Multiples of 3 and 5

Problem 1

If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23.

Find the sum of all the multiples of 3 or 5 below 1000.

Answer: **233168**

Completed on Wed, 15 Jul 2015, 12:06

Even Fibonacci numbers

Problem 2

Each new term in the Fibonacci sequence is generated by adding the previous two terms. By starting with 1 and 2, the first 10 terms will be:

1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

By considering the terms in the Fibonacci sequence whose values do not exceed four million, find the sum of the even-valued terms.

Answer: **4613732**

Completed on Thu, 16 Jul 2015, 19:00

Largest prime factor

Problem 3

The prime factors of 13195 are 5, 7, 13 and 29.

What is the largest prime factor of the number 600851475143 ?

Answer:

Confirmation Code:

Largest palindrome product

Problem 4

A palindromic number reads the same both ways. The largest palindrome made from the product of two 2-digit numbers is 9009 = 91 × 99.

Find the largest palindrome made from the product of two 3-digit numbers.

Answer:

Confirmation Code:

Smallest multiple

Problem 5

2520 is the smallest number that can be divided by each of the numbers from 1 to 10 without any remainder.

What is the smallest positive number that is evenly divisible by all of the numbers from 1 to 20?

Answer:

Confirmation Code:

Sum square difference

Problem 6

The sum of the squares of the first ten natural numbers is,

$$1^2 + 2^2 + ... + 10^2 = 385$$

The square of the sum of the first ten natural numbers is,

$$(1 + 2 + ... + 10)^2 = 55^2 = 3025$$

Hence the difference between the sum of the squares of the first ten natural numbers and the square of the sum is 3025 − 385 = 2640.

Find the difference between the sum of the squares of the first one hundred natural numbers and the square of the sum.

Answer:

Confirmation Code:

10001st prime

Problem 7

By listing the first six prime numbers: 2, 3, 5, 7, 11, and 13, we can see that the 6th prime is 13.

What is the 10 001st prime number?

Answer:

Confirmation Code:

Largest product in a series

Problem 8

The four adjacent digits in the 1000-digit number that have the greatest product are 9 × 9 × 8 × 9 = 5832.

```
73167176531330624919225119674426574742355349194934
96983520312774506326239578318016984801869478851843
85861560789112949495459501737958331952853208805511
12540698747158523863050715693290963295227443043557
66896648950445244523161731856403098711121722383113
62229893423380308135336276614282806444486645238749
30358907296290491560440772390713810515859307960866
70172427121883998797908792274921901699720888093776
65727333001053367881220235421809751254540594752243
52584907711670556013604839586446706324415722155397
53697817977846174064955149290862569321978468622482
83972241375657056057490261407972968652414535100474
82166370484403199890008895243450658541227588666881
16427171479924442928230863465674813919123162824586
```

```
17866458359124566529476545682848912883142607690042
24219022671055626321111109370544217506941658960408
07198403850962455444362981230987879927244284909188
84580156166097919133875499200524063689912560717606
05886116467109405077541002256983155200055935729725
71636269561882670428252483600823257530420752963450
```

Find the thirteen adjacent digits in the 1000-digit number that have the greatest product. What is the value of this product?

Answer:

Confirmation Code:

Special Pythagorean triplet

Problem 9

A Pythagorean triplet is a set of three natural numbers, $a < b < c$, for which,

$$a^2 + b^2 = c^2$$

For example, $3^2 + 4^2 = 9 + 16 = 25 = 5^2$.

There exists exactly one Pythagorean triplet for which $a + b + c = 1000$.
Find the product abc.

Answer:

Confirmation Code:

Summation of primes

Problem 10

The sum of the primes below 10 is $2 + 3 + 5 + 7 = 17$.

Find the sum of all the primes below two million.

Answer:

Confirmation Code:

Largest product in a grid

Problem 11

In the 20×20 grid below, four numbers along a diagonal line have been marked in red.

```
08 02 22 97 38 15 00 40 00 75 04 05 07 78 52 12 50 77 91 08
49 49 99 40 17 81 18 57 60 87 17 40 98 43 69 48 04 56 62 00
81 49 31 73 55 79 14 29 93 71 40 67 53 88 30 03 49 13 36 65
52 70 95 23 04 60 11 42 69 24 68 56 01 32 56 71 37 02 36 91
22 31 16 71 51 67 63 89 41 92 36 54 22 40 40 28 66 33 13 80
24 47 32 60 99 03 45 02 44 75 33 53 78 36 84 20 35 17 12 50
32 98 81 28 64 23 67 10 26 38 40 67 59 54 70 66 18 38 64 70
67 26 20 68 02 62 12 20 95 63 94 39 63 08 40 91 66 49 94 21
24 55 58 05 66 73 99 26 97 17 78 78 96 83 14 88 34 89 63 72
21 36 23 09 75 00 76 44 20 45 35 14 00 61 33 97 34 31 33 95
78 17 53 28 22 75 31 67 15 94 03 80 04 62 16 14 09 53 56 92
16 39 05 42 96 35 31 47 55 58 88 24 00 17 54 24 36 29 85 57
86 56 00 48 35 71 89 07 05 44 44 37 44 60 21 58 51 54 17 58
19 80 81 68 05 94 47 69 28 73 92 13 86 52 17 77 04 89 55 40
04 52 08 83 97 35 99 16 07 97 57 32 16 26 26 79 33 27 98 66
88 36 68 87 57 62 20 72 03 46 33 67 46 55 12 32 63 93 53 69
04 42 16 73 38 25 39 11 24 94 72 18 08 46 29 32 40 62 76 36
20 69 36 41 72 30 23 88 34 62 99 69 82 67 59 85 74 04 36 16
20 73 35 29 78 31 90 01 74 31 49 71 48 86 81 16 23 57 05 54
01 70 54 71 83 51 54 69 16 92 33 48 61 43 52 01 89 19 67 48
```

The product of these numbers is 26 × 63 × 78 × 14 = 1788696.

What is the greatest product of four adjacent numbers in the same direction (up, down, left, right, or diagonally) in the 20×20 grid?

Answer:

Confirmation Code:

Highly divisible triangular number

Problem 12

The sequence of triangle numbers is generated by adding the natural numbers. So the 7^{th} triangle number would be 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28. The first ten terms would be:

1, 3, 6, 10, 15, 21, 28, 36, 45, 55, ...

Let us list the factors of the first seven triangle numbers:

```
 1: 1
 3: 1,3
 6: 1,2,3,6
10: 1,2,5,10
15: 1,3,5,15
21: 1,3,7,21
28: 1,2,4,7,14,28
```

We can see that 28 is the first triangle number to have over five divisors.

What is the value of the first triangle number to have over five hundred divisors?

Answer:

Confirmation Code:

Large sum

Problem 13

Work out the first ten digits of the sum of the following one-hundred 50-digit numbers.

```
37107287533902102798797998220837590246510135740250
46376937677490009712648124896970078050417018260538
74324986199524741059474233309513058123726617309629
91942213363574161572522430563301811072406154908250
23067588207539346171171980310421047513778063246676
89261670696623633820136378418383684178734361726757
28112879812849979408065481931592621691275889832738
44274228917432520321923589422876796487670272189318
47451445736001306439091167216856844588711603153276
70386486105843025439939619828917593665686757934951
62176457141856560629502157223196586755079324193331
64906352462741904929101432445813822663347944758178
92575867718337217661963751590579239728245598838407
58203565325359399008402633568948830189458628227828
80181199384826282014278194139940567587151170094390
35398664372827112653829987240784473053190104293586
86515506006295864861532075273371959191420517255829
```

7169388870771546649911559348760353292171497005 6938
5437007057682668462462149565007647178729443837 7604
5328265410875682844319119063469403785521777929 5145
3612327252500029607107508256381565671088525835 0721
4587657617241097644733911060721826523687722363 6045
1742370690585186066044820762120981328786073396 9412
8114266041808683061932846081119106155694051268 9692
5193432545172838864191804704929321505864256304 9483
6246722164843507620172791803994469300473295634 0691
1573244438690812579451408905770622942919710792 8209
5503768752567877309186254074496984450833039368 2126
1833638482533015468619612434876768129753437594 6515
8038628759287849020152168555482871720121925776 6954
7818283375799310361474035685644909552709786479 7581
1672632010043689784255353992093183744149780686 0984
4840309812907779179908821879532736447567559084 8030
8708698755139271185451707854416185242432069315 0332
5995940689575653678210707492696653767632623544 7210
6979395067965269474259770973916669376304263398 7085
4105268470829908521139942736573411618276031500 1271
6537860736150108085700914993951255702819874600 4375
3582903531743471732693212357815498262974255273 7307
9495375976510530594696606768315657437716740187 5275
8890280257173322961917666871381993181104877019 0271
2526768027607800301367868099252546340106163286 6526
3627021854049770558562994658063623799314074625 5962
2407448690823117497779236546625724692332281091 7141
9143028819710328859780666976089293863828502533 3403
3441306557801612781592181500556186883646842009 0470
2305308117281643048762379196984248725503663878 4583
1148769693215490281042402013833512446218144177 3470
6378329949063625966649858761822122522551248676 4533
6772018697169854431241957240991395900895231005 8822
9554825530026352078153229679624948164195386821 8774
7608532713228572311042480345612486769706450799 5236
3777424253541129168427686553892620502491032657 2967
2370191327572567528565324825826546309220705859 6522
2979886027225833191312637514734199488953476574 5501
1849570145487928898485682772607771372140379887 9715
3829820378303147352772158034814451349137322665 1381
3482954382919991818027891652243102739225112286 9539
4095795306640523263253804410005965493915987959 3635
2974615218550237130764225512118369380358038858 4903
4169811622207297718615823667842468915799353296 1922
6246795719440126904387710727504810239089552359 7457
2318970677254791506150550495392297953090112996 7519
8618808822587531452958409925120382900940777077 5672
1130673970830472448381653387350234084564705807 7308
8295917476714036319800818712901187549131054712 6581
9762333104481838626951545633492636657289756340 0500
4284628018351707052783183942588214552122725125 0327
5512160354698120058176216521282765275169129689 7789
3223819573432933994643750190783694576588335239 9886
7550616496518477518073816883786109152735792970 1337
6217784275219262340194239963916804498399317331 2731
3292418570714734956691667468763466091503591467 7504
9951867143023521962889489010242332511691361962 6622

```
73267460800591547471830798392868535206946944540724
76841822524674417161514036427982273348055556214818
97142617910342598647204516893989422179826088076852
87783646182799346313767754307809363333018982642090
10848802521674670883215120185883543223812876952786
71329612474782464538636993009049310363619763878039
62184073572399794223406235393808339651327408011116
66627891981488087797941876876144230030984490851411
60661826293682836764744779239180335110989069790714
85786944089552990653640447425576083659976645795096
66024396409905389607120198219976047599490197230297
64913982680032973156037120041377903785566085089252
16730939319872750275468906903707539413042652315011
94809377245048795150954100921645863754710598436791
78639167021187492431995700641917969777599028300699
15368713711936614952811305876380278410754449733078
40789923115535562561142322423255033685442488917353
44889911501440648020369068063960672322193204149535
41503128880339536053299340368006977710650566631954
81234880673210146739058568557934581403627822703280
82616570773948327592232845941706525094512325230608
22918802058777319719839450180888072429661980811197
77158542502016545090413245809786882778948721859617
72107838435069186155435662884062257473692284509516
20849603980134001723930671666823555245252804609722
53503534226472524250874054075591789781264330331690
```

Answer:

Confirmation Code:

Longest Collatz sequence

Problem 14

The following iterative sequence is defined for the set of positive integers:

$n \to n/2$ (n is even)
$n \to 3n + 1$ (n is odd)

Using the rule above and starting with 13, we generate the following sequence:

$$13 \to 40 \to 20 \to 10 \to 5 \to 16 \to 8 \to 4 \to 2 \to 1$$

It can be seen that this sequence (starting at 13 and finishing at 1) contains 10 terms. Although it has not been proved yet (Collatz Problem), it is thought that all starting numbers finish at 1.

Which starting number, under one million, produces the longest chain?

NOTE: Once the chain starts the terms are allowed to go above one million.

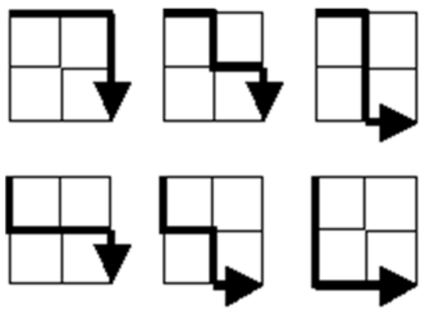
Answer:

Confirmation Code:

Lattice paths

Problem 15

Starting in the top left corner of a 2×2 grid, and only being able to move to the right and down, there are exactly 6 routes to the bottom right corner.



How many such routes are there through a 20×20 grid?

Answer:

Confirmation Code:

Power digit sum

Problem 16

$2^{15} = 32768$ and the sum of its digits is $3 + 2 + 7 + 6 + 8 = 26$.

What is the sum of the digits of the number 2^{1000}?

Answer:

Confirmation Code:

Number letter counts

Problem 17

If the numbers 1 to 5 are written out in words: one, two, three, four, five, then there are 3 + 3 + 5 + 4 + 4 = 19 letters used in total.

If all the numbers from 1 to 1000 (one thousand) inclusive were written out in words, how many letters would be used?

NOTE: Do not count spaces or hyphens. For example, 342 (three hundred and forty-two) contains 23 letters and 115 (one hundred and fifteen) contains 20 letters. The use of "and" when writing out numbers is in compliance with British usage.

Answer:

Confirmation Code:

Maximum path sum I

Problem 18

By starting at the top of the triangle below and moving to adjacent numbers on the row below, the maximum total from top to bottom is 23.

3
7 4
2 **4** 6
8 5 **9** 3

That is, 3 + 7 + 4 + 9 = 23.

Find the maximum total from top to bottom of the triangle below:

```
                            75
                          95 64
                        17 47 82
                      18 35 87 10
                    20 04 82 47 65
                  19 01 23 75 03 34
                88 02 77 73 07 63 67
              99 65 04 28 06 16 70 92
            41 41 26 56 83 40 80 70 33
          41 48 72 33 47 32 37 16 94 29
        53 71 44 65 25 43 91 52 97 51 14
      70 11 33 28 77 73 17 78 39 68 17 57
    91 71 52 38 17 14 91 43 58 50 27 29 48
  63 66 04 68 89 53 67 30 73 16 69 87 40 31
04 62 98 27 23 09 70 98 73 93 38 53 60 04 23
```

NOTE: As there are only 16384 routes, it is possible to solve this problem by trying every route. However, Problem 67, is the same challenge with a triangle containing one-hundred rows; it cannot be solved by brute force, and requires a clever method! ;o)

Answer:

Confirmation Code:

Counting Sundays

Problem 19

You are given the following information, but you may prefer to do some research for yourself.

- 1 Jan 1900 was a Monday.
- Thirty days has September,
  April, June and November.
  All the rest have thirty-one,
  Saving February alone,
  Which has twenty-eight, rain or shine.
  And on leap years, twenty-nine.
- A leap year occurs on any year evenly divisible by 4, but not on a century unless it is divisible by 400.

How many Sundays fell on the first of the month during the twentieth century (1 Jan 1901 to 31 Dec 2000)?

Answer:

Confirmation Code:

Factorial digit sum

Problem 20

$n!$ means $n \times (n - 1) \times ... \times 3 \times 2 \times 1$

For example, $10! = 10 \times 9 \times ... \times 3 \times 2 \times 1 = 3628800$,
and the sum of the digits in the number $10!$ is $3 + 6 + 2 + 8 + 8 + 0 + 0 = 27$.

Find the sum of the digits in the number $100!$

Answer:

Confirmation Code:

Amicable numbers

Problem 21

Let d(n) be defined as the sum of proper divisors of n (numbers less than n which divide evenly into n).
If d(a) = b and d(b) = a, where $a \neq b$, then a and b are an amicable pair and each of a and b are called amicable numbers.

For example, the proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110; therefore d(220) = 284. The proper divisors of 284 are 1, 2, 4, 71 and 142; so d(284) = 220.

Evaluate the sum of all the amicable numbers under 10000.

Answer:

Confirmation Code:

Names scores

Problem 22

Using names.txt (right click and 'Save Link/Target As...'), a 46K text file containing over five-thousand first names, begin by sorting it into alphabetical order. Then working out the alphabetical value for each name, multiply this value by its alphabetical position in the list to obtain a name score.

For example, when the list is sorted into alphabetical order, COLIN, which is worth 3 + 15 + 12 + 9 + 14 = 53, is the 938th name in the list. So, COLIN would obtain a score of 938 × 53 = 49714.

What is the total of all the name scores in the file?

Answer:

Confirmation Code:

Non-abundant sums

Problem 23

A perfect number is a number for which the sum of its proper divisors is exactly equal to the number. For example, the sum of the proper divisors of 28 would be 1 + 2 + 4 + 7 + 14 = 28, which means that 28 is a perfect number.

A number n is called deficient if the sum of its proper divisors is less than n and it is called abundant if this sum exceeds n.

As 12 is the smallest abundant number, 1 + 2 + 3 + 4 + 6 = 16, the smallest number that can be written as the sum of two abundant numbers is 24. By mathematical analysis, it can be shown that all integers greater than 28123 can be written as the sum of two abundant numbers. However, this upper limit cannot be reduced any further by analysis even though it is known that the greatest number that cannot be expressed as the sum of two abundant numbers is less than this limit.

Find the sum of all the positive integers which cannot be written as the sum of two abundant numbers.

Answer:

Confirmation Code:

Lexicographic permutations

Problem 24

A permutation is an ordered arrangement of objects. For example, 3124 is one possible permutation of the digits 1, 2, 3 and 4. If all of the permutations are listed numerically or alphabetically, we call it lexicographic order. The lexicographic permutations of 0, 1 and 2 are:

012   021   102   120   201   210

What is the millionth lexicographic permutation of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9?

Answer:

Confirmation Code:

1000-digit Fibonacci number

Problem 25

The Fibonacci sequence is defined by the recurrence relation:

$F_n = F_{n-1} + F_{n-2}$, where $F_1 = 1$ and $F_2 = 1$.

Hence the first 12 terms will be:

$F_1 = 1$
$F_2 = 1$
$F_3 = 2$
$F_4 = 3$
$F_5 = 5$
$F_6 = 8$
$F_7 = 13$
$F_8 = 21$

$F_9 = 34$
$F_{10} = 55$
$F_{11} = 89$
$F_{12} = 144$

The 12th term, F_{12}, is the first term to contain three digits.

What is the index of the first term in the Fibonacci sequence to contain 1000 digits?

Answer:

Confirmation Code:

Reciprocal cycles

Problem 26

A unit fraction contains 1 in the numerator. The decimal representation of the unit fractions with denominators 2 to 10 are given:

$^1/_2$ = 0.5
$^1/_3$ = 0.(3)
$^1/_4$ = 0.25
$^1/_5$ = 0.2
$^1/_6$ = 0.1(6)
$^1/_7$ = 0.(142857)
$^1/_8$ = 0.125
$^1/_9$ = 0.(1)
$^1/_{10}$ = 0.1

Where 0.1(6) means 0.166666..., and has a 1-digit recurring cycle. It can be seen that $^1/_7$ has a 6-digit recurring cycle.

Find the value of $d < 1000$ for which $^1/_d$ contains the longest recurring cycle in its decimal fraction part.

Answer:

Confirmation Code:

Quadratic primes

Problem 27

Euler discovered the remarkable quadratic formula:

$$n^2 + n + 41$$

It turns out that the formula will produce 40 primes for the consecutive values $n = 0$ to 39. However, when $n = 40$, $40^2 + 40 + 41 = 40(40 + 1) + 41$ is divisible by 41, and certainly when $n = 41$, $41^2 + 41 + 41$ is clearly divisible by 41.

The incredible formula $n^2 - 79n + 1601$ was discovered, which produces 80 primes for the consecutive values $n = 0$ to 79. The product of the coefficients, −79 and 1601, is −126479.

Considering quadratics of the form:

$n^2 + an + b$, where $|a| < 1000$ and $|b| < 1000$

where $|n|$ is the modulus/absolute value of n
e.g. $|11| = 11$ and $|-4| = 4$

Find the product of the coefficients, a and b, for the quadratic expression that produces the maximum number of primes for consecutive values of n, starting with $n = 0$.

Answer:

Confirmation Code:

Number spiral diagonals

Problem 28

Starting with the number 1 and moving to the right in a clockwise direction a 5 by 5 spiral is formed as follows:

21	22	23	24	**25**
20	**7**	8	**9**	10
19	6	**1**	2	11
18	**5**	4	**3**	12
17	16	15	14	**13**

It can be verified that the sum of the numbers on the diagonals is 101.

What is the sum of the numbers on the diagonals in a 1001 by 1001 spiral formed in the same way?

Answer:

Confirmation Code:

Distinct powers

Problem 29

Consider all integer combinations of a^b for $2 \le a \le 5$ and $2 \le b \le 5$:

$2^2=4$, $2^3=8$, $2^4=16$, $2^5=32$
$3^2=9$, $3^3=27$, $3^4=81$, $3^5=243$
$4^2=16$, $4^3=64$, $4^4=256$, $4^5=1024$
$5^2=25$, $5^3=125$, $5^4=625$, $5^5=3125$

If they are then placed in numerical order, with any repeats removed, we get the following sequence of 15 distinct terms:

4, 8, 9, 16, 25, 27, 32, 64, 81, 125, 243, 256, 625, 1024, 3125

How many distinct terms are in the sequence generated by a^b for $2 \le a \le 100$ and $2 \le b \le 100$?

Answer:

Confirmation Code:

Digit fifth powers

Problem 30

Surprisingly there are only three numbers that can be written as the sum of fourth powers of their digits:

$1634 = 1^4 + 6^4 + 3^4 + 4^4$
$8208 = 8^4 + 2^4 + 0^4 + 8^4$
$9474 = 9^4 + 4^4 + 7^4 + 4^4$

As $1 = 1^4$ is not a sum it is not included.

The sum of these numbers is 1634 + 8208 + 9474 = 19316.

Find the sum of all the numbers that can be written as the sum of fifth powers of their digits.

Answer:

Confirmation Code:

Coin sums

Problem 31

In England the currency is made up of pound, £, and pence, p, and there are eight coins in general circulation:

1p, 2p, 5p, 10p, 20p, 50p, £1 (100p) and £2 (200p).

It is possible to make £2 in the following way:

1×£1 + 1×50p + 2×20p + 1×5p + 1×2p + 3×1p

How many different ways can £2 be made using any number of coins?

Answer:

Confirmation Code:

Digit cancelling fractions

Problem 33

The fraction $^{49}/_{98}$ is a curious fraction, as an inexperienced mathematician in attempting to simplify it may incorrectly believe that $^{49}/_{98} = {}^{4}/_{8}$, which is correct, is obtained by cancelling the 9s.

We shall consider fractions like, $^{30}/_{50} = {}^{3}/_{5}$, to be trivial examples.

There are exactly four non-trivial examples of this type of fraction, less than one in value, and containing two digits in the numerator and denominator.

If the product of these four fractions is given in its lowest common terms, find the value of the denominator.

Answer:

Confirmation Code:

Digit factorials

Problem 34

145 is a curious number, as 1! + 4! + 5! = 1 + 24 + 120 = 145.

Find the sum of all numbers which are equal to the sum of the factorial of their digits.

Note: as 1! = 1 and 2! = 2 are not sums they are not included.

Answer:

Confirmation Code:

Circular primes

Problem 35

The number, 197, is called a circular prime because all rotations of the digits: 197, 971, and 719, are themselves prime.

There are thirteen such primes below 100: 2, 3, 5, 7, 11, 13, 17, 31, 37, 71, 73, 79, and 97.

How many circular primes are there below one million?

Answer:

Confirmation Code:

Double-base palindromes

Problem 36

The decimal number, 585 = 1001001001_2 (binary), is palindromic in both bases.

Find the sum of all numbers, less than one million, which are palindromic in base 10 and base 2.

(Please note that the palindromic number, in either base, may not include leading zeros.)

Answer:

Confirmation Code:

Truncatable primes

Problem 37

The number 3797 has an interesting property. Being prime itself, it is possible to continuously remove digits from left to right, and remain prime at each stage: 3797, 797, 97, and 7. Similarly we can work from right to left: 3797, 379, 37, and 3.

Find the sum of the only eleven primes that are both truncatable from left to right and right to left.

NOTE: 2, 3, 5, and 7 are not considered to be truncatable primes.

Answer:

Confirmation Code:

Pandigital multiples

Problem 38

Take the number 192 and multiply it by each of 1, 2, and 3:

192 × 1 = 192
192 × 2 = 384
192 × 3 = 576

By concatenating each product we get the 1 to 9 pandigital, 192384576. We will call 192384576 the concatenated product of 192 and (1,2,3)

The same can be achieved by starting with 9 and multiplying by 1, 2, 3, 4, and 5, giving the pandigital, 918273645, which is the concatenated product of 9 and (1,2,3,4,5).

What is the largest 1 to 9 pandigital 9-digit number that can be formed as the concatenated product of an integer with (1,2, ... , n) where $n > 1$?

Answer:

Confirmation Code:

Integer right triangles

Problem 39

If p is the perimeter of a right angle triangle with integral length sides, {a,b,c}, there are exactly three solutions for $p = 120$.

{20,48,52}, {24,45,51}, {30,40,50}

For which value of $p \leq 1000$, is the number of solutions maximised?

Answer:

Confirmation Code:

Champernowne's constant

Problem 40

An irrational decimal fraction is created by concatenating the positive integers:

0.123456789101112131415161718192021...

It can be seen that the 12th digit of the fractional part is 1.

If d_n represents the n^{th} digit of the fractional part, find the value of the following expression.

$$d_1 \times d_{10} \times d_{100} \times d_{1000} \times d_{10000} \times d_{100000} \times d_{1000000}$$

Answer:

Confirmation Code:

Pandigital prime

Problem 41

We shall say that an *n*-digit number is pandigital if it makes use of all the digits 1 to *n* exactly once. For example, 2143 is a 4-digit pandigital and is also prime.

What is the largest *n*-digit pandigital prime that exists?

Answer:

Confirmation Code:

Coded triangle numbers

Problem 42

The n^{th} term of the sequence of triangle numbers is given by, $t_n = ½n(n+1)$; so the first ten triangle numbers are:

1, 3, 6, 10, 15, 21, 28, 36, 45, 55, ...

By converting each letter in a word to a number corresponding to its alphabetical position and adding these values we form a word value. For example, the word value for SKY is 19 + 11 + 25 = 55 = t_{10}. If the word value is a triangle number then we shall call the word a triangle word.

Using words.txt (right click and 'Save Link/Target As...'), a 16K text file containing nearly two-thousand common English words, how many are triangle words?

Answer:

Confirmation Code:

Sub-string divisibility

Problem 43

The number, 1406357289, is a 0 to 9 pandigital number because it is made up of each of the digits 0 to 9 in some order, but it also has a rather interesting sub-string divisibility property.

Let d_1 be the 1^{st} digit, d_2 be the 2^{nd} digit, and so on. In this way, we note the following:

- $d_2d_3d_4$=406 is divisible by 2
- $d_3d_4d_5$=063 is divisible by 3
- $d_4d_5d_6$=635 is divisible by 5
- $d_5d_6d_7$=357 is divisible by 7
- $d_6d_7d_8$=572 is divisible by 11
- $d_7d_8d_9$=728 is divisible by 13
- $d_8d_9d_{10}$=289 is divisible by 17

Find the sum of all 0 to 9 pandigital numbers with this property.

Answer:

Confirmation Code:

Pentagon numbers

Problem 44

Pentagonal numbers are generated by the formula, $P_n=n(3n-1)/2$. The first ten pentagonal numbers are:

1, 5, 12, 22, 35, 51, 70, 92, 117, 145, ...

It can be seen that $P_4 + P_7 = 22 + 70 = 92 = P_8$. However, their difference, 70 − 22 = 48, is not pentagonal.

Find the pair of pentagonal numbers, P_j and P_k, for which their sum and difference are pentagonal and $D = |P_k - P_j|$ is minimised; what is the value of D?

Answer:

Confirmation Code:

Triangular, pentagonal, and hexagonal

Problem 45

Triangle, pentagonal, and hexagonal numbers are generated by the following formulae:

Triangle	$T_n=n(n+1)/2$	1, 3, 6, 10, 15, ...
Pentagonal	$P_n=n(3n-1)/2$	1, 5, 12, 22, 35, ...
Hexagonal	$H_n=n(2n-1)$	1, 6, 15, 28, 45, ...

It can be verified that $T_{285} = P_{165} = H_{143} = 40755$.

Find the next triangle number that is also pentagonal and hexagonal.

Answer:

Confirmation Code:

Goldbach's other conjecture

Problem 46

It was proposed by Christian Goldbach that every odd composite number can be written as the sum of a prime and twice a square.

$9 = 7 + 2\times1^2$
$15 = 7 + 2\times2^2$
$21 = 3 + 2\times3^2$
$25 = 7 + 2\times3^2$
$27 = 19 + 2\times2^2$
$33 = 31 + 2\times1^2$

It turns out that the conjecture was false.

What is the smallest odd composite that cannot be written as the sum of a prime and twice a square?

Answer:

Confirmation Code:

Distinct primes factors

Problem 47

The first two consecutive numbers to have two distinct prime factors are:

$14 = 2 \times 7$
$15 = 3 \times 5$

The first three consecutive numbers to have three distinct prime factors are:

$644 = 2^2 \times 7 \times 23$
$645 = 3 \times 5 \times 43$
$646 = 2 \times 17 \times 19$.

Find the first four consecutive integers to have four distinct prime factors. What is the first of these numbers?

Answer:

Confirmation Code:

Self powers

Problem 48

The series, $1^1 + 2^2 + 3^3 + ... + 10^{10} = 10405071317$.

Find the last ten digits of the series, $1^1 + 2^2 + 3^3 + ... + 1000^{1000}$.

Answer:

Confirmation Code:

Prime permutations

Problem 49

The arithmetic sequence, 1487, 4817, 8147, in which each of the terms increases by 3330, is unusual in two ways: (i) each of the three terms are prime, and, (ii) each of the 4-digit numbers are permutations of one another.

There are no arithmetic sequences made up of three 1-, 2-, or 3-digit primes, exhibiting this property, but there is one other 4-digit increasing sequence.

What 12-digit number do you form by concatenating the three terms in this sequence?

Answer:

Confirmation Code:

Consecutive prime sum

Problem 50

The prime 41, can be written as the sum of six consecutive primes:

$$41 = 2 + 3 + 5 + 7 + 11 + 13$$

This is the longest sum of consecutive primes that adds to a prime below one-hundred.

The longest sum of consecutive primes below one-thousand that adds to a prime, contains 21 terms, and is equal to 953.

Which prime, below one-million, can be written as the sum of the most consecutive primes?

Answer:

Confirmation Code:

Prime digit replacements

Problem 51

By replacing the 1st digit of the 2-digit number *3, it turns out that six of the nine possible values: 13, 23, 43, 53, 73, and 83, are all prime.

By replacing the 3rd and 4th digits of 56**3 with the same digit, this 5-digit number is the first example having seven primes among the ten generated numbers, yielding the family: 56003, 56113, 56333, 56443, 56663, 56773, and 56993. Consequently 56003, being the first member of this family, is the smallest prime with this property.

Find the smallest prime which, by replacing part of the number (not necessarily adjacent digits) with the same digit, is part of an eight prime value family.

Answer:

Confirmation Code:

Permuted multiples

Problem 52

It can be seen that the number, 125874, and its double, 251748, contain exactly the same digits, but in a different order.

Find the smallest positive integer, x, such that $2x$, $3x$, $4x$, $5x$, and $6x$, contain the same digits.

Answer:

Confirmation Code:

Combinatoric selections

Problem 53

There are exactly ten ways of selecting three from five, 12345:

123, 124, 125, 134, 135, 145, 234, 235, 245, and 345

In combinatorics, we use the notation, ${}^5C_3 = 10$.

In general,

$${}^nC_r = \frac{n!}{r!(n-r)!}, \text{where } r \le n,\ n! = n\times(n-1)\times\ldots\times3\times2\times1, \text{ and } 0! = 1.$$

It is not until $n = 23$, that a value exceeds one-million: ${}^{23}C_{10} = 1144066$.

How many, not necessarily distinct, values of ${}^nC_r$, for $1 \le n \le 100$, are greater than one-million?

Answer:

Confirmation Code:

Poker hands

Problem 54

In the card game poker, a hand consists of five cards and are ranked, from lowest to highest, in the following way:

- **High Card**: Highest value card.
- **One Pair**: Two cards of the same value.
- **Two Pairs**: Two different pairs.
- **Three of a Kind**: Three cards of the same value.
- **Straight**: All cards are consecutive values.
- **Flush**: All cards of the same suit.
- **Full House**: Three of a kind and a pair.
- **Four of a Kind**: Four cards of the same value.
- **Straight Flush**: All cards are consecutive values of same suit.
- **Royal Flush**: Ten, Jack, Queen, King, Ace, in same suit.

The cards are valued in the order:
2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, Ace.

If two players have the same ranked hands then the rank made up of the highest value wins; for example, a pair of eights beats a pair of fives (see example 1 below). But if two ranks tie, for example, both players have a pair of queens, then highest cards in each hand are compared (see example 4 below); if the highest cards tie then the next highest cards are compared, and so on.

Consider the following five hands dealt to two players:

Hand	**Player 1**	**Player 2**	**Winner**
1	5H 5C 6S 7S KD Pair of Fives	2C 3S 8S 8D TD Pair of Eights	Player 2
2	5D 8C 9S JS AC Highest card Ace	2C 5C 7D 8S QH Highest card Queen	Player 1
3	2D 9C AS AH AC Three Aces	3D 6D 7D TD QD Flush with Diamonds	Player 2
4	4D 6S 9H QH QC Pair of Queens Highest card Nine	3D 6D 7H QD QS Pair of Queens Highest card Seven	Player 1
5	2H 2D 4C 4D 4S Full House With Three Fours	3C 3D 3S 9S 9D Full House with Three Threes	Player 1

The file, poker.txt, contains one-thousand random hands dealt to two players. Each line of the file contains ten cards (separated by a single space): the first five are Player 1's cards and the last five are Player 2's cards. You can assume that all hands are valid (no invalid characters or repeated cards), each player's hand is in no specific order, and in each hand there is a clear winner.

How many hands does Player 1 win?

Answer:

Confirmation Code:

Lychrel numbers

Problem 55

If we take 47, reverse and add, 47 + 74 = 121, which is palindromic.

Not all numbers produce palindromes so quickly. For example,

> 349 + 943 = 1292,
> 1292 + 2921 = 4213
> 4213 + 3124 = 7337

That is, 349 took three iterations to arrive at a palindrome.

Although no one has proved it yet, it is thought that some numbers, like 196, never produce a palindrome. A number that never forms a palindrome through the reverse and add process is called a Lychrel number. Due to the theoretical nature of these numbers, and for the purpose of this problem, we shall assume that a number is Lychrel until proven otherwise. In addition you are given that for every number below ten-thousand, it will either (i) become a palindrome in less than fifty iterations, or, (ii) no one, with all the computing power that exists, has managed so far to map it to a palindrome. In fact, 10677 is the first number to be shown to require over fifty iterations before producing a palindrome: 4668731596684224866951378664 (53 iterations, 28-digits).

Surprisingly, there are palindromic numbers that are themselves Lychrel numbers; the first example is 4994.

How many Lychrel numbers are there below ten-thousand?

NOTE: Wording was modified slightly on 24 April 2007 to emphasise the theoretical nature of Lychrel numbers.

Answer:

Confirmation Code:

Powerful digit sum

Problem 56

A googol (10^{100}) is a massive number: one followed by one-hundred zeros; 100^{100} is almost unimaginably large: one followed by two-hundred zeros. Despite their size, the sum of the digits in each number is only 1.

Considering natural numbers of the form, a^b, where a, b < 100, what is the maximum digital sum?

Answer:

Confirmation Code:

Square root convergents

Problem 57

It is possible to show that the square root of two can be expressed as an infinite continued fraction.

$$\sqrt{2} = 1 + 1/(2 + 1/(2 + 1/(2 + \dots))) = 1.414213\dots$$

By expanding this for the first four iterations, we get:

1 + 1/2 = 3/2 = 1.5
1 + 1/(2 + 1/2) = 7/5 = 1.4
1 + 1/(2 + 1/(2 + 1/2)) = 17/12 = 1.41666...
1 + 1/(2 + 1/(2 + 1/(2 + 1/2))) = 41/29 = 1.41379...

The next three expansions are 99/70, 239/169, and 577/408, but the eighth expansion, 1393/985, is the first example where the number of digits in the numerator exceeds the number of digits in the denominator.

In the first one-thousand expansions, how many fractions contain a numerator with more digits than denominator?

Answer:

Confirmation Code:

Spiral primes
Problem 58

Starting with 1 and spiralling anticlockwise in the following way, a square spiral with side length 7 is formed.

```
37 36 35 34 33 32 31
38 17 16 15 14 13 30
39 18  5  4  3 12 29
40 19  6  1  2 11 28
41 20  7  8  9 10 27
42 21 22 23 24 25 26
43 44 45 46 47 48 49
```

It is interesting to note that the odd squares lie along the bottom right diagonal, but what is more interesting is that 8 out of the 13 numbers lying along both diagonals are prime; that is, a ratio of 8/13 ≈ 62%.

If one complete new layer is wrapped around the spiral above, a square spiral with side length 9 will be formed. If this process is continued, what is the side length of the square spiral for which the ratio of primes along both diagonals first falls below 10%?

Answer:

Confirmation Code:

XOR decryption
Problem 59

Each character on a computer is assigned a unique code and the preferred standard is ASCII (American Standard Code for Information Interchange). For example, uppercase A = 65, asterisk (*) = 42, and lowercase k = 107.

A modern encryption method is to take a text file, convert the bytes to ASCII, then XOR each byte with a given value, taken from a secret key. The advantage with the XOR function is that using the same encryption key on the cipher text, restores the plain text; for example, 65 XOR 42 = 107, then 107 XOR 42 = 65.

For unbreakable encryption, the key is the same length as the plain text message, and the key is made up of random bytes. The user would keep the encrypted message and the encryption key in different locations, and without both "halves", it is impossible to decrypt the message.

Unfortunately, this method is impractical for most users, so the modified method is to use a password as a key. If the password is shorter than the message, which is likely, the key is repeated cyclically throughout the message. The balance for this method is using a sufficiently long password key for security, but short enough to be memorable.

Your task has been made easy, as the encryption key consists of three lower case characters. Using cipher.txt (right click and 'Save Link/Target As...'), a file containing the encrypted ASCII codes, and the knowledge that the plain text must contain common English words, decrypt the message and find the sum of the ASCII values in the original text.

Answer:

Confirmation Code:

Prime pair sets

Problem 60

The primes 3, 7, 109, and 673, are quite remarkable. By taking any two primes and concatenating them in any order the result will always be prime. For example, taking 7 and 109, both 7109 and 1097 are prime. The sum of these four primes, 792, represents the lowest sum for a set of four primes with this property.

Find the lowest sum for a set of five primes for which any two primes concatenate to produce another prime.

Answer:

Confirmation Code:

Cyclical figurate numbers

Problem 61

Triangle, square, pentagonal, hexagonal, heptagonal, and octagonal numbers are all figurate (polygonal) numbers and are generated by the following formulae:

Triangle	$P_{3,n}=n(n+1)/2$	1, 3, 6, 10, 15, ...
Square	$P_{4,n}=n^2$	1, 4, 9, 16, 25, ...
Pentagonal	$P_{5,n}=n(3n-1)/2$	1, 5, 12, 22, 35, ...
Hexagonal	$P_{6,n}=n(2n-1)$	1, 6, 15, 28, 45, ...
Heptagonal	$P_{7,n}=n(5n-3)/2$	1, 7, 18, 34, 55, ...
Octagonal	$P_{8,n}=n(3n-2)$	1, 8, 21, 40, 65, ...

The ordered set of three 4-digit numbers: 8128, 2882, 8281, has three interesting properties.

1. The set is cyclic, in that the last two digits of each number is the first two digits of the next number (including the last number with the first).
2. Each polygonal type: triangle ($P_{3,127}$=8128), square ($P_{4,91}$=8281), and pentagonal ($P_{5,44}$=2882), is represented by a different number in the set.
3. This is the only set of 4-digit numbers with this property.

Find the sum of the only ordered set of six cyclic 4-digit numbers for which each polygonal type: triangle, square, pentagonal, hexagonal, heptagonal, and octagonal, is represented by a different number in the set.

Answer:

Confirmation Code:

Cubic permutations

Problem 62

The cube, 41063625 (345^3), can be permuted to produce two other cubes: 56623104 (384^3) and 66430125 (405^3). In fact, 41063625 is the smallest cube which has exactly three permutations of its digits which are also cube.

Find the smallest cube for which exactly five permutations of its digits are cube.

Answer:

Confirmation Code:

Powerful digit counts

Problem 63

The 5-digit number, $16807=7^5$, is also a fifth power. Similarly, the 9-digit number, $134217728=8^9$, is a ninth power.

How many *n*-digit positive integers exist which are also an *n*th power?

Answer:

Confirmation Code:

Odd period square roots

Problem 64

All square roots are periodic when written as continued fractions and can be written in the form:

$$\sqrt{N} = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \ldots}}}$$

For example, let us consider $\sqrt{23}$:

$$\sqrt{23} = 4 + \sqrt{23} - 4 = 4 + \cfrac{1}{\cfrac{1}{\boxed{\sqrt{23}-4}}} = 4 + \cfrac{1}{1 + \cfrac{\sqrt{23} - 3}{7}}$$

If we continue we would get the following expansion:

$$\sqrt{23} = 4 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{}}}}$$

$8 + \ldots$

The process can be summarised as follows:

$$a_0 = 4, \quad \frac{1}{\boxed{\sqrt{23}-4}} = \frac{\sqrt{23}+4}{7} = 1 + \frac{\sqrt{23}-3}{7}$$
$$a_1 = 1, \quad \frac{7}{\boxed{\sqrt{23}-3}} = \frac{7(\sqrt{23}+3)}{14} = 3 + \frac{\sqrt{23}-3}{2}$$
$$a_2 = 3, \quad \frac{2}{\boxed{\sqrt{23}-3}} = \frac{2(\sqrt{23}+3)}{14} = 1 + \frac{\sqrt{23}-4}{7}$$
$$a_3 = 1, \quad \frac{7}{\boxed{\sqrt{23}-4}} = \frac{7(\sqrt{23}+4)}{7} = 8 + \sqrt{23}-4$$
$$a_4 = 8, \quad \frac{1}{\boxed{\sqrt{23}-4}} = \frac{\sqrt{23}+4}{7} = 1 + \frac{\sqrt{23}-3}{7}$$
$$a_5 = 1, \quad \frac{7}{\boxed{\sqrt{23}-3}} = \frac{7(\sqrt{23}+3)}{14} = 3 + \frac{\sqrt{23}-3}{2}$$
$$a_6 = 3, \quad \frac{2}{\boxed{\sqrt{23}-3}} = \frac{2(\sqrt{23}+3)}{14} = 1 + \frac{\sqrt{23}-4}{7}$$
$$a_7 = 1, \quad \frac{7}{\boxed{\sqrt{23}-4}} = \frac{7(\sqrt{23}+4)}{7} = 8 + \sqrt{23}-4$$

It can be seen that the sequence is repeating. For conciseness, we use the notation $\sqrt{23} = [4;(1,3,1,8)]$, to indicate that the block (1,3,1,8) repeats indefinitely.

The first ten continued fraction representations of (irrational) square roots are:

$\sqrt{2}=[1;(2)]$, period=1
$\sqrt{3}=[1;(1,2)]$, period=2
$\sqrt{5}=[2;(4)]$, period=1
$\sqrt{6}=[2;(2,4)]$, period=2
$\sqrt{7}=[2;(1,1,1,4)]$, period=4
$\sqrt{8}=[2;(1,4)]$, period=2
$\sqrt{10}=[3;(6)]$, period=1
$\sqrt{11}=[3;(3,6)]$, period=2
$\sqrt{12}= [3;(2,6)]$, period=2
$\sqrt{13}=[3;(1,1,1,1,6)]$, period=5

Exactly four continued fractions, for $N \leq 13$, have an odd period.

How many continued fractions for $N \leq 10000$ have an odd period?

Answer:

Confirmation Code:

Convergents of e

Problem 65

The square root of 2 can be written as an infinite continued fraction.

$$\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \dots}}}}$$

The infinite continued fraction can be written, $\sqrt{2} = [1;(2)]$, (2) indicates that 2 repeats *ad infinitum*. In a similar way, $\sqrt{23} = [4;(1,3,1,8)]$.

It turns out that the sequence of partial values of continued fractions for square roots provide the best rational approximations. Let us consider the convergents for $\sqrt{2}$.

$$1 + \frac{1}{2} = 3/2$$

$$1 + \cfrac{1}{2 + \cfrac{1}{2}} = 7/5$$

$$1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2}}} = 17/12$$

$$1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2}}}} = 41/29$$

Hence the sequence of the first ten convergents for $\sqrt{2}$ are:

1, 3/2, 7/5, 17/12, 41/29, 99/70, 239/169, 577/408, 1393/985, 3363/2378, ...

What is most surprising is that the important mathematical constant,
$e = [2; 1,2,1, 1,4,1, 1,6,1 , \dots , 1,2k,1, \dots]$.

The first ten terms in the sequence of convergents for *e* are:

2, 3, 8/3, 11/4, 19/7, 87/32, 106/39, 193/71, 1264/465, 1457/536, ...

The sum of digits in the numerator of the 10^{th} convergent is 1+4+5+7=17.

Find the sum of digits in the numerator of the 100^{th} convergent of the continued fraction for *e*.

Answer:

Confirmation Code:

Diophantine equation

Problem 66

Consider quadratic Diophantine equations of the form:

$$x^2 - Dy^2 = 1$$

For example, when D=13, the minimal solution in *x* is $649^2 - 13 \times 180^2 = 1$.

It can be assumed that there are no solutions in positive integers when D is square.

By finding minimal solutions in *x* for D = {2, 3, 5, 6, 7}, we obtain the following:

$3^2 - 2 \times 2^2 = 1$
$2^2 - 3 \times 1^2 = 1$
$\mathbf{9}^2 - 5 \times 4^2 = 1$
$5^2 - 6 \times 2^2 = 1$
$8^2 - 7 \times 3^2 = 1$

Hence, by considering minimal solutions in *x* for D ≤ 7, the largest *x* is obtained when D=5.

Find the value of D ≤ 1000 in minimal solutions of *x* for which the largest value of *x* is obtained.

Answer:

Confirmation Code:

Maximum path sum II
Problem 67

By starting at the top of the triangle below and moving to adjacent numbers on the row below, the maximum total from top to bottom is 23.

3
7 4
2 **4** 6
8 5 **9** 3

That is, 3 + 7 + 4 + 9 = 23.

Find the maximum total from top to bottom in triangle.txt (right click and 'Save Link/Target As...'), a 15K text file containing a triangle with one-hundred rows.

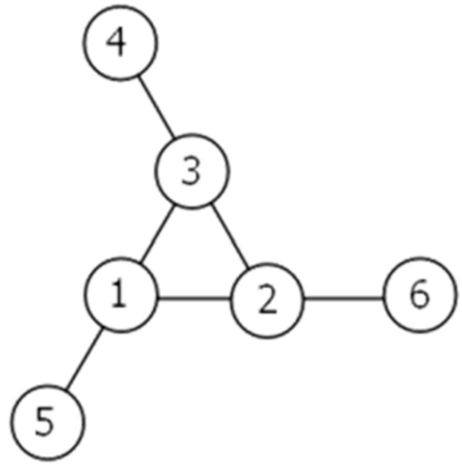
NOTE: This is a much more difficult version of Problem 18. It is not possible to try every route to solve this problem, as there are 2^{99} altogether! If you could check one trillion (10^{12}) routes every second it would take over twenty billion years to check them all. There is an efficient algorithm to solve it. ;o)

Answer:

Confirmation Code:

Magic 5-gon ring
Problem 68

Consider the following "magic" 3-gon ring, filled with the numbers 1 to 6, and each line adding to nine.



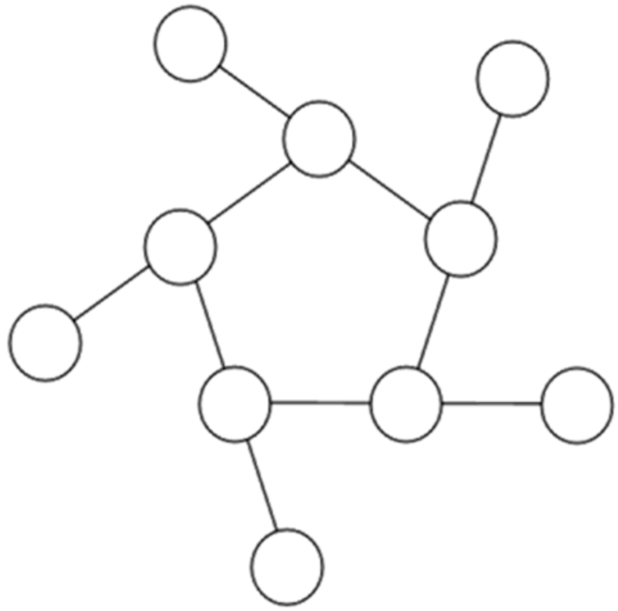
Working **clockwise**, and starting from the group of three with the numerically lowest external node (4,3,2 in this example), each solution can be described uniquely. For example, the above solution can be described by the set: 4,3,2; 6,2,1; 5,1,3.

It is possible to complete the ring with four different totals: 9, 10, 11, and 12. There are eight solutions in total.

Total	**Solution Set**
9	4,2,3; 5,3,1; 6,1,2
9	4,3,2; 6,2,1; 5,1,3
10	2,3,5; 4,5,1; 6,1,3
10	2,5,3; 6,3,1; 4,1,5
11	1,4,6; 3,6,2; 5,2,4
11	1,6,4; 5,4,2; 3,2,6
12	1,5,6; 2,6,4; 3,4,5
12	1,6,5; 3,5,4; 2,4,6

By concatenating each group it is possible to form 9-digit strings; the maximum string for a 3-gon ring is 432621513.

Using the numbers 1 to 10, and depending on arrangements, it is possible to form 16- and 17-digit strings. What is the maximum **16-digit** string for a "magic" 5-gon ring?



Answer:

Confirmation Code:

Totient maximum

Problem 69

Euler's Totient function, φ(*n*) [sometimes called the phi function], is used to determine the number of numbers less than *n* which are relatively prime to *n*. For example, as 1, 2, 4, 5, 7, and 8, are all less than nine and relatively prime to nine, φ(9)=6.

n	**Relatively Prime**	**φ(*n*)**	***n*/φ(*n*)**
2	1	1	2
3	1,2	2	1.5
4	1,3	2	2
5	1,2,3,4	4	1.25
6	1,5	2	3
7	1,2,3,4,5,6	6	1.1666...
8	1,3,5,7	4	2
9	1,2,4,5,7,8	6	1.5
10	1,3,7,9	4	2.5

It can be seen that n=6 produces a maximum $n/\varphi(n)$ for $n \leq 10$.

Find the value of $n \leq 1,000,000$ for which $n/\varphi(n)$ is a maximum.

Answer:

Confirmation Code:

Totient permutation

Problem 70

Euler's Totient function, $\varphi(n)$ [sometimes called the phi function], is used to determine the number of positive numbers less than or equal to n which are relatively prime to n. For example, as 1, 2, 4, 5, 7, and 8, are all less than nine and relatively prime to nine, $\varphi(9)=6$.
The number 1 is considered to be relatively prime to every positive number, so $\varphi(1)=1$.

Interestingly, $\varphi(87109)=79180$, and it can be seen that 87109 is a permutation of 79180.

Find the value of n, $1 < n < 10^7$, for which $\varphi(n)$ is a permutation of n and the ratio $n/\varphi(n)$ produces a minimum.

Answer:

Confirmation Code:

Ordered fractions

Problem 71

Consider the fraction, n/d, where n and d are positive integers. If $n<d$ and HCF(n,d)=1, it is called a reduced proper fraction.

If we list the set of reduced proper fractions for $d \leq 8$ in ascending order of size, we get:

1/8, 1/7, 1/6, 1/5, 1/4, 2/7, 1/3, 3/8, **2/5**, 3/7, 1/2, 4/7, 3/5, 5/8, 2/3, 5/7, 3/4, 4/5, 5/6, 6/7, 7/8

It can be seen that 2/5 is the fraction immediately to the left of 3/7.

By listing the set of reduced proper fractions for $d \leq 1{,}000{,}000$ in ascending order of size, find the numerator of the fraction immediately to the left of 3/7.

Answer:

Confirmation Code:

Counting fractions

Problem 72

Consider the fraction, n/d, where n and d are positive integers. If $n<d$ and HCF(n,d)=1, it is called a reduced proper fraction.

If we list the set of reduced proper fractions for $d \leq 8$ in ascending order of size, we get:

1/8, 1/7, 1/6, 1/5, 1/4, 2/7, 1/3, 3/8, 2/5, 3/7, 1/2, 4/7, 3/5, 5/8, 2/3, 5/7, 3/4, 4/5, 5/6, 6/7, 7/8

It can be seen that there are 21 elements in this set.

How many elements would be contained in the set of reduced proper fractions for $d \leq$ 1,000,000?

Answer:

Confirmation Code:

Counting fractions in a range

Problem 73

Consider the fraction, n/d, where n and d are positive integers. If $n<d$ and HCF(n,d)=1, it is called a reduced proper fraction.

If we list the set of reduced proper fractions for $d \leq 8$ in ascending order of size, we get:

1/8, 1/7, 1/6, 1/5, 1/4, 2/7, 1/3, **3/8, 2/5, 3/7**, 1/2, 4/7, 3/5, 5/8, 2/3, 5/7, 3/4, 4/5, 5/6, 6/7, 7/8

It can be seen that there are 3 fractions between 1/3 and 1/2.

How many fractions lie between 1/3 and 1/2 in the sorted set of reduced proper fractions for $d \leq 12{,}000$?

Answer:

Confirmation Code:

Digit factorial chains

Problem 74

The number 145 is well known for the property that the sum of the factorial of its digits is equal to 145:

1! + 4! + 5! = 1 + 24 + 120 = 145

Perhaps less well known is 169, in that it produces the longest chain of numbers that link back to 169; it turns out that there are only three such loops that exist:

169 → 363601 → 1454 → 169
871 → 45361 → 871
872 → 45362 → 872

It is not difficult to prove that EVERY starting number will eventually get stuck in a loop. For example,

69 → 363600 → 1454 → 169 → 363601 (→ 1454)
78 → 45360 → 871 → 45361 (→ 871)
540 → 145 (→ 145)

Starting with 69 produces a chain of five non-repeating terms, but the longest non-repeating chain with a starting number below one million is sixty terms.

How many chains, with a starting number below one million, contain exactly sixty non-repeating terms?

Answer:

Confirmation Code:

Singular integer right triangles

Problem 75

It turns out that 12 cm is the smallest length of wire that can be bent to form an integer sided right angle triangle in exactly one way, but there are many more examples.

12 cm: (3,4,5)
24 cm: (6,8,10)
30 cm: (5,12,13)
36 cm: (9,12,15)
40 cm: (8,15,17)
48 cm: (12,16,20)

In contrast, some lengths of wire, like 20 cm, cannot be bent to form an integer sided right angle triangle, and other lengths allow more than one solution to be found; for example, using 120 cm it is possible to form exactly three different integer sided right angle triangles.

120 cm: (30,40,50), (20,48,52), (24,45,51)

Given that L is the length of the wire, for how many values of L ≤ 1,500,000 can exactly one integer sided right angle triangle be formed?

Answer:

Confirmation Code:

Counting summations

Problem 76

It is possible to write five as a sum in exactly six different ways:

4 + 1
3 + 2
3 + 1 + 1
2 + 2 + 1
2 + 1 + 1 + 1
1 + 1 + 1 + 1 + 1

How many different ways can one hundred be written as a sum of at least two positive integers?

Answer:

Confirmation Code:

Prime summations

Problem 77

It is possible to write ten as the sum of primes in exactly five different ways:

7 + 3
5 + 5
5 + 3 + 2
3 + 3 + 2 + 2
2 + 2 + 2 + 2 + 2

What is the first value which can be written as the sum of primes in over five thousand different ways?

Answer:

Confirmation Code:

Coin partitions

Problem 78

Let p(*n*) represent the number of different ways in which *n* coins can be separated into piles. For example, five coins can be separated into piles in exactly seven different ways, so p(5)=7.

```
OOOOO

OOOO   O

OOO   OO

OOO   O   O

OO   OO   O

OO   O   O   O

O   O   O   O   O
```

Find the least value of *n* for which p(*n*) is divisible by one million.

Answer:

Confirmation Code:

Passcode derivation

Problem 79

A common security method used for online banking is to ask the user for three random characters from a passcode. For example, if the passcode was 531278, they may ask for the 2nd, 3rd, and 5th characters; the expected reply would be: 317.

The text file, keylog.txt, contains fifty successful login attempts.

Given that the three characters are always asked for in order, analyse the file so as to determine the shortest possible secret passcode of unknown length.

Answer:

Confirmation Code:

Square root digital expansion

Problem 80

It is well known that if the square root of a natural number is not an integer, then it is irrational. The decimal expansion of such square roots is infinite without any repeating pattern at all.

The square root of two is 1.41421356237309504880..., and the digital sum of the first one hundred decimal digits is 475.

For the first one hundred natural numbers, find the total of the digital sums of the first one hundred decimal digits for all the irrational square roots.

Answer:

Confirmation Code: