School of Mathematics, Thapar University
Operations Research (UMA-019)
Tutorial Sheet 4

1. Write the duals of the following problems:

(i) Max $z = x_1 - 2x_2 + 4x_3 - 3x_4$, s/t $x_1 + x_2 - 3x_3 + x_4 = 9$,
$3x_1 + 5x_2 + 2x_3 - 7x_4 \le 5$, $x_1 - 3x_2 + 5x_4 \ge 8, x_1, x_2, x_3, x_4 \ge 0$

(ii) Min $z = 2x_1 + x_2 + x_3$, s/t $x_1 + x_2 - x_3 \ge 1, -2x_1 + x_3 \le 0, x_1 - x_2 + x_3 = 2,$
$x_1 \ge 0, x_2 \le 0$

(iii) Min $z = 6x_1 + 3x_2$, s/t $6x_1 - 3x_2 + x_3 \ge 2, 3x_1 + 4x_2 + x_3 \ge 5,$
$x_1, x_2, x_3 \ge 0.$

(iv) Max $z = x_1 + x_2$, s/t $2x_1 + x_2 = 5, 3x_1 - x_2 = 6,$
x_1, x_2 is unrestricted.

2. If a linear programming problem has an unbounded solution then show that its dual is infeasible.

3. If a (primal) LPP is feasible and its dual is infeasible, then show that the primal is unbounded.

4. Show that the following problem and its dual are infeasible.

Max $z = 8x_1 + 6x_2$, s/t $2x_1 - x_2 \ge 2, -4x_1 + 2x_2 \ge 1$, $x_1, x_2 \ge 0$

5. Write the dual of the problem: Max $z = x_1 + 2x_2 + x_3$,
s/t $x_1 + x_2 - x_3 \le 2$, $x_1 - x_2 + x_3 = 1$, $2x_1 + x_2 + x_3 \ge 2$, $x_1 \ge 0$, $x_2 \le 0$

and using the duality theory show that maximum of z cannot exceed one.

6. Show by inspection that the dual of the problem:

Max $z = -2x_1 + 3x_2 + 5x_3$, s/t $x_1 - x_2 + x_3 \le 15$, $x_1, x_2, x_3 \ge 0$

is infeasible. What can you say about the solution of the primal?

7 (i) Solve the following problem graphically. Write its dual. Then using the complementary slackness theorem obtain the solution of the dual problem.

Maximize $z = 2x_1 + 3x_2$
Subject to $x_1 + x_2 \le 3$, $2x_1 + 3x_2 \ge 3$, $-x_1 + x_2 \le 0$, $x_1 \le 2$, $x_1, x_2 \ge 0$.

(ii) Write the dual of the problem:

Minimize $z = x_1 + 2x_2 + 3x_3 + 4x_4$,
Subject to $x_1 + 2x_2 + 2x_3 + 3x_4 \ge 30, 2x_2 + 3x_3 + 2x_4 \ge 40$ $x_1, x_2, x_3, x_4 \ge 0$.

Solve the dual graphically. Then using the complementary slackness theorem obtain the solution of the above problem.

8. Describe the dual simplex method. Using it solve:

(i) $Min\ z = 2x_1 + x_2,\ s/t\ 3x_1 + x_2 \geq 3,\ 4x_1 + 3x_2 \geq 6, x_1 + 2x_2 \leq 3, x_1, x_2 \geq 0.$

(ii) $\text{Min } z = x_1 + 4x_2 + 3x_4,\ s/t\ x_1 + 2x_2 - x_3 + x_4 \geq 3$
$s/t\ x_1 + 2x_2 - x_3 + x_4 \geq 3, -2x_1 + x_2 + 4x_3 + x_4 \geq 2,\ x_1, x_2, x_3,\ x_4 \geq 0$

(iii) $Min\ z = 5x_1 + 6x_2,\ s/t\ x_1 + x_2 \geq 2,\ 4x_1 + x_2 \geq 4,\ \ x_1, x_2 \geq 0.$

(iv) $Min\ z = 4x_1 + 2x_2,\ s/t\ x_1 + x_2 = 1,\ 3x_1 - x_2 \geq 2,\ \ x_1, x_2 \geq 0.$