambda\alpha} = 0$ for all $\lambda \neq \alpha$

now therefore we can say

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\lambda} \left(\partial_\mu g_{\lambda\nu} + \partial_\nu g_{\mu\lambda} - \partial_\lambda g_{\mu\nu}\right)$$

for $\lambda = 0$

① $\mu = 0, \nu = 0$

$$\Gamma^{0}_{00} = \frac{1}{2} g^{00} \left(\partial_0 g_{00} + \partial_0 g_{00} - \partial_0 g_{00}\right)$$

we know $\partial_0 = \frac{\partial}{\partial t}$

$$\Gamma^{0}_{00} = \frac{1}{2}\, 1 \Big(\underbrace{\frac{\partial}{\partial t}(1) + \frac{\partial}{\partial t}(1) - \frac{\partial}{\partial t}(1)}_{0}\Big)$$

$$\boxed{\Gamma^{0}_{00} = 0}$$

② $\mu = 0, \nu = 1$

$$\Gamma^{0}_{01} = \frac{1}{2} g^{00} \left(\partial_0 g_{01} + \partial_1 g_{00} - \partial_0 g_{01}\right)$$

$$= \frac{1}{2} \Big(\frac{\partial}{\partial t}(0) + \underbrace{\frac{\partial}{\partial r}(1)}_{0} - \frac{\partial}{\partial t}(0)\Big) = 0$$

$$\boxed{\Gamma^0_{01} = 0}$$

we know $\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu}$

$$\Rightarrow \boxed{\Gamma^0_{10} = 0}$$

③ $\mu = 0, \nu = 2$

$$\Gamma^0_{02} = \tfrac{1}{2} g^{00} \left(\partial_0 g_{02} + \partial_2 g_{00} - \partial_0 g_{02} \right)$$

$$= \tfrac{1}{2} \left(\frac{\partial}{\partial t}(0) + \underbrace{\frac{\partial}{\partial \theta}(1)}_{0} - \frac{\partial}{\partial t}(0) \right)$$

$$\boxed{\Gamma^0_{02} = 0}$$

$$\Rightarrow \boxed{\Gamma^0_{20} = 0}$$

④ $\Gamma^0_{03} = \tfrac{1}{2} g^{00} \left(\partial_0 g_{03} + \partial_3 g_{00} - \partial_0 g_{03} \right)$

$$= \tfrac{1}{2} \left(\frac{\partial}{\partial t}(0) + \underbrace{\frac{\partial}{\partial \phi}(1)}_{0} - \frac{\partial}{\partial t}(0) \right)$$

$$\boxed{\Gamma^0_{03} = 0}$$

$$\Rightarrow \boxed{\Gamma^0_{30} = 0}$$

⑤ $\mu = 1, \nu = 0$

$$\Gamma^0_{10} = \tfrac{1}{2} g^{00} \left(\partial_1 g_{00} + \partial_0 g_{10} - \partial_0 g_{10} \right)$$

$$= \tfrac{1}{2} \left(\frac{\partial}{\partial t}(0) + \frac{\partial}{\partial r}(1) - \frac{\partial}{\partial t}(0) \right)$$

$$\Gamma^0_{10} = 0$$

hence $\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu}$

⑥ $\mu=1, \nu=1$

$$\Gamma^0_{11} = \tfrac{1}{2} g^{00}\left(\partial_1 g_{01} + \partial_1 g_{10} - \partial_0 g_{11}\right)$$

$$= \tfrac{1}{2}(1)\left(\frac{\partial}{\partial r}(0) + \frac{\partial}{\partial r}(0) - \frac{\partial}{\partial t}\left(\frac{-R^2(t)}{1-kr^2}\right)\right)$$

$$\boxed{\Gamma^0_{11} = \frac{R(t)\dot{R}(t)}{1-kr^2}}$$

⑦ $\mu=1, \nu=2$

$$\Gamma^0_{12} = \tfrac{1}{2} g^{00}\left(\partial_1 g_{02} + \partial_2 g_{10} - \partial_0 g_{12}\right)$$

$$= \tfrac{1}{2}\left(\frac{\partial}{\partial r}(0) + \frac{\partial}{\partial \theta}(0) - \frac{\partial}{\partial t}(0)\right)$$

$$\boxed{\Gamma^0_{12} = 0}$$

$$\Rightarrow \boxed{\Gamma^0_{21} = 0}$$

⑧ $\mu=1, \nu=3$

$$\Gamma^0_{13} = \tfrac{1}{2} g^{00}\left(\partial_1 g_{03} + \partial_3 g_{10} - \partial_0 g_{13}\right)$$

$$= \tfrac{1}{2}\left(\frac{\partial}{\partial r}(0) + \frac{\partial}{\partial \phi}(0) - \frac{\partial}{\partial t}(0)\right)$$

$$\boxed{\Gamma^0_{13} = 0}$$

$$\Rightarrow \boxed{\Gamma^0_{31} = 0}$$

⑨ $\mu=2, \nu=2$

$$\Gamma^0_{22} = \tfrac{1}{2} g^{00}\left(\partial_2 g_{02} + \partial_2 g_{20} - \partial_0 g_{22}\right)$$

$$= \tfrac{1}{2}\left(\frac{\partial}{\partial \theta}(0) + \frac{\partial}{\partial \theta}(0) - \frac{\partial}{\partial t}\left(-R^2(t)r^2\right)\right)$$

$$\boxed{\Gamma^0_{22} = R(t)\dot{R}(t)r^2}$$

⑩ $\mu = 2, \nu = 3$

$$\Gamma^0_{23} = \tfrac{1}{2} g^{00} \left(\partial_2 g_{03} + \partial_3 g_{20} - \partial_0 g_{23} \right)$$

$$= \tfrac{1}{2} \left(\frac{\partial}{\partial \theta}(0) + \frac{\partial}{\partial \phi}(0) - \frac{\partial}{\partial t}(0) \right)$$

$$\boxed{\Gamma^0_{23} = 0}$$

$$\Rightarrow \boxed{\Gamma^0_{32} = 0}$$

⑪ $\mu = 3, \nu = 3$

$$\Gamma^0_{33} = \tfrac{1}{2} g^{00} \left(\partial_3 g_{03} + \partial_3 g_{30} - \partial_0 g_{33} \right)$$

$$= \tfrac{1}{2} \left(\frac{\partial}{\partial \phi}(0) + \frac{\partial}{\partial \phi}(0) - \frac{\partial}{\partial t}\left(-R^2(t) r^2 \sin^2\theta \right) \right)$$

$$\boxed{\Gamma^0_{33} = R(t) \dot{R}(t) r^2 \sin^2\theta}$$

for $\lambda = 1$

① $\mu = 0, \nu = 0$

$$\Gamma^1_{00} = \tfrac{1}{2} g^{11} \left(\partial_0 g_{10} + \partial_0 g_{01} - \partial_1 g_{00} \right)$$

$$= \tfrac{1}{2} \left(- \frac{1 - kr^2}{R(t)} \right) \left(\frac{\partial}{\partial t}(0) + \frac{\partial}{\partial t}(0) - \underbrace{\frac{\partial}{\partial r}(1)}_{0} \right)$$

$$\boxed{\Gamma^1_{00} = 0}$$

② $\mu = 0, \nu = 1$

$$\Gamma^1_{01} = \tfrac{1}{2} g^{11} \left(\partial_0 g_{11} + \partial_1 g_{01} - \partial_1 g_{01} \right)$$
$$= \tfrac{1}{2} g^{11} \left(\frac{\partial}{\partial t}(g_{11}) + \frac{\partial}{\partial r}(0) - \frac{\partial}{\partial r}(0) \right)$$

$$\Gamma^1_{01} = 1/2 \left(-\frac{1-kr^2}{R^2(t)}\right)\left(\frac{\partial}{\partial t}\left(-\frac{R^2(t)}{1-kr^2}\right)\right)$$

$$= \left(\frac{1-kr^2}{R^2(t)}\right)\left(\frac{R(t)\dot{R}(t)}{1-kr^2}\right)$$

$$\boxed{\Gamma^1_{01} = \frac{\dot{R}(t)}{R(t)}}$$

$$\Rightarrow \boxed{\Gamma^1_{10} = \frac{\dot{R}(t)}{R(t)}}$$

③ $\mu=0, \nu=2$

$$\Gamma^1_{02} = 1/2\, g^{11}\left(\partial_0 g_{12} + \partial_2 g_{01} - \partial_1 g_{02}\right)$$

$$= 1/2\, g^{11}\left(\frac{\partial}{\partial t}(0) + \frac{\partial}{\partial \theta}(0) - \frac{\partial}{\partial r}(0)\right)$$

$$\boxed{\Gamma^1_{02} = 0}$$

$$\Rightarrow \boxed{\Gamma^1_{20} = 0}$$

④ $\mu=0, \nu=3$

$$\Gamma^1_{03} = 1/2\, g^{11}\left(\partial_0 g_{13} + \partial_3 g_{01} - \partial_1 g_{03}\right)$$

$$= 1/2\, g^{11}\left(\frac{\partial}{\partial t}(0) + \frac{\partial}{\partial \phi}(0) - \frac{\partial}{\partial r}(0)\right)$$

$$\boxed{\Gamma^1_{03} = 0}$$

$$\Rightarrow \boxed{\Gamma^1_{30} = 0}$$

⑤ $\mu=1, \nu=1$

$$\Gamma^1_{11} = 1/2\, g^{11}\left(\partial_1 g_{11} + \cancel{\partial_1 g_{11}} - \cancel{\partial_1 g_{11}}\right)$$

$$= 1/2\left(-\frac{1-kr^2}{R^2(t)}\right)\left(\frac{\partial}{\partial r}\left(-\frac{R^2(t)}{1-kr^2}\right)\right)$$

$$\Gamma^1_{11} = \frac{1}{2}\left(\frac{1-kr^2}{\cancel{R^2(t)}}\right)\left(\cancel{R^2(t)}\right)\frac{\partial}{\partial r}\left(\frac{1}{1-kr^2}\right)$$

$$= \frac{1}{\cancel{2}}\left(\cancel{1-kr^2}\right)\left(-\frac{1}{(1-kr^2)^{\cancel{2}}}\right)\left(-\cancel{2}kr\right)$$

$$\boxed{\Gamma^1_{11} = \frac{kr}{1-kr^2}}$$

⑥ $\mu=1, \nu=2$

$$\Gamma^1_{12} = \frac{1}{2}g^{11}\left(\partial_1 g_{12} + \partial_2 g_{11} - \partial_1 g_{12}\right)$$

$$= \frac{1}{2}\left(-\frac{1-kr^2}{R^2(t)}\right)\left(\frac{\partial}{\partial r}(0) + \underbrace{\frac{\partial}{\partial \theta}\left(-\frac{R^2(t)}{1-kr^2}\right)}_{0} + \frac{\partial}{\partial r}(0)\right)$$

$$\boxed{\Gamma^1_{12} = 0}$$

$$\Rightarrow \boxed{\Gamma^1_{21} = 0}$$

⑦ $\mu=1, \nu=3$

$$\Gamma^1_{13} = \frac{1}{2}g^{11}\left(\partial_1 g_{13} + \partial_3 g_{11} - \partial_1 g_{13}\right)$$

$$= \frac{1}{2}\left(-\frac{1-kr^2}{R^2(t)}\right)\left(\frac{\partial}{\partial r}(0) + \underbrace{\frac{\partial}{\partial \phi}\left(-\frac{R^2(t)}{1-kr^2}\right)}_{0} + \frac{\partial}{\partial r}(0)\right)$$

$$\boxed{\Gamma^1_{13} = 0}$$

$$\Rightarrow \boxed{\Gamma^1_{31} = 0}$$

⑧ $\mu=2, \nu=2$

$$\Gamma^1_{22} = \frac{1}{2}g^{11}\left(\partial_2 g_{12} + \partial_2 g_{21} + \left(-\partial_1 g_{22}\right)\right)$$

$$= \frac{1}{2}\left(-\frac{1-kr^2}{R^2(t)}\right)\left(\frac{\partial}{\partial \theta}(0) + \frac{\partial}{\partial \theta}(0) - \frac{\partial}{\partial r}\left(-R^2(t)r^2\right)\right)$$

$$\Gamma^1_{22} = \cancel{1/2}\left(-\frac{1-kr^2}{\cancel{R^2(t)}}\right)\left(\cancel{2}\cancel{R^2(t)}\,r\right)$$

$$\boxed{\Gamma^1_{22} = -r(1-kr^2)}$$

(9) $\mu=2,\ \nu=3$

$$\Gamma^1_{23} = 1/2\, g^{11}\left(\partial_2 g_{13} + \partial_3 g_{21} - \partial_1 g_{23}\right)$$

$$= 1/2\, g^{11}\left(\frac{\partial}{\partial\theta}(0) + \frac{\partial}{\partial\phi}(0) - \frac{\partial}{\partial r}(0)\right)$$

$$\boxed{\Gamma^1_{23} = 0}$$

$$\Rightarrow \boxed{\Gamma^1_{32} = 0}$$

(10) $\mu=3,\ \nu=3$

$$\Gamma^1_{33} = 1/2\, g^{11}\left(\partial_3 g_{13} + \partial_3 g_{31} - \partial_1 g_{33}\right)$$

$$= 1/2\left(-\frac{(1-kr^2)}{R^2(t)}\right)\left(\frac{\partial}{\partial\phi}(0) + \frac{\partial}{\partial\phi}(0) - \frac{\partial}{\partial r}\left(R^2(t)\, r^2 \sin^2\theta\right)\right)$$

$$= \cancel{1/2}\left(-\frac{(1-kr^2)}{\cancel{R^2(t)}}\right)\left(\cancel{2}\cancel{R^2(t)}\, r \sin^2\theta\right)$$

$$\boxed{\Gamma^1_{33} = -r\sin^2\theta\,(1-kr^2)}$$

for $\lambda=2$

(1) $\mu=0,\ \nu=0$

$$\Gamma^2_{00} = 1/2\, g^{22}\left(\partial_0 g_{20} + \partial_0 g_{02} - \partial_2 g_{00}\right)$$

$$= 1/2\, g^{22}\left(\frac{\partial}{\partial t}(0) + \frac{\partial}{\partial t}(0) - \underbrace{\frac{\partial}{\partial\theta}(1)}_{0}\right)$$

$$\boxed{\Gamma^2_{00} = 0}$$

② $\mu = 0, \nu = 1$

$$\Gamma^2_{01} = \frac{1}{2} g^{22} \left(\partial_0 g_{21} + \partial_1 g_{02} - \partial_2 g_{01} \right)$$

$$= \frac{1}{2} g^{22} \left(\frac{\partial}{\partial t}(0) + \frac{\partial}{\partial r}(0) - \frac{\partial}{\partial \theta}(0) \right)$$

$$\boxed{\Gamma^2_{01} = 0}$$

$$\hookrightarrow \boxed{\Gamma^2_{10} = 0}$$

③ $\mu = 0, \nu = 2$

$$\Gamma^2_{02} = \frac{1}{2} g^{22} \left(\partial_0 g_{22} + \partial_2 g_{02} - \partial_2 g_{02} \right)$$

$$= \frac{1}{2} \left(-\frac{1}{R^2(t) r^2} \right) \left(\frac{\partial}{\partial t} \left(-R^2(t) r^2 \right) \right)$$

$$= \frac{1}{\cancel{2}} \left(-\frac{1}{R^{\cancel{2}}(t) \cancel{r^2}} \right) \left(-\cancel{2} \cancel{R(t)} \dot{R}(t) \cancel{r^2} \right.$$

$$\boxed{\Gamma^2_{02} = \frac{\dot{R}(t)}{R(t)}}$$

$$\hookrightarrow \boxed{\Gamma^2_{20} = \frac{\dot{R}(t)}{R(t)}}$$

④ $\mu = 0, \nu = 3$

$$\Gamma^2_{03} = \frac{1}{2} g^{22} \left(\partial_0 g_{23} + \partial_3 g_{02} - \partial_2 g_{03} \right)$$

$$= \frac{1}{2} g^{22} \left(\frac{\partial}{\partial t}(0) + \frac{\partial}{\partial \phi}(0) - \frac{\partial}{\partial \theta}(0) \right)$$

$$\boxed{\Gamma^2_{03} = 0}$$

$$\hookrightarrow \boxed{\Gamma^2_{30} = 0}$$

⑤ $\mu = 1, \nu = 1$

$$\Gamma^2_{11} = \frac{1}{2} g^{22} \left(\partial_1 g_{21} + \partial_1 g_{12} - \partial_2 g_{11} \right)$$

$$\Gamma^2_{11} = \frac{1}{2} g^{22} \left(\frac{\partial}{\partial r}(0) + \frac{\partial}{\partial r}(0) - \underbrace{\frac{\partial}{\partial \theta}\left(-\frac{1-kr^2}{R^2(t)} \right)^{-1}}_{0} \right)$$

$$\boxed{\Gamma^2_{11} = 0}$$

$\hookrightarrow \because g_{11}$ is independent of θ

⑥ $\mu = 1, \nu = 2$

$$\Gamma^2_{12} = \frac{1}{2} g^{22} \left(\partial_1 g_{22} + \partial_2 g_{12} - \partial_2 g_{12} \right)$$

$$= \frac{1}{2} \left(-\frac{1}{R^2(t) r^2} \right) \left(\frac{\partial}{\partial r} \left(-R^2(t)(r^2) \right) \right)$$

$$= \frac{1}{\cancel{2}} \frac{1}{\cancel{R^2(t)} r^{\cancel{2}}} \left(\cancel{R^2(t)} (\cancel{2} \cancel{r}) \right)$$

$$\boxed{\Gamma^2_{12} = 1/r}$$

$$\hookrightarrow \boxed{\Gamma^2_{21} = 1/r}$$

⑦ $\mu = 1, \nu = 3$

$$\Gamma^2_{13} = \frac{1}{2} g^{22} \left(\partial_1 g_{23} + \partial_3 g_{12} - \partial_2 g_{13} \right)$$

$$= \frac{1}{2} g^{22} \left(\frac{\partial}{\partial r}(0) + \frac{\partial}{\partial \phi}(0) - \frac{\partial}{\partial \theta}(0) \right)$$

$$\boxed{\Gamma^2_{13} = 0}$$

$$\hookrightarrow \boxed{\Gamma^2_{31} = 0}$$

⑧ $\mu = 2, \nu = 2$

$$\Gamma^2_{22} = \frac{1}{2} g^{22} \left(\partial_2 g_{22} + \cancel{\partial_2 g_{22}} - \cancel{\partial_2 g_{22}} \right)$$

$$= \frac{1}{2} g^{22} \left(\underbrace{\frac{\partial}{\partial \theta} \left(-R^2(t) r^2 \right)}_{0} \right) = 0$$

$$\boxed{\Gamma^2_{22} = 0}$$

⑨ $\mu = 2, \nu = 3$

$$\Gamma^2_{23} = \tfrac{1}{2} g^{22} \left(\partial_2 \cancel{g_{23}} + \partial_3 g_{22} - \partial_2 \cancel{g_{23}} \right)$$

$$= \tfrac{1}{2} g^{22} \left(\underbrace{\frac{\partial}{\partial \phi} \left(-R^2(t) r^2 \right)}_{0} \right)$$

$$\boxed{\Gamma^2_{23} = 0}$$

$$\hookrightarrow \boxed{\Gamma^2_{32} = 0}$$

⑩ $\Gamma^2_{33} = \tfrac{1}{2} g^{22} \left(\partial_3 g_{23} + \partial_3 g_{32} - \partial_2 g_{33} \right)$

$$= \tfrac{1}{2} \left(-\frac{1}{R^2(t) r^2} \right) \left(\frac{\partial}{\partial \phi}(0) + \frac{\partial}{\partial \phi}(0) - \frac{\partial}{\partial \theta} \left(-R^2(t) r^2 \sin^2\theta \right) \right)$$

$$= \tfrac{1}{\cancel{2}} \left(-\frac{1}{\cancel{R^2(t) r^2}} \right) \left(\cancel{2} \sin\theta \cos\theta \, \cancel{R^2(t) r^2} \right)$$

$$\boxed{\Gamma^2_{33} = \sin\theta \cos\theta}$$

for $\lambda = 3$

① $\mu = 0, \nu = 0$

$$\Gamma^3_{00} = \tfrac{1}{2} g^{33} \left(\partial_0 g_{30} + \partial_0 g_{03} + (-\partial_3 g_{00}) \right)$$

$$= \tfrac{1}{2} g^{33} \left(\frac{\partial}{\partial t}(0) + \frac{\partial}{\partial t}(0) - \underbrace{\frac{\partial}{\partial \phi}(1)}_{0} \right)$$

$$\boxed{\Gamma^3_{00} = 0}$$

② $\mu = 0, \nu = 1$

$$\Gamma^3_{01} = \tfrac{1}{2} g^{33} \left(\partial_0 g_{31} + \partial_1 g_{03} - \partial_3 g_{01} \right)$$

$$= \frac{1}{2} g^{33} \left(\frac{\partial}{\partial t}(0) + \frac{\partial}{\partial r}(0) - \frac{\partial}{\partial \phi}(0)\right)$$

$$\boxed{\Gamma^3_{01} = 0}$$

$$\hookrightarrow \boxed{\Gamma^3_{10} = 0}$$

③ $\mu = 0, \nu = 2$

$$\Gamma^3_{02} = \frac{1}{2} g^{33} \left(\partial_0 g_{32} + \partial_2 g_{03} - \partial_3 g_{02}\right)$$

$$= \frac{1}{2} g^{33} \left(\frac{\partial}{\partial t}(0) + \frac{\partial}{\partial \theta}(0) - \frac{\partial}{\partial \phi}(0)\right)$$

$$\boxed{\Gamma^3_{02} = 0}$$

$$\hookrightarrow \boxed{\Gamma^3_{20} = 0}$$

④ $\mu = 0, \nu = 3$

$$\Gamma^3_{03} = \frac{1}{2} g^{33} \left(\partial_0 g_{33} + \partial_3 \cancel{g_{03}} - \partial_3 \cancel{g_{03}}\right)$$

$$= \frac{1}{2} \left(-\frac{1}{R^2(t) r^2 \sin^2\theta}\right) \left(\frac{\partial}{\partial t}\left(-R^2(t) r^2 \sin^2\theta\right)\right)$$

$$= \frac{1}{\cancel{2}} \left(\frac{1}{R^{\cancel{2}}(t) \cancel{r^2} \cancel{\sin^2\theta}}\right) \left(\cancel{2}\dot{R}(t) \cancel{R(t)} \cancel{r^2} \cancel{\sin^2\theta}\right)$$

$$\boxed{\Gamma^3_{03} = \frac{\dot{R}(t)}{R(t)}}$$

$$\hookrightarrow \boxed{\Gamma^3_{30} = \frac{\dot{R}(t)}{R(t)}}$$

⑤ $\mu = 1, \nu = 1$

$$\Gamma^3_{11} = \frac{1}{2} g^{33} \left(\partial_1 g_{31} + \partial_1 g_{13} - \partial_3 g_{11}\right)$$

$$= \frac{1}{2} g^{33} \left(\frac{\partial}{\partial r}(0) + \frac{\partial}{\partial r}(0) - \underbrace{\frac{\partial}{\partial \phi}\left(\frac{-R^2(t)}{1 - kr^2}\right)}_{0}\right) = 0$$

$$\boxed{\Gamma^3_{11} = 0}$$