

Functional Dependencies

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What is Functional Dependency

A functional dependency is a constraint that specifies the relationship between two sets of attributes where one set can accurately determine the value of other sets.

It is denoted as $X \rightarrow Y$, where X is a set of attributes that is capable of determining the value of Y . The attribute set on the left side of the arrow, X is called **Determinant**, while on the right side, Y is called the **Dependent**.

Functional dependencies are used to mathematically express relations among database entities and are very important to understand advanced concepts in Relational Database System.

Example:

roll_no	name	dept_name	dept_building
42	abc	CO	A4
43	pqr	IT	A3the
44	xyz	CO	A4
45	xyz	IT	A3

From This table some valid functional dependencies:

$\text{roll_no} \rightarrow \{ \text{name}, \text{dept_name}, \text{dept_building} \}$

Here, roll_no can determine values of fields name, dept_name and dept_building, hence a valid Functional dependency

$\text{roll_no} \rightarrow \text{dept_name}$, Since, roll_no can determine whole set of {name, dept_name, dept_building}, it can determine its subset dept_name also.

$\text{dept_name} \rightarrow \text{dept_building}$, Dept_name can identify the dept_building accurately.

$\text{roll_no} \rightarrow \text{name}$,

$\{ \text{roll_no}, \text{name} \} \twoheadrightarrow \{ \text{dept_name}, \text{dept_building} \}$, etc.

Here are some invalid functional dependencies:

$\text{name} \rightarrow \text{dept_name}$ Students with the same name can have different dept_name, hence this is not a valid functional dependency.

$\text{dept_building} \rightarrow \text{dept_name}$ There can be multiple departments in the same building, For example, in the above table departments ME and EC are in the same building B2, hence $\text{dept_building} \rightarrow \text{dept_name}$ is an invalid functional dependency.

More invalid functional dependencies: $\text{name} \rightarrow \text{roll_no}$, $\{\text{name}, \text{dept_name}\} \rightarrow \text{roll_no}$, $\text{dept_building} \rightarrow \text{roll_no}$, etc.

Armstrong's axioms/properties of functional dependencies:

Reflexivity: If Y is a subset of X , then $X \rightarrow Y$ holds by reflexivity rule

For example, $\{\text{roll_no}, \text{name}\} \rightarrow \text{name}$ is valid.

Augmentation: If $X \rightarrow Y$ is a valid dependency, then $XZ \rightarrow YZ$ is also valid by the augmentation rule.

For example, If $\{\text{roll_no}, \text{name}\} \rightarrow \text{dept_building}$ is valid, hence $\{\text{roll_no}, \text{name}, \text{dept_name}\} \rightarrow \{\text{dept_building}, \text{dept_name}\}$ is also valid. \rightarrow

Transitivity: If $X \rightarrow Y$ and $Y \rightarrow Z$ are both valid dependencies, then $X \rightarrow Z$ is also valid by the Transitivity rule.

For example, $\text{roll_no} \rightarrow \text{dept_name}$ & $\text{dept_name} \rightarrow \text{dept_building}$, then $\text{roll_no} \rightarrow \text{dept_building}$ is also valid.

Types of Functional dependencies in DBMS:

1. Trivial functional dependency
2. Non-Trivial functional dependency
3. Multivalued functional dependency
4. Transitive functional dependency

1. Trivial Functional Dependency

In **Trivial Functional Dependency**, a dependent is always a subset of the determinant.

i.e. If $X \rightarrow Y$ and **Y is the subset of X**, then it is called trivial functional dependency

For example,

roll_no	name	age
42	abc	17
43	pqr	18
44	xyz	18

Here, $\{\text{roll_no, name}\} \rightarrow \text{name}$ is a trivial functional dependency, since the dependent **name** is a subset of determinant set **$\{\text{roll_no, name}\}$**

Similarly, $\text{roll_no} \rightarrow \text{roll_no}$ is also an example of trivial functional dependency.

2. Non-trivial Functional Dependency

In **Non-trivial functional dependency**, the dependent is strictly not a subset of the determinant.

i.e. If $X \rightarrow Y$ and **Y is not a subset of X**, then it is called Non-trivial functional dependency.

For example

roll_no	name	age
42	abc	17
43	pqr	18
44	xyz	18

Here, **roll_no \rightarrow name** is a non-trivial functional dependency, since the dependent **name** is **not a subset of** determinant **roll_no**.
Similarly, **{roll_no, name} \rightarrow age** is also a non-trivial functional dependency, since **age** is **not a subset of {roll_no, name}**.

3. Multivalued Functional Dependency

In **Multivalued functional dependency**, entities of the dependent set are **not dependent on each other**.

i.e. If $a \rightarrow \{b, c\}$ and there exists **no functional dependency** between **b and c**, then it is called a **multivalued functional dependency**.

For example

roll_no	name	age
42	abc	17
43	pqr	18
44	xyz	18

Here, $\text{roll_no} \rightarrow \{\text{name}, \text{age}\}$ is a multivalued functional dependency, since the dependents **name & age** are **not dependent** on each other (i.e. $\text{name} \rightarrow \text{age}$ or $\text{age} \rightarrow \text{name}$ doesn't exist !)

4. Transitive Functional Dependency

In transitive functional dependency, dependent is indirectly dependent on determinant.

i.e. If $a \rightarrow b$ & $b \rightarrow c$, then according to axiom of transitivity, $a \rightarrow c$. This is a **transitive functional dependency**

For example

Here, $\text{enrol_no} \rightarrow \text{dept}$ and $\text{dept} \rightarrow \text{building_no}$,

Hence, according to the axiom of transitivity, $\text{enrol_no} \rightarrow \text{building_no}$ is a valid functional dependency. This is called Transitive functional dependency.

enrol_no	name	dept	building_no
42	abc	CO	4
43	pqr	EC	2
44	xyz	IT	1
45	abc	EC	2