Merge Sort algorithm

relational formula will be

$$T(n) = 2T(n/2) + n - 1$$

ignore -1,

So $T(n) = 2T(n/2) + n$ — ①

Putting $n = n/2$

$$T(n/2) = 2T(n/2^2) + n/2$$ — ②

Put 2 eqn in 1 eqn

$$T(n) = 2[2T(n/2^2) + n/2] + n$$

$$= 2^2 T(n/2^2) + 2n/2 + n$$

$$T(n) = 2^2 T(n/2^2) + 2n$$ — ③

Putting $n = n/2^2$ in eqn 1

$$T(n/2^2) = 2T(n/2^2) + n/2^2$$ — ④

Putting 4 eqn in 3 eqn

$$T(n) = 2^2[2T(n/2^2) + n/2^2] + 2n$$

$$T(n) = 2^3 T(n/2^3) + 3n$$

From eq¹, eq³, eq⁵ ... we get

$$T(n) = 2^i T(n/2^i) + n$$ ---- ⑥

From stopping Condition :

$\frac{n}{2^i} = 1$ And $T(n/2^i) = 0$

$n = 2^i$

Apply cog both sides

$\log n = \log 2^i$

$\log n = i \log 2$

$\frac{\log n}{\log 2} = i$

$\log_2^n = i$

From 6 eqn

$T(n) = 2^i T(n/2^i) + n$

$= 2^i \times 0 + \log_2^n \cdot n$

$= T(n) = n \log n.$