

Hierarchical methods.

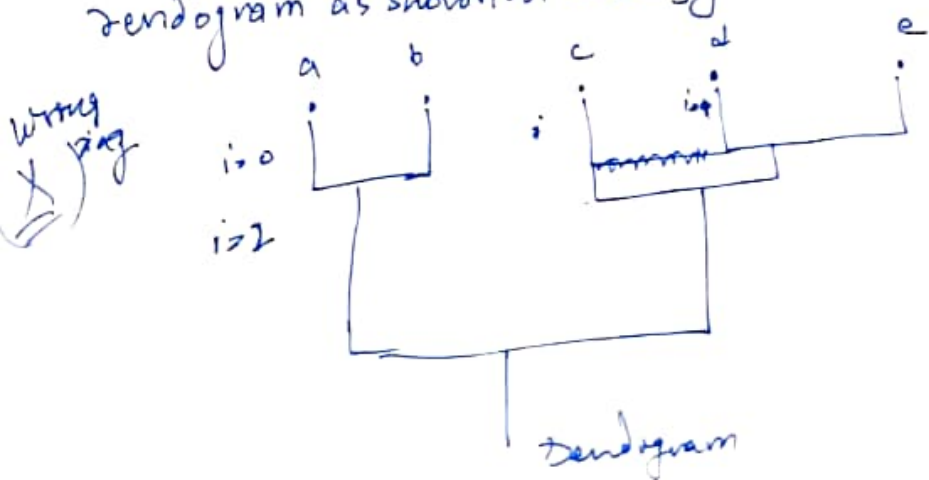
* The hierarchical agglomerative clustering methods are most commonly used. The basic steps followed in this type of Hierarchical methods or general algorithm are.

- 1 * Find the two closest objects + merge them into cluster.
- 2 * Find + merge the next two closest points, where a point is either an individual object or a cluster of objects.
- 3 * If more than one cluster remain, again return to step 2.

* Agglomerative Algorithm :- * It follows bottom-up strategy.

- * According to some similarity measure (ED), the merging is done by choosing the closest clusters first.
- * A dendrogram, which is a tree like structure, which is used to represent hierarchical clustering.
- * Individual objects are represented by leaf nodes + clusters are represented by root nodes.

This representation is known as dendrogram as shown in below fig.



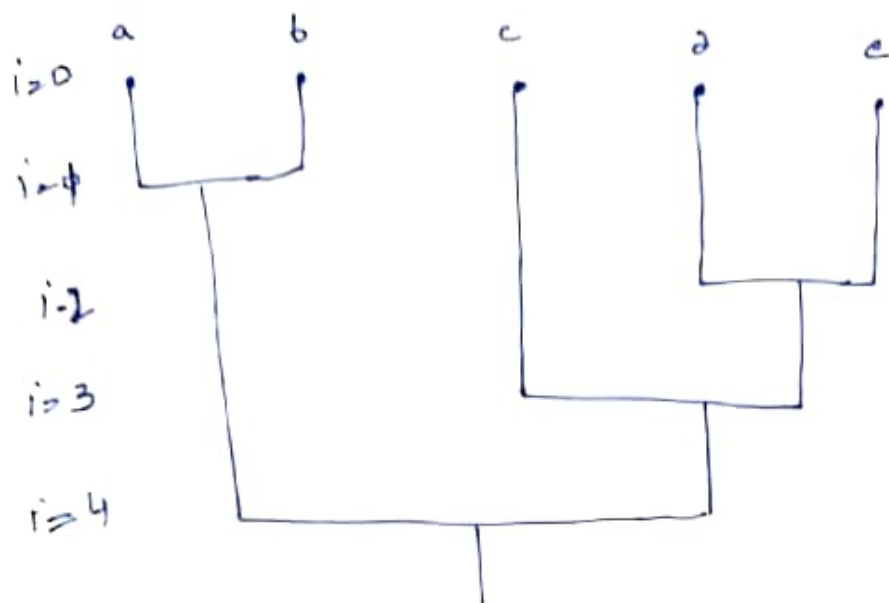


Fig 1. Dendrogram

Distance measure / similarity measure.

Min dist: $\text{dist}_{\min}(C_i, C_j) = \min_{p \in C_i, p' \in C_j} \{ |p - p'| \}$

This is called nearest-neighbour clustering algorithm.

Max dist: $\text{dist}_{\max}(C_i, C_j) = \max_{p \in C_i, p' \in C_j} \{ |p - p'| \}$

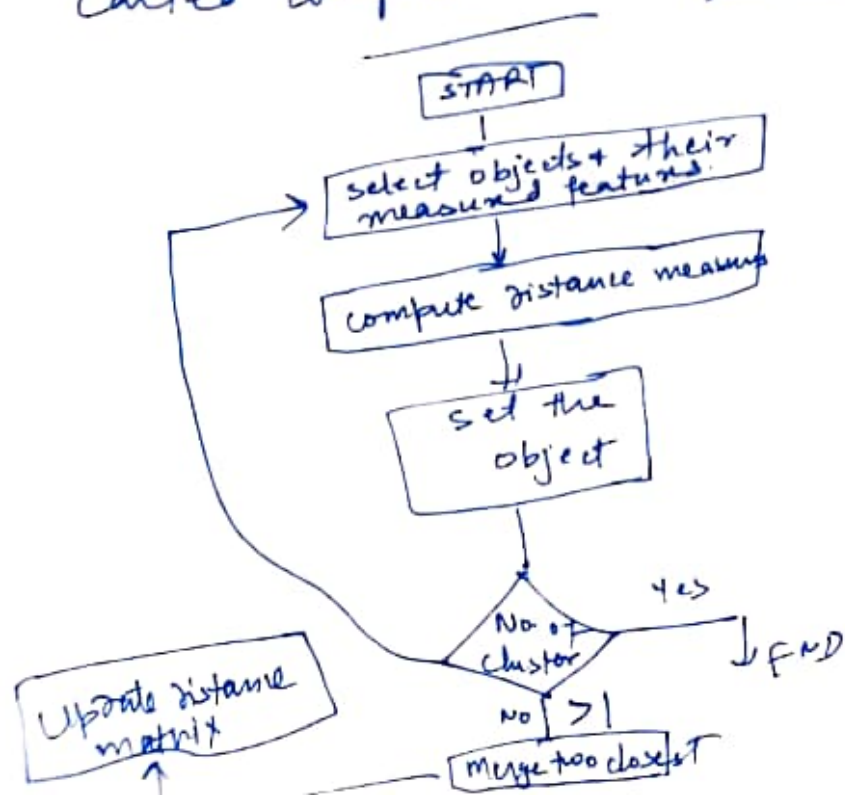
Two objects or Two points

two clusters.

Mean dist: $\text{dist}_{\text{mean}}(C_i, C_j) = |m_i - m_j| \quad \{x\}$

Avg dist: $\text{dist}_{\text{Avg}}(C_i, C_j) = \frac{1}{n_i n_j} \sum_{p \in C_i, p' \in C_j} |p - p'|$

- * when an algorithm uses the min. distance $d_{\min}(C_i, C_j)$ to measure the distance b/w clusters, it is called nearest-neighbor clustering algorithm.
- * If the clustering process is terminated, when the distance b/w the nearest clusters exceeds user-defined threshold, it is called - Single linkage algorithm.
- * Agglomerative hierarchical clustering algorithm with min. distance measure is called as minimum spanning tree algorithm.
- * An algorithm that uses the max. distance $d_{\max}(C_i, C_j)$ to measure the distance b/w clusters is called farthest-neighbor clustering algo. If clustering is terminated when the max. distance exceeds a user-defined threshold, it is called complete-linkage algorithm.



Agglomerative Alg. :- Single link.

Find the clusters using single link technique. Use Euclidean distance as similarity measure & draw the dendrogram.

Sample No.	X	Y
P ₁	0.40	0.53
P ₂	0.22	0.38
P ₃	0.35	0.32
P ₄	0.26	0.19
P ₅	0.08	0.41
P ₆	0.45	0.30

Distance matrix

$$d[(x,y), (a,b)] = \sqrt{(x-a)^2 + (y-b)^2}$$

$$\begin{aligned} \text{Euclidean distance } d(P_1, P_2) &= \sqrt{(0.4 - 0.22)^2 + (0.53 - 0.38)^2} \\ &= \sqrt{(0.18)^2 + (0.15)^2} \\ &= \sqrt{0.0324 + 0.0225} \end{aligned}$$

$$\begin{aligned} &= 0.23 \\ \text{Distance matrix: } & \begin{matrix} & P_1 & P_2 & P_3 & P_4 & P_5 & P_6 \end{matrix} \\ & \begin{matrix} P_1 & 0 & 0.23 & & & & \\ P_2 & 0.23 & 0 & & & & \\ P_3 & 0.29 & & 0 & & & \\ P_4 & 0.34 & & & 0 & & \\ P_5 & 0.34 & & & & 0 & \\ P_6 & 0.24 & & & & & 0 \end{matrix} \end{aligned}$$

$$\begin{aligned} 114 \quad d(P_1, P_3) &= \sqrt{(0.4 - 0.35)^2 + (0.53 - 0.32)^2} \\ &= \sqrt{(0.05)^2 + (0.2)^2} = \sqrt{0.0025 + 0.0441} \\ &= 0.22 \end{aligned}$$

$$\begin{aligned} d(P_1, P_4) &= \sqrt{(0.4 - 0.26)^2 + (0.53 - 0.19)^2} \\ &= \sqrt{(0.14)^2 + (0.34)^2} = \sqrt{0.0196 + 0.1156} \\ &= 0.37. \end{aligned}$$

$$\begin{aligned} d(P_1, P_5) &= \sqrt{(0.4 - 0.08)^2 + (0.53 - 0.41)^2} \\ &= \sqrt{(0.32)^2 + (0.12)^2} \\ &= \sqrt{0.1024 + 0.0144} = 0.34. \end{aligned}$$

$$\begin{aligned} d(P_1, P_6) &= \sqrt{(0.4 - 0.45)^2 + (0.53 - 0.30)^2} \\ &= \sqrt{(0.05)^2 + (0.23)^2} = 0.24. \end{aligned}$$

start for P_2