Tutorial - 4

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Q1) $T(n) = 3T(n/2) + n^2$

$\rightarrow$ $T(n) = aT(n/b) + f(n^2)$

$a \geq 1, b \geq 1$

On compairing

$a = 3, b = 2, f(n) = n^2$

Now $c = \log_b a = \log_2 3 = 1.584$

$n^2 = n^{1.584} < n^2$

$\therefore f(n) > n^c$

$\therefore T(n) = O(n^2)$

Q2) $T(n) = 4T(n/2) + n^2$

$\rightarrow$ $a \geq 1, b > 1$

$a = 4, b = 2, f(n) = n^2$

$c = \log_2 4 = 2$

$n^c = n^2 = f(n) = n^2$

$\therefore T(n) = O(n^2 \log_2 n)$

Q3) $T(n) = T(n/2) + 2^n$

$a = 1$

$b = 2$

$f(n) = 2^n$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$T(n) = O(2^n)$

Q4) $T(n) = 2^n T(n/2) + n^n$

$\rightarrow a = 2^n$

$b = 2 \quad f(n) = n^2$

$c = \log_b a = \log_2 2^n$

$= n$

$n^c \Rightarrow n^n$

$f(n) = n^c$

$f(n) = \theta(n^2 \log_2 n)$

Q5) $T(n) = 16T(n/4) + n$

$\rightarrow a = 16, b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 (4)^2 = 2 \log_4 4$

$= 2$

$n^c \Rightarrow n^2$

$f(n) < n^c$

$\therefore T(n) = \theta(n^2)$

Q6) $T(n) = 2T(n/2) + n \log n$

$a = 2, b = 2$

$f(n) = n \log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$n \log n > n$

$f(n) > n^c$

$T(n) = O(n \log n)$

$T(n) = 2T(n/2) + n\log n$

$a = 2, b = 2, f(n) = n/\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$\therefore \frac{n}{\log n} < n$

$\therefore f(n) < n^c$

$\therefore T(n) = O(n)$

Q8) $T(n) = 2T(n/4) + n^{0.51}$

$\rightarrow a = 2, b = 4, f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$\therefore n^c = n^{0.5}$

$n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\therefore T(n) = \Theta(n^{0.51})$

Q9) $T(n) = 0.5T(n/2) + 1/n$

$\rightarrow a = 0.5, b = 2$

$a \geq 1$ but here a is 0.5

so we cannot apply Master's

Theorem

Q10 $T(n) = 16T(n/4) + n!$

$\rightarrow a = 16, b = 4, f(n) = n!$

$\therefore c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

As $n! > n^2$

$\therefore T(n) = \Theta(n!)$

Q11) $4T(n/2) + \log n$

$\rightarrow a = 4, b = 2, f(n) = \log n$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) = \log n$

$\therefore \log n < n^2$

$f(n) < n^c$

$T(n) = O(n^c)$

$= O(n^2)$

Q12 $T(n) = sqrt(n) + T(n/2) + \log n$

$\rightarrow a\sqrt{n}, b=2$

$c = \log_b a \neq \log_2 \sqrt{n} = \frac{1}{2}\log_2 n$

$\therefore \frac{1}{2}\log_2 n < \log(n)$

$\therefore f(n) > n^c$

$T(n) = \Theta(f(n))$

$= \Theta(\log(n))$

Q13) $T(n) = 3T(n/2) + n$

$\rightarrow a = 3, b = 2 \quad f(n) = n$

$c = \log_b a = \log_2 3 = 1.5849$

$n^c = n^{1.5484}$

$n < n^{1.5849}$

$\Rightarrow f(n) < n^c$

$T(n) = \Theta(n^{1.5849})$

Q14) $T(n) = 3T(n/3) + sqrt(n)$

$\rightarrow a = 3, b = 3$

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

As $sqrt(n) < n$

$f(n) < n^c$

$T(n) = O(n)$

Q15) $T(n) = 4T(n/2) + n$

$a = 4, b = 2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$n < n^2$ (for any constant)

$f(n) < n^c$

$T(n) = O(n^2)$