Huazhong University of Science and Technology

AUT5951

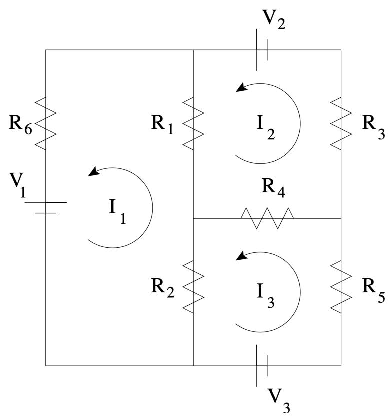
Homework 1 - Foundations of Data Science

Problem 1

Consider the circuit depicted in the figure. By using the two following facts: (i) the voltage drop across a resistor is $V = IR$ and (ii) the sum of all voltage drops in a closed loop sum to zero. The currents I_1, I_2 and I_3 are determined from the 3×3 system:

$$
\begin{aligned}
R_6 I_1 + R_1(I_1 - I_2) + R_2(I_1 - I_3) &= V_1 \\
R_3 I_2 + R_4(I_2 - I_3) + R_1(I_2 - I_1) &= V_2 \\
R_5 I_3 + R_4(I_3 - I_2) + R_2(I_3 - I_1) &= V_3
\end{aligned}
$$

where $R_1 = 20$, $R_2 = 15$, $R_3 = 25$, $R_4 = 20$, $R_5 = 30$, $R_6 = 40$, $V_2 = 0$, $V_3 = 200$, and V_1 will be variable. In this form, the associated matrix is strictly diagonal dominant.



(a) Vary V_1 from 0 to 100 in steps of 2 (i.e. $V_1 = 0, 2, 4, \ldots, 100$) and calculate I_1, I_2 and I_3 as a function of increasing V_1 by solving the system with the standard backslash (MATLAB) command (e.g., `x = A \ y` gives $x = A^{-1}y$). Save your results in a matrix of 3 columns and 51 rows where the first, second and third column are I_1, I_2 and I_3 respectively.

(b) Repeat part (a), but now solve it with two additional methods: Jacobi Iterations and Gauss-Seidel Iterations. For the iteration methods, begin with the guess $(I_1, I_2, I_3) = (0, 0, 0)$. This will give you two additional matrices of size 3 columns by 51 rows for the Jacobi and Gauss-Seidel respectively.

(c) For the two iteration methods, what is the **average** number of iterations required to solve the given equation with accuracy 10^{-6}. The accuracy constraint should be based upon looking at the norm of the difference between successive iterations, i.e. $\|x_{n+1} - x_n\|_\infty < 10^{-6}$. Save the two answers (for Jacobi first and Gauss-Seidel second) as a row vector with two components.