TDA Technical Guide

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Introduction

In this guide, the model fitting of TDA is clarified. TDA (Traffic Dynamics Analyser) is a tool that extracts data from the PeMS database (https://dot.ca.gov/programs/traffic-operations/mpr/pems-source), processes this data and generates graphs that visualize the macroscopic traffic dynamics (traffic density, vehicle speed and traffic flow). It also fits the data to established models.

The code has been created and used for the following article (Rohaert et al., 2022):

> Rohaert, A., Kuligowski, E., Ardinge, A., Wahlqvist, J., Gwynne, S., Kimball, A., Benichou, N., & Ronchi, E. (2022). Traffic dynamics during the 2019 Kincade wildfire evacuation. [Submitted for Peer-Review to an International Journal.].

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This guide follows the following structure:

1. Nomenclature

Quantities

k	density (veh/km/lane)
q	flow (veh/h/lane)
v	speed (km/h)
a, b, c	parameters of the model by Van Aerde and Rakha
c	kinematic wave speed at jam density in the model by del Castillo & Benítez (km/h)

Subscripts

$\square_c$	quantity $\square$ at peak capacity, critical quantity
$\square_f$	quantity $\square$ in entirely sparse traffic, free-flow situation
$\square_j$	quantity $\square$ in entirely jammed traffic, total congestion

Units

h	hours
km	kilometres
lane	number of lanes
m	metre
mi	miles
pce	passenger car equivalents
veh	number of vehicles

2. Speed-density and flow-density models

TDA includes a set of well-known traffic models that can be fitted on the data. In this section, the models are expressed as a relationship between traffic density and vehicle speed (see Equation 2 till 7). The corresponding traffic flow can be obtained by multiplying those two quantities (see Equation 1):

$$q = k\,v \tag{1}$$

Equation 2 presents the parabolic model by Greenshield (Greenshield, 1935):

$$v = v_f\left(1 - \frac{k}{k_j}\right) \tag{2}$$

Equation 3 presents the exponential model by Underwood (Underwood, 1961)

$$v = v_f e^{\left(-\frac{k}{k_c}\right)} \tag{3}$$

Equation 4 presents the North-Western model by Drake et al (Drake et al., 1965) :

$$v = v_f e^{\left(-\frac{1}{2}\left(\frac{k}{k_c}\right)^2\right)} \tag{4}$$

Equation 5 presents the bi-linear model by Daganzo (Daganzo, 1994):

$$v = \begin{cases} v_f, & 0 \le k \le k_c \\ q_c\left(\dfrac{k_j - k}{k_j - k_c}\right)\dfrac{1}{k}, & k_c \le k \le k_j \end{cases} \tag{5}$$

Equation 6 presents the model by Van Aerde and Rakha (Van Aerde & Rakha, 1995; Wu & Rakha, 2009):

$$k = \frac{1}{a + \dfrac{b}{v_f - v} + c\,v}\ with \left\{\begin{aligned} a &= v_f\left(2v_c - v_f\right)/k_j v_c^2 \\ b &= v_f\left(v_c - v_f\right)^2/k_j v_c^2 \\ c &= 1/(v_c k_c) - v_f/k_j v_c^2 \end{aligned}\right\} \tag{6}$$

Equation 7 presents the model by del Castillo & Benítez (Del Castillo & Benítez, 1995):

$$v = v_f\left(1 - e^{\left(\frac{c}{vf}\left(1 - \frac{k_j}{k}\right)\right)}\right) \tag{7}$$

In all those models, v, k and $q = k\,v$ are the variables and all other quantities are constant parameters (which differ for each road type).

3. Model fitting techniques

TDA fits the models by minimizing the root mean squared error (RMSE) of the speed predictions, given the density observations and corresponding speed observations. The minimization of the RMSE towards the model parameters is executed by means of Powell's conjugate direction (gradient-free) method (Powell, 1964). To do so, the minimize function of the SciPy Optimise package is used (*SciPy User Guide v1.9.1*, 2022).

4. Reliable, accurate and balanced data

Naturally, the result of the curve fitting depends on all selected data points. It is the user's responsibility to provide reliable data point. More information about the reliability of the data is provided by Chen (2003, Chapter 5). (Rohaert et al., 2022) describes a process of data selection, as well as the accuracy of the data.

Moreover, the fitting technique treats all datapoints with the same importance. This means that the model fitting is more accurate where more data is present. In the example below, 92% of all datapoints have a traffic density below 20 veh/km/lane. Consequently, the Greenshield model is not able to provide accurate prediction for higher densities. However, due to the higher flexibility of the Van Aerde & Rakha model, it follows the trend of the data more precisely, even though the data is poorly balanced.

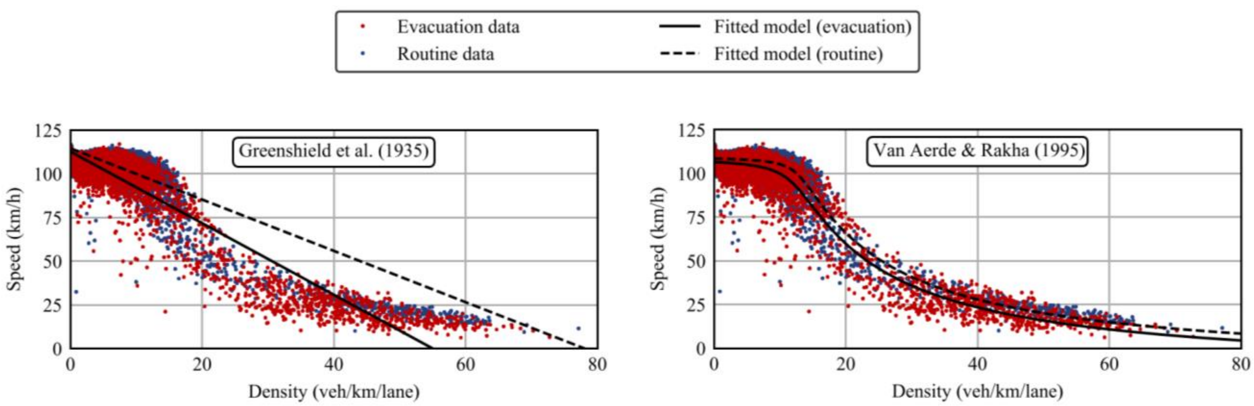

Figure 1: Example of the limitations of fitting models on poorly balanced datasets (Rohaert et al., 2022).

5. References

Chen, C. (2003). *Freeway Performance Measurement System (PeMS)*. Institute of Transportation Studies, University of California.

Daganzo, C. F. (1994). The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory. *Transportation Research Part B: Methodological*, *28*(4), 269–287. https://doi.org/10.1016/0191-2615(94)90002-7

Del Castillo, J. M., & Benítez, F. G. (1995). On the functional form of the speed-density relationship—Part I: General theory. *Transportation Research Part B: Methodological*, *29*(5), 373–389. https://doi.org/10.1016/0191-2615(95)00008-2

Drake, J., Schofer, J., & May, A. (1965). A statistical analysis of speed-density hypotheses. *Traffic Flow and Transportation*, 53–87.

Greenshield, B. D. (1935). A study of Traffic Capacity. *Proceedings of the Fourteenth Annual Meeting of the Highway Research Board Held at Washington, D.C. December 6-7, 1934. Part I*, *14*, 448–477.

Powell, M. J. D. (1964). An efficient method for finding the minimum of a function of several variables without calculating derivatives. *The Computer Journal*, *7*(2), 155–162. https://doi.org/10.1093/comjnl/7.2.155

Rohaert, A., Kuligowski, E., Ardinge, A., Wahlqvist, J., Gwynne, S., Kimball, A., Benichou, N., & Ronchi, E. (2022). Traffic dynamics during the 2019 Kincade wildfire evacuation. *[Submitted for Peer-Review to an International Journal.]*.

SciPy User Guide v1.9.1. (2022). https://docs.scipy.org/doc/scipy/tutorial/index.html

Underwood, R. T. (1961). Speed, volume and density relationships. *Quality and Theory of Traffic Flow. A Symposium, Yale University Bureau of Highway Traffic*, 141–188.

Van Aerde, M., & Rakha, H. (1995). *Multivariate calibration of single regime speed-flow-density relationships*. 334–341. https://doi.org/10.1109/VNIS.1995.518858

Wu, N., & Rakha, H. (2009). Derivation of Van Aerde Traffic Stream Model from Tandem-Queuing Theory. *Transportation Research Record: Journal of the Transportation Research Board*, *2124*(1), 18–27. https://doi.org/10.3141/2124-02