School of Mathematics and Computer Applications, TU
Optimization Techniques (UMA-031)
Tutorial Sheet-6
(Transportation problem)

1. Iron ore is to be transported from three mines to four steel mills situated in different cities. Find the minimum cost transportation schedule given the following cost matrix:

		Steel Mills				
		A	B	C	D	Ore Available
Mines	I	14	56	48	27	13
	II	82	35	21	81	19
	III	99	31	71	63	16
Ore required		7	14	21	6	

2. In a flood relief operation, there are four bases of operations B_i $(i=1,2,3,4)$ from where air crafts can take relief materials to three targets T_j $(j=1,2,3)$. Because of the difference in air crafts, range to target and flying altitudes, the relief material (in tons) per aircraft from any base that can be delivered to any target differs according to following table:

	T_1	T_2	T_3
B_1	8	6	5
B_2	6	6	6
B_3	10	8	4
B_4	8	6	4

The daily sortie capacity of each of the four bases is 150 sorties per day and the daily requirement of sorties on each target is 200. Find the allocation of sorties that maximizes the total tonnage over all the targets. If the problem has alternative solutions, find one.

3 In the following transportation problem, the penalty costs per unit of unsatisfied demand are 5, 3, and 2 for destinations 1, 2, and 3 respectively.

	D1	D2	D3	Availability
S1	5	1	7	10
S2	6	4	6	80
S2	3	2	5	15
Demand	75	20	50	

(1) Determine the optimal solution.
(2) Formulate above transportation as an LPP.

4. Consider the following transportation problem for minimum transportation cost.

	D_1	D_2	D_3	D_4	D_5	Availability
O_1	20	19	14	21	16	40
O_2	15	20	13	19	16	60
O_3	18	15	18	20	10	70
Requirement	30	40	50	40	60	

Find an optimal solution of this transportation problem under the following restrictions.

(1) There is no transportation from origin O_3 to destination D_5.
(2) The Origin O_1 supplies exactly 20 units to Destination D_4.
(3) The Destination D_2 receives atlest 10 units from Origin O_2.

5. Solve the following transportation problem for minimum cost starting with the degenerate basis $x_{12} = 30$, $x_{21} = 40$, $x_{32} = 20$, $x_{43} = 60$

	D_1	D_2	D_3	Availability
O_1	4	5	2	30
O_2	4	1	3	40
O_3	3	6	2	20
O_4	2	3	7	60
Demand	40	50	60	

6 A company has four plants producing the same product. The production cost differs from one plant to another as do the cost of raw materials. There are five regional warehouses. Sales price at each is different. The maximum sales, capacity, unit transportation costs etc. are given in the following table. Determine the transportation schedule which maximizes the over all profit.

		Plants					
		1	2	3	4	Sales	
		Maximum					
Production cost (Rs.)		15	18	14	13	Price	Sales
Raw material cost (Rs.)		10	9	12	8	(Rs.)	
Warehouse							
Transportation cost (Rs.)	1	3	9	5	4	34	80
	2	1	7	4	5	32	110
	3	5	8	3	6	31	150
	4	7	3	8	2	31	100
	5	4	5	6	7	31	150
Capacity		150	200	175	100		

①.

Solutions for Transportation problem:-

Q-1:

	A	B	C	D	
1	14	56	48	27	13
2	82	35	21	81	19
3	99	31	71	63	16
	7	14	21	6.	

→ This is a minimization problem.

→ This is a balanced problem.

→ Use LCM to find the Initial B.F.S :-

14 ~~7~~	56	48	27 6	~~13~~ ~~6~~.
82	35	21 19	81	~~19~~
99	31 14	71 2	63 0.	~~16~~
~~7~~	~~14~~	~~21~~ ~~2~~	~~6~~ ~~0~~	

→ Apply U.V method on this:- (for optimality)

	$V_1=14$	$V_2=-5$	$V_3=35$	$V_4=27$	
$u_1=0$	14 7	56 -61	48 -13	27 6	13
$u_2=-16$	82 -84	35 -56	21 19	81 -70	19
$u_3=36$	99 -49	31 14	71 2	63 0	16
	7	14	21	6.	

All the $u_i+v_j-c_{ij} \leq 0$ for non-basic cells ⇒ optimal soln

Soln $x_{11}=7$, $x_{14}=6$, $x_{23}=19$,

$x_{32}=14$, $x_{33}=6$, $x_{34}=0$

$z = 1235$

Q-2

8	6	5	150
6	6	6	150
10	8	4	150
8	6	4	150
200	200	200.	

→ This is a balanced Transportation problem

→ This is a maximization problem, so Convert to minimization first

Maxima of the matrix = 10.

→ Initial BFS. using VAM

2 50	4 50	5 50	~~150~~ 100	②	②	1	①
4	4	4 150	~~150~~	0	0	0	0
0 150	2	6	~~150~~	②	–	–	–
2	4 150	6	~~150~~	②	②	②	–
~~200~~ ~~50~~	~~200~~ 50	~~200~~ 50					
②	②	1					
②	0	1					
–	0	1					
–	0	①					

→ U-V method for optimal soln :-

	$V_1 = 2$	$V_2 = 4$	$V_3 = 5$	
$u_1 = 0$	2 50	4 50	5 50	150
$u_2 = -1$	4 -3	4 -1	4 150	150
$u_3 = -2$	0 150	2 0	6 -3	150
$u_4 = 0$	2 0	4 150.	6 -1	150
	200	200	200.	

All the $u_i + v_j - c_{ij} \leq 0$, for non-basic

$\Rightarrow$ optimal soln.

Soln. is $x_{11} = 50$, $x_{12} = 50$, $x_{13} = 50$, $x_{23} = 150$,

$x_{31} = 150$, $x_{42} = 150$

Max. relief material transported is :- $\underline{\underline{4250}}$

→ $u_i + v_j - c_{ij}$ for (3,2) & (4,1) is zero,

Hence alternative optima exists. Choose any of these to be entering and proceed :-

2 50 + θ	4 50 − θ	5 50	150
4 -3	4 -1	4 150	150
0 150 − θ	2 0 θ	6 -3	150
2 0	4 150	6 -1	150
200	200	200	

$\theta = \min\{50, 150\} = 50$

2 100	4	5 50	150
4	4	4 150	150
0 100	2 50	6	150
2	4 150	6	150
200	200	200	

Soln. is :- $x_{11} = 100$, $x_{13} = 50$, $x_{23} = 150$,

$x_{31} = 100$, $x_{32} = 50$, $x_{42} = 150$

$z = \underline{\underline{4250}}$.

Q-3

5	1	7	10
6	4	6	80
3	2	5	15
75	20	50	

→ This is a ~~maximization~~ minimization problem.

→ $\Sigma a_i = 105$, $\Sigma b_j = 145$ ⇒ Unbalanced problem and the demand is more than the supply so we add additional row. The penalities for unsatisfied demand given act as cost of the row added, hence the bal. TP. is given by:-

VAM for Initial BFS.

$\boxed{5}$	$\boxed{1}$ 10	$\boxed{7}$	~~10~~	④ X
$\boxed{6}$ 60	$\boxed{4}$ 10	$\boxed{6}$ 10.	80	2 2 2
$\boxed{3}$ 15	$\boxed{2}$	$\boxed{5}$	~~15~~	1 1 1
$\boxed{5}$	$\boxed{3}$	$\boxed{2}$ 40.	~~40~~.	1 1
~~75~~ 60	~~20~~ 10.	~~80~~. 10.		
2.	1	3		
2	1	③		
③.	2	1		

$\sum a_i < \sum b_j$

=> unbalanced

=) add additional row with cost 5, 3, 2 (given penalties),

u-v method.

	$v_1 = 3$	$v_2 = 1$	$v_3 + 3$
$u_1 = 0$	$\boxed{5}$ -2	$\boxed{1}$ 10	$\boxed{7}$ -4
$u_2 = +3$	$\boxed{6}$ 60	$\boxed{4}$ 10.	$\boxed{6}$ 10
$u_3 = 0$	$\boxed{3}$ 15	$\boxed{2}$ -1	$\boxed{5}$ -2
$u_4 = -1$	$\boxed{5}$ -3	$\boxed{3}$ -3	$\boxed{2}$ 40.

⇒ Optimal as all $u_i + v_j - c_{ij} \le 0$ for non-basic variable.

→ **Soln!-** $x_{12} = 10$, $x_{21} = 60$, $x_{22} = 10$, $x_{33} = 10$,

$x_{31} = 15$, $x_{43} = 40$, $z = \underline{\underline{595}}$

(ii) Formulation of the LPP!-

$$\text{Min } z = 5x_{11} + x_{12} + 7x_{13} + 6x_{21} + 4x_{22} + 6x_{23} + 3x_{31} + 2x_{32} + 5x_{33}$$

st.

$$x_{11} + x_{12} + x_{13} = 10$$

$$x_{21} + x_{22} + x_{23} = 80$$

$$x_{31} + x_{32} + x_{33} = 15$$

$x_{11} + x_{21} + x_{31} \le 75$

$x_{12} + x_{22} + x_{32} \le 20$

$x_{13} + x_{23} + x_{33} \le 80$

(∵ ≤ constrai

∴ the dema

is more

than supply).

$x_{ij} \ge 0$, $i = 1, 2, 3$ & $j = 1, 2, 3$.

Q-4

20	19	14	21	16	40
15	20	13	19	16	60
18	15	18	20	10	70.
30	40	50	40	60.	

→ This is a minimization problem.

→ $\Sigma a_i = 170$, $\Sigma b_j = 220$.

⇒ unbalanced prob; add addition row with cost (0, 0, 0, 0, 0) and RHS = 50.

→ Modified table is :-

20	19	14	21	16	40
15	20	13	19	16	60
18	15	18	20	10	70
0	0	0	0	0	50.
30	40	50	40	60.	

(i) There is no transportation from Origin O_3 to destination D_5.

ie No allocation is allowed on route (3,5)

ie we need to force $x_{35} = 0$.

For this choose $c_{35} = M$.
↓
large +ve number

(this will be −M for max. problem

→ Initial soln by LCM

20	19	14	21 30	16 10	~~40~~ ~~30~~
15 10	20	13 50	19	16	~~60~~ ~~10~~
18 20	15 40	18	20 10	M	~~70~~ ~~30~~ ~~10~~.
0	0	0	0	0 50	~~50~~.
~~30~~ ~~20~~	~~40~~	~~50~~	~~40~~	~~60~~ ~~10~~	

	$v_1 = 19$	$v_2 = 16$	$v_3 = 17$	$v_4 = 21$	$v_5 = 16$	
$u_1 = 0$	$\boxed{20}$ $\boxed{-1}$	$\boxed{19}$ $\boxed{-3}$	$\boxed{14}$ $\boxed{3}$	$\boxed{21}$ $30-\theta$	$\boxed{16}$ $10+\theta$	40
$u_2 = -4$	$\boxed{15}$ 10	$\boxed{20}$ $\boxed{-8}$	$\boxed{13}$ 50	$\boxed{19}$ $\boxed{-2}$	$\boxed{16}$ $\boxed{-4}$	60
$u_3 = -1$	$\boxed{18}$ 20	$\boxed{15}$ 40	$\boxed{18}$ $\boxed{-2}$	$\boxed{20}$ 10	$\boxed{M}$ $\boxed{15-M}$	70
$u_4 = -16$	$\boxed{0}$ $\boxed{3}$	$\boxed{0}$ $\boxed{0}$	$\boxed{0}$ $\boxed{1}$	$\boxed{0}$ $\boxed{5}$ θ	$\boxed{0}$ $50-\theta$	50
	30	40	50	40	60	

x_{44} enters, $\theta = \min\{30, 50\} = 30$. & x_{14} leaves

	$v_1 = 14$	$v_2 = 11$	$v_3 = 12$	$v_4 = 16$	$v_5 = 16$	
$u_1 = 0$	$\boxed{20}$ $\boxed{-6}$	$\boxed{19}$ $\boxed{-8}$	$\boxed{14}$ $\boxed{-2}$	$\boxed{21}$ $\boxed{-5}$	$\boxed{16}$ 40	40
$u_2 = 1$	$\boxed{15}$ $10-\theta$	$\boxed{20}$ $\boxed{-8}$	$\boxed{13}$ 50.	$\boxed{19}$ $\boxed{-2}$	$\boxed{16}$ $\boxed{1}$ θ	60
$u_3 = 4$	$\boxed{18}$ $20+\theta$	$\boxed{15}$ 40	$\boxed{18}$ $\boxed{-2}$	$\boxed{20}$ $10-\theta$	$\boxed{M}$ $\boxed{20-M}$	70
$u_4 = -16$	$\boxed{0}$ $\boxed{-2}$	$\boxed{0}$ $\boxed{-5}$	$\boxed{0}$ $\boxed{-4}$	$\boxed{0}$ $30+\theta$	$\boxed{0}$ $20-\theta$	50.
	30	40	50	40	60.	

x_{25} enters, $\theta = \min\{20, 10, 10\} = 10$, Both x_{21} & x_{34} become zero and we can choose any one to leave

	$v_1=15$	$v_2=12$	$v_3=13$	$v_4=16.$	$v_5=16$	
$u_1=0$	$\boxed{20}$ $\boxed{-5}$	$\boxed{19}$ $\boxed{-7}$	$\boxed{14}$ $\boxed{-1}$	$\boxed{21}$ $\boxed{-5}$	$\boxed{16}$ 40	40
$u_2=0$	$\boxed{15}$ 0	$\boxed{20}$ $\boxed{-8}$	$\boxed{13}$ 50	$\boxed{19}$ $\boxed{-3}$	$\boxed{16}$ 10	60
$u_3=3$	$\boxed{18}$ 30	$\boxed{15}$ 40.	$\boxed{18}$ $\boxed{-2}$	$\boxed{20}$ $\boxed{-1}$	$\boxed{M}$ $\boxed{19-M}$	70
$u_4=-16$	$\boxed{0}$ $\boxed{-1}$	$\boxed{0}$ $\boxed{-4}$	$\boxed{0}$ $\boxed{-3}$	$\boxed{0}$ 40	$\boxed{0}$ 10	50.
	30	40	50	40	60	

All $u_i+v_j-c_{ij} \leq 0$ for non-basic cells

$\Rightarrow$ optimal soln.

Soln:- $x_{15}=40$, $x_{21}=0$, $x_{23}=50$, $x_{25}=10$

$x_{31}=30$, $x_{32}=40$, $x_{44}=40$, $x_{45}=10$

$$z = \underline{\underline{2590}}$$

(II) The Origin O_1 Supplies exactly 20 units to destination D_4.

i.e $x_{14}=20$, and no more allocation is allowed on the route (1, 4). Hence we block it by using cost of M (large +ve number) instead of 21 and Subtract 20 from a_1 + b_4

The ~~f~~ modified table is :-

→ I.B.F.S. using VAM

20	19	14 20	M	16	~~20~~	2 2 2 ⑤
15 0	20 30	13 30	19	16	~~60~~ ~~30~~	2 2 2 2 2 ⑤
18	15 10	18	20	10 60	~~70~~ ~~10~~	5 5 5 3 3 3
0 30	0	0	0 20	0	~~50~~ ~~30~~ 0	0 0 – –
~~30~~ 0	~~40~~ 30	~~50~~ 30	~~20~~	~~60~~		
15	15	13	⑲	10		
⑮	⑮	13	–	10		
3	4	1	–	⑥		
3	4	1	–	–		
3	⑤	⑤	–	–		
3	⑤	–	–	–		

→ Initial Basic feasible soln is

	$v_1=16$	$v_2=21$	$v_3=14$	$v_4=16$	$v_5=16$	
$u_1=0$	20 -4	19 2	14 20	M 16-M	16 0	20
$u_2=-1$	15 0 +θ	20 30 -θ	13 30	19 -5	16 -1	60
$u_3=-6$	18 -8	15 10	18 -10	20 -10	10 60	70
$u_4=-16$	0 30 -θ	0 5 θ	0 -2	0 20	0 0	50
	30	40	50	20	60	

⑦

x_{42} enters, $\theta = \min\{30, 30\} = 30$, x_{41} leaves

	$V_1 = 16$	$V_2 = 21$	$V_3 = 14$	$V_4 = 21$	$V_5 = 16$	
$u_1 = 0$	20 -4	19 θ 2	14 $20-\theta$	M $21-M$	16 0	20
$u_2 = -1$	15 30	20 $0-\theta$	13 $30+\theta$	19 1	16 -1	60
$u_3 = -6$	18 -8	15 10	18 -10	20 -5	10 60	70
$u_4 = -21$	0 -5	0 30	0 -7	0 20	0 -5	50
	30	40	50	20	60	

x_{12} enters, $\theta = \min\{0, 20\} = 0$, x_{22} leaves.

	$V_1 = 16$	$V_2 = 19$	$V_3 = 14$	$V_4 = 19$	$V_5 = 14$	
$u_1 = 0$	20 -4	19 0	14 20	M $19-M$	16 -2	20
$u_2 = -1$	15 30	20 -2	13 30	19 -1	16 -3	60
$u_3 = -4$	18 -6	15 10	18 -8	20 -5	10 60	70
$u_4 = -19$	0 -3	0 30	0 -5	0 20	0 -5	50
	30	40	50	20	60	

All $u_i + v_j - C_{ij} \leq 0$ for non-basic cells.
$\Rightarrow$ Optimal soln

Soln. in $x_{12} = 0$, $x_{13} = 20$, $x_{21} = 30$, $x_{23} = 30$, $x_{32} = 10$, $x_{35} = 60$
$x_{42} = 30$, $x_{34} = 20$, $x_{14} = 20$, $z = 2290$//

(iii) Destination D_2 recieves atleast 10 units form O_2. ie allocate $x_{22} = 10$ and keep route (2,2) for more allocations. Modified Table is

$\boxed{x_{22}=10}$

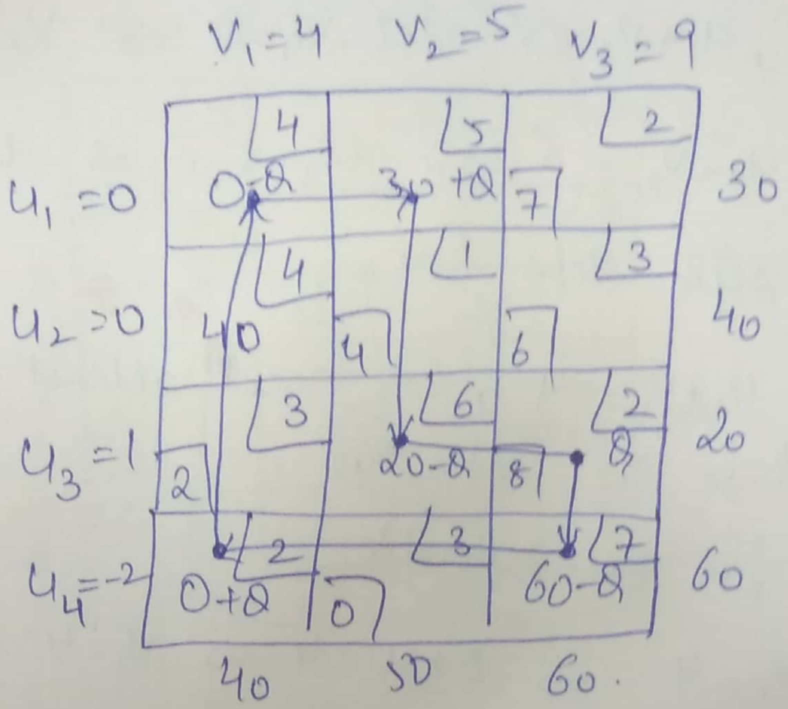
20	19	14	21	16	40
15	20	13	19	16	50
18	15	18	20	10	70
0	0	0	0	0	50
30	30	50	40	60	

→ Proceed as before to solve the problem, and soln. is given by:-

~~x_{12}~~ $x_{12} = 10$, $x_{13} = 30$, $x_{21} = 30$, $x_{23} = 20$,

$x_{32} = 10$, $x_{35} = 60$, $x_{42} = 10$, $x_{44} = 40$

$\boxed{x_{22} = 10}$, $z = \underline{\underline{2270}}$



	$V_1 = 4$	$V_2 = 5$	$V_3 = 9$	
$u_1 = 0$	4 $0-\theta$	5 $30+\theta$	2 7	30
$u_2 = 0$	4 40	1 4	3 6	40
$u_3 = 1$	3 2	6 $20-\theta$	2 8 θ	20
$u_4 = -2$	2 $0+\theta$	3 0	7 $60-\theta$	60
	40	50	60.	

→ This is a balanced TP & a minimization problem.

→ We are asked to find the optimal soln starting with the degenerate soln.

$x_{12} = 30$, $x_{21} = 40$, $x_{32} = 20$, $x_{43} = 60$

total allocation is 150.

⇒ $m+n-1 = 4+3-1 = 6$ = No. of Basic variables, but we are given only 4, hence the other two are equal to zero and we need to find the place (cells) which should be zero and basic.

→ We shall use the fact that all basic Cells are ~~atleast~~ Connected to atleast one more. And we should be able to solve the eqns. for u_i's & v_j's by assuming ~~only~~ value of only one of these.

Using $(1,2)$, $u_1+v_2=5$, let $u_1=0 \Rightarrow v_2=5$

$(3,2)$, $u_3+v_2=6 \Rightarrow u_3+5=6 \Rightarrow u_3=1$

And these eqn. stops here,

form $(2,1)$, $u_2+v_1=4$, we need atleast one of these to solve this.

Let $x_{11}=0$, a basic variable

$(1,1) \Rightarrow u_1+v_1=4 \Rightarrow 0+v_1=4 \Rightarrow v_1=4$

$(2,1) \Rightarrow u_2=0$

We are left with u_4 & v_3 and the eqn. which we can use for these (ie is only one which is for $(4,3)$, Hence we need to put one more zero, to find either of u_4 or v_3, ~~using~~ Let $x_{41}=0$ (basic cell) $\Rightarrow u_4+v_1=2$

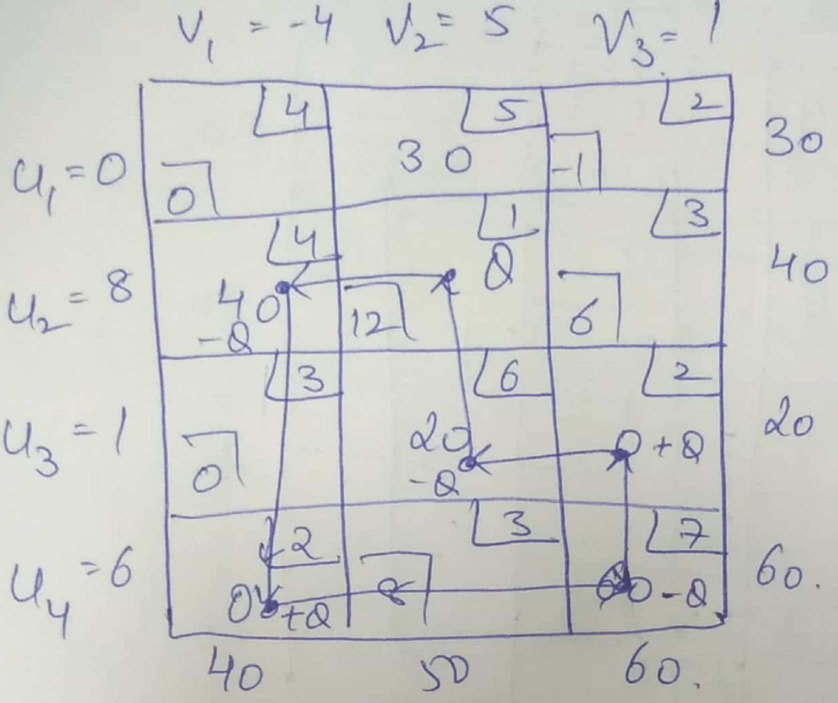
$\Rightarrow u_4=-2$.

$(4,3) \Rightarrow u_4+v_3=7 \Rightarrow v_3=9$.

This gives us our starting B.F.S.

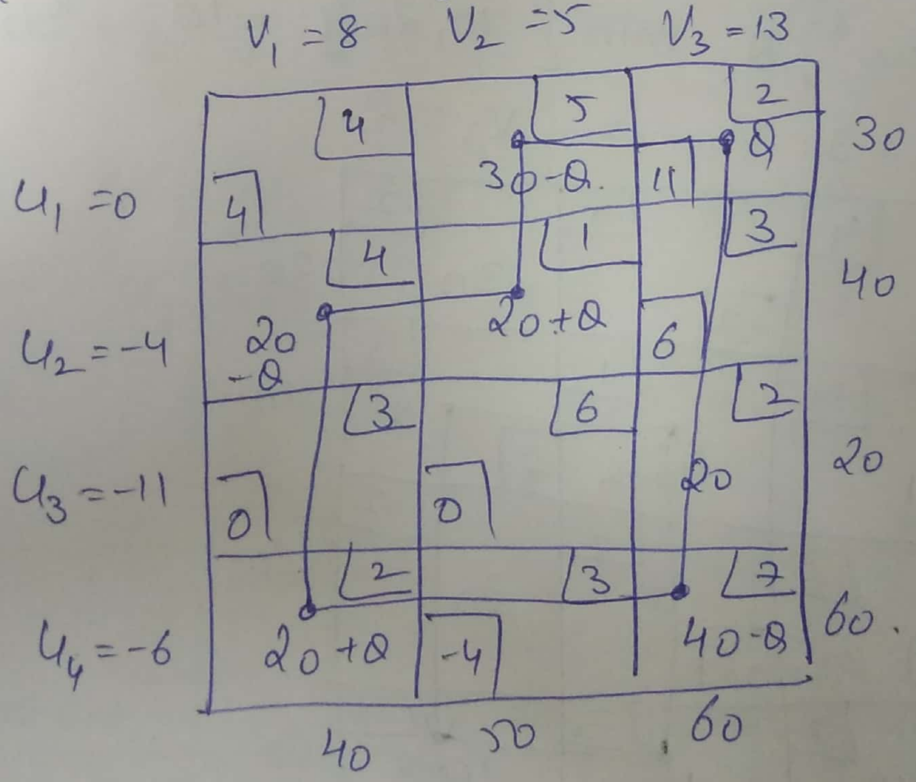
→ ~~Applying~~ Calculating $u_i+v_j-C_{ij}$ for ~~basi~~ non-basic cells we get x_{33} as the entering variable.

$\theta = \min\{0, 20, 60\} = 0$, x_{41} leaves.



	$V_1=-4$	$V_2=5$	$V_3=1$	
$u_1=0$	4 0	5 30	2 -1	30
$u_2=8$	4 40 $-\theta$	1 θ 12	3 6	40
$u_3=1$	3 0	6 20 $-\theta$	2 0 $+\theta$	20
$u_4=6$	2 0 $+\theta$	3 8	7 60 $-\theta$	60.
	40	50	60.	

x_{22} enters, $\theta = \min\{40, 60, 20\} = 20$, x_{32} leaves.



	$V_1=8$	$V_2=5$	$V_3=13$	
$u_1=0$	4 4	5 30 $-\theta$.	2 11 θ	30
$u_2=-4$	4 20 $-\theta$	1 20 $+\theta$	3 6	40
$u_3=-11$	3 0	6 0	2 20	20
$u_4=-6$	2 20 $+\theta$	3 -4	7 40 $-\theta$	60.
	40	50	60	

x_{13} enters, $\theta = \min\{40, 20, 30\} = 20.$, x_{21} leaves.

	$v_1=-3$	$v_2=5$	$v_3=2$	
$u_1=0$	$\boxed{4}$ $\boxed{-7}$	$\boxed{5}$ $10-\theta$	$\boxed{2}$ $20+\theta$	30
$u_2=-4$	$\boxed{4}$ $\boxed{-11}$	$\boxed{1}$ 40	$\boxed{3}$ $\boxed{-5}$	40
$u_3=0$	$\boxed{3}$ $\boxed{0}$	$\boxed{6}$ $\boxed{-1}$	$\boxed{2}$ 20	20
$u_4=5$	$\boxed{2}$ 40	$\boxed{3}$ θ $\boxed{7}$	$\boxed{7}$ $20-\theta$	60
	40	50	60	

x_{42} enters, $\theta = \min\{10, 20\} = 10$, x_{12} leaves

	$v_1=-3$	$v_2=-2$	$v_3=2$	
$u_1=0$	$\boxed{4}$ $\boxed{-7}$	$\boxed{5}$ $\boxed{-7}$	$\boxed{2}$ 30	30
$u_2=3$	$\boxed{4}$ $\boxed{-4}$	$\boxed{1}$ $40-\theta$	$\boxed{3}$ θ $\boxed{2}$	40
$u_3=0$	$\boxed{3}$ $\boxed{0}$	$\boxed{6}$ $\boxed{-8}$	$\boxed{2}$ 20	20
$u_4=5$	$\boxed{2}$ 40	$\boxed{3}$ $10+\theta$	$\boxed{7}$ $10-\theta$	60
	40	50	60.	

x_{23} enters, $\theta = \min\{10, 40\} = 10$, x_{43} leaves.

	$v_1=-1$	$v_2=0$	$v_3=2$	
$u_1=0$	$\boxed{4}$ $\boxed{-5}$	$\boxed{5}$ $\boxed{-5}$	$\boxed{2}$ 30	30
$u_2=1$	$\boxed{4}$ $\boxed{-4}$	$\boxed{1}$ 30	$\boxed{3}$ 10	40
$u_3=0$	$\boxed{3}$ $\boxed{-4}$	$\boxed{6}$ $\boxed{-6}$	$\boxed{2}$ 20	20
$u_4=3$	$\boxed{2}$ 40	$\boxed{3}$ 20	$\boxed{7}$ $\boxed{-2}$	60
	40	50	60.	

All $u_i+v_j-c_{ij}$ for non-basic cells $\leq 0 \Rightarrow$ optimal

<u>Soln</u>

$x_{43}=30, \quad x_{22}=30, \quad x_{23}=10,$

$x_{33}=20, \quad x_{41}=40, \quad x_{42}=20$

$z = \underline{300}$

<u>Q6</u>.

		Plants					
		1	2	3	4		
Prod. cost		15	18	14	13.		
Raw material cost		10	9	12	8.		
Ware house						Sales (availibity)	S.P.
	1	3	9	5	4	80	34
	2	1	7	4	5	110	32
	3	5	8	3	6	150	31
	4	7	3	8	2	100	31
	5	4	5	6	7	150.	31
(demand) Capacity		150	200	175	100.		

Net profit for each cell = S.P − Prod. Cost − Raw mat. cost − transportation cost

Profit matrix

34−15−10−3 = 6	34−18−9 −9 = −2	34−14−12 −5 = 3	34−13−8 −4 = 9	80
32−15−10 −1 = 6	32−18−9 −7 = −2	32−14−12 − 4 = 2	32−13−8 −5 = 6	110
31−15−10 −5 = 1	31 −18−9 −8 = −4	31−14−12 − 3 = 2	31−13−8 −6 = 4	150
31−15−10 −7 = −1	31 −18−9 −3 = 1	31−14−12 − 8 = −3	31−13−8 −2 = 8	100
31−15−10 −4 = 2	31−18−9 −5 = −1	31−14−12 −6 = −1	31−13−8 −7 = 3	150.
150	200	175	100	

→ This is maximization prob., Convert to minimization

maxima = 9.

3	11	6	0	80
3	11	7	3	100
8	13	7	5	150
10	8	12	1	100
7	10	10	6	150.
150	200	175	100.	

→ This is an unbalanced problem, Hence balancing,

→ Initial B.F.S using VAM.

3 80	11	6	0 0	~~80~~	3 3 ③ ③ - -
3 70	11 15	7 25	3	~~40~~ ~~40~~ ~~15~~ 0	0 0 0 4 ④ 4 ④
8	13	7 150	5	~~150~~	2 2 2 1 1 ⑥ -
10	8	12	1 100	~~100~~	7 ⑦ - - - - -
7	10 150	10	6	~~150~~	1 1 1 ③ 3 0 0
0	0 35	0	0	~~35~~	0 - - - - - -
~~150~~ ~~70~~	~~200~~ ~~165~~	~~175~~ ~~25~~	~~100~~ ~~0~~		
3	⑧	6	0		
0	2	1	1		
0	1	1	③		
0	1	1	-		
④	1	0	-		
-	1	0	-		
-	1	3	~~3~~		

→ u-v method:-

	$v_1=3$	$v_2=11$	$v_3=7$	$v_4=0$	
$u_1=0$	3 80−θ	11 0	6 11	0 0+θ	80
$u_2=0$	3 70+θ	11 15−θ	7 25	3 -3	110
$u_3=0$	8 -5	13 -2	7 150	5 -5	150
$u_4=1$	10 -6	8 4 θ	12 -4	1 100−θ	100
$u_5=-1$	7 -5	10 150	10 -4	6 -7	150
$u_6=-11$	0 -8	0 35	0 -4	0 -11	35
	150	200	175	100	

x_{42} enters, $\theta = \min\{100, 80, 15\} = 15$, x_{22} leaves.

	$v_1=3$	$v_2=7$	$v_3=7$	$v_4=0$	
$u_1=0$	3 65−θ	11 -4	6 1 θ	0 15	80
$u_2=0$	3 85+θ	11 -4	7 25−θ	3 -3	110
$u_3=0$	8 -5	13 -6	7 150.	5 -5	150
$u_4=1$	10 -6	8 15	12 -4	1 85	100
$u_5=3$	7 -1	10 150.	10 0	6 -3	150
$u_6=-7$	0 -4	0 35	0 0	0 -7	35
	150	200	175	100	

x_{13} enters

$\theta = \min\{25, 65\}$
$= 25$

x_{23} leaves.

	$v_1=3$	$v_2=7$	$v_3=6$	$v_4=0$	
$u_1=0$	$\boxed{3}$ 40	$\boxed{11}$ $\boxed{-4}$	$\boxed{6}$ 25	$\boxed{0}$ 15	80
$u_2=0$	$\boxed{3}$ 110	$\boxed{11}$ $\boxed{-4}$	$\boxed{7}$ $\boxed{-1}$	$\boxed{3}$ $\boxed{-3}$	110
$u_3=1$	$\boxed{8}$ $\boxed{-4}$	$\boxed{13}$ $\boxed{-5}$	$\boxed{7}$ 150	$\boxed{5}$ $\boxed{-4}$	150
$u_4=1$	$\boxed{10}$ $\boxed{-6}$	$\boxed{8}$ 15	$\boxed{12}$ $\boxed{-5}$	$\boxed{1}$ 85	100
$u_5=3$	$\boxed{7}$ $\boxed{-1}$	$\boxed{10}$ 150	$\boxed{10}$ $\boxed{-1}$	$\boxed{6}$ $\boxed{-3}$	100
$u_6=-7$	$\boxed{0}$ $\boxed{-4}$	$\boxed{0}$ 35	$\boxed{0}$ $\boxed{-1}$	$\boxed{0}$ $\boxed{-7}$	35
	150	200	175	100	

All $u_i+v_j-c_{ij} \leq 0$ $\Rightarrow$ optimal.

Soln. is $x_{11}=40$, $x_{13}=25$, $x_{14}=15$

$x_{21}=110$, $x_{33}=150$, $x_{42}=15$, $x_{44}=85$

$x_{52}=150$, $\boxed{x_{62}=35}$ → not shipped unsatisfied demand, hence not counted

$z = 6\times40 + 3\times25 + 9\times15 + 6\times110 +$

$2\times150 + 4\times15 + 8\times85 + (-10)\times150$

~~$+ 2\times35$~~ $= 1955$

in optimal value