School of Mathematics, Thapar University
Operations Research (UMA-019)
Tutorial Sheet 2

1. Write the standard form of the LPP

(i) Max $Z = 2x_1 + x_2 + x_3$
s. t. $x_1 - x_2 + 2x_3 \geq 2$, $|2x_1 + x_2 - x_3| \leq 4$, $3x_1 - 2x_2 - 7x_3 \leq 3$
$x_1, x_3 \geq 0$, $x_2 \leq 0$

(ii) Max $Z = x_1 + 2x_2 - x_3$
s. t. $x_1 + x_2 - x_3 \leq 5$, $-x_1 + 2x_2 + 3x_3 \geq -4$, $2x_1 + 3x_2 - 4x_3 \geq 3$, $x_1 + x_2 + x_3 = 2$,
$x_1 \geq 0$, $x_2 \geq p, x_3$ is unrestricted in sign.

(iii) Min $Z = 2x_1 - x_2 + 2x_3$
s. t. $-x_1 + x_2 + x_3 = 4$, $-x_1 + x_2 - x_3 \leq 6$, $x_1 \leq 0$, $x_2 \geq 0, x_3$ is unrestricted in sign.

(iv) Max $Z = 2x_1 - 2x_2 + 3x_3$
s.t. $x_1 + x_2 - x_3 \leq 5$, $x_1 - x_2 + 2x_3 \geq 2$, $3x_1 - 2x_2 - 7x_3 \leq 3$
,
$x_1 \leq 0$, x_2, x_3 are unrestricted in sign.

2. Examine whether the following sets are convex or not:
(a) $\{(x_1, x_2) : x_1 x_2 \leq 1\}$ (b) $\{(x_1, x_2) : x_1^2 + x_2^2 < 1\}$ (c) $\{(x_1, x_2) : x_1^2 + x_2^2 \geq 3\}$

(d) $\{(x_1, x_2) : 4x_1 \geq x_2^2\}$ (e) $\{(x_1, x_2) : 0 < x_1^2 + x_2^2 \leq 4\}$
(f) $\{(x_1, x_2) : x_2 - 3 \geq -x_1^2, x_1, x_2 \geq 0\}$
(g) $\{(x_1, x_2) : 2x_1 + 5x_2 \leq 20, x_1 + 2x_2 \geq 6\}$ (h) $\{(x_1, x_2) : x_1 x_2 \geq 4, x_1, x_2 \geq 0\}$

3. Prove that intersection of any collection (finite or infinite) of convex sets in R^n is a convex set.

4. Show that the set of all optimal solutions of a linear programming problem is a convex set.

5. Find all the extreme points of the set $S = \{(x_1, x_2) \mid x_1 + 2x_2 \geq -2, -x_1 + x_2 \leq 4, x_1 \leq 4\}$ and represent the point (2, 3) as the convex combination of the extreme points of S.

6. Prove that half space $\{\mathbf{X} \in \mathbf{R}^n : \mathbf{a}^{\mathrm{T}}\mathbf{X} \geq \alpha\}$ is a convex set.

7. Let $\mathbf{S}_1$ and $\mathbf{S}_2$ be two disjoint nonempty set in R^n. Then show that the set
$S = \{X_1 - X_2 : X_1 \in S_1, X_2 \in S_2\}$ is convex.

8. Linearize the following objective function: Max $z = \min\{ |2x_1 + 5x_2|, |7x_1 - 3x_2| \}$

9. Solve the following linear programming problems graphically and state what your solution indicate.
(i) $Max\ z = 5x_1 + 3x_2$, s/t $3x_1 + 5x_2 \leq 15$, $5x_1 + 2x_2 \leq 10$, $x_1, x_2 \geq 0$.

(ii) $Min\ z = 2x_1 + 3x_2,\ s/t\ x_1 + x_2 \le 4,\ 6x_1 + 2x_2 \ge 8,\ x_1 + 5x_2 \ge 4,\ x_1 \le 3,\ x_2 \le 3,\ x_1, x_2 \ge 0.$

(iii) $Max\ z = 2x_1 + 2x_2,\ s/t\ x_1 - x_2 \ge -1,\ -0.5x_1 + x_2 \le 2,\ x_1, x_2 \ge 0.$

(iv) $Max\ z = -3x_1 + 2x_2,\ s/t\ x_1 - x_2 \le 0,\ x_1 \le 3,\ x_1, x_2 \ge 0.$

(v) $Max\ z = -x_1 + 2x_2,\ s/t\ x_1 - x_2 \ge -1,\ -0.5x_1 + x_2 \le 2,\ x_1, x_2 \ge 0.$

(vi) $Max\ z = 3x_1 - 2x_2,\ s/t\ x_1 + x_2 \le 1,\ 2x_1 + 2x_2 \ge 4,\ x_1, x_2 \ge 0.$

(vii) $Max\ z = x_1 + x_2,\ s/t\ x_1 - x_2 \ge 0,\ 3x_1 - x_2 \le -3,\ x_1, x_2 \ge 0.$

10. Consider the following LPP

$Max\ z = -4x_1 + 6x_2,\ s/t\ 2x_1 - 3x_2 \ge -6,\ -x_1 + x_2 \le 1,\ x_1, x_2 \ge 0.$

Show graphically that the variable can be increased indefinitely while the optimal value of the objective function remains constant.