EDAN95

Applied Machine Learning

Lecture 4: Linear Classification and Feed-Forward Networks

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Content

Overview and practice of the major neural network architectures:

- Feed forward
- Convolutional
- Embeddings
- Recurrent
- LSTM

We will use:

- keras, https://keras.io/, an powerful API to design and train network, and
- scikit-learn, http://scikit-learn.org/stable/, a general purpose machine-learning toolkit.

The Book

Machine Learning with Python
by François Chollet (creator of Keras)
All the code examples, as well as
explanations are available from
github:

https://github.com/fchollet/deep-learning-with-python-notebook

with Pvthon

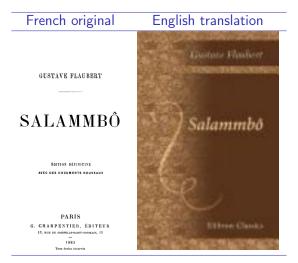
Chollet should publish a second edition soon: https://www.manning.com/books/deep-learning-with-python-second-edition

Some Definitions

- Machine learning always starts with data sets: a collection of objects or observations.
- Machine-learning algorithms can be classified along two main lines: supervised and unsupervised classification.
- Supervised algorithms need a training set, where the objects are described in terms of attributes and belong to a known class or have a known output.
- The performance of the resulting classifier is measured against a test set.
- \bullet We can also use *N*-fold cross validation, where the test set is selected randomly from the training set *N* times, usually 10.
- Unsupervised algorithms consider objects, where no class is provided.
- Unsupervised algorithms learn regularities in data sets.

A Data Set: Salammbô

A corpus is a collection – a body – of texts.



Supervised Learning

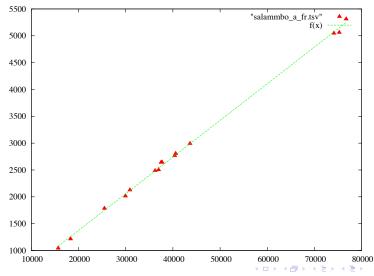
Letter counts from Salammbô

Chapter	French		English	
	# characters	# A	# characters	# A
Chapter 1	36,961	2,503	35,680	2,217
Chapter 2	43,621	2,992	42,514	2,761
Chapter 3	15,694	1,042	15,162	990
Chapter 4	36,231	2,487	35,298	2,274
Chapter 5	29,945	2,014	29,800	1,865
Chapter 6	40,588	2,805	40,255	2,606
Chapter 7	75,255	5,062	74,532	4,805
Chapter 8	37,709	2,643	37,464	2,396
Chapter 9	30,899	2,126	31,030	1,993
Chapter 10	25,486	1,784	24,843	1,627
Chapter 11	37,497	2,641	36,172	2,375
Chapter 12	40,398	2,766	39,552	2,560
Chapter 13	74,105	5,047	72,545	4,597
Chapter 14	76,725	5,312	75,352	4,871
Chapter 15	18,317	1,215	18,031	1,119

Data set: https://github.com/pnugues/ilppp/tree/master/programs/ch04/salammbo

Supervised Learning: Regression

Letter count from Salammbô in French



Models

We will assume that data sets are governed by functions or models. For instance given the set:

$$\{(x_i,y_i)|0< i\leqslant N\},$$

there exists a function such that:

$$f(x_i) = y_i$$
.

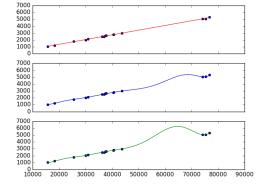
Supervised machine learning algorithms will produce hypothesized functions or models fitting the data.

Notations

- x, the vector representing an observation (or sample, or example);
 in Salammbô, an observation is the number of letters in a chapter. We have 15 observations;
- y, the observed response (or target, or output);
 in Salammbô, the number of As in a chapter. We have 15 responses;
- \hat{y} , the value predicted by the model;
- w, the weights or parameters of the model, so that $\mathbf{w} \cdot \mathbf{x} = \hat{\mathbf{y}}$
- X, the matrix of all the observations
- y, the vector of all the responses

Selecting a Model

Often, multiple models can fit a data set: Three polynomials of degree: 1, a straight line, 8, and 9 to fit the *Salammbô* dataset.



A general rule in machine learning is to prefer the simplest hypotheses, here the lower polynomial degrees. Otherwise, the model can **overfit** the data. In our case, the optimal model w has two parameters: (w_0, w_1) .

Process and Algorithms

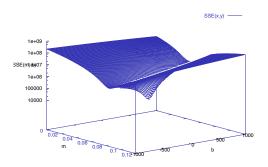
What are the optimal values of w?

The model should minimize the difference between the predicted values and the observed values: This is called the **loss**

For Salammbô, the loss is the mean of the squared errors (MSE):

$$\frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2$$

Visualizing the Loss



To minimize this loss, the solver applies a *stochastic gradient descent* (SGD), or a variant of it, that finds a sequence of model parameters that will reduce the loss.

Keras provides a set of optimizers: sgd, rmsprop, adam, nadam, etc

Minimizing the Loss

The loss function is convex and has a unique minimum.

The loss reaches a minimum when the partial derivatives are zero:

$$\frac{\partial Loss}{\partial m} = \sum_{i=1}^{q} \frac{\partial}{\partial m} (y_i - (mx_i + b))^2 = -2 \sum_{i=1}^{q} x_i (y_i - (mx_i + b)) = 0$$

$$\frac{\partial Loss}{\partial b} = \sum_{i=1}^{q} \frac{\partial}{\partial b} (y_i - (mx_i + b))^2 = -2 \sum_{i=1}^{q} (y_i - (mx_i + b)) = 0$$

The Gradient Descent

The gradient descent is a numerical method to find the minimum of $f(x_0, x_1, x_2, ..., x_n) = y$, when there is no analytical solution. Let us denote $x = (x_0, x_1, x_2, ..., x_n)$

We derive successive approximations to find the minimum of f:

$$f(x_1) > f(x_2) > ... > f(x_k) > f(x_{k+1}) > .. > min$$

Points in the neighborhood of x are defined by x+v with ||v|| small Given x, find v subject to f(x) > f(x+v)

The Gradient Descent (Cauchy, 1847)

Using a Taylor expansion: $f(x+v) = f(x) + v \cdot \nabla f(x) + \dots$

The gradient is a direction vector corresponding to the steepest slope:

$$\nabla f(x_0, x_1, x_2, ..., x_n) = \left(\frac{\partial f}{\partial x_0}, \frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, ..., \frac{\partial f}{\partial x_n}\right).$$

f(x+v) reaches a minimum or a maximum when v and $\nabla f(x)$ are colinear:

- Steepest ascent: $v = \alpha \nabla f(x)$,
- Steepest descent: $v = -\alpha \nabla f(x)$,

where $\alpha > 0$.

We have then: $f(x - \alpha \nabla f(x)) \approx f(x) - \alpha ||\nabla f(x)||^2$.

The inequality:

$$f(x) > f(x - \alpha \nabla f(x))$$

enables us to move one step down to the minimum.

We then use the iteration:

$$\mathsf{x}_{k+1} = \mathsf{x}_k - \alpha_k \nabla f(\mathsf{x}_k).$$



Computing the Gradient

Modern machine-learning platform use an automatic differentiation algorithm.

- For a description of it in Tensorflow, see https: //www.tensorflow.org/tutorials/customization/autodiff
- For a more elaborate description: http://www.cs.toronto.edu/ ~rgrosse/courses/csc421_2019/slides/lec06.pdf
- For a description of the tf.gradients class: https://www.tensorflow.org/api_docs/python/tf/gradients

Keras provides a set of optimizers: sgd, rmsprop, adam, nadam, etc

The Matrices

	Γ1	36961		[2533.22]		2503		913.26	
	1	43621		2988.11		2992	;se =	15.14	
	1	15694		1080.65	;y=	1042		1493.86	
	1	36231		2483.36		2487		13.25	
	1	29945	$; w = \begin{bmatrix} 8.7253 \\ 0.0683 \end{bmatrix}; \hat{y} =$	2054.02		2014		1601.31	
	1	40588		2780.95		2805		578.40	
	1	75255		5148.76		5062		7527.51	
X =	1	37709		2584.31		2643		3444.53	
	1	30899	2119.18		2126		46.57		
	1	25486		1749.46		1784		1193.04	
	1	37497		2569.83		2641		5065.18	
	1	40398	105	2767.97		2766		38920	
	1	74105		5070.21		5047		538.909	
	1	76725		5249.16		5312		3948.29	
	1	18317		1259.81		1215		2007.53	

Code Example

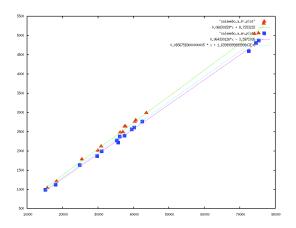
Jupyter Notebook: 1.1-datasetandregression.ipynb

Classification Dataset

Data set for binary classification: Salammbô in French (1) and English (0)

	# char.	# A	class (y)	# char.	# A	class (y)
Chapter 1	36,961	2,503	1	35,680	2,217	0
Chapter 2	43,621	2,992	1	42,514	2,761	0
Chapter 3	15,694	1,042	1	15,162	990	0
Chapter 4	36,231	2,487	1	35,298	2,274	0
Chapter 5	29,945	2,014	1	29,800	1,865	0
Chapter 6	40,588	2,805	1	40,255	2,606	0
Chapter 7	75,255	5,062	1	74,532	4,805	0
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Chapter 12	40,398	2,766	1	39,552	2,560	0
Chapter 13	74,105	5,047	1	72,545	4,597	0
Chapter 14	76,725	5,312	1	75,352	4,871	0
Chapter 15	18,317	1,215	1	18,031	1,119	0

Supervised Learning: Regression and Classification



Given the data set, $\{(x_i, y_i) | 0 < i \le N\}$ and a model f:

- Classification: f(x) = y is discrete,
- Regression: f(x) = y is continuous.

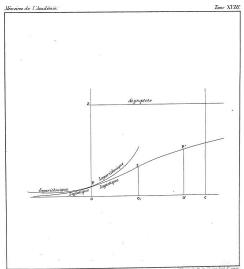
Berkson's Dataset (1944)

Binary classification with probabilities

Drug	Number	Survive	Die	Mortality	Expected
concentration	exposed	Class 0	Class 1	rate	mortality
40	462	352	110	.2359	.2206
60	500	301	199	.3980	.4339
80	467	169	298	.6380	.6085
100	515	145	370	.7184	.7291
120	561	102	459	.8182	.8081
140	469	69	400	.8529	.8601
160	550	55	495	.9000	.8952
180	542	43	499	.9207	.9195
200	479	29	450	.9395	.9366
250	497	21	476	.9577	.9624
300	453	11	442	.9757	.9756

Table: A data set. Adapted and simplified from the original article that described how to apply logistic regression to classification by Joseph Berkson, Application of the Logistic Function to Bio-Assay. *Journal of the American Statistical Association* (1944).

Classification with Probabilities: The Logistic Curve (Verhulst)



$$Logistic(x) = \frac{1}{1 + e^{-x}}$$

$$\hat{y}(x) = Logistic(w \cdot x)$$

= $\frac{1}{1 + e^{-w \cdot x}}$

The logistic curve is also called a sigmoid

Multiple Classes: Types of Iris



Iris virginica



Iris setosa



Iris versicolor

Courtesy Wikipedia

Fisher's Iris Dataset (1936)

180 MULTIPLE MEASUREMENTS IN TAXONOMIC PROBLEMS

Table I

	Iris e	setosa		1	Iris ve	rsicolor		Iris virginica				
Sepal length	Sepal width	Petal length	Petal width	Sepal length	Sepal width	Petal length	Petal width	Sepal length	Sepal width	Petal length	Petal width	
5.1	3.5	1.4	0.2	7.0	3.2	4.7	1.4	6.3	3.3	6.0	2.5	
4.9	3.0	1.4	0.2	6.4	3.2	4.5	1.5	5.8	2.7	5.1	1.9	
4.7	3.2	1.3	0.2	6.9	3.1	4.9	1.5	7.1	3.0	5.9	2.1	
4.6	3.1	1.5	0.2	5.5	2.3	4.0	1.3	6.3	2.9	5.6	1.8	
5.0	3.6	1.4	0.2	6.5	2.8	4.6	1.5	6.5	3.0	5.8	2.2	
5.4	3.9	1.7	0.4	5.7	2.8	4.5	1.3	7.6	3.0	6.6	2.1	
4.6	3.4	1.4	0.3	6.3	3.3	4.7	1.6	4.9	2.5	4.5	1.7	
5.0	3.4	1.5	0.2	4.9	2.4	3.3	1.0	7.3	2.9	6.3	1.8	
4.4	2.9	1-4	0.2	6.6	2.9	4.6	1.3	6.7	2.5	5.8	1.8	
4.9	3.1	1.5	0.1	5.2	2.7	3.9	1.4	7.2	3.6	6.1	2.5	
5.4	3.7	1.5	0.2	5.0	2.0	3.5	1.0	6.5	3.2	5.1	2.0	
4.8	3.4	1.6	0.2	5.9	3.0	4.2	1.5	6.4	2.7	5.3	1.9	
4.8	3.0	1.4	0.1	6.0	2.2	4.0	1.0	6.8	3.0	5.5	2.1	
4.3	3.0	1-1	0.1	6.1	2.9	4.7	1.4	5.7	2.5	5.0	2.0	
5.8	4.0	1.2	0.2	5.6	2.9	3.6	1.3	5.8	2.8	5.1	2.4	
5.7	4.4	1.5	0.4	6.7	3.1	4.4	1.4	6-4	$3 \cdot 2$	5.3	2.3	
5.4	3.9	1.3	0.4	5.6	3.0	4.5	1.5	6.5	3.0	5.5	1.8	
5.1	3.5	1.4	0.3	5.8	2.7	4.1	1.0	7.7	3.8	6.7	2.2	
5.7	3.8	1.7	0.3	6.2	2.2	4.5	1.5	7.7	2.6	6.9	$2\cdot3$	
5.1	3.8	1.5	0.3	5.6	2.5	3.9	1.1	6.0	$2 \cdot 2$	5.0	1.5	
5.4	3.4	1.7	0.2	5.9	3.2	4.8	1.8	6.9	$3 \cdot 2$	5.7	$2 \cdot 3$	
5.1	3.7	1.5	0.4	6.1	2.8	4.0	1.3	5.6	2.8	4.9	2.0	
			0.0	0.0	0.~						0.0	

Categorical Attributes. After Quinlan (1986)

Object		Attributes									
	Outlook	Temperature	Humidity	Windy							
1	Sunny	Hot	High	False	Ν						
2	Sunny	Hot	High	True	Ν						
3	Overcast	Hot	High	False	P						
4	Rain	Mild	High	False	P						
5	Rain	Cool	Normal	False	P						
6	Rain	Cool	Normal	True	Ν						
7	Overcast	Cool	Normal	True	P						
8	Sunny	Mild	High	False	Ν						
9	Sunny	Cool	Normal	False	P						
10	Rain	Mild	Normal	False	P						
11	Sunny	Mild	Normal	True	P						
12	Overcast	Mild	High	True	P						
13	Overcast	Hot	Normal	False	P						
14	Rain	Mild	High	True	N						

Linear Classification

We represent classification using a threshold function (a variant of the signum function):

$$H(w \cdot x) = \begin{cases} 1 & \text{if } w \cdot x \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

The classification function associates P with 1 and N with 0.

We want to find the separating hyperplane:

$$\hat{y}(x) = H(w \cdot x)$$

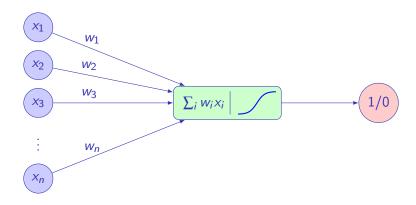
= $H(w_0 + w_1x_1 + w_2x_2 + ... + w_nx_n),$

given a data set of q examples: $DS = \{(1, x_1^j, x_2^j, ..., x_n^j, y^j) | j: 1...q\}.$

We use $x_0 = 1$ to simplify the equations.

For a binary classifier, y has then two possible values $\{0, 1\}$ corresponding in our example to $\{French, English\}$.

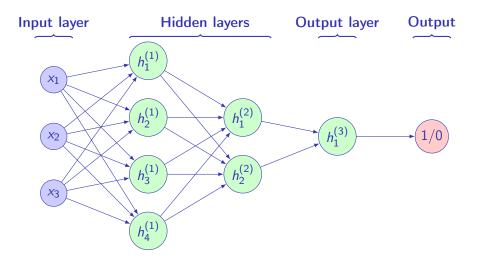
Logistic Regression as a Neural Network



Code Example

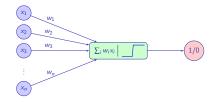
Jupyter Notebook: 1.2-salammboclassification.ipynb, first part

Neural Networks

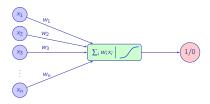


Activation Functions

The perceptron



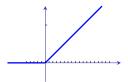
Logistic regression



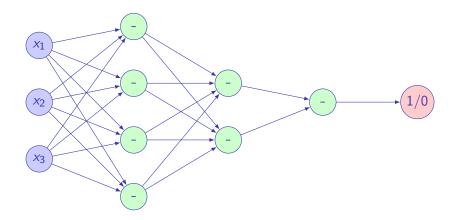
Activation Functions

Rectified linear unit (ReLU), where

$$reLU(x) = max(0,x).$$



Neural Networks with Hidden Layers

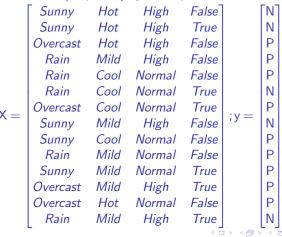


Code Example

Jupyter Notebook: 1.2-salammboclassification.ipynb, second part

Matrix Notation

- A feature vector (predictors): x, and feature matrix: X;
- The class: y and the class vector: y;
- The predicted class (response): \hat{y} , and predicted class vector: \hat{y}



Converting Symbolic Attributes into Numerical Vectors

Linear classifiers are numerical systems.

Symbolic – nominal – attributes are mapped onto vectors of binary values.

This is called a one-hot encoding

A conversion of the weather data set.

Object		Attributes									Class
		Outlook			Temperature			Humidity		Windy	
	Sunny	Overcast	Rain	Hot	Mild	Cool	High	Normal	True	False	
1	1	0	0	1	0	0	1	0	0	1	N
2	1	0	0	1	0	0	1	0	1	0	N
3	0	1	0	1	0	0	1	0	0	1	P
4	0	0	1	0	1	0	1	0	0	1	P
5	0	0	1	0	0	1	0	1	0	1	P
6	0	0	1	0	0	1	0	1	1	0	N
7	0	1	0	0	0	1	0	1	1	0	P
8	1	0	0	0	1	0	1	0	0	1	N
9	1	0	0	0	0	1	0	1	0	1	P
10	0	0	1	0	1	0	0	1	0	1	P
11	1	0	0	0	1	0	0	1	1	0	P
12	0	1	0	0	1	0	1	0	1	0	P
13	0	1	0	1	0	0	0	1	0	1	P
14	0	0	1	0	1	0	1	0	1	0	N

Loss

The loss function is defined as $L(y,\hat{y})$ with $\hat{y} = h(x)$, where x is the vector of attributes, h the classifier, and y, the correct value.

Absolute value loss
$$L_1(y,\hat{y}) = |y-\hat{y}|$$

Squared error loss $L_2(y,\hat{y}) = (y-\hat{y})^2$
0/1 loss $L_{0/1}(y,\hat{y}) = 0$ if $y=\hat{y}$ else 1
Binary crossentropy
Categorical crossentropy

For Keras, see here: https://keras.io/losses/ for the available losses

Empirical Loss

We compute the empirical loss of a classifier h on a set of examples E using the formula:

$$Loss(L, E, h) = \frac{1}{N} \sum_{E} L(y, h(x)).$$

For continuous functions:

Loss(L, E, h) =
$$\frac{1}{N} \sum_{F} (y - h(x))^{2}$$
.

Understanding the Loss: Entropy

Information theory models a text as a sequence of symbols.

Let $x_1, x_2, ..., x_N$ be a discrete set of N symbols representing the characters. The information content of a symbol is defined as

$$I(x_i) = -\log_2 P(x_i) = \log_2 \frac{1}{P(x_i)},$$

where

$$P(x_i) = \frac{Count(x_i)}{\sum_{j=1}^{n} Count(x_j)}.$$

Entropy, the average information content, is defined as:

$$H(X) = -\sum_{x \in X} P(x) \log_2 P(x),$$

By convention: $0 \log_2 0 = 0$.

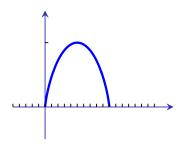


Understanding the Entropy

For a two-class set, we set:

$$x = \frac{p}{p+n}$$
 and $\frac{n}{p+n} = 1-x$.

$$I(x) = -x \log_2 x - (1-x) \log_2 (1-x)$$
 with $x \in [0,1]$.



The entropy reaches a maximum when there are as many positive as negative examples in the data set. It is minimal when the set consists of either positive or negative examples.

Entropy of a Text

The entropy of the text is

$$\begin{array}{lll} H(X) & = & -\sum\limits_{x \in X} P(x) \log_2 P(x). \\ & = & -P(A) \log_2 P(A) - P(B) \log_2 P(B) - \dots \\ & & -P(Z) \log_2 P(Z) - P(\grave{A}) \log_2 P(\grave{A}) - \dots \\ & & -P(\ddot{Y}) \log_2 P(\ddot{Y}) - P(blanks) \log_2 P(blanks). \end{array}$$

Entropy of Gustave Flaubert's *Salammbô* in French is H(X) = 4.39.

Cross-Entropy

Common losses in classification: binary or multinomial (categorical) crossentropy.

The cross entropy of M on P is defined as:

$$H(P,M) = -\sum_{x \in X} P(x) \log_2 M(x).$$

We have the inequality $H(P) \leq H(P, M)$.

The difference is called the Kullback-Leibler divergence.

			Entropy	Cross entropy	Difference			
Salammbô,	chapters	1-14,	4.39481	4.39481	0.0			
training set								
Salammbô, c	hapter 15, t	est set	4.34937	4.36074	0.01137			
Notre Dame	de Paris, tes	st set	4.43696	4.45507	0.01811			
Nineteen Eig	<i>hty-Four</i> , te	st set	4.35922	4.82012	0.46090			

Example of Cross Entropy

In practice, we use the mean and the natural logarithm:

$$H(P,M) = -\frac{1}{|X|} \sum_{x \in X} P(x) \log M(x),$$

where P is the truth, and M is the prediction of the model, a probability in the case of logistic regression.

Binary classification

Observations	1	2	3	4	5	6
Dose	140	300	140	160	140	250
Observed class	0	1	1	1	1	1
Model prediction	0.3487	0.9964	0.8557	0.9056	0.8557	0.9882
Model prediction	0.6513					
$-P(x)\log M(x)$:	0.4287	0.0036	0.1559	0.0992	0.1559	0.0119
Model prediction	0.6513 0.4287	0.0036		0.0992		

Mean = 0.14252826

Code Example

Jupyter Notebook: 1.3-logisticregression.ipynb

Multiple Categories

We can generalize logistic regression to multiple categories.

We use then the softmax function:

$$P(y = i|x) = \frac{e^{-w_i \cdot x}}{\sum_{j=1}^{C} e^{-w_j \cdot x}},$$

that defines the probability of an observation represented by x to belong to class i.

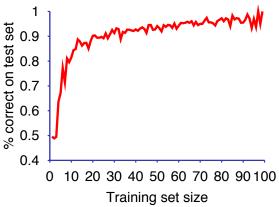
Again, we use stochastic gradient descent to compute the weights: w.

Code Example

Jupyter Notebook: 1.4-multiclass.ipynb

Learning Curve

The classical evaluation technique uses a training set and a test set. Generally, the larger the training set, the better the performance. This can be visualized with a learning curve. From the textbook, Stuart Russell and Peter Norvig, *Artificial Intelligence*, 3rd ed., 2010, page 703.



Overfitting

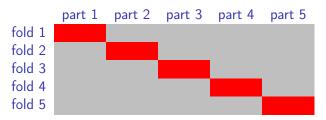
- When two classifiers have equal performances on a specific test set, the simplest one is supposed to be more general
- A small network is always preferable to a larger one.
- Complex classifiers may show an overfit to the training data and have poor performance when the data set changes.
- We assess the overfit by drawing the loss and accuracy curves for the training set and a separate validation set. See the examples in the Keras book.

Evaluation

- The standard evaluation procedure is to train the classifier on a training set and evaluate the performance on a test set.
- When we have only one set, we divide it in two subsets: the training set and the test set (or holdout data).
- The split can be 90–10 or 80–20
- This often optimizes the classifier for a specific test set and creates an overfit

Cross Validation

- A N-fold cross validation mitigates the overfit
- The set is partitioned into N subsets, N = 5 for example, one of them being the test set (red) and the rest the training set (gray).
- The process is repeated N times with a different test set: N folds



At the extreme, leave-one-out cross-validation

Model Selection

- Validation can apply to one classification method
- We can use it to select a classification method and its parametrization.
- Needs three sets: training set, development set, and test set.

Measuring Quality: The Confusion Matrix

A task in natural language processing: Identify the parts of speech (POS) of words.

Example: The can rusted

- The human: *The*/art (DT) *can*/noun (NN) *rusted*/verb (VBD)
- The POS tagger: *The*/art (DT) *can*/modal (MD) *rusted*/verb (VBD)

↓Correct	Tagger $ o$									
	DT	IN	JJ	NN	RB	RP	VB	VBD	VBG	VBN
DT	99.4	0.3	_	_	0.3	_	_	-	_	_
IN	0.4	97.5	_	_	1.5	0.5	_	_	_	_
JJ	_	0.1	93.9	1.8	0.9	_	0.1	0.1	0.4	1.5
NN	-	_	2.2	95.5	_	_	0.2	_	0.4	_
RB	0.2	2.4	2.2	0.6	93.2	1.2	_	_	_	_
RP	_	24.7	_	1.1	12.6	61.5	_	_	_	_
VB	_	_	0.3	1.4	_	_	96.0	_	_	0.2
VBD	-	_	0.3	_	_	_	_	94.6	_	4.8
VBG	-	_	2.5	4.4	_	_	_	_	93.0	_
VBN	-	_	4.6	_	_	_	_	4.3	-	90.6

After Franz (1996, p. 124)

