Numerical on Multiple Linear Regression (MLR)

Data set of following type

Y (D.V)	X_1	X_2
-2.7	4	7
4.5	5	6
2.5	5	7
10.5	5	2
6.7	3	2

$\rightarrow$ I.Vs

A.T. Eqn. of MLR

$Y = b_0 + b_1 X_1 + b_2 X_2$ — (1)

Our objective is to estimate the approximate value of $b_0, b_1 \,\&\, b_2$

$\therefore$ (A) — $b_0 = \bar{Y} - b_1 \bar{X}_1 - b_2 \bar{X}_2$

(B) — $$b_1 = \frac{(\sum x_2^2)\cdot(\sum x_1 y) - (\sum x_1 \cdot x_2)\cdot(\sum x_2 y)}{(\sum x_1^2)(\sum x_2^2) - (\sum x_1 \cdot x_2)^2}$$

(C) — $$b_2 = \frac{(\sum x_1^2)\cdot(\sum x_2 y) - (\sum x_1 \cdot x_2)(\sum x_1 y)}{(\sum x_1^2)(\sum x_2^2) - (\sum x_1 \cdot x_2)^2}$$

Use this in (A) (B) & (C)

Now, $\sum x_i^2 = \sum X_i^2 - \frac{(\sum X_i)^2}{N}$

Let $i=1$, then $\sum x_1^2 = \sum X_1^2 - \frac{(\sum X_1)^2}{N}$

$i=2$, then $\sum x_2^2 = \sum X_2^2 - (\sum X_2)^2/N$.

* Also, $\sum x_i y = \sum X_i Y - \frac{(\sum X_i)(\sum Y)}{N}$

Let, i=1, then $\sum x_1 y = \sum X_1 Y - \frac{(\sum X_1)(\sum Y)}{N}$

i=2, then $\sum x_2 y = \sum X_2 Y - \frac{(\sum X_2)(\sum Y)}{N}$

* Also, $\sum x_1 \cdot x_2 = \sum X_1 X_2 - (\sum X_1)\cdot(\sum X_2)/N$

↑ Use above in Eqⁿ. (A), (B) & (C).

Y	X_1	X_2	X_1^2	X_2^2	$X_1 Y$	$X_2 Y$	$X_1 X_2$
-2.7	4	7	16	49	-10.8	-18.9	28
4.5	5	6	25	36	22.5	27	30
2.5	5	7	25	49	12.5	17.5	35
10.5	5	2	25	4	52.5	21	10
6.7	3	2	9	4	20.1	20.1	6

$\sum Y = 21.5$ $\sum X_1 = 22$ $\sum X_2 = 24$ $\sum X_1^2 = 100$ $\sum X_2^2 = 142$ $\sum X_1 Y = 96.8$ $\sum X_2 Y = 60$ $\sum X_1 X_2 = 109$

Now, Let's ~~us~~ get started by putting the calculated values in formula's.

Consider

$$\sum x_1^2 = \sum X_1^2 - \frac{(\sum X_1)^2}{N}$$

$$= 100 - \frac{(22)^2}{5}$$

$$= \frac{500 - 484}{5} = \frac{16}{5} = 3.2$$

Consider

$$\sum x_2^2 = \sum X_2^2 - \frac{(\sum X_2)^2}{N}$$

$$= 142 - \frac{(24)^2}{5}$$

$$= \frac{710 - 576}{5} = \frac{134}{5}$$

$$= 26.8$$

Consider

$$\sum x_1 y = \sum X_1 Y - \frac{(\sum X_1)(\sum Y)}{N}$$

$$= 96.8 - \frac{22 \times 21.5}{5}$$

$$= 96.8 - 94.6$$

$$= 2.2$$

Consider

$$\sum x_2 y = \sum X_2 Y - \frac{\sum X_2 \cdot \sum Y}{N}$$

$$= 60 - \frac{24 \times 21.5}{5}$$

$$= -43.2$$

Now, $$\sum x_1 \cdot x_2 = \sum X_1 \cdot X_2 - \frac{\sum X_1 \cdot \sum X_2}{N}$$

$$= 109 - \frac{22 \cdot 24}{5}$$

$$= 109 - 105.6$$

$$= 3.4$$

Now consider the Eq^n. (B)

$$b_1 = \frac{(\sum x_2)^2 \cdot (\sum x_1 y) - (\sum x_1 \cdot x_2) \cdot (\sum x_2 \cdot y)}{(\sum x_1^2)(\sum x_2^2) - (\sum x_1 \cdot x_2)^2}$$

$$= \frac{26.8 \times (2.2) - (3.4) \times (-43.2)}{(3.2) \cdot (26.8) - (3.4)^2}$$

$$= \frac{58.96 + 146.88}{85.76 - 11.56} = \frac{205.84}{74.2} = 2.774$$

$$\therefore \boxed{b_1 = 2.774}$$

Similarly $b_2 = \dfrac{(\sum x_1^2)\cdot(\sum x_2 y) - (\sum x_1 \cdot x_2)\cdot(\sum x_1 y)}{(\sum x_1^2)\cdot(\sum x_2^2) - (\sum x_1 \cdot x_2)^2}$

$$\therefore b_2 = \frac{(3.2)\cdot(-43.2) - (3.4)(2.2)}{(3.2)\cdot(26.8) - (3.4)^2}$$

$$= \frac{-138.24 - 7.48}{85.76 - 11.56} = \frac{-145.72}{74.2}$$

$$\boxed{b_2 = -1.964}$$

$\therefore$ Eqn (A) becomes

$$b_0 = \overline{Y} - b_1\overline{X}_1 - b_2\overline{X}_2$$

$$b_0 = \frac{1}{5}\left[21.5 - (2.774)\cdot(22) + (1.964)\cdot 24\right]$$

$$b_0 = \frac{1}{5}\left[21.5 - 61.028 + 47.136\right]$$

$$b_0 = \frac{7.608}{5} = 1.522$$

Eq^n. (1) become

$$Y = b_0 + b_1 X_1 + b_2 X_2$$

$$Y = 1.522 + 2.744 X_1 + (-1.964) \cdot X_2$$

$$\Rightarrow Y = 1.522 + 2.744 X_1 - 1.964 X_2$$

is the required multiple Linear regression Eq^n.

Factor Analysis Model
Extracting Commonfactor
Determining number of factors
Transformation of factor analysis
factor Scores.