Dark Matter

Introduction :

It is a kind of matter that does not interact with electromagnetic radiations in any way, making it practically invisible to any kind of telescopic observations.

However its presence can be inferred from its gravitational effect on visible matter, radiations, structure of universe. So their presence can be deduced from gravimetry or gravitational lensing.

It makes up 27% of mass-energy content of universe. It is the most abundantly found mass content. (visible matter 4.8%)

Evidences :

① Radial velocity dispersion of galaxies in coma cluster :

Frit Zwicky in 1933 studied the radial velocity dispersion of galaxies in Coma cluster using virial theorem.

Virial theorem

$$2\langle T\rangle = -\langle U\rangle$$

T : ~~kinetic~~ kinetic energy

U : potential energy

From Newton's law

$$\vec{F_i} = m_i \frac{d^2 \vec{r_i}}{dt^2}$$

multiplying with position vector (dot product)

$$\vec{r}_i \cdot \vec{F}_i = m_i \, \vec{r}_i \, \frac{d^2\vec{r}_i}{dt^2}$$

from $\quad \frac{d^2}{dt^2}(\vec{r}^{\,2}) = 2\left(\frac{d\vec{r}}{dt} \cdot \frac{d\vec{r}}{dt}\right) + 2\left(\frac{d^2\vec{r}}{dt^2}\right)\vec{r}$

$$\vec{r}\,\frac{d^2\vec{r}}{dt^2} = \tfrac{1}{2}\,\frac{d^2(\vec{r}^{\,2})}{dt^2} - (\cancel{2}\,v^2)/\cancel{2}$$

$$\vec{r}\,\frac{d^2\vec{r}}{dt^2} = \tfrac{1}{2}\,\frac{d^2(\vec{r}^{\,2})}{dt^2} - v^2$$

substituting back into equation

$$\vec{r}_i \cdot \vec{F}_i + m_i v_i^2 = \tfrac{1}{2}\,\frac{d^2}{dt^2}(m_i \vec{r}_i^{\,2})$$

taking time average

$$\langle \vec{r}_i \cdot \vec{F}_i \rangle + \langle m_i v_i^2 \rangle = \left\langle \tfrac{1}{2}\,\frac{d^2}{dt^2}(m_i \vec{r}_i^{\,2}) \right\rangle$$

$\because$ it is a ~~(gravitationally)~~ gravitationally bound, steady-state system

$$\left\langle \frac{d^2}{dt^2}(\quad) \right\rangle = 0$$

$$\Rightarrow \langle \vec{r}_i \cdot \vec{F}_i \rangle + \langle m_i v_i^2 \rangle = 0$$

as m_i is a constant wrt time

$$\langle \vec{r}_i \cdot \vec{F}_i \rangle = -\, m_i \langle v_i^2 \rangle$$

as the force acting is gravitational force

$$\vec{F}_i = -\sum_{j \neq i} \frac{G m_i m_j (\vec{r}_i - \vec{r}_j)}{|\vec{r}_i - \vec{r}_j|^3}$$

$\vec{r_i} \cdot \vec{F_i}$ gives just the radial part of the gravitational force

$$\vec{r_i} \cdot \vec{F_i} = -\sum_{j \neq i} \frac{G m_i m_j}{|\vec{r_i} - \vec{r_j}|^3} \vec{r_i} \cdot (\vec{r_i} - \vec{r_j})$$

$$= -\sum_{j \neq i} \frac{G m_i m_j}{|\vec{r_i} - \vec{r_j}|^3} r_i^2 - \underbrace{(\vec{r_i} \cdot \vec{r_j})}_{\text{symmetrical}}.$$

$$\Rightarrow \sum_i \vec{r_i} \cdot \vec{F_i} = -1/2 \sum_{i \neq j} \frac{G m_i m_j}{|\vec{r_i} - \vec{r_j}|}$$

taking time average

$$\langle \vec{r_i} \cdot \vec{F_i} \rangle = -\sum_{j<i} \frac{G m_i m_j}{r_{ij}}$$

where $r_{ij} = |\vec{r_i} - \vec{r_j}|$

$$\Rightarrow m_i \langle v_i^2 \rangle = \sum_{j<i} \frac{G m_i m_j}{r_{ij}}$$

summing over all galaxies

$$\sum_i m_i \langle v_i^2 \rangle = \sum_i \sum_{j<i} \frac{G m_i m_j}{r_{ij}}$$

$$\sum_i m_i = M_{tot}$$

the average squared velocity

$$\langle v^2 \rangle = 1/M_{tot} \sum_i m_i \langle v_i^2 \rangle$$

$$\Rightarrow M_{tot} \langle v^2 \rangle = \underbrace{\sum_i \sum_{j<i} \frac{G m_i m_j}{r_{ij}}}_{\text{total gravitational potential (U) energy}}$$

evaluating U for a symmetric mass distributions give us

$$|U| = \frac{3}{5} \frac{G M_{tot}^2}{R_{tot}}$$

putting this back into the equation

$$M_{tot} \langle v^2 \rangle = \frac{3}{5} \frac{G M_{tot}^2}{R_{tot}}$$

$$\Rightarrow \boxed{M_{tot} = \frac{5}{3} \frac{R_{tot} \langle v^2 \rangle}{G}}$$

observed velocity dispersion and R_{tot} for Coma cluster is

$\langle v^2 \rangle \sim 10^{15}\ cm^2/s^2$; $R_{tot} = 1 Mpc = 3.09 \times 10^{24} cm$

hence

$$M_{tot} = \frac{5}{3} \frac{(3.09 \times 10^{24})(10^{15})}{6.6743 \times 10^{-11}} \left(\frac{cm \cdot cm^2/s^2}{m^3/kg\, s^2} \right)$$

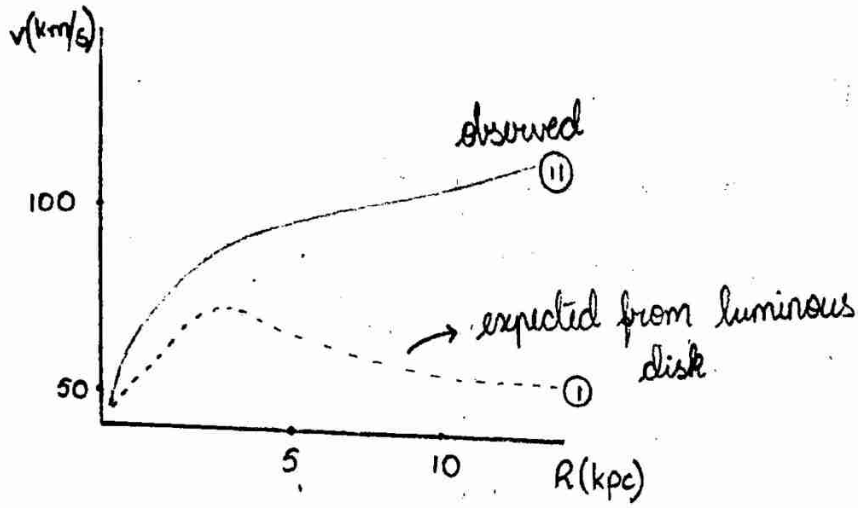
now solving the above gives us $\rightarrow M_{tot} \sim 4.5 \times 10^{13} M_{\odot}$

but $M_{tot} \sim 8.5 \times 10^{7} M_{\odot}$ is the total observed mass or rather the mass calculated from stellar light

He called the matter that is contributing to the mass difference or additional mass as Dark Matter ("Dunkel Materie") because it is not visible or non-luminous.

② Galactic rotation curve :

Vera Rubin in 1970 studied the angular motion of galaxies. She made a galactic rotation curve for various galaxies.



ON applying the equation of motion on visible part / disk we get

$$\frac{m v^2(r)}{r} = \frac{G M(r) m}{r^2}$$

$$M(r) = 4/3 \pi r^3 \rho$$

hence $v^2(r) = 4/3 \pi r^2 G$

$$\Rightarrow v^2(r) \propto r^2$$

$$\boxed{v(r) \propto r}$$

∴ as r increases velocity should increase with in range of the visible galaxy cluster

once we go beyond it no mass was expected to be present

∴ mass becomes constant

$$\therefore \frac{mv^2(r)}{r} = \frac{GMm}{r^2}$$

$$v^2(r) = \frac{GM}{r}$$

$$\Rightarrow v(r) \propto 1/r^{1/2}$$

so the velocity was expected to fall as the distance increases

But in the observed curve (II) the velocity increases upto a point and remains constant or increases slightly. This shows that there is mass extending beyound the visible galaxy. But as the mass is invisible, Vera Rubin came to the same conclusion as Frit Zwicky regarding the existance of Dark Matter.

She proposed that the galaxies are embeded in large, massive, unseen halos of matter, known today as dark matter hallos. (ex: Isothermal spehere, Navarro-Frenk-White, Burket profiles) (NFW)

③ Formation of Bullet cluster observed in 2006

The event of merging of two galaxies that happened ~~(in)~~ 3.7 billion years ago was observed by Chandra X-ray observatory in years 2002 to 2006.

The merged cluster now known as Bullet cluster was formed when a larger cluster experienced a high-speed collision from a smaller fast-moving subcluster (the 'bullet') that passed through it.

We cannot actually call it a merged cluster yet but it is the snap-shortly after the collision has occurred.

While observing this, they found that the visible matter had the impact of this collision, but on checking the results of gravitational lensing, a large amount of non-luminous mass seemed to have passed through without any or minimal interaction.

This lump of mass was recognised as Dark Matter by the observers

④ Cosmological Evidence

The Cosmic Microwave Background Radiation is the after glow of Big Bang, whose anisotropies encode information about the early universe's composition.

If universe contained only baryonic matter

· Due to Photon - Baryon coupling:

In the early universe, photons and baryons are expected to be bound or coupled tightly, behaving like a fluid that supports acoustic (sound) waves.

· Resistance to collapse:

The photonic pressure resists baryonic gravitational collapse untill recombination.

This supresses the density fluctuations

Due to these features of specturm were expected to be as follows

• Peak amplitudes : The first acoustic peak would have been due to baryonic compression and "loading" but subsequent peaks should have been much weaker due to no extra gravitational potential wells.

• Peak spacing : The distance between peaks would not match observations because baryons alone cannot explain all the properties and structures of early universe.

If only baryons existed the cosmic structures would have been formed much later and with less efficiency and it would not be same as we see today.

• The Overall anisotropies : The effect of temperature fluctuations would be greater and the detailed multipole structure would have been very different.

The observed spectrum

• On decomposing the CMB anisotropies into a spectrum of multipoles. showing series of acoustic peaks at well-defined positions and amplitudes, "The Angular Power Spectrum" we observe the following features

→ Multiple Peaks : All the peaks (3) are promitent

- 1st peak compression
- second / third peak rarefaction

→ Observed amplitutes and spacing: They were different from the predictions of models that focus only on existance of baryonic matter

But they are similar to the ones which consider both baryonic matter and significant cold dark matters' presence

→ The small one part (in 100,000 parts) fluctuations or anisotropies in amplitude of CMB due to Temperature are consistent with predictions of universes dominated by dark matter

∴ The anisotropies observed in CMBR spectrum is an evidence of existence and aboundance of dark matter.

The reasons:

Essential characteristic of Dark Matter:

→ No interaction with light or photon - baryon fluid.

→ It exerts gravitational influence, creating deep potential wells predictively even before recombination.

These help in creation of the specific peak structure in CMB angular power spectrum, as these wells allow baryons to recombine leading to structure formation.

The Lambda-CDM (cold dark matter) cosmological model including both dark matter and baryonic matter, fits the observed spectrum exactly

And through detailed model fitting we can find the constiuent ratios.

The best fit model gives following ratio

- Baryonic matter : $\Omega_b \simeq 0.049 (\sim 4.9\%)$
- Dark matter : $\Omega_{DM} \simeq 0.268 (\sim 26.8\%)$
- Dark energy : $\Omega_\Lambda \simeq 0.688 (\sim 68.3\%)$

where $\Omega_i = \frac{\rho_i}{\rho_{crit}}$

where ρ_i is density of component i
and ρ_{crit} is critical density for a flat universe

And this inturn helps us find Dark Matter Relic Density

$$\text{Relic Density} = \Omega_{DM} h^2 \simeq 0.1199 \pm 0.0022$$

where h is huble constant.

Epochs of Universe:

① Planck Epoch ($0 \sim 10^{-43}$ seconds)

→ The universe was in an extremely hot, dense state where all fundamental forces (gravity, electromagnetism, weak and strong nuclear forces) were believed to be unified.

→ Here quantum gravity effects are considered to be dominante and all the known physics laws break down is this phase.

→ The scale of physics at this epoch is Planck scale.

② Grand Unification Epoch ($\sim 10^{-43}$ to 10^{-36} seconds)

→ Gravity is assumed to get separated from the other fundamental forces.

→ This epoch ends with strong force separating from electroweak force.

→ Cosmic inflation is believed to begin at the end of this epoch, causing rapid exponential expansion.

③ Inflationary Epoch ($\sim 10^{-36}$ to 10^{-32} seconds)

→ The universe was believed to be expanding exponentially by a factor of about 10^{26} in magnitude.

→ Quantum fluctuations during inflation seed the tiny perturbations that grow into large scale structures.

→ sets the initial conditions for the observable universe's homogenity and flatness.

④ Electroweak Epoch ($\sim 10^{-36}$ to 10^{-12} seconds)

→ Electromagnetic and weak forces are unified.

→ The universe is considered to be still hot and dense.

→ Particle interactions shape the early matter content.

⑤ Quark Epoch ($\sim 10^{-12}$ to 10^{-6} seconds)

→ The universe in this phase is predicted to cool down just enough for quarks, leptons and gluons to exist freely in a quark-gluon plasma.

→ It assumed that may be matter - antimatter asymmetry was set during this phase.

⑥ Hadron Epoch ($\sim 10^{-6}$ to 1 second)

→ Quarks combine into hadrons.

→ Matter - antimatter annihilation is predicted to reduce particle density, leaving an excess of matter.

⑦ Lepton Epoch (~1 to 10 seconds)

→ Leptons are assumpted to dominate the universe's mass - energy.

→ neutrinos decouple and stream freely.

⑧ Photon Epoch (~10 seconds to 380,000 years)

→ Universe is dominated by photons, electrons and nuclei forming a plasma state.

→ Photons constantly scatter off electrons, preventing the universe from being transperent.

→ Dark Matter decouples from radiation and begins gravitationally clumping as it doesnot react / interact with photons.

⑨ (~~[illegible]~~) Recombination Epoch (~380,000 years)

→ Electrons combine with protons and neuclie to form neutral atoms.

→ This makes universe invisible as the long wavelength photons then existed could not scatter them and started traveling freely. These photons are now observed as Cosmic Microwave Background Radiation (CMBR).

→ This also marks the end of Photon Epoch.

⑩ Cosmic Dark Ages (~380,000 years to 150 million years)

→ It is the phase where neutral Hydrogen gas is expected to dominate.

→ No source of luminosity, hence the universe is dark.

→ Dark matter's gravitational wells, attracking baryonic matter, grow.

⑪ Reionization Epoch (~150 million years to 1 billion years)

→ First stars, galaxies and quasars (quasi-stellar radio sources) formed

→ Thier radiation reionizing the neutral hydrogen, making universe transparent for ultraviolet light.

→ structure fromation begins in earnest, guided by dark matter distribution.

⑫ Galaxy Epoch (~1 billion years to present)

→ Galaxies, galaxy clusters and large-scale cosmic structures form.

(~~→ Thier radiation~~)

→ The universe continues expanding and cooling.

→ Dark energy becomes dominant around 6 billion years ago, accelerating the expansion.

Dark Energy:

→ Dark Energy is a mysterious form of energy that permeates all of space and exerts a repulsive gravitational effect.
This causes expansion to accelerate rather than decelerate.

→ It is not clumped up or clustered like dark matter and matter but it is roughly uniformly distributed through-out space.

First Evidence:

→ Dark energy's discovery came in 1998 from observing distant Type 1a supernovae, which showed the univers's expansion is unexpectedly accelerating instead of slowing down due to gravity.

→ This was supported by multiple independent observations including Cosmic Microwave Background anisotropies, large - scale structure surveys and galaxy cluster measurements.

Theoritical Interpretation:

· The simplest interpretation of it is cosmological constant, introduced by Einstein, representing a constant energy density inherent to empty space, the vaccum energy.

· Mathematically the energy density of dark energy remains fixed

$$\rho_{\text{dark energy}} \propto a^0$$

where a is cosmic scale factor.

→ Dark energy currently constitutes about 68 - 69% of total mass-energy content of the universe

The Thermal Freeze out

Thermal freeze out is a key process in the early universe that determines the present day (relic) aboundance of certain particles - especially dark matter.

Lets now discuss about it

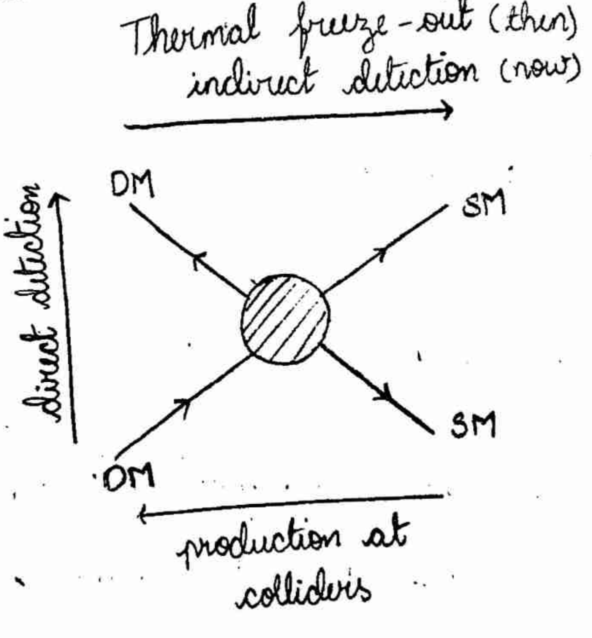
→ Dark Matter was assumed to be in equilibrium with hot plasma of standard particles in the early universe via $2 \leftrightarrow 2$ interaction.

There were two processes occuring

① interaction

② expansion

When universe was very hot interaction rate dominated the rate of expansion



$m_{DM}, m_{SM} \ll T$

$\Rightarrow x \ll 1$

$\Gamma \gg H$

where Γ is interaction rate

H is expansion rate

then

$$DM \; DM \longleftrightarrow SM \; SM$$

dark matter particles in thermal equilibrium with standard model particles.

But as the universe cools down, both rates become comparible

$m_{SM} < T < m_{DM}$

i.e. $x_f \sim x$

$\Gamma \sim H$

gives

$DM\, DM \longrightarrow SM\, SM$

the dark matter particles convert into standard model particles but not the other way round

Hence breaking the equilibrium

Finally as it cools down beyound a threshold, rate of expansion starts to dominate. In this phase Dark Matter freezes out.

That is it stops interacting and it's aboundance per comoving volume is fixed.

$m_{DM} \gg T$ i.e. $x \gg x_f$

$H \gg \Gamma$

$\therefore$ DM becomes relic

I now connect the cosmological relic density to the microscopic physics of early universe using Boltzmann equation. This helps us theoretically calculate the relic abundance.

Constructing Boltzmann Equations:

Introduction:

We use Boltzmann equation, to quantitatively describe the evolution of dark Matter particle abundance in early universe. It governs how the phase-space distribution function $f(x,p)$ changes with time, position and momentum for particles moving in the expanding, curved space-time of cosmology.

general form of Boltzmann equation

$$\hat{\mathcal{L}}[f] = C[f]$$

where:

$f \rightarrow$ phase space distribution function

$\hat{\mathcal{L}} \rightarrow$ Liouville operator

$c \rightarrow$ collision term

① Liouville operator

→ It discribes the evolution of the destribution function along particle paths determined by phase-space geometry. For a general relativistic formework, the particles follow geodesic paths, which are the straightest possible trajectories in curved space time

Solving the LHS

$$\hat{\mathcal{L}}[f(p^{\mu}, x^{\mu})]$$

where p^{μ} and x^{μ} are covariant vector components in four dimensional space-time

$$\hat{L}[f(p^\mu, x^\mu)] = \frac{df(p^\mu, x^\mu)}{d\tau}$$

where τ is an affine parameter along the particle trajectory

In this we choose proper time along particle's worldline as the affine parameter

$$\Rightarrow \hat{L}[f(p^\mu, x^\mu)] = \sum_i \left[\frac{\partial f}{\partial p^i}\left(\frac{\partial p^i}{\partial \tau}\right) + \frac{\partial f}{\partial x^i}\left(\frac{\partial x^i}{\partial \tau}\right)\right]$$

$$= \sum_i \left[\frac{\partial f}{\partial p^i}(\dot{p}^i) + \frac{\partial f}{\partial x^i}(\dot{x}^i)\right]$$

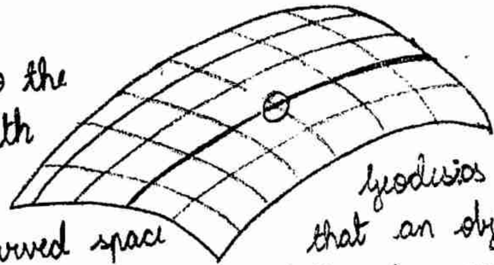
we use geodesic equation to solve further

(Deriving geodesic equation)

$\delta S = 0$ for actual path

where S is action

it is also the straightest path that can be followed in curved space time



geodesics are paths that an object will follow in curved space time when there is no external force

$$S = \int d\lambda (L)$$

L is lagrangian

deriving lagrangian from element of space-time (line element)

$$ds^2 = g_{ij}(x)\, dx^i dx^j$$

where $g_{ij}(x)$ is the metric tensor

parameterising the path wrt λ

$$ds^2 = g_{ij}(x) \frac{dx^i}{d\lambda} \frac{dx^j}{d\lambda} (d\lambda)^2$$

$$ds = \left(g_{ij}(x) \frac{dx^i}{d\lambda} \frac{dx^j}{d\lambda}\right)^{1/2} d\lambda$$